Idea: test C-CAPM on survey data

Motivation: avoid the “joint test” of the model and rational expectations. Focus on the model.
The Standard Consumption Based Asset Pricing Model

CRRA utility: $C_t^{1-\gamma} / (1 - \gamma)$

Assume: excess return ($R^e_t$) and consumption growth ($\Delta c_t$) have bivariate normal distribution

$$E(R^e_t) = \text{Cov}(R^e_t, \Delta c_t)\gamma$$  (1)

$$= \text{Corr}(R^e_t, \Delta c_t)\sigma(R^e_t)\sigma(\Delta c_t)\gamma$$  (2)
Ex post data (US, 1952–2005) and the “equity premium puzzle”

\[
E(R_t^e) = \text{Corr}(R_t^e, \Delta c_t) \sigma(R_t^e) \sigma(\Delta c_t) \gamma
\]

\[
\begin{array}{cccc}
0.06 & \text{0.17 or 0.33} & 0.14 & 0.01 \\
\end{array}
\]

Requires risk aversion (\(\gamma\)) to be very high...(here \(\gamma \geq 43\))
Relation Between Survey Data and the Asset Pricing Equation

Conditional moments (survey) → unconditional moments (asset pricing equation)

Asset pricing equation holds for each individual investor, if beliefs about $\Delta c_{it}$. Data is on $\Delta c_t$. Must assume

1. Idiosyncratic risk is not priced
2. Aggregation of beliefs is not complicated. From theory: “representative” investor has
   (a) mean of individual means
   (b) variance = f(individual variance, disagreement). $\gamma = 1$ vs $\gamma = 10$
Warning: no single (and consistent) database...a mosaic of evidence
Expected Stock Return

S&P Industrials (–1990), S&P 500 Composite (1990–)

June and December surveys: index levels in 7 and 13 months ahead. Problems with “base values”. Focus on expectations for 7-13 months (“forward” return)

Only the capital gain part of returns: assume dividend yield is (on average) correctly guessed

Aggregation (“mean of individual means”): cross-sectional median in $t$

Conditional $\rightarrow$ unconditional: $E E_{t-1} R^e_t$
Figure 1: Expected capital gains on the S&P index in excess of a riskfree rate, %.
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Survey expectations</td>
<td>−1.1</td>
<td>1.5</td>
<td>−0.4</td>
</tr>
<tr>
<td>S&amp;P Industrials, ex post</td>
<td>3.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500, ex post</td>
<td></td>
<td>5.3</td>
<td></td>
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<tr>
<td>S&amp;P combined, ex post</td>
<td></td>
<td></td>
<td>3.8</td>
</tr>
<tr>
<td>Dividend yield S&amp;P 500</td>
<td>4.0</td>
<td>2.1</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table 1: **Average capital gains in excess of a riskfree rate, dividend yield, annualised %**.

⇒ $E(R_i^e) \approx 3\% - 3.5\%$, or half the historical mean excess return

$$E(R_i^e) = \text{Corr}(R_i^e, \Delta c_t) \sigma(R_i^e) \sigma(\Delta c_t) \gamma$$

$$0.5 \times 0.06$$
Plausible?

(1) Volatile market, hard to “learn”. Avg forecast error=3%, Std(forecast error)=16%, $T = 100$. $t$-test of avg error=0:

$$t = \frac{3}{16/\sqrt{100}} = 1.9$$

(2) Figure 2: 2 prolonged periods of consistent underestimation: 1954–1955 (+70%) and 1995–1999 (+300%). Korean war and dot-com booms.
Figure 2: Expected and ex post capital gains on the S&P index in excess of a risk-free rate, %.
Consumption Growth Volatility

Survey of Professional Forecasters: survey of economic experts

*Subjective*: SPF asks for probabilities of GDP growth rates:

1. Survey is about output growth: assume results carry over to consumption growth
2. Estimate a variance from the “histogram” (for each forecaster and period)
3. Aggregation issues are disregarded: cross-sectional median of variance (of active in $t$)
4. Conditional→unconditional: assume no predictability

\[
\text{unconditional Std} = \sqrt{\text{time-average conditional variance}}
\]
Ex post:

Forecast error Std (using consensus forecasts)
<table>
<thead>
<tr>
<th></th>
<th>4 quarters</th>
<th>3 quarters</th>
<th>2 quarters</th>
<th>1 quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey</td>
<td>0.8</td>
<td>0.7</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Ex post</td>
<td>1.1</td>
<td>0.9</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 2: Standard deviation of year-on-year output growth, %.

⇒ \( \sigma(\Delta c_t) \approx 0.6 \times \text{historical std (forecasters underestimate uncertainty)} \)

\[
\mathbb{E}(R_t^e) = \text{Corr}(R_t^e, \Delta c_t)\sigma(R_t^e)\sigma(\Delta c_t)\gamma_{0.6 \times 0.01}
\]
Stock Return Volatility

No survey data

Lots of S&P 500 options (daily data 1985–2005, different times to expiration)

For each trade date: average IV of 2 puts and 2 calls closest to atm

Aggregation: already done by the market...

Conditional→unconditional: assume unpredictable returns

unconditional Std = \sqrt{\text{time-average conditional variance}}


<table>
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<tr>
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<th>4 quarters</th>
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<th>1 quarter</th>
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<tr>
<td>Ex post</td>
<td>16.8</td>
<td>16.7</td>
<td>16.7</td>
<td>16.6</td>
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</tbody>
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Table 3: Standard deviation of S&P 500 returns, annualised %.

⇒ $\sigma(R_t^e) = 1.15 \times \text{historical volatility}$

\[
E(R_t^e) = \text{Corr}(R_t^e, \Delta c_t) \frac{\sigma(R_t^e)}{1.15 \times 0.14} \sigma(\Delta c_t) \gamma
\]
Plausible?

(1) Stochastic, non-priced volatility (Hull-White): add approx 0.5 to IV

(2) Stochastic, priced volatility (Heston): perhaps. But Figure 3...IV overestimates more at low realised volatility. Really risk premia (since low vol≈stable vol)? Perhaps expectation errors?
Figure 3: Standard deviation of the S&P 500 index, annualised %. 

Standard deviation of S&P500, 6–month horizon

%
Implications for the Equity Premium

\[
\begin{align*}
E(R_t^e) &= \text{Corr}(R_t^e, \Delta c_t) \cdot \sigma(R_t^e) \cdot \sigma(\Delta c_t) \cdot \gamma. \\
&= 0.5 \times 0.06 \cdot ? \times 1.15 \times 0.14 \times 0.6 \times 0.01
\end{align*}
\]

\[
\Rightarrow \gamma \text{ with survey data need to be } 0.7 \text{ of } \gamma \text{ in ex post data (30 instead of 43)}
\]

Progress, but no clear victory for C-CAPM