Leaders, Followers, and Risk Dynamics in Industry Equilibrium*

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August 3, 2011

Abstract

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Abstract

We study the distinct impacts of own and rival actions on risk and return when firms strategically compete in the product market. Contrary to simple intuition, a competitor’s options to adjust capacity reduce own-firm risk. For example, if a rival possesses a growth option, an increase in industry demand directly enhances profits but also encourages value-reducing competitor expansion. The rival option thus acts as a natural hedge. Within the industry, we obtain endogenous differences in expected returns. In a leader-follower equilibrium, own-firm and competitor risks and required returns move together through contractions and oppositely during expansions, providing testable new predictions.

JEL Classification: G31, G12, D43.

Keywords: Real options; dynamics of firm and industry risk; asset pricing; corporate investment; valuation; cost-of-capital.
1 Introduction

How does the ability of a firm to expand or contract its operations impact the risk and required return of a product market rival? Casual intuition suggests that competitor growth opportunities threaten a firm’s future market-share and profits, making it riskier today. Such reasoning would seem to be consistent with the famous statement by Hicks (1935) that “the best of all monopoly profits is a quiet life.” In recent literature, Hou and Robinson (2006) and other authors use related intuition to deduce that the possibility of rival actions should raise own-firm risks.¹

We show how this intuition can fail. Using a standard model of dynamic duopoly in a homogeneous output market, we demonstrate that the threat of competitor expansion makes a firm less risky today. Moreover, firm risk decreases as the likelihood of competitor expansion increases. These surprising results come from distinguishing the effect of competition on value from the effect of competition on risk. The threat of a rival’s expansion does indeed decrease own value, as the casual intuition suggests, by taking away expected future market share and depressing price. However, the negative valuation effect of the rival growth option creates a natural hedge for industry shocks. When a competitor possesses a growth option, good news about the product market is partially offset by the bad news that rival expansion is more likely. Conversely, bad news about industry demand is counterbalanced by a decline in the threat of competitor capacity additions. All else equal, rival growth options reduce own-firm risk.

Similar economic forces also reduce firm risk when a rival possesses a contraction option. In this case, industry demand shocks are partially offset by opposite movements in the likelihood of rival asset sales, again providing a natural hedge that reduces own-firm risk. Hence, both rival growth and contraction options reduce own firm risk, providing a counterexample

¹Hou and Robinson study the empirical relationship between industry structure and average stock returns and state: “If barriers to entry in product markets insulate some firms from aggregate demand shocks... industries with high barriers to entry [will be] associated with lower equilibrium stock returns.” Related literature includes Ali, Klasa, and Yeung (2008), Irvine and Pontiff (2009), and Peress (2010).
to the common view that low risk and return are the consequence of a “quiet life” provided by barriers to entry.

Our analysis also sheds new light on the common practice of estimating firm risk as a composite of the historical risks of industry peers.\(^2\) We show that risk and expected return should vary over time with the position of a firm as a “leader” or “follower” relative to its rival in absorbing industry demand shocks by expansion or contraction. In particular, during a market upturn a wedge is driven between firms’ expected returns as the market leader’s risk increases due to growth option leverage, while the follower’s risk drops because of the rival hedging effect. By contrast, during a market downturn firms’ expected returns tend to move together as the market leader’s anticipated contraction reduces both its own and the follower’s risks. These results provide novel empirical predictions, and suggest that the recent literature examining the link between product market competition and industry returns in the cross-section should be extended to consider the dynamics of own-firm and rival risk around expansions and contractions.


\(^2\)The widely used Ibbotson Beta Book provides estimates of beta based on a peer group that depends on industry classification, and the use of industry competitors to proxy for own-firm risk is discussed in finance textbooks such as Brealey and Myers (2001), and Ross, Westerfield, and Jaffe (1996).
tical and adjustment costs are convex, all firms make symmetric infinitesimal investments or disinvestments, and the distinct risk impacts of own and rival actions are not apparent. We consider the case of oligopoly with potentially asymmetric firms and lumpy investment, which permits that a leader and follower arise in equilibrium. Our study fills an important gap in the finance literature, since it is the first that permits analysis of the separate effects of own-firm versus rival real options on the dynamics of risk and required return.3

Lambrecht and Perraudin (2003) consider investment by firms in a winner-take-all environment. Their primary innovation is to permit asymmetric information about competitor’s investment costs, which implies that the market value of a firm jumps upward when a firm gains control of the market by investing, and market value falls to zero when a competitor invests. Their key results thus relate to the resolution of incomplete information and related announcement effects, which impact higher moments of returns such as skewness. In their model agents are risk neutral and firms always earn the risk-free rate of return. By contrast, we have no asymmetric information, but derive implications for betas and expected returns in a variety of industry stages, according to the capital levels of firms and their available expansion or contraction options.

Other research in the real options literature analyzes equilibrium exercise of expansion or contraction opportunities in a duopoly setting, but does not investigate risk dynamics. Examples that relate most closely to the framework we consider include Smets (1991), Dixit and Pindyck (1994), Grenadier (1996), Huisman and Kort (1999), Boyer, Lasserre, Marriotti, and Moreaux (2004), Murto (2004), and Pawlina and Kort (2006).

Our framework emphasizes the importance of the product market demand elasticity in determining the boundary between simultaneous-exercise equilibrium and leader-follower

equilibria, providing an empirically measurable link between product market characteristics and risk dynamics. For high demand elasticities, simultaneous exercise can be supported for large adjustment cost asymmetries. By contrast, when demand elasticities are low, even arbitrarily small adjustment cost asymmetries lead to leader-follower exercise as the unique equilibrium outcome. For contraction options, no simultaneous-move equilibria exist.

Lumpy investment is a key assumption of our model, and a well-known endogenous consequence of non-convex adjustment costs (e.g., Dixit and Pindyck, 1994; Stokey, 2009). Empirical literature including Cooper and Haltiwanger (2006) provides strong evidence of investment spikes at the plant level, and we therefore believe that a theoretical framework consistent with non-convex adjustment costs is essential to understand the evolution of firm risk. In a standard model with convex adjustment costs, sequential lumpy investment would disappear. Because of the distinct risk effects of own and rival decisions, and the asynchronicity of actions in a leader-follower equilibrium, our framework implies substantial endogenous time-series and cross-sectional variation in expected returns at the firm level.

The importance of non-convexities for our theoretical implications is consistent with the recent literature on R&D. In research subsequent to our own, Bena and Garlappi (2011) adopt a patent race framework consistent with non-convexities, close to the assumptions of our model. In their model, as in ours, exogenously heterogeneous firms optimally time lumpy investments in order to gain better access to a product market, and the implications for risk

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4A number of natural variations of the model we use would not fundamentally change the relationship that we show between product market conditions and risk dynamics. For example, variable costs of production give a role for operating flexibility, which should dampen the magnitude of the risk dynamics we show, but would not qualitatively change our results. Similarly, alternative functional forms, such as using linear rather than isoelastic demand, would have quantitative but not qualitative effects.

5The model would then correspond to a traditional capital accumulation game in which equilibrium investment is chosen in incremental units on the basis of expected marginal revenues and costs. Investment in a traditional capital accumulation game with convex adjustment costs critically depends on the strategy space available. When firms employ Markov as opposed to open-loop strategies, excess investment and preemptive investment commonly occur (Reynolds, 1987; Fudenberg and Tirole, 1991; Dockner et al., 2000).

6In earlier work, Garlappi (2004) assumes continuous and infinitesimal research costs with the option to mothball at any time. In this model the mechanism driving risk as well as risk dynamics differ substantially from our model. Risk is time-varying in Garlappi’s model only when instantaneous exploration costs have a fixed component, as with operating leverage, while our risk dynamics are driven by option leverage. Firms exercise real options by making lumpy investments, whereas firms make infinitesimal investments in Garlappi’s model and random success in R&D, determined by nature, drives the major changes in risk.
are broadly similar to our model.\textsuperscript{7} Thus, non-convex costs, whether in capacity investments or R&D outlays, robustly drive similar risk dynamics for leaders and followers.\textsuperscript{8}

The framework we adopt importantly overcomes the difficulties with subgame perfection noted by Back and Paulson (2009) in the model of Grenadier (2002) and in other recent papers such as Aguerrevere (2003, 2009) analyzing equilibrium stopping-time games. Back and Paulson show that while the equilibrium discussed by Grenadier is an “open-loop” Nash equilibrium, it does not satisfy the standard subgame perfection requirement of a Markov perfect “closed-loop” equilibrium.\textsuperscript{9} All of the equilibria we consider satisfy subgame perfection and hence form closed-loop equilibria, but, unlike work based on the Grenadier (2002) equilibrium, option values remain positive because expansion opportunities are finite.

Section 2 describes the general model. In Section 3, we analyze the simplest case where one firm is a strategic dummy, and show the risk-reducing effects of rival growth options. Section 4 presents the leader-follower equilibrium where firms with asymmetric costs may expand or contract. Section 5 concludes.

\section{The Asymmetric Duopoly Model}

We present a model in which two strategically interacting firms compete in output levels in a homogeneous goods market, and have options to invest or disinvest in capacity.

\subsection{Industry Demand, Production, and Investment}

Let $Q_1^t$ and $Q_2^t$ denote the output rates of firm one and firm two at instant $t$, and define the industry output rate $Q_t = Q_1^t + Q_2^t$. The homogeneous good price is determined by the

\textsuperscript{7}In particular, in their model as in ours, the risk of the leader falls after the leader’s investment while the risk of the follower rises, and as the follower’s investment threshold approaches, the leader’s risk declines while the follower’s risk increases.

\textsuperscript{8}Bena and Garlappi find empirical evidence from patent filings consistent with the predictions of this model. Bustamante (2010) similarly considers extensions of the framework we use and discusses empirical evidence.

\textsuperscript{9}Subgame perfection rules out strategies involving precommitments that are not credible. In the strategies described by Grenadier, firms have an incentive to preempt investment by their rivals but do not do so.
iso-elastic inverse demand curve

$$P_t = X_t Q_t^{\gamma-1},$$

(1)

where $0 < \gamma < 1$, and $X_t$ is an exogenous state variable that represents the level of industry-wide demand. The dynamics of $X_t$ are specified by

$$dX_t = gX_t dt + \sigma X_t dW_t,$$

(2)

where $dW_t$ is the increment of a Wiener process, $g$ is the constant drift, and $\sigma^2$ the variance.

Firm $i$ produces output at time $t$ using installed capital $K^i_t$ where $i \in \{1, 2\}$. Any capital level $K^i_t$ is associated with a maximum output level $Q(K^i_t) \geq Q^i_t$. For simplicity, capital levels take one of three discrete values: $K^i_t \in \{\kappa_0, \kappa_1, \kappa_2\}$, where $\kappa_0 < \kappa_1 < \kappa_2$, and $Q(\kappa_0) < Q(\kappa_1) < Q(\kappa_2)$. Costs of production for firm $i$ at date $t$ are given by the increasing function $F^i_t = f(K^i_t)$. This cost structure emphasizes operating leverage, since total expenditures depend only on the installed capital level $K^i_t$. Given the three possible capital levels, there are also three possible levels of fixed operating costs: $F^i_t \in \{f_0, f_1, f_2\}$, where $f_0 < f_1 < f_2$.

To move from one capital state to another, the firm may incur costs or generate cash flows from buying or selling the productive asset, inclusive of adjustment costs. To capture this idea, we specify for each firm a matrix of discrete transition costs:

$$\Lambda^i = \begin{bmatrix} 0 & \lambda^i_{01} & \lambda^i_{02} \\ \lambda^i_{10} & 0 & \lambda^i_{12} \\ \lambda^i_{20} & \lambda^i_{21} & 0 \end{bmatrix}.$$ 

The instantaneously incurred lump-sum cost for firm $i$ to move from capital level $\kappa_m$ to $\kappa_n$ is given by $\lambda^i_{mn}$. The only source of heterogeneity across firms is that $\Lambda^1$ and $\Lambda^2$ need not be identical. We assume that at date zero, each firm is endowed with $K^i_0 = \kappa_1$ units of capital.

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10The assumption that the potential output levels $Q(\kappa_j)$ are the same for firms 1 and 2 is not essential, and is made here for notational convenience. The arguments in the Appendix are valid when the output levels $Q(\kappa_j)$ differ across firms $i$, hence permitting asymmetric revenue functions.
We finally define indicator variables $D_{imn}t$ that take the value one at the instant when firm $i$ switches from capital level $\kappa_m$ to $\kappa_n$, and zero elsewhere. We denote by $D_{i}t$ the matrix of investment decisions $D_{imn}t$.

2.2 Output, Investment Strategies, and Equilibrium

The economy described above is a dynamic game between firms 1 and 2. At each instant, the managers of the two firms choose output rates $Q_{i}t$ and make investment decisions $D_{i}t$ knowing the complete history of the game denoted by $\Phi_t = ([Q_{1s}, Q_{2s}, K_{1s}^{1}, K_{2s}^{2}]_{s<t}, [X_{s}]_{s\leq t})$.

We define the payoff to firm $i$ as the present value of the expected discounted future cash flows. The cash flows at time $t$ derive from revenues in excess of fixed costs $\pi_{i}t \equiv P_{t}Q_{i}t - F_{i}t$ and from lumpy investment costs related to the decision $D_{i}t$. We assume the absence of agency conflicts, so that manager $i$ maximizes the value function

$$V_{i}t \equiv E_{t}\int_{t}^{\infty}e^{-r(s-t)}\frac{M_{t+s}}{M_{t}}\left[\pi_{i}t+s ds + 1'\left(D_{i}t+s * \Lambda^{i}\right)1\right], \quad (3)$$

where $1' = [1, 1, 1]$, $*$ represents element-by-element multiplication, and the pricing kernel $M_{t}$ satisfies $M_{0} = 1$ and $dM_{t} = \frac{\mu - r}{\sigma} M_{t}dW_{t}$.\footnote{\$\mu$ is the constant drift of the dynamics of a traded asset that is perfectly correlated with the demand shocks $X_{t}$. This traded asset together with a risk free bond allows us to define a risk neutral measure for the demand dynamics with the drift $r - \delta$ where $\delta$ is defined as $\delta \equiv \mu - g > 0$.}

Given the Markov structure of this environment, it is natural to restrict attention to Markov strategies. Manager $i$ can then take actions $Q_{i}t$ and $D_{i}t$ that depend only on the most recently observed values of the payoff relevant state variables $X_{t}$ and $K_{t-} \equiv (K_{t-}^{1}, K_{t-}^{2})$, where $K_{t-} \equiv \lim_{s\uparrow t} K_{s}$. A pure strategy Markov-perfect equilibrium (MPE) of the game is a pair of strategies $(Q_{i}t, D_{i}t), i = 1, 2$, such that the value functions (3) are maximized in every state $(K_{t-}, X_{t})$ given the equilibrium strategy of the rival.

It is straightforward to show that any MPE must have quantity choices equal to static Cournot equilibrium output levels. Given our assumption that demand is sufficiently elastic
(implied by $\gamma > 0$) and the absence of marginal costs, all firms produce at full capacity. Hence, any MPE strategy requires $Q^i_t = Q^i(K^i_t)$.\(^{12}\) The instantaneous profit functions

$$
\pi^i_t = X_t \left[ Q^1(K^1_t) + Q^2(K^2_t) \right]^{\gamma-1} Q^i(K^i_t) - F^i_t
$$

are fully determined by the current capital levels $K^1_t$ and $K^2_t$ and industry demand $X_t$. To aid exposition, define for $m, n \in \{0, 1, 2\}$ the \textit{revenue factors}

$$
R^i_{1m} \equiv \left[ Q^1(\kappa_m) + Q^2(\kappa_n) \right]^{\gamma-1} Q^1(\kappa_m), \\
R^i_{2m} \equiv \left[ Q^1(\kappa_m) + Q^2(\kappa_n) \right]^{\gamma-1} Q^2(\kappa_n),
$$

We can then write the profit of firm $i$ as $\pi^i_t(K^1_t = \kappa_m, K^2_t = \kappa_n, X_t) = X_t R^i_{nm} - F^i_t$.

Given the determination of the instantaneous output choices $Q^i_t$, we henceforth focus on the dynamic game of option exercise involving the investment decisions $D^i_t$. A Markov strategy is characterized by exercise boundaries that for each player $i$ and capital state $K^i_{t-}$ specify regions of the state variable $X_t$ at which player $i$ will change his capital level to a new state. We use standard backward induction to derive MPE of the dynamic game.

\section{Rival Growth Options and Risk}

This section considers the simplest case of the model in Section 2. We assume that one rival is \textit{flexible}, and begins with one option to either expand or contract, while the other rival is \textit{inflexible} and has no ability to change its capital level. This scenario allows us to isolate the two sources of option risk, own and rival, that can occur in a real options duopoly.

\(^{12}\)Instantaneous suboptimal actions are ruled out by Markov perfect equilibrium, which requires that all players' strategies must depend only on payoff relevant state variables.
Adjustment costs are:

\[ \Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ S & 0 & -I \\ -\infty & -\infty & 0 \end{bmatrix}, \]

\[ \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix}, \]

where \( S, I > 0 \). Firm 1, the flexible firm, thus begins at capital level \( \kappa_1 \). It can expand to \( \kappa_2 \) by paying the investment cost \( I \), or contract to \( \kappa_0 \) receiving the salvage value \( S \). Once firm 1 expands or contracts, it has no further options to change capacity. Firm 2 begins at capital level \( \kappa_1 \) and has no real options.

The exercise decisions and valuations are given by

**Proposition 1:** Let \( X_E > 0 \) and \( X_C < X_E \) be the expansion and contraction boundaries of the flexible firm, characterized in the Appendix. The flexible firm value prior to option exercise is:

\[
V^1(K_t, X_t) = \underbrace{R^{11}_t X_t / \delta}_{\text{assets in place: } V^1_A(K_t, X_t)} + \underbrace{B_1^1 X_t^{\nu_1} + B_2^1 X_t^{\nu_2}}_{\text{option value: } V^1_O(K_t, X_t)} - \underbrace{f(K_t^1) / r}_{\text{operating leverage: } V^1_F(K_t)},
\]

where \( B_1^1 \) and \( B_2^1 \) are positive constants determined by the boundary conditions, and \( \nu_1 > 1 \) and \( \nu_2 < 0 \) are the roots of the characteristic equation given in the Appendix. The value of the inflexible firm prior to option exercise is:

\[
V^2(K_t, X_t) = \underbrace{R^{21}_t X_t / \delta}_{\text{assets in place: } V^2_A(K_t, X_t)} + \underbrace{B_1^2 X_t^{\nu_1} + B_2^2 X_t^{\nu_2}}_{\text{strategic externality: } V^2_C(K_t, X_t)} - \underbrace{f(K_t^2) / r}_{\text{operating leverage: } V^2_F(K_t)},
\]

where \( B_1^1 \leq 0, B_2^1 \geq 0 \) are determined by the value matching conditions at the rival exercise.
As in standard real option models, (e.g., McDonald and Siegel, 1985, 1986), the flexible firm value consists of assets in place and its own option value. The real option has two components related to the growth opportunity and contraction option respectively. The positivity of the constants \( B_1^1 \) and \( B_2^1 \) reflects that ownership of these options is value-enhancing, and the positive and negative signs of the roots \( \nu_1 \) and \( \nu_2 \) reflect that growth option value increases, while contraction option value decreases, with increases in the underlying asset.

The inflexible firm value includes an externality imposed by the rival firm options. The value of this externality has two components, related to the rival growth and contraction options. The negative sign of \( B_2^2 \) reflects that rival expansion options reduce value, while \( B_2^2 \geq 0 \) follows from the value enhancing effect of competitor contraction. We note that contraction options impact own and rival-firm values with the same sign, whereas expansion options have opposite valuation impacts on a firm and its rivals.

These valuation effects have implications for risk.

**Proposition 2.** The dynamic betas for the flexible and inflexible firm are:

\[
\beta^i(K_t, X_t) = 1 + \frac{f_1/r}{V^i(K_t, X_t)} + \left\{ (\nu_1 - 1) \frac{B_1^i X_1^{\nu_1}}{V^i(K_t, X_t)} + (\nu_2 - 1) \frac{B_2^i X_2^{\nu_2}}{V^i(K_t, X_t)} \right\}
\]

prior to option exercise and \( \beta^i(K_t, X_t) = 1 + V^i_F(K_t)/V^i(K_t, X_t) \) afterwards.

The betas for the flexible and inflexible firms consist of three parts. By assumption the revenue beta is equal to 1. The second component is operating leverage, which always increases risk, and the final term for both firms arises from the flexible firm’s real options. Although the structure of beta for both firms is similar, the economic interpretation is very different. The flexible firm’s risk depends only on its own decisions, whereas the inflexible firm has no decisions to make and its risk is determined entirely by industry effects.

The flexible firm risk is higher because of its option to expand, since \( \nu_1 > 1 \) and \( B_1^1 \geq 0 \),
and lower because of its option to contract since $\nu_2 < 0$ and and $B^1_2 \geq 0$. By contrast, the inflexible firm risk is reduced by either type of competitor real option, since $B^2_1 \leq 0$ and $B^2_2 \geq 0$. This simple example illustrates two important points, which we now discuss in more detail.

First, rival real options, either expansion or contraction, reduce risk. Intuitively, a competitor’s investment decisions act as a natural hedge against variations in the exogenous state variable. For example, good news about demand going up will be partially offset by the bad news that the competitor is closer to expanding. Figure 1 gives a graphical presentation of this hedging argument. Before the flexible firm exercises her option, industry demand is indicated by the downward sloping curve $D$ and the industry supplies output at the full-capacity level $Q_1$. Consider now an increase in demand to the level $D'$ that induces the flexible firm to exercise her growth option. The corresponding increase in industry supply causes prices to increase less than to the level $P^*$ corresponding to the old supply curve. Prices rise more moderately to $P_2$ instead of $P^*$, and the dampening in profits caused by the increase in industry supply after a positive demand shock creates a natural hedge.

The second implication of the example in this section is that expansion options have an oppositely signed impact on own-firm and rival risk, while contraction options affect both firms’ risk in the same direction. These risk implications follow from the valuation impacts of own and rival real options. Contraction options of both one’s own firm and rivals create a hedge against adverse moves in underlying fundamentals. By contrast, own-firm expansion opportunities amplify risk, whereas rival expansion opportunities mitigate the potential for upside gain.

Figure 2 shows the own and rival risk effects discussed above. For simplicity, we assume the inflexible firm has no operating leverage. In the figure, $X_C$ is the critical level of demand at which the flexible firm shrinks and $X_E$ is critical level at which the flexible firm expands. The diagram illustrates that rival real options reduce risk, and that real options can cause own and competitor risks to move together or in opposite directions. As demand increases
and the growth option becomes more important, the flexible firm’s risk increases while the inflexible firm’s risk decreases. By contrast, when demand decreases and the contraction option is more valuable, own and rival firm risk tend to move together. The next section investigates the robustness of these results when both firms possess growth options and exercise is strategic.

4 Dynamic Risk in Asymmetric Industry Equilibrium

We now permit both firms to have real options to adjust capacity. We consider two cases: 1) both firms have expansion options, and 2) both firms have contraction options.

4.1 Equilibrium Exercise of Expansion Options

In the case where both firms have a single growth option, adjustment costs are

\[
\Lambda^1 \equiv \begin{bmatrix}
0 & -\infty & -\infty \\
-\infty & 0 & -I \\
-\infty & -\infty & 0
\end{bmatrix}, \quad \Lambda^2 \equiv \begin{bmatrix}
0 & -\infty & -\infty \\
-\infty & 0 & -\rho I \\
-\infty & -\infty & 0
\end{bmatrix},
\]

where \( \rho \geq 1 \) so that the expansion costs of firm 1 are lower than those of firm 2. The Appendix shows that in all Markov-perfect equilibria the low-cost firm invests at least as soon as the high-cost firm. Industry structure can therefore be in one of three phases: a juvenile industry where neither firm has exercised its growth option, an adolescent industry where the “leader” has exercised and the “follower” has not, and a mature industry where both firms have expanded. Figure 3 depicts the different industry stages. All three industry stages occur for a finite period of time in a leader-follower equilibrium, whereas in a simultaneous equilibrium the industry structure jumps immediately from juvenile to mature.

Following Pawlina and Kort (2006),\(^{13}\) determining payoffs under different strategies pro-

\(^{13}\)See also Smets (1991), Dixit and Pindyck (1994), and Grenadier (1996), who consider leader-follower
ceeds by backward induction, and allows determination of the type of equilibrium. We assume that the initial demand state \( X_0 \) is strictly less than the leader trigger level of firm 1, which ensures that the juvenile industry state occurs for a finite period of time in equilibrium. The Appendix shows that four different types of equilibria exist: (i) simultaneous equilibrium, (ii) non-preemptive leader-follower equilibrium, (iii) preemptive leader-follower equilibrium, and (iv) random leader-follower equilibrium.

Figure 4 diagrams the equilibrium outcomes, for different levels of demand elasticity, inversely related to \( \gamma \), and the cost asymmetry \( \rho \). For high demand elasticity (low \( \gamma \)), the simultaneous equilibrium can be supported for large cost asymmetries, whereas under low demand elasticity even small cost asymmetries produce leader-follower behavior. Demand elasticity relates to simultaneous exercise because of the impact of investment on the profits generated by assets in place. When demand elasticity is high, expanding output has a large negative effect on assets-in-place. The incentive to preemptively invest as a leader is then small, and the firms are more willing to wait to invest, which supports simultaneous exercise.

The valuation equations for leader-follower equilibria, given in the Appendix, show that both firm values are composed of the growing perpetuity value of the assets in place assuming constant industry structure, the perpetuity value of the fixed costs, the own-firm option value, and the externality imposed by the rival option. Let \( X^1_L \) denote the leaders trigger, which takes the value \( X^1_L^{LN} \) in a non-preemptive equilibrium and \( X^1_L^{LP} \) in a preemptive equilibrium, where \( X^1_L^{LN} \) and \( X^1_L^{LP} \) are described in the Appendix. The leader’s value function in a juvenile

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\(^{14}\)Grenadier (1996) discusses outcomes when the initial state exceeds the leader trigger in the symmetric case.
industry can then be written:

\[
V^1(K_t, X_t) = \frac{R^{11}_{t}}{\delta} X_t + \underbrace{\left[ X^1_L - \left( 1 - \frac{1}{\nu_1} \right) X^1_{LN} \right]}_{\text{assets in place}} \underbrace{\left[ \frac{R^{11}_{21} - R^{11}_{11}}{\delta} \left( \frac{X_t}{X^1_L} \right)^{\nu_1} \right]}_{\text{own growth option}} \\
+ \frac{X^2_F}{\delta} [R^{22}_{21} - R^{22}_{21}] \left( \frac{X_t}{X^2_F} \right)^{\nu_1} - \frac{f_1}{r}.
\]

The rival value adjustment is always negative, consistent with the price-reducing effect of competitor expansion. The leader’s own growth option value is proportional to a weighted sum of the two triggers \(X^1_L\) and \(X^1_{LN}\). In a non-preemptive equilibrium \(X^1_L = X^1_{LN}\) it is straightforward to observe that the leader’s own growth option value must be positive since \(\nu_1 > 1\). In a preemptive equilibrium, equation (4) interestingly shows that both the preemptive and non-preemptive triggers enter into the valuation equation, and the relative sizes of the two triggers as well as \(\nu_1\) determine the sign of the own growth option value. For most parameters, the own growth option value is positive, but for example if the risk-free rate and hence \(\nu_1\) are large then the leader’s own growth option value can be negative.\(^{15}\)

The follower’s value in a juvenile industry similarly can be written:

\[
V^2(K_t, X_t) = \frac{R^{21}_{11}X_t}{\delta} + \underbrace{\left[ X^1_L - \left( 1 - \frac{1}{\nu_1} \right) X^1_{LN} \right]}_{\text{assets in place}} \underbrace{\left[ \frac{R^{11}_{21} - R^{11}_{11}}{\delta} \left( \frac{X_t}{X^1_L} \right)^{\nu_1} \right]}_{\text{own growth option}} \\
+ \frac{X^1_L}{\delta} [R^{22}_{21} - R^{22}_{21}] \left( \frac{X_t}{X^1_L} \right)^{\nu_1} - \frac{f_1}{r}.
\]

In contrast to the leader, the follower’s own-option effect is always positive and the rival value-adjustment is negative.

In a simultaneous-exercise equilibrium, let \(X^i_S\) denote the common exercise trigger given in the Appendix. The valuation equations of each firm are given by:

\(^{15}\)One set of parameters for which the leader own growth option value is negative is \(\gamma = 0.5, \sigma = 0.1, q_1 = 2, q_2 = 10, f_1 = f_2 = 0, r_f = 1.0, \delta = 0.88, I = 100.\)
\[ V^i(K_t, X_t) = \frac{R_{11}^i}{\delta} X_t + \frac{X_S^i}{\delta \nu_1} [R_{22}^i - R_{11}^i] \left( \frac{X_t}{X_S^i} \right)^{\nu_1} - \frac{f_1}{r} . \]  

\hspace{1cm} (5)

Hence, in contrast to the leader-follower equilibrium, under simultaneous exercise only a single option component appears, and a separate rival-firm value adjustment is not apparent. Of course, both own-firm and rival effects are implicitly embedded within the growth option component of (5), but there is not a unique decomposition of the change in profits \( R_{22}^i - R_{11}^i \), and the own and rival effects are therefore not separately identified.\(^{16}\) The inability to separately distinguish own and rival effects in the simultaneous exercise case highlights the value of developing the risk implications of the leader-follower equilibrium, which is the central contribution of our paper.

We can summarize the risk implications of the different equilibria by decomposing the value functions according to:

\[ V^i(K_t, X_t) = V^i_A(K_t, X_t) + V^i_O(K_t, X_t) + V^i_C(K_t, X_t) + V^i_F(K_t), \]

where \( V^i_A(K_t, X_t) \) is the value of the assets in place, \( V^i_O(K_t) \) the operating leverage, \( V^i_C(K_t, X_t) \) is the own-option component of value, and \( V^i_F(K_t, X_t) \) is the rival-option component of value.\(^{17}\) We then show:

**Proposition 3.** In all pure strategy equilibria, systematic firm risks for the leader and the

\(^{16}\)For example, one possible decomposition for firm 1 is to designate \( R_{22}^{1} - R_{12}^{1} \) as the own growth option component and \( R_{12}^{1} - R_{11}^{1} \) as the rival effect. On the other hand, it is equally sensible to view \( R_{22}^{1} - R_{21}^{1} \) as the competitor effect and \( R_{21}^{1} - R_{11}^{1} \) as the own effect.

\(^{17}\)We acknowledge that the own and rival components can interact, particularly in the pre-emption equilibrium where the follower real option directly influences the leader trigger. However, conditional on the leader and follower triggers the decomposition into own and rival components is natural.
follower are given by

\[ \beta^i(K_t, X_t) = 1 + \frac{V^o_i(K_t, X_t) + V^c_i(K_t, X_t)}{V^i(K_t, X_t)}(\nu_1 - 1) + \frac{V^e_i(K_t)}{V^i(K_t, X_t)}. \]  

(6)

Systematic firm risk is thus driven by a firm’s operating leverage, its own growth options, and the risk reducing effects of rival growth options.

We emphasize several important points regarding Proposition 3. First, the own growth option and rival growth option components of value enter additively into the second term in (6). Hence, the risk effects of own and rival growth options are identical when normalized by dollar values, which provides a remarkable simplification. Second, since \( \nu_1 > 1 \) and \( V^c_i(K_t, X_t) < 0 \), rival growth options always reduce risk, independent of whether the equilibrium is simultaneous, pre-emptive, or non-preemptive. Third, when \( V^o_i(K_t, X_t) > 0 \), which always holds for non-preemptive equilibria, own-firm expansion options increase risk. Perhaps most importantly, only in the simultaneous equilibrium can we uniquely sign the sum \( V^o_i(K_t, X_t) + V^c_i(K_t, X_t) \) everywhere in the state space. In a simultaneous exercise equilibrium the sum is guaranteed to be positive and the cumulative effect of growth options is to increase risk, as in Aguerrevere (2009). However, in a leader-follower equilibrium \( V^o_i(K_t, X_t) + V^c_i(K_t, X_t) \) can generally not be uniquely signed, implying that the cumulative effect of industry growth options will alternately increase or decrease risk for a given firm at different times.

Figure 5 displays the evolution of risk for the different types of equilibria. In Panels A and B the expansion-cost asymmetries are large (\( \rho \) equals 2.0 and 1.3 respectively) and the equilibrium type is non-preemptive. In Panel C the cost asymmetry is moderate (1.1) and the equilibrium is preemptive. In panel D costs are identical (\( \rho = 1 \)) and the equilibrium is simultaneous.

Moving from Panel A to B, the follower trigger moves forward closer to the leader trigger, but has no strategic impact on the leader decision of when to exercise, consistent with the
nature of the non-preemptive equilibrium. In Panel C, the follower trigger moves close enough
to the leader trigger that the follower would have an incentive to preempt the leader. The
leader therefore itself preempt the preemptive investment of the follower, by moving forward
its trigger to \( X_{LP}^1 \). Finally, in Panel D the firms are sufficiently symmetric and the option
value of waiting relative to acting as a leader sufficiently large that a simultaneous exercise
equilibrium can be sustained.

The risk dynamics of the two firms in the leader-follower equilibria in Panels A-C differ
markedly from the simultaneous exercise equilibrium in Panel D. For a leader-follower
equilibrium, prior to the leader’s exercise the leader’s risk increases more steeply than the
follower. Immediately upon the exercise of the leader growth option, the leader risk drops
discretely and the follower risk jumps upwards, reversing the risk-ordering of the two firms.
The two firms’ risk loadings continue to move apart until the follower growth option is ex-
ercised, and no growth options remain. By contrast, under simultaneous exercise in Panel
D, the risk of both firms increases equally and always is above one until the exercise trigger
and then drops to the level of the cash flow beta. The dynamics of risk in a leader-follower
equilibrium thus differ dramatically from the simultaneous exercise case.

4.2 Equilibrium Exercise of Contraction Options

We now assume that each firm has a single contraction option. Capital adjustment costs are
specified by

\[
\Lambda^1 = \begin{bmatrix}
0 & -\infty & -\infty \\
S & 0 & -\infty \\
-\infty & -\infty & 0
\end{bmatrix}, \quad \Lambda^2 = \begin{bmatrix}
0 & -\infty & -\infty \\
\rho S & 0 & -\infty \\
-\infty & -\infty & 0
\end{bmatrix},
\]

where \( 0 < \rho \leq 1 \) so that firm 1 has the high and firm 2 the low salvage value. This implies
that firm 1 has an incentive to contract earlier. Our interest again lies in equilibrium play
of the two rivals, and in the Appendix we derive the pure strategy equilibria. In contrast to
the case of expansion options, leader-follower exercise is the unique equilibrium, following Murto (2004). We note that preemption does not play a role in contractions because any rival reduction in output increases rather than reduces firm value. Hence, for symmetric firms the follower value exceeds the leader value.

Denote the leader contraction trigger by $X_C^1$ and the follower trigger by $X_C^2 < X_C^1$. The value function for the leader is

$$V^1(K_t, X_t) = \frac{R_{11}}{\delta} X_t + \frac{X_C^1}{\delta \nu^2} [R_{01}^1 - R_{11}^1] \left( \frac{X_t}{X_C^1} \right)^{\nu^2} + \frac{X_C^2}{\delta} [R_{00}^1 - R_{01}^1] \left( \frac{X_t}{X_C^2} \right)^{\nu^2} \left[ R_{01}^1 - R_{11}^1 \right] \left( \frac{X_t}{X_C^1} \right)^{\nu^2} - \frac{f_1}{r} + \text{operating leverage},$$

and the value function for the follower is

$$V^2(K_t, X_t) = \frac{R_{11}}{\delta} X_t + \frac{X_C^1}{\delta \nu} [R_{00}^2 - R_{01}^2] \left( \frac{X_t}{X_C^2} \right)^{\nu^2} + \frac{X_C^2}{\delta} [R_{01}^2 - R_{11}^1] \left( \frac{X_t}{X_C^2} \right)^{\nu^2} \left[ R_{01}^2 - R_{11}^1 \right] \left( \frac{X_t}{X_C^1} \right)^{\nu^2} - \frac{f_1}{r} + \text{operating leverage}.$$  

The value functions are composed of the value of assets-in-place net of fixed costs, the value of fixed costs, and the own-firm and rival option effects. The own-firm contraction option corresponds to a put and has positive value, consistent with the product of $\nu^2 < 0$ and $[R_{11}^1 - R_{01}^1] < 0$. The rival value adjustment also has a positive value, consistent with the increased market price induced by lower industry output.

As in the expansion case, the value functions can be written as:

$$V^i(K_t, X_t) = V_A^i(K_t, X_t) + V_O^i(K_t, X_t) + V_C^i(K_t, X_t) + V_F^i(K_t, X_t).$$
In contrast to expansion options, the rival effect for downsizing is positive $V_C^i(K_t, X_t) > 0$. Risk dynamics follow from the valuation equations.

**Proposition 4.** *Systematic firm risks for both firms are*

$$
\beta^i(K_t, X_t) = 1 + \frac{V_O^i(K_t, X_t) + V_C^i(K_t, X_t)}{V^i(K_t, X_t)}(\nu_2 - 1) + \frac{V_F^i(K_t)}{V^i(K_t, X_t)},
$$

*for all industry states $K_t$, where $\nu_2 < 0$ and $V_O^i(K_t, X_t), V_C^i(K_t, X_t) > 0$.*

As in the case of expansion options, the own-firm and rival values of contractions appear additively in the numerator of the second term, again implying that own and competitor contraction options have the same risk implications when normalized by dollar values. In contrast to the case of expansion options the signs of $V_O^i$ and $V_C^i$ are always positive, which combined with $\nu_2 < 0$ implies that contraction options, whether own or rival, always reduce risk.

Figure 6 summarizes the risk dynamics. In Panel A the degree of salvage value asymmetry is large with $\rho = 0.1$, and in the remaining three panels $\rho$ progressively increases until reaching $\rho = 0.99999$. As $\rho$ increases and the follower salvage value increases, its contraction trigger moves closer to the leader’s. Unlike the expansion case, the increase in the follower’s trigger has no strategic impact on the leader’s exercise, which always occurs at the same level of demand. Even in the case where the salvage value is almost one, the difference in the leader and follower triggers is discrete. This occurs because the exit of the leader raises the incentives of the follower to delay, implying that the two triggers cannot be arbitrarily close together.

The risk dynamics of the two firms in the contraction equilibrium differ, with each firm’s risk dropping faster prior to its own capacity reduction. However, consistent with Proposition 4, both contraction options reduce risk for both firms, and the firms’ risks always move in the same direction.
5 Conclusion

We study the dynamics of risk and required return in an environment where firms with access to a common product market strategically interact, with options to expand or contract output. We find that a competitor’s options to adjust capacity, whether expansion or contraction, reduce own-firm risk through a natural hedging channel. Product market improvements increase the probability of near-term rival expansion, which provides an offsetting decrease in own-firm value. Conversely, negative demand shocks induce competitor contraction, reducing the decline in own-firm value. These results provide a counterexample to the common view that the actions of product market rivals generally increase risk, and that low risk and return are a consequence of the “quiet life” provided by barriers to entry.

The empirical implications of our analysis are broad. In the presence of non-convex adjustment costs and imperfect competition, theory predicts:

- Own firm risk declines with the probability of rival firm expansion or contraction.
- During expansions, own-firm and rival risk tend to move oppositely; industry peer betas are poor proxies for own-firm risk.
- During contractions, own-firm and rival risk tend to move together; industry peer betas are good proxies for own-firm risk.

Importantly, empirical proxies for the drivers of risk in our model are readily available. For example, the predictions above can be tested by examining the dynamics of firm risk around own-firm and rival investment “spikes,” where spikes are defined as in Cooper and Haltiwanger (2006) by abnormally large increases in investment levels. We therefore anticipate future empirical work testing these predictions.
Appendix

The first section of the Appendix contains proofs of the propositions in the main text. The second section gives details of the expansion game discussed in Section 4.1, and the final section gives details of the contraction game discussed in Section 4.2.

A Proofs of Propositions from the Main Text

A.1 Proof of Proposition One

Standard arguments (e.g., Carlson, Fisher, and Giamarino, 2004), imply that demand dynamics under the risk neutral measure are

\[ dX_t = (r - \delta)X_t dt + \sigma X_t d\hat{W}_t, \]

(7)

where \( r > \delta \equiv \mu - g > 0 \). Following Dixit and Pindyck (1994) the continuation value of the flexible firm is obtained from the Bellman equation

\[ \frac{1}{2} \sigma^2 X^2 V_{XX} + (r - \delta) X V_X - r V + X R_1 - f_1 = 0, \]

(8)

with the boundary conditions

\[
\begin{align*}
V^1(X_E) &= \frac{R^1_{21} X_E}{\delta} - I - \frac{f_2}{r} , \\
V^1(X_C) &= \frac{R^1_{01} X_C}{\delta} + S - \frac{f_0}{r} , \\
V^1_X(X_E) &= \frac{R^1_{31}}{\delta} , \\
V^1_X(X_C) &= \frac{R^1_{30}}{\delta} .
\end{align*}
\]

The first two equations ensure value matching at the instants of expansion and contraction, respectively. The last two equations guarantee smooth pasting which is a requirement for optimality.

The solution to this system of equations satisfies

\[ V^1(K_t, X_t) = \frac{R^1_{11} X_t}{\delta} - \frac{f_1}{r} + B^1_1 X_t^{\nu_1} + B^1_2 X_t^{\nu_2} , \]

where \( B^1_1 \) and \( B^1_2 \) solve

\[
\begin{align*}
(1 - \nu_1) B^1_1 X_{E}^{\nu_1} + (1 - \nu_2) B^1_2 X_{E}^{\nu_2} &= -I - \frac{f_2 - f_1}{r} , \\
(1 - \nu_1) B^1_1 X_{C}^{\nu_1} + (1 - \nu_2) B^1_2 X_{C}^{\nu_2} &= S - \frac{f_0 - f_1}{r} ,
\end{align*}
\]

(9)
and the constants $\nu_1 > 1$ and $\nu_2 < 0$ are the positive and negative roots to the characteristic equation $\sigma^2 \nu (\nu - 1)/2 + (r - \delta) \nu - r = 0$, satisfying

$$
\nu_{1,2} = \frac{1}{2} - \frac{r - \delta}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{r - \delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}.
$$

For given positive values of $X_E$ and $X_C$ we observe that $B^1_1, B^1_2 > 0$. There is no convenient analytical solution for $X_E$ and $X_C$ due to nonlinearity of the system of equations, but the values $X_E > X_C$ are easily found numerically.

The continuation value of the inflexible firm is derived using $X_E$ and $X_C$ as exogenous barriers (see Dixit, 1993). The valuation equation satisfies

$$
\frac{1}{2} \sigma^2 X^2 V^2_{XX} + (r - \delta) XV^2_X - rV^2 + X R^2_{11} - f_1 = 0,
$$

with two boundary conditions that ensure value matching:

$$
V^2(X_E) = \frac{R^2_{21} X_E}{\delta} - \frac{f_1}{r},
$$

$$
V^2(X_C) = \frac{R^2_{01} X_C}{\delta} - \frac{f_1}{r}.
$$

Smooth pasting conditions are not needed because the inflexible firm does not choose the boundary levels $X_E$ and $X_C$ and hence these values are not optimizing to the inflexible firm value.

The value function for the inflexible firm is given by

$$
V^2(K_t, X_t) = \frac{R^2_{11} X_t}{\delta} - \frac{f_1}{r} + B^2_1 X^\nu_1 + B^2_2 X^\nu_2,
$$

where $B^2_1$ and $B^2_2$ are the solutions to the equations

$$
B^2_1 X^\nu_1 + B^2_2 X^\nu_2 = \frac{R^2_{21} - R^2_{11}}{\delta} X_E,
$$

$$
B^2_1 X^\nu_1 + B^2_2 X^\nu_2 = \frac{R^2_{01} - R^2_{11}}{\delta} X_C.
$$

It is straightforward to observe that $B^2_1 < 0$ and $B^2_2 > 0$.

### A.2 Proof of Propositions Two, Three, and Four

Following the arguments in Carlson, Fisher, and Giammarino (2004) betas are given by

$$
\beta^i(K_t, X_t) = \frac{\partial V^i(K, X)}{\partial X} \frac{X}{V(K, X)}.
$$

Taking partial derivatives and substituting firm values into this expression gives the results.
B Expansion Options

We detail the outcomes of the expansion game in Section 4.1 and summarize the value functions of the two firms.

**Proposition:** The MPE of the expansion game are characterized by:

1. **Simultaneous Equilibrium:** There exists a value $\rho^{**}(\gamma, q_1, q_2, f_1, f_2, v_1) > 0$ such that for all $1 \leq \rho < \rho^{**}$ the unique MPE involves simultaneous exercise with a trigger that maximizes the low-cost firm’s value. There exist no other simultaneous investment MPE. Hence if $\rho^{**} < 1$, no simultaneous exercise equilibria exist.

2. **Non-Preemptive Leader-Follower Equilibrium:** There exists a value $\rho^{*}(\gamma, q_1, q_2, f_1, f_2, v_1) > 1$ such that for all $\rho > \max[\rho^*, \rho^{**}]$, the unique MPE results in the high-cost firm acting as the follower with trigger $X^2_F$ and the low-cost firm acting as the leader with trigger $X^1_{LN} < X^2_F$, where the triggers given in the Appendix are identical to those obtained if the roles of leader and follower were predetermined prior to the beginning of the game, and the follower could not threaten to preempt the leader’s investment.

3. **Preemptive Leader-Follower Equilibrium:** For $\rho$ satisfying $\rho^{*} \geq \rho \geq \max[\rho^*, 1]$, the unique MPE results in the high-cost firm acting as the follower with trigger $X^2_F$ and the low-cost firm acting as the leader with trigger $X^1_{LP} < X^2_F$, where the trigger of the leader $X^1_{LP} \leq X^1_{LN}$ is determined as the indifference point of the high-cost firm between acting as a leader or a follower. Hence the threat of the high-cost firm to preemptively expand causes the low-cost firm to itself preemptively invest just at the instant when the high-cost firm’s preemption threat becomes credible. The leader’s expansion deters growth of the follower in the region between $X^1_{LP}$ and $X^2_F$.

4. **Random Leader-Follower Equilibrium:** If $\rho = 1$ and $\rho^{**} < 1$, no pure strategy MPE is possible. To obtain a mixed strategy MPE requires expanding the strategy space as discussed in Fudenberg and Tirole (1985) and Huisman and Kort (1999). In the mixed strategy equilibrium the leader is randomly chosen at instant $X^1_{LP} = X^2_{LP}$, and the other firm becomes the follower exercising at $X^1_F = X^2_F$.

**Proof:** The arguments build on Pawlina and Kort (2006). We provide detailed logic to keep the proof self contained, and extend their proof along the following dimensions: (i) we permit operating leverage; (ii) we accommodate the case where $\rho = 1$ so that firms 1 and 2 are identical ex ante; and (iii) we provide formulas for the value functions for all $X \in (0, \infty)$ and all industry stages juvenile, adolescent, and mature.

The structure of the proof is to provide in Part 1 the value functions of firm 1 and 2 under different strategies:

\[18\text{See also Thijssen, Huisman, and Kort (2002) and Boyer, Lasserre, Mariotti, and Moreaux (2004).}\]

\[19\text{See also the paper by Mason and Weeds (2010) in which non-preemptive, preemptive and simultaneous-move equilibria in a simple real option game are discussed.}\]
i) Nonpreemptive leader-follower: firm 1 expands before firm 2 at a demand level that satisfies firm 1’s smooth pasting condition.

ii) Preemptive leader-follower: firm 1 expands before firm 2 but at a demand level forced by firm 2’s preemption threat. Firm 1’s exercise will not satisfy smooth pasting because the exercise is not at an unconstrained optimal level.

iii) Simultaneous move: both firms expand simultaneously.

iv) Off-equilibrium: firm 2 leads.

Following Part 1, in Parts 2-4, we provide conditions under which equilibrium holds.

**Part 1: Value function calculations**

i) Non-preemptive leader-follower: The value of firm 1 satisfies

\[
\frac{1}{2} \sigma^2 X^2 V^1_{XX} + (r - \delta) XV^1_X - rV^1 + XR^1_{11} - f_1 = 0, \quad (11)
\]

with the boundary conditions

\[
V^1(X^1_L) = \frac{R^1_{21} X^1_L}{\delta} - \frac{f_2}{r} - I + B (X^1_L)^{\nu_1} \quad (12)
\]

\[
V^1(X^2_F) = \frac{R^1_{22} X^2_F}{\delta} - \frac{f_2}{r} \quad (13)
\]

\[
V^1_X(X^1_L) = \frac{R^1_{21}}{\delta} + B \nu_1 (X^1_L)^{\nu_1-1} \quad (14)
\]

where \(X^1_L\) is the exercise trigger optimally set by firm 1, \(X^2_F > X^1_L\) is the trigger level when the follower exercises the option, and the constant \(B\) is calculated through backward induction using the follower’s trigger. The solution is

\[
V^1_{L}(X) = \frac{R^1_{11} X_t}{\delta} - \frac{f_1}{r} + \frac{(R^1_{21} - R^1_{11}) X^1_L}{\delta \nu_1} \left( \frac{X}{X^1_L} \right)^{\nu_1} \nonumber \\
+ \frac{(R^1_{22} - R^1_{21}) X^2_F}{\delta} \left( \frac{X}{X^2_F} \right)^{\nu_1}, \quad (15)
\]

where

\[
X^1_L = X^1_{LN} = \frac{\nu_1}{\nu_1 - 1} \frac{\delta(f_2 - f_1 + r I)}{r(R^1_{21} - R^1_{11})}. \quad (16)
\]

The value of firm 2 satisfies:

\[
\frac{1}{2} \sigma^2 X^2 V^2_{XX} + (r - \delta) XV^2_X - rV^2 + XR^2_{11} - f_1 = 0, \quad (17)
\]
with the boundary conditions

\[ V^2(X^1_L) = \frac{R^2_{21}X^1_L}{\delta} - \frac{f_1}{r} + C \left( X^1_L \right)^{\nu_1} \]  \hspace{1cm} (18)

\[ V^2(X^2_F) = \frac{R^2_{22}X^2_F}{\delta} - \frac{f_2}{r} - \rho \]  \hspace{1cm} (19)

\[ V^2_X(X^2_F) = \frac{R^2_{22}}{\delta}, \]  \hspace{1cm} (20)

where \( C \) is a constant. We obtain

\[ V^2_F(X) = \frac{R^1_{11}X_t}{\delta} - \frac{f_1}{r} + \left( R^2_{22} - R^2_{21} \right) \left( \frac{X}{X^1_L} \right)^{\nu_1} \]

\[ + \left( R^2_{22} - R^2_{21} \right) \left( \frac{X}{X^2_F} \right)^{\nu_1}, \]  \hspace{1cm} (21)

where

\[ X^2_F = \frac{\nu_1}{\nu_1 - 1} \frac{\delta(f_2 - f_1 + r\rho)}{r(R^2_{22} - R^2_{21})}. \]  \hspace{1cm} (22)

\[ ii) \ Preemptive \ leader \ follower: \] In a preemptive equilibrium, firm 1 chooses a trigger \( X^1_L \) that does not satisfy the smooth pasting condition (14). The firm value is

\[ V^1(X) = \frac{R^1_{11}X}{\delta} - \frac{f_1}{r} + \left( R^1_{22} - R^1_{11} \right) \left( \frac{X}{X^1_L} \right)^{\nu_1} \]

\[ + \left[ \frac{(R^1_{21} - R^1_{11})X^1_L}{\delta} - \frac{f_2 - f_1 + r\rho}{r} \right] \left( \frac{X}{X^1_L} \right)^{\nu_1}. \]  \hspace{1cm} (23)

\[ iii) \ Simultaneous \ move: \] If both firms exercise simultaneously at the trigger level \( X^i_S \) the value functions satisfy

\[ V^i_S(X) = \frac{R^i_{11}X}{\delta} - \frac{f_1}{r} + \frac{(f_2 - f_1 + r\rho)}{r(\nu_1 - 1)} \left( \frac{X}{X^i_S} \right)^{\nu_1} \]  \hspace{1cm} (24)

with the trigger levels

\[ X^i_S = \frac{\nu_1}{\nu_1 - 1} \frac{\delta(f_2 - f_1 + r\rho)}{r(R^2_{22} - R^2_{11})}. \]  \hspace{1cm} (25)

\[ iv) \ Off-equilibrium, \ firm \ 2 \ leads: \] If firm 2 does not act as the follower and instead exercises as the leader its value function is

\[ V^2_L(X) = \frac{R^2_{11}X}{\delta} - \frac{f_1}{r} + \left( \frac{R^2_{22} - R^2_{11}}{\delta} \right) \left( \frac{X}{X^2_L} \right)^{\nu_1} \]

\[ + \left( \frac{R^2_{22} - R^2_{12}}{\delta} \right) \left( \frac{X}{X^1_F} \right)^{\nu_1} \]  \hspace{1cm} (26)

with \( X^2_L \) as the leader’s trigger for firm 2 and \( X^1_F \) the follower’s trigger for firm 1.
Part 2: Conditions for preemptive and non-preemptive equilibria: In a preemptive equilibrium, firm 1 expands at the instant when firm 2 is indifferent between acting as the leader or the follower. The difference between firm 2’s value with immediate exercise and its value as a follower is given by:

\[
G(X, \rho) \equiv \frac{(R_{12}^2 - R_{21}^2)X}{\delta} - \frac{(f_2 - f_1 + rI)}{r} + \frac{R_{22}^2 - R_{21}^2}{\delta} \frac{X^1_1}{r} \left( \frac{X}{X_s} \right)^{\nu_1} - \frac{(R_{22}^2 - R_{21}^2)X^2_1}{\delta \nu_1} \left( \frac{X}{X^2_F} \right)^{\nu_1}.
\]

(27)

This function is strictly concave in \(X\), since \(\nu_1 > 1\), \((R_{22}^2 - R_{21}^2) < 0\), and \((R_{22}^2 - R_{21}^2) > 0\). It therefore has a unique maximum which is characterized by \(\partial G(X, \rho)/\partial X = 0\). Denoting this value \(\hat{X}\), the function \(G\) has zero, one, or two real roots, depending on whether \(G(\hat{X}, \rho) < 0\), \(G(\hat{X}, \rho) = 0\), or \(G(\hat{X}, \rho) > 0\), respectively. In addition, \(G\) can be seen to be strictly decreasing in \(\rho\) by substituting \(X^2_F\) from equation (22) into (27).

Let the couple \((X^*, \rho^*)\) solve the system of equations

\[
G(X^*, \rho^*) = 0,
\]

(28)

\[
\frac{\partial G(X^*, \rho^*)}{\partial X} = 0.
\]

(29)

For any \(\rho \geq \rho^*\) the follower does not have an incentive to become the leader, since there are no real solutions to \(G(X, \rho) = 0\), and in equilibrium firm 1 acts as the leader and firm 2 acts as the follower. For \(\rho < \rho^*\) firm 2 has an incentive to become the leader. This incentive exists for all values of \(X\) in the interval \([X^1_{LP}, X^2_F]\), where \(X^1_{LP}\) is the smallest solution to \(G(X, \rho) = 0\). If the leader’s investment trigger satisfies \(X^1_L < X^1_{LP}\) the follower value of firm 2 exceeds its leader value and firm 2 does not have an incentive to change its follower role. If, however, \(X^1_L > X^1_{LP}\) the follower has an incentive to preempt the leader, which in turn causes the leader to choose \(X^1_{LP}\) as the investment trigger.

Part 3: Conditions for simultaneous equilibrium: Since \(I^1 = I < \rho I = I^2\) the only candidate for a simultaneous equilibrium is trigger level \(X^*_S < X^2_S\). In a simultaneous equilibrium: (i) the value of firm 1 being the leader has to be smaller than moving simultaneously with firm 2, and (ii) firm 2 has to find it profitable to move simultaneously with firm 1 and not to wait and act as the follower. The difference between the firm 1 leader value assuming immediate exercise and the value from waiting for simultaneous exercise is given by

\[
\Delta(X, \rho) = \frac{(R_{12}^1 - R_{11}^1)X}{\delta} \frac{f_2 - f_1 + rI}{r} + \frac{R_{22}^1 - R_{21}^1}{\delta} \frac{X^1_1}{r} \left( \frac{X}{X^2_F} \right)^{\nu_1} \]

\[
\quad - \frac{(R_{22}^1 - R_{21}^1)X^1_S}{\delta \nu_1} \left( \frac{X}{X^1_S} \right)^{\nu_1}.
\]

(30)

This function is strictly concave in \(X\), since \(\nu_1 > 1\), \((R_{22}^1 - R_{21}^1) < 0\), and \((R_{22}^1 - R_{21}^1) > 0\). It therefore has a unique maximum which is characterized by \(\partial \Delta(X, \rho)/\partial X = 0\). In addition, \(\Delta\) can be seen to be strictly decreasing in \(\rho\) by substituting \(X^2_F\) from equation (22) into (30).

Following the logic that was used in Part 2 above, it can be shown that there exists \(X^{**}\)
and \( \rho^* \) such that \( \Delta(X^*, \rho^*) = 0 \) and \( \Delta X(X^*, \rho^*) = 0 \), implying that for all \( \rho < \rho^* \) firm 1 prefers the simultaneous exercise equilibrium. (If \( \rho^* < 1 \) a simultaneous equilibrium is not possible.) Comparing equations (22) and (25), \( R^2_{21} < R^2_{11} \) implies that \( X^*_F < X^*_S \), thus if the conditions for optimal simultaneous exercise are satisfied for firm 1, they are also satisfied for firm 2.

**Part 4: Random leader follower equilibrium:** If \( \rho = 1 \) and \( \rho^* < 1 \), none of the pure strategy equilibria above are possible. To define a mixed strategy equilibrium requires an expansion of the strategy space following Fudenberg and Tirole (1985) and Huisman and Kort (1999). In such a mixed strategy equilibrium the leader is chosen randomly at the trigger level \( X^*_L \) while the other firm acts as the follower. ■

For the reader’s convenience we summarize the leader and follower value functions for asymmetric and symmetric exercise. The value function for the leader in a preemptive or non-preemptive equilibrium is given by:

\[
V^1(K_t, X_t) = \begin{cases} 
\frac{R^1_1 X_t}{\delta} - \frac{f_1}{r} + \left[ \frac{(R^1_{21} - R^1_{11}) X_t}{\delta} - \frac{(f_2 - f_1 + r I)}{r} \right] \left( \frac{X_t}{X^*_L} \right)^{\nu_1} \\
\frac{X^2_F}{r} \left[ R^1_{22} - R^1_{21} \right] \left( \frac{X_t}{X^*_F} \right)^{\nu_1} \\
\frac{R^2_{21} X_t}{\delta} - \frac{f_2}{r} + \frac{X^2_F}{r} \left[ R^1_{22} - R^1_{21} \right] \left( \frac{X_t}{X^*_F} \right)^{\nu_1} \\
\frac{R^2_{12} X_t}{\delta} - \frac{f_2}{r} 
\end{cases}
\]

with \( X^*_L \) either being the preemptive or the non-preemptive trigger. The value function of the follower is given by:

\[
V^2(K_t, X_t) = \begin{cases} 
\frac{R^2_1 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + r I}{r(\nu_1 - 1)} \left( \frac{X_t}{X^*_F} \right)^{\nu_1} + \frac{X^*_L}{\delta} \left[ R^2_{21} - R^2_{11} \right] \left( \frac{X_t}{X^*_L} \right)^{\nu_1} \\
\frac{R^2_2 X_t}{\delta} - \frac{f_2}{r} + \frac{f_2 - f_1 + r I}{r(\nu_1 - 1)} \left( \frac{X_t}{X^*_F} \right)^{\nu_1} \\
\frac{R^2_{22} X_t}{\delta} - \frac{f_2}{r} 
\end{cases}
\]

with \( X^*_F \) as the optimal follower trigger.

Under simultaneous exercise the value functions are:

\[
V^i_S(K_t, X_t) = \begin{cases} 
\frac{R^1_1 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + r I^i}{r(\nu_1 - 1)} \left( \frac{X_t}{X^*_S} \right)^{\nu_1} X_t \leq X^*_S, \\
\frac{R^2_1 X_t}{\delta} - \frac{f_2}{r} X_t > X^*_S, 
\end{cases}
\]

with the expansion trigger \( X^*_S = \nu_1 \delta (f_2 - f_1 + r I^i) / [(\nu_1 - 1) r (R^i_{22} - R^i_{11})] \).

**C Contraction Options**

We detail the outcomes of the contraction option game in Section 4.2. and summarize the firms’ value functions.
Proposition: For every $0 < \rho < 1$ there exists a unique MPE of the contraction game in which the high salvage value firm acts as the leader with trigger $X^L_C$ and the low salvage value firm acts as the follower with trigger $X^H_1$. If $\rho = 1$, there exist two pure strategy MPE, one in which firm 1 acts as the leader and firm 2 as the follower, and in the other firm 2 acts as the leader and firm 1 as the follower. No equilibrium exists with positive probability of simultaneous contraction.

Proof: The value function of firm 2 acting as a follower is

$$V^2_F(K_t, X_t) = \begin{cases} \frac{R^2_{0t}X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + r\rho S}{r(1 - \nu_2)} \left( \frac{X_t}{X^C_1} \right)^{\nu_2} & X_t \geq X^2_C, \\ \frac{R^2_{0t}X_t}{\delta} - \frac{f_0}{r} + \rho S & X_t \leq X^2_C, \end{cases}$$

(31)

where

$$X^2_C = \frac{\nu_2\delta(f_1 - f_0 + r\rho S)}{(1 - \nu_2)r[R^2_{00} - R^2_{01}]}$$

is the contraction trigger when firm 2 acts as the follower. Now assume, instead, that firm 2 having the smaller salvage value acts as the leader. The value function of firm 2 at the time when it contracts as the leader is

$$V^2_L(K_t, X_t) = \begin{cases} \frac{R^2_{0t}X_t}{\delta} - \frac{f_0}{r} + \rho S + \frac{X^1_{1,F}[R^2_{00} - R^2_{10}]}{X^C_1} \left( \frac{X_t}{X^C_1} \right)^{\nu_2} & X_t \geq X^1_{1,F}, \\ \frac{R^2_{0t}X_t}{\delta} - \frac{f_0}{r} + \rho S & X_t \leq X^1_{1,F}, \end{cases}$$

(33)

where

$$X^1_{1,F} = \frac{\nu_2\delta(f_1 - f_0 + r\rho S)}{(1 - \nu_2)r[R^1_{00} - R^1_{10}]}$$

(34)

is the trigger when firm 1 contracts as the follower and firm 2 acts as the leader. Given our assumptions on the revenue functions it follows that trigger (34) is strictly greater than trigger (32) for $0 < \rho < 1$. From this property and the value functions (31) and (33) it can be shown that

$$G(X_t, \rho) \equiv V^2_L(X_t, \rho) - V^2_F(X_t, \rho) \leq 0$$

holds for all $X_t$. Hence, firm 2 never has an incentive to become the leader. Therefore sequential exercise of contraction options is the unique pure strategy MPE for $\rho < 1$. Sequential exercise of firms from the industry has been shown in the paper by Murto (2004) for asymmetric firms, as well. His model differs from ours, however, since firms have to pay a cost when exiting the market that is smaller than the current fixed costs instead of receiving a positive salvage value. In case of symmetric firms, $\rho = 1$, Proposition 2 in Murto (2004) can be applied which establishes the existence of two pure strategy MPE.

For the reader’s convenience, we summarize the value functions of the leader and follower. The leader’s value function is

$$V^1_L(K_t, X_t) = \begin{cases} \frac{R^1_{1t}X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + rS}{r(1 - \nu_2)} \left( \frac{X_t}{X^C_1} \right)^{\nu_2} + \frac{X^2_C}{X^C_1} [R^1_{00} - R^1_{01}] \left( \frac{X_t}{X^C_1} \right)^{\nu_2} & X_t > X^1_C, \\ \frac{R^1_{1t}X_t}{\delta} - \frac{f_0}{r} + \frac{X^2_C}{X^C_1} [R^1_{00} - R^1_{01}] \left( \frac{X_t}{X^C_1} \right)^{\nu_2} & X^2_C \leq X_t \leq X^1_C, \\ \frac{R^1_{1t}X_t}{\delta} - \frac{f_0}{r} & X_t < X^2_C, \end{cases}$$

28
with the optimal trigger $X_C^1 = \frac{\nu_2 \delta (f_1 - f_0 + rS)}{(1 - \nu_2) r |R_{01} - R_{11}|}$. The follower’s value function is

$$V_F^2(K_t, X_{t}) = \begin{cases} 
\frac{R_{11} X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + r\rho S}{r(1 - \nu_2)} \left( \frac{X_t}{X_C^2} \right)^{\nu_2} + \frac{X_C^2}{\delta} \frac{R_{01}^2 - R_{11}^2}{R_{00}^2 - R_{01}^2} \left( \frac{X_t}{X_C^2} \right)^{\nu_2} & X_t \geq X_C^1, \\
\frac{R_{01}^2 X_t}{\delta} - \frac{f_0}{r} + \frac{f_1 - f_0 + r\rho S}{r(1 - \nu_2)} \left( \frac{X_t}{X_C^2} \right)^{\nu_2} & X_C^2 \leq X_t \leq X_C^1, \\
\frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{r} & X_t < X_C^2. 
\end{cases}$$

with the contraction trigger $X_C^2 = \frac{\nu_2 \delta (f_1 - f_0 + rS)}{(1 - \nu_2) r |R_{00} - R_{01}|}$ that satisfies the inequality $X_C^1 > X_C^2$.

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$^{20}$Derivation of the leader’s value function follows standard arguments using corresponding value matching and smooth pasting conditions.
References


[34] McDonald, R., and D. Siegel, 1985, Investment and the valuation of firms when there is an option to shut down. *International Economic Review* 26, 331-49.


Figure 1: The hedging effect of a rival real option. The figure illustrates price responses to demand shocks with and without an expansion response by a rival. With current industry output fixed at $Q_1$, an upward shift in demand from $D$ to $D'$ will result in an increase in the product price from $P_1$ to $P^*$. If demand crosses the threshold for investment by the rival, an increase in industry output to $Q_2$ dampens the price response, so that prices increase only to $P_2$. 
Figure 2: Risk dynamics of the flexible and inflexible firm. This figure represents the relationship between the level of the demand state $X_t$ and the systematic risk $\beta$ of firms within an industry. The solid curve illustrates risk dynamics for the flexible firm. Flexible firm risk derives from assets in place, a contraction option, and an expansion option when $X_c < X_t < X_e$ and from assets in place alone otherwise. The risk-reducing effect of the contraction option is largest (smallest) and the risk-increasing effect of the expansion option is smallest (largest) when $X_t = X_c$ ($X_t = X_e$). Exercising either option causes a discontinuous change in risk to $\beta = 1$. The dashed curve illustrates risk dynamics for the inflexible firm. Rival options unambiguously decrease inflexible firm risk, so that the inflexible firm’s beta is relatively low at both of the critical points $X_c$ and $X_e$. The parameter values are $\gamma = 0.5$, $\sigma = 0.2$, $q_0 = 1$, $q_1 = 2$, $q_2 = 10$, $f_0 = f_1 = f_2 = 0$, $r_f = 0.05$, $\delta = 0.03$, $S = 50$, and $I = 500$. 
Figure 3: Timeline of the duopoly investment game. This figure illustrates the endogenous timing of industry expansions in a leader-follower equilibrium.
Figure 4: Equilibrium regions in the expansion game. This figure shows how inverse demand elasticity $\gamma$ and investment cost differentials $\rho$ relate to the equilibrium predicted by the expansion game. The simultaneous-move equilibrium occurs in the region $1 \leq \rho \leq \rho^{**}$, the preemptive leader-follower equilibrium exists in the region $\max\{\rho^{**},1\} < \rho \leq \rho^*$, and the non-preemptive leader-follower equilibrium exists in the region $\rho \geq \max\{\rho^*,\rho^{**}\}$. The random-leader equilibrium is the only possibility when $\rho = 1$ and $\rho^{**} < 1$, indicated by asterisks. The parameter values used for the figure are $\sigma = 0.2$, $q_1 = 2$, $q_2 = 10$, $f_1 = f_2 = 0$, $r_f = 0.05$, $\delta = 0.03$, and $I = 500$. 
Figure 5: Risk dynamics in expansions. This figure illustrates leader (dashed line) and follower (solid line) risk dynamics in the expansion game. In Panels A and B the equilibrium is non-preemptive leader-follower, in Panel C it is preemptive leader-follower, and in Panel D it is simultaneous. In Panels A-C, leader risk is above (below) follower risk prior to (following) leader expansion. Leader and follower firm risk dynamics are identical only in the simultaneous equilibrium of Panel D. The parameter values are $\gamma = 0.5$, $\sigma = 0.2$, $q_1 = 2$, $q_2 = 10$, $f_1 = f_2 = 0$, $r_f = 0.05$, $\delta = 0.03$, and $I = 500$. 

Figure 6: Risk dynamics in contractions. This figure illustrates leader (dashed line) and follower (solid line) risk dynamics in the contraction game for various values of $\rho$. Leader risk is below (above) follower risk prior to (following) leader contraction. The parameter values are $\gamma = 0.5$, $\sigma = 0.2$, $q_0 = 1$, $q_1 = 2$, $f_0 = f_1 = 0$, $r_f = 0.05$, $\delta = 0.03$, and $S = 50$. 