Incentives to Innovate and the Decision to Go Public or Private*

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Abstract

We model the impact of public and private ownership structures on firms’ incentives to choose innovative projects. Innovation requires the exploration of new ideas with potential advantages but unknown probability of success. We show that it is optimal to go public when firms wish to exploit the current technology and to go private when firms wish to explore new ideas. This result follows from the fact that privately-held companies are less transparent to outside investors than publicly-held ones. In private firms, insiders can time the market by choosing an early exit strategy when they learn bad news. This option makes insiders more tolerant to failures and thus more inclined to choose innovative projects. However, in public firms this option is less valuable because there is less information asymmetry concerning cash flows. In such firms, prices of publicly-traded securities react quickly to good news, providing insiders with incentives to choose conventional but safer projects, in order to cash in early when good news arrive. Extensions to the model allow us to incorporate other drivers of the decision to go public or private, such as liquidity and cost of capital considerations. Our model rationalizes recent evidence linking private equity to innovation and creative destruction and also generates new predictions concerning the determinants of going public and going private decisions.

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1. Introduction

Privately-held companies are more innovative. Using patent citation data, Lerner, Sorensen and Stromberg (2008) find that firms invest in more influential innovations after being acquired by private equity funds. Popov and Roosenboom (2008) find that private equity investment increases the number of patents in a panel sample of firms from 21 European countries. There is also evidence that private equity is associated with corporate restructurings and turnarounds, changes in strategic direction, and "creative destruction" (i.e. Schumpeterian innovation). Davis et al. (2008) find that firms acquired by private equity funds fire workers and shut down many existing establishments, but also that they create new jobs in new greenfield establishments and engage more both in mergers and acquisitions and in divestitures. They conclude that private equity is a catalyst for creative destruction.

In this paper we develop a model in which a firm’s ownership structure—the choice between being public or private—affects corporate insiders’ incentives to innovate. Our key contribution is to show that private ownership creates incentives towards innovation, while public ownership creates incentives against innovation. Thus, our model can rationalize the link between private ownership and innovation. Because we allow for an endogenous choice of ownership structure, the model also provides a novel explanation for going public and going private decisions. We find that the decision to go public or private is affected by the relative profitability of innovative versus conventional projects.

We develop the main ideas in a model in which a risk-neutral insider, who can choose between an innovative project and a conventional project (exploitation of existing ideas or exploration of new ones), may choose to liquidate his stake early by trading on the basis of his private information. We first show that, under private ownership, if the insider can time the market by choosing an early exit strategy when he learns bad news, he becomes more tolerant to failures and thus more inclined to choose the innovative project. Exiting early constitutes a real option to the insider. This option is valuable only when the insider’s actions do not fully reveal his private information. An early exit does not fully reveal the
insider’s private information for two reasons. First, cash flows of private firms may not be fully observable, thus outside investors may be poorly informed about interim cash flows generated by projects. Second, the insider may suffer a liquidity shock, thus outside investors cannot know whether trading is motivated by information or liquidity.

Under public ownership, cash flows are observable and thus an early exit after bad news is not profitable. There is no tolerance to failures in public companies. Furthermore, market prices of public securities react quickly to good news. This is known to create incentives for short-termist behavior (Stein, 1989). A rational pressure towards quick results arises in our model because good news are quickly incorporated into market prices. Thus, the insider may prefer the conventional project to the innovative one because the former has a higher probability of an early success. When the choice of projects is not perfectly observable and the market expects innovation to occur, the market will erroneously attribute a high value to good news coming from a firm that chose the conventional project. Clearly, this situation cannot be an equilibrium. We show that the unique equilibrium involves choosing the conventional project with some positive probability, even when innovation is ex ante efficient.

Our model shows that incentives in public firms are biased towards conventional projects, while incentives in private firms are biased towards innovative projects. Consequently, the optimal structure of ownership—public or private—may change with the firm’s life cycle depending on whether exploration of new ideas or exploitation of existing ones is optimal.

We interpret our model as a theory of the evolution of ownership structures. It is usually believed that innovation is very important early in a firm’s life. Our model thus predicts that it is optimal to start private to maximize incentives to innovate. Our model would also view going private decisions as complements to risky restructurings. Every time a firm needs to reinvent itself, it makes sense to do it out of the public eye. Major company restructurings involving radical changes in strategy are departures from the conventional, and thus more properly motivated under private equity.
Our theory fits well with some anecdotal accounts of going private decisions. An example is the 2007 acquisition of Bausch & Lomb, an eye care products company, by Warburg Pincus, a private equity firm. The (friendly) acquisition was triggered by problems associated with Bausch & Lomb’s recall of its contact solution in 2006 because of links to *fusarium keratitis*, a rare fungal eye infection. Not only the company had to deal with corporate liability lawsuits, it also had its brand seriously tarnished. Bausch & Lomb’s CEO justified going private as a means to give the company flexibility to focus on the long run: "As a private company, Bausch & Lomb will have greater flexibility to focus on our long-term strategic direction to be a global leader in providing innovative and technologically advanced eye health products to eye care professionals and consumers."\(^1\)

Our model would explain this example as follows. A negative shock (lawsuits) destroys much of the brand value of B&L. The company then needs to invest in new products and strengthen its brand. These are new rather than conventional projects, with risky but potentially positive returns in the long run, and as such more easily motivated under a private ownership structure.

Our model is closely related to two different theoretical literatures: (1) models of interactions between stock prices and incentives in firms and (2) models of the decision to go public or private.


Our model is closely related to the work of Stein (1989), who develops a model of rational managerial short-termism driven by the stock market. In his model, firms take actions to boost current earnings at the cost of lower future earnings in an attempt to mislead the

\(^1\)CEO Ron Zarrella; quote from FOXnews.com (Associated Press), May 16, 2007.
market. However, in equilibrium the market is not fooled but managers are stuck with an inefficient strategy. The same logic is present in our model. If the firm is public, a manager may choose the conventional project even when the innovative project has a higher NPV, because the former has a higher probability of generating high earnings in the short run. But our model also shows the other side of the story. If the firm is private and thus free from pressure to boost current earnings, it will normally put too much emphasis on future cash flows. Without the stock market punishing short-run falls in earnings, managers rationally become biased towards innovative projects, which are risky but very profitable if successful. This bias may lead them to *inefficient long-termism*: innovation may be chosen even when it is inferior to conventional methods. Thus, our model provides a more balanced view of market incentives: while managers of public firms may focus excessively on current earnings, managers of private firms may focus excessively on future earnings. The best structure thus depends on the nature of the set of projects available to the firm at the time.

Our paper is also related to a large literature on the choice between public and private structures, including Boot, Gopalan, and Thakor (2006), Shah and Thakor (1988), Chemmanur and Fulghieri (1999), Zingales (1995), and Pagano and Roel (1998). None of these papers consider incentives for innovation as a determinant of ownership structures.

More closely related to our model is the work of Maksimovic and Pichler (2001). In their model, firms may choose between a new or an existing technology and then decide whether to finance future rounds of investment with either public or private offerings. Public offerings are assumed to be cheaper, but they reveal information about the industry’s viability to potential competitors. Thus, firms may strategically choose to delay finance or resort to private offerings to prevent entry. Their model is concerned with the effect of technological uncertainty at the industry level on the mode and timing of financing. Our model is concerned with the effect of the financing mode (private or public) on firms’ internal incentives to choose between different technologies. Thus, our model allows us to address a very different question: Should the decision to go public or private depend on the perceived profitability
of new versus old technologies?

The paper is structured as follows. We present the basic model in Section 2, discuss the going public or private decision in Section 3, develop extensions in Section 4, and conclude with a discussion of empirical applications in Section 5. All proofs are in the Appendix.

2. The basic model

2.1. Setup

A risk-neutral insider initially holds all shares of a firm. The insider has no initial wealth, is protected by limited liability, and has outside utility normalized to zero. We view the insider as a manager-entrepreneur who founded the firm and owns it fully. Because the identity of the manager making the decisions is not important in our model, we assume that the founder stays as manager regardless of how many shares he sells to other investors. The results are identical if the founder is replaced by a newly-hired professional manager.

2.1.1. Technology

The insider has to decide between two projects, projects 1 and 2, at two consecutive dates, dates 0 and 1. Each project has two possible outcomes: success or failure. Success yields earnings $S$ and failure yields earnings $F$, $S > F$. We call project 1 exploitation of existing ideas and project 2 exploration of new ideas (this terminology is from March, 1991). This setup is from Manso (2006).

If the insider chooses project 1, the conventional project, there is a probability $p$ of success. The probability $p$ is known to everyone. If the insider chooses project 2, the innovative project, the probability of success is $p_2$. The probably $p_2$ is unknown, but the insider updates his expectations about $p_2$ after learning the outcome of the project undertaken. We assume that

$$E [p_2|F] < E [p_2] < E [p_2|S].$$  (1)
The expectation of success $p_2$ is higher if the project was successful in period 1. It is only possible to learn about $p_2$ if the insider chooses project 2.

The insider would only consider choosing the innovative project if it has a chance of improving upon the old method. Thus we need to assume also that $E[p_2|S] > p$ to eliminate the trivial case in which project 1 always strictly dominates project 2. On the other hand, the insider would always choose the innovative project if $E[p_2]$, the unconditional probability of success before ever trying the project, was higher than $p$. The interesting case is when $E[p_2] < p$. To economize on algebra and notation, define $\delta$ and $\theta$ such that $\delta p = E[p_2]$ and $\theta p = E[p_2|S]$. Our assumptions imply that $0 < \delta < 1$ and $1 < \theta < 1/p$. To summarize,

$$\delta p = E[p_2] < p < E[p_2|S] = \theta p. \tag{2}$$

Equations (1) and (2) summarize all characteristics of project 2. From (1), project 2 is exploratory because it is only possible to know more about the new method by trying it out. From (2), project 2 is promising because its probability of success is higher than the probability of success of project 1 if project 2 is successful in period 1. We can think of radical methods that look unlikely to work but that would greatly improve the current method if they do work. The interpretation of $\delta$ and $\theta$ is that a method is more radical the smaller $\delta$ is and the higher $\theta$ is.

Total profits (gross of any initial investment costs) are given by the undiscounted sum of earnings of the two dates, $\pi = x_1 + x_2$, where $x_i$ is equal to $F$ or $S$. We assume that earnings are only liquid in date 2. That is, earnings $x_1$ are known at date 1 but dividends based on $x_1$ are paid at date 2 (as when sales are on trade credit so that earnings $x_1$ are simply accounts receivables). More generally, we wish to capture a situation in which it is possible to learn a signal $x_1$ at date 1 about future profits of the firm, although such cash flows have not yet materialized. We call $x_1$ earnings in date 1 for simplicity of exposition, but it is better understood as "a signal in date 1 about a fraction of the earnings in date 2."
Figure 1. Technology: Earnings and probabilities associated with each initial project choice.

The insider makes an initial investment $I$, paid in cash, to produce positive earnings by investing in either project. Without this initial investment, all earnings are equal to zero regardless of the project chosen.

The insider may switch from one project to the other after observing $x_1$. If the insider initially chooses to exploit the old method, the option to switch has zero value. If the initial choice is to explore the new method, however, to maximize firm value the insider switches to project 1 after observing $x_1 = F$. The option to switch is valuable under exploration. If the new method is tried out but fails, the insider returns to the old method. Figure 1 provides a visual summary of the technology taking into account the option to switch.

Under exploitation (project 1), the expected market value of the firm (gross of initial investment costs) is

$$\hat{v}_1 = 2 \left[ pS + (1 - p) F \right].$$

We always write the value of the firm gross of initial investment costs, unless we say otherwise. The value of the firm takes into account the two periods of operation. To simplify notation,
we make $F = 0$ (without loss of generality), so that

$$\hat{v}_1 = 2pS. \tag{4}$$

If the insider chooses exploration (project 2), in case of success in date 2, the firm continues to use the new method. In case of failure, the firm returns to the old method (project 1). The expected market value of the firm under exploration is

$$\hat{v}_2 = p \{1 + \delta [1 + p (\theta - 1)]\} S. \tag{5}$$

Because expected cash flows are always linear in $S$, the value of $S$ plays no role in most of the analysis that follow. Thus, to simplify notation further we let $v_i \equiv \hat{v}_i / S$, $i = 1, 2$, which is the expected value of the firm per unit of $S$. That is equivalent to setting $S = 1$. In sum, the technological assumptions imply

$$v_1 = 2p, \quad \tag{6}$$
$$v_2 = p \{1 + \delta [1 + p (\theta - 1)]\}, \quad \text{and} \quad \tag{7}$$
$$v_2 - v_1 = p \{\delta [1 + p (\theta - 1)] - 1\}. \quad \tag{8}$$

The innovative project (project 2) is ex ante preferable to the conventional project (project 1) if and only if $v_2 - v_1 \geq 0$. In general, we have that

$$v_2 - v_1 \leq 0 \text{ if and only if } \delta [1 + p (\theta - 1)] \leq 1. \quad \tag{9}$$

### 2.1.2. Liquidity and financial market frictions

The key financial market friction in our model is the existence of a demand for liquid assets due to (unmodeled) borrowing constraints. We model the insider’s preference for liquid
assets by assuming that he has a utility function as in Diamond and Dibvyg (1983),

$$U(c_1, c_2) = \begin{cases} c_1 & \text{with probability } \mu, \\ c_2 & \text{with probability } 1 - \mu, \end{cases}$$

(10)

where $c_t$ is consumption at date $t$. This reduced-form approach is common in microeconomic models of liquidity shocks (see e.g. Freixas and Rochet, 1997). With probability $\mu$, a liquidity shock forces the insider to consume at date 1. With probability $1 - \mu$, there is no liquidity shock and dividends and consumption are synchronized at date 2. We can think of liquidity shocks as representing different types of consumers. Insiders that do not suffer a liquidity shock are called late consumers. Insiders that suffer a liquidity shock are early consumers.

For liquidity shocks to have a real impact on decisions, we need to assume that the insider faces personal borrowing constraints. The assumption of limited liability eliminates uncollateralized borrowing. The assumption of zero initial wealth means that the insider has no initial collateral. We need to assume further that the insider cannot borrow using the securities issued against the firm’s cash flows as collateral.\(^2\)

Liquid securities such as cash can be stored from one period to the following at no cost. There is no discounting nor systematic risk in the economy.

2.1.3. Project financing

The insider must sell securities backed by future cash flows to finance the initial investment $\hat{I}$, as the insider has no initial wealth. The insider may sell securities to private or public investors. The initial investment $\hat{I}$ is observable to all and contractible. Thus, the investment $\hat{I}$ must occur for sure if the insider sells securities to raise funds for investments. To simplify notation, we let $I \equiv \hat{I}/S$ denote the investment cost as a proportion of $S$.

We initially assume that the only securities available are share contracts. This is for sim-

\(^2\)Although we state this as an assumption, it is possible to endogenize borrowing constraints fully by introducing additional moral hazard considerations to the problem.
plicity of exposition. Capital structure choices are relevant in our model (that is, the model is not in an Modigliani-Miller world), but they do not change the qualitative results concerning the choice between private and public ownership structures. To show the robustness of the results, we discuss later the introduction of other securities such as debt.

2.1.4. Ownership structure: differences between private and public structures

The key results of our model only depend on one difference between private and public ownership. We assume that, under public ownership, interim earnings $x_1$ are observable by everyone. In contrast, under private ownership, only the insider and the incumbent private investors observe $x_1$. Future private investors also do not observe $x_1$. These assumptions are meant to capture the fact that public companies are more transparent. They are subject, for example, to tighter regulatory disclosure requirements such as quarterly earnings reporting and comprehensive annual reports, to analyst coverage and, perhaps most importantly, to the aggregation of dispersed information into the stock price.

For the sake of realism and to permit the analysis of different trade-offs, we also allow for other differences between the two structures, such as the cost of capital and liquidity costs. These enrich the model but are not necessary for any of the qualitative results linking innovation incentives and going public or private decisions.

We assume that there are transaction costs associated with raising funds for investment through an IPO. Administrative and underwriting costs are usually about 7%-11% of the IPO proceedings. More importantly, IPO underpricing can create much higher costs. Seasoned Equity Offerings (SEOs) are less costly, but discounts are also common, with a typical negative stock price reaction after announcements of equity offerings of 3% (Asquith and Mullins, 1986).

We capture the costs of issuing public equity by a parameter $c_1 \in (0, 1)$, such that each dollar sold in public offerings yields only $c_1$ to the firm. A high $c_1$ means a low discount. For example, if IPO markets are hot, raising investment funds from public markets is relatively
easy, thus a hot IPO market is equivalent to a high $c_1$.

Raising capital through private equity also involves transaction costs. We denote by $c_2 \in (0, 1)$ the discount factor associated with private securities. This parameter is likely to change with changes in the institutional environment and the state of the economy. For example, when interest rates are relatively low, private equity funds can borrow cheaply and thus going private becomes less costly for the firm, as required returns fall. Private equity booms are thus associated with high levels of $c_2$.

We make no assumptions with respect to the relative cost of public equity capital $c_2 - c_1$. Thus, our model allows for situations in which funds for investment are cheaper if financed by public securities ($c_1 > c_2$) as well as cases in which being private reduces the cost of capital ($c_1 < c_2$).

A traditional justification for going public is to create liquidity for insiders’ shareholdings. For example, a founder may value the option of selling his stake quickly on the market should the need arise. If the firm is privately held, the founder may have to negotiate with a few private investors. Especially in cases in which the founder suffers a liquidity shock and needs to sell quickly, his bargaining power may be compromised if the firm is private. In contrast, in public markets the founder may more easily be able to sell his own shares through organized markets (provided compliance with insider trading regulations). To capture a potential liquidity advantage of public equity, we assume that each dollar in shares sold by the insider in date 1 (the liquidity shock period) yields only $k \leq 1$ if the company is private. No such discount happens if the firm is public. In most of the analysis that follows, for simplicity we assume no liquidity discount when the insider sells his own shares ($k = 1$). In the robustness section, we fully consider the case in which $k < 1$.

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3 The fact that measured underpricing is larger in hot IPO markets does not imply that it is more costly to go public in hot markets. Underpricing is measured with respect to the price of the shares allocated to initial investors, which are likely to be higher in hot markets.

4 In Chemmanur and Fulghieri’s (1999) model of the decision to go public, the cost of capital in public firms reflects the trade-off between the liquidity of public securities and the information production costs associated with the duplication of monitoring efforts by public investors. Our model can incorporate such effects in reduced form by changing $c_1$ and $c_2$ according to which effect dominates.
2.1.5. The structure of information and timing of events

At date 0, the insider decides to sell a fraction $1 - \alpha$ of the shares to either private investors or public markets. The insider needs to raise at least $I$ in cash to pay for the initial investment cost. After paying $I$, the insider chooses project 1 or 2. Outsiders cannot observe which project was chosen.

At date 1, the insider observes the first realization of earnings $x_1 \in \{0, S\}$ and then chooses a project (1 or 2), which again is unobservable by outsiders. The insider learns his type. If the insider is an early consumer, he sells all shares he owns $\alpha$ regardless of the market valuation of his shares. If the insider is a late consumer, he may sell some of the shares or keep them until date 2. After observing whether the insider places orders to sell or keeps the shares, the market forms a price for the shares.

At date 2, the second-period earnings $x_2 \in \{0, S\}$ are realized, the shareholders receive dividends $x_1 + x_2$, and the firm is liquidated. The liquidation value is normalized to zero. Figure 2 illustrates the time line.
2.2. Private ownership

We solve the problem by working backwards. We start the analysis by first considering the case of private ownership; that is, at date 0 the insider has chosen to sell $1 - \alpha$ shares to private investors. We take $\alpha$ as exogenous for now and then work backwards to find the optimal $\alpha$ as a function of the parameters of the model.

After $1 - \alpha$ shares are sold, at the end of date 0 the insider chooses either project 1 or 2. Recall that project choice is private information to insiders only. The intuition is that, although investments may be observable, the insider has unique information or expertise that allows him to have a more precise estimate of the probabilities of success in each project. This is a natural assumption, which is consistent with the view that managers’ unique expertise may be essential for investment decisions.

Let $\sigma \in [0,1]$ be the probability that the insider chooses project 2 (innovation). We allow from the outset for the possibility of equilibria involving mixed strategies. Intuitively, an equilibrium with strictly mixed strategies could also be interpreted as the choice of an "intermediate project," which is more innovative than project 1 but not as radical as project 2. Our goal in this section is to compute the equilibrium project choice $\sigma^*$ under private ownership.

2.2.1. Selling behavior at date 1

Recall that at the end of date 1, after observing $x_1$, the insider chooses whether to keep his shares or sell them. We assume that the current private investors would not want to buy out the insider (for example, the private investors may also suffer a liquidity shock that is perfectly correlated with the insider’s liquidity shock). Thus, if the insider sells, the buyers are either new private investors or public investors in an IPO. Because the identity of the new investors is irrelevant in our model, we simply say that the insider sells shares to the market.

We first consider how the market updates its beliefs after observing the insider selling
shares in date 1. Let \( m \) be the posterior probability that the insider had a liquidity shock conditional on the insider selling shares at date 1. The probability \( m \) is a measure of market optimism. A low \( m \) means that the market believes that the insider is selling shares because the firm is overvalued, while a high \( m \) means that the market believes that a liquidity shock is probably the main reason why the insider sells. If we treat \( m \) as an exogenous parameter, we can perform comparative statics with respect to market beliefs; a low \( m \) is equivalent to a "cold market" while a high \( m \) is equivalent to a "hot market." On the other hand, by treating \( m \) as endogenous, hot and cold markets still exist, but they are driven by fundamentals rather than sentiment.

A liquidity shock forces the insider to sell his shares. Without a liquidity shock, the insider chooses whether to sell or not. We thus need to characterize when an insider without a liquidity shock chooses to sell. The following lemma describes the insider’s behavior when earnings are \( x_1 = S \).

**Lemma 1** *In the private ownership case, a late-consumer insider never sells shares at date 1 after observing \( x_1 = S \).*

Intuitively, a late-consumer insider would only sell shares in date 1 if he believes these shares are overvalued. Market rationality rules out excessive overvaluation (i.e. shares sold at prices that are not compatible with the selling behavior of the insider), thus prices at date 1 are never high enough to entice an insider with good news to sell rather than keep his shares.

From Lemma 1, the insider never sells if \( x_1 = S \). Now define \( b \in [0, 1] \) as the probability that the insider sells shares after observing \( x_1 = 0 \).\(^5\) By Bayes’s rule, rational market beliefs imply

\[
m = \Pr(\text{shock} \mid \text{sell}) = \frac{\Pr(\text{sell} \mid \text{shock}) \Pr(\text{shock})}{\Pr(\text{sell})}.
\]

\(^5\)Because \( b \) can only be non-zero if \( x_1 = 0 \), whether project 1 or 2 were chosen is immaterial for the decision to sell, thus \( b \) does not need to be conditional on project choice.
The inputs for this formula are as follow. In an equilibrium where the probability of choosing project 2 is $\sigma$, the unconditional probability of selling shares at date 1 is given by

$$\text{Pr}(\text{sell}) = \mu + (1 - \mu) [1 - p + \sigma p (1 - \delta)] b. \quad (12)$$

Conditional on having a liquidity shock, the insider sells with probability $\text{Pr}(\text{sell} \mid \text{shock}) = 1$. Since the probability of a liquidity shock is $\mu$, we have that

$$m(\sigma, b) = \frac{\mu}{\mu + (1 - \mu) [1 - p + \sigma p (1 - \delta)] b}. \quad (13)$$

The equilibrium value of shares if the market holds rational beliefs is

$$V(\sigma, b) = m(\sigma, b) [\sigma v_2 + (1 - \sigma) v_1] + [1 - m(\sigma, b)] p. \quad (14)$$

After a failure, the best option is to switch to project 1. Conditional on $x_1 = 0$, the expected value of the firm is $p$. A necessary condition for selling shares with positive probability is

$$V(\sigma, b) \geq p. \quad (15)$$

The next lemma characterizes the equilibrium behavior of the insider if $x_1 = 0$.

**Lemma 2** In the private ownership case, a late-consumer insider sells shares with probability 1 at date 1 after observing $x_1 = 0$.

Intuitively, after $x_1 = 0$ the insider always prefers to sell because the market always assigns a strictly positive probability to $x_1 = S$ when the insider sells. This belief is rational because the insider could be an early consumer who is forced to sell.

From Lemma 2, the equilibrium $b^*$ is 1. We define the value of shares sold at date 1 in an equilibrium with $\sigma$ by $V(\sigma) \equiv V(\sigma, 1)$.

A key aspect of the private ownership case is the possibility of selling shares in date 1
after observing a failure $x_1 = 0$. A late-consumer insider only sells his shares in date 1 if they are overvalued. This may happen in equilibrium because the market does not observe $x_1$ and thus cannot distinguish between a liquidity-motivated sale and an opportunistic one. This information asymmetry creates a valuable option for a late-consumer insider.

Let $T (\sigma) \equiv V (\sigma) - p$ denote the (intrinsic) value of the option to exit early for a late-consumer insider conditional on $x_1 = 0$. Selling his shares is a real option to the insider. The value of the underlying asset is $V (\sigma)$ – the market value of shares in equilibrium – while the option’s exercise price (i.e. the opportunity cost of selling in date 1) is $p$. Notice that $T (\sigma)$ is a function of both the fundamental parameters and the equilibrium strategy and beliefs.

2.2.2. Project choice in date 0

Now that we know how the insider behaves and how the market sets the price of shares in date 1, we can go back to date 0 to analyze the choice between the two projects. Suppose that the market expects project 2 to be chosen with probability $\sigma$. We first determine the expected value as of date 0 of each share held by the insider if he chooses project 2:

$$ q_2 (\sigma) \equiv \mu V (\sigma) + (1 - \mu) [(1 - \delta p) (p + T (\sigma)) + \delta p (1 + \theta p)] . \quad (16) $$

To understand this expression, recall that at date 0 the insider does not yet know his type. He knows that that with probability $\mu$ he will be an early consumer and be forced to sell. With probability $1 - \mu$ he is a late consumer and has the option to sell after a failure.

Similarly, the expected value of one share when the insider chooses project 1 at date 0 while the market expects that project 2 is chosen with probability $\sigma$ is:

$$ q_1 (\sigma) \equiv \mu V (\sigma) + (1 - \mu) [(1 - p) (p + T (\sigma)) + \theta p (1 + p)] . \quad (17) $$

For an equilibrium with a positive probability of exploration (project 2) to occur ($\sigma > 0$), choosing project 2 at date 0 must be incentive compatible for the insider. That is, we need
\( q_2 (\sigma) \geq q_1 (\sigma) \) to guarantee incentive compatibility. Similarly, if \( \sigma < 1 \) in equilibrium, we need that \( q_2 (\sigma) \leq q_1 (\sigma) \). The next proposition summarizes the incentive compatibility constraints.

**Proposition 1 Incentive Compatibility under Private Ownership.**

1. An equilibrium in which the insider chooses project 2 (exploration of new ideas) with strictly positive probability \( (\sigma > 0) \) exists only if the following incentive compatibility condition holds:

\[
v_2 - v_1 + p(1 - \delta)T(\sigma) \geq 0.
\]  

(18)

2. An equilibrium in which the insider chooses project 1 (exploitation of old ideas) with strictly positive probability \( (\sigma < 1) \) exists only if the following incentive compatibility condition holds:

\[
v_2 - v_1 + p(1 - \delta)T(\sigma) \leq 0.
\]  

(19)

The main intuition for the incentive effects of private ownership on innovation can be grasped from the incentive compatibility (IC) condition 18. The first part of this condition, \( v_2 - v_1 \), illustrates the efficiency incentives for choosing the innovative project. We call this the *efficiency effect*. This effect is fully determined by the technology and it can be either positive or negative. In a first best world, the efficiency effect would determine which project is chosen.

Proposition 1 shows that there is a second force pushing towards innovation, which is given by

\[
p(1 - \delta)T(\sigma) = (1 - \delta p)T(\sigma) - (1 - p)T(\sigma) \geq 0.
\]  

(20)

Because the innovative project has a higher probability of failure than the conventional one, the expected value of the option to exit early is higher when innovation is chosen (\((1 - \delta p)T(\sigma) \geq (1 - p)T(\sigma))


The value of the option to exit early $T(\sigma)$ reflects the fact that the private ownership structure displays a higher degree of tolerance to failure than the first-best benchmark. Tolerance to failure has been shown to be a key feature of optimal incentive schemes for innovation (Manso, 2006). The key insight of our model is that tolerance to failure is more valuable for innovation because the option to exit early is exercised more often. Thus, the tolerance-to-failure effect is always positive.

The option to exit early nudges the insider towards choosing the more innovative project. When innovation is efficient from an technological perspective ($v_2 - v_1 \geq 0$), this extra incentive towards innovation is not necessary; the IC is not binding.

More interesting is the case of $v_2 - v_1 < 0$. This is a situation in which innovation is inefficient. Proposition 2 below shows that, for a set of parameters, innovation may be chosen despite being the inefficient choice. This happens because the option value of an early exit is strictly positive. If the tolerance-to-failure effect dominates the (negative) efficiency effect, the private ownership structure creates incentives to innovate even when it would be optimal to choose the conventional project.

The next proposition fully characterizes all equilibria under private ownership for any given set of fundamental parameters.

**Proposition 2** A unique equilibrium always exists and is given by:

1. If $v_1 - v_2 \leq 0$, then $\sigma^* = 1$.

2. If $v_1 - v_2 > 0$, then $\sigma^*$ is uniquely given by

$$
\sigma^* = \begin{cases}
1 & \text{if } \frac{v_1 - v_2}{(1-\delta)} \leq T(1) \\
T^{-1} \left( \frac{v_1 - v_2}{p(1-\delta)} \right) & \text{if } \frac{v_1 - v_2}{(1-\delta)} \in (T(1), T(0)) \\
0 & \text{if } \frac{v_1 - v_2}{(1-\delta)} \geq T(0)
\end{cases}
$$

Figure 3 illustrates the three possible cases when $v_1 - v_2 > 0$. The flat dashed lines represent different values for $v_1 - v_2$. The $R_1$ line represents a case in which $v_1 - v_2$ is
Figure 3. Equilibrium $\sigma^*$ when $v_1 - v_2 > 0$.

sufficiently high. In such a case, the (absolute value of the) efficiency effect is large and dominates the the tolerance-to-failure effect, implying that the first-best action $\sigma^* = 0$ is chosen in equilibrium. The $R_2$ line represents an intermediate value for $v_1 - v_2$, for which a given probability of innovation $\sigma^* \in (0, 1)$ makes the insider indifferent between projects 1 and 2. That means that the efficiency effect is fully offset by the tolerance-to-failure effect, and the unique equilibrium must involve some inefficient amount of innovation. Finally, the $R_3$ line is a case where $v_1 - v_2$ is positive but small, so that the option to exit early is so valuable to the insider compared to $v_1 - v_2$ that the insider always makes the inefficient project choice in equilibrium.

In sum, our model shows that the private ownership structure is biased towards innovation. This bias is welcome when $v_1 - v_2 < 0$ but may lead to inefficiencies if $v_1 - v_2 > 0$. Earnings opacity, typical in privately-owned firms, gives an exit option to the insider. It is profitable to sell before a bad signal of the fundamental value of the firm becomes public. The exit option is available regardless of the project chosen, innovative or conventional. But the exit option is more valuable under innovation because the probability of failure is higher.
2.2.3. Comparative statics

When $v_2 - v_1 \geq 0$, changes in the fundamental parameters do not affect the equilibrium, as long as $v_2 - v_1$ remains non-negative. Thus, the interesting case for comparative statics is when $v_1 - v_2 > 0$.

Suppose that we have $\sigma^* \in (0, 1)$, that is, the $R_2$ case in Figure 3. To perform comparative statics with respect to the fundamental parameters of the model, first define

$$g (\sigma, \mu, \theta, \delta) = m (\sigma, 1) [p + \sigma (v_2 - v_1)] + (1 - \delta) p (v_2 - v_1).$$

(22)

We have that

$$\frac{\partial g}{\partial \sigma} (\sigma^*, \mu, \theta, \delta) = \frac{\partial m (\sigma^*, 1)}{\partial \sigma} [p + \sigma^* (v_2 - v_1)] + (v_2 - v_1) m (\sigma^*, 1) < 0$$

because $\frac{\partial m(\sigma^*, 1)}{\partial \sigma} < 0$ and $v_2 - v_1 < 0$. Define $\Delta \equiv \frac{\partial g}{\partial \sigma} (\sigma^*, \mu, \theta, \delta)$. Then

$$\frac{\partial \sigma^*}{\partial \mu} = -\frac{\frac{\partial m}{\partial \mu} [p + \sigma^* (v_2 - v_1)]}{\Delta} > 0,$$

(23)

because $\frac{\partial m}{\partial \mu} > 0$. Intuitively, by making it easier for a late-consumer insider to disguise his trade as a liquidity shock, innovation becomes more attractive and in equilibrium there is more of it.

Increases in $\delta$ and $\theta$ increase the NPV of innovation. Thus, we would expect that the equilibrium amount of innovation should also increase. This is indeed true:

$$\frac{\partial \sigma^*}{\partial \theta} = -\frac{p^2 \delta [\sigma^* m (\sigma^*, 1) + 1 - \delta]}{\Delta} > 0,$$

(24)

$$\frac{\partial \sigma^*}{\partial \delta} = -\frac{\frac{\partial m}{\partial \delta} [p + \sigma^* (v_2 - v_1)] + [\sigma^* \mu p + (1 - \delta) p^2] [1 + p (\theta - 1)] m (\sigma^*, 1) + p (v_1 - v_2)}{\Delta} > 0.$$

(25)
2.2.4. The value of being private

Now we compute the expected value of the firm to the insider taking into account the value of shares initially sold to private investors. Because the insider needs to raise $I$ to finance the investment, if the market expects $\sigma^*$ to occur in equilibrium, then the revenue from selling shares must satisfy

$$(1 - \alpha) [\sigma^* v_2 + (1 - \sigma^*) v_1] \geq \frac{I}{c_1}. \quad (26)$$

Due to the trading costs implied by $c_1 < 1$, the insider will only sell the minimum number of shares that allows him to invest. To avoid uninteresting cases in which the investment can never be financed, we assume that $I \in (0, c_1 \min \{v_1, v_2\})$. That is, the firm’s cost of capital is never so high so that funds for investment cannot be raised. Under this assumption, the equilibrium insider’s stake is uniquely given by

$$\alpha^* = 1 - \frac{I}{c_1 [\sigma^* v_2 + (1 - \sigma^*) v_1]}. \quad (27)$$

Finally, the ex ante value of the firm to the insider under private ownership is given by

$$W_{private} = \alpha^* [\sigma^* q_2 (\sigma^*) + (1 - \sigma^*) q_1 (\sigma^*)] = \sigma^* v_2 + (1 - \sigma^*) v_1 - \frac{I}{c_1}. \quad (28)$$

The first two terms of this expression represent the expected outcome from the project decision and the third term is the initial investment cost. Notice that $W_{private}$ may differ from the first best in a full information, frictionless economy both because the equilibrium level of innovation $\sigma^*$ is excessive compared to the first best (our results also imply that under private ownership there is never too little innovation) and because raising funds for investing is costly ($c_1 < 1$), which generates deadweight costs.
2.3. Public ownership

In this section we consider the case of public ownership, i.e. there are \(1 - \alpha\) of shares floating in the market. As in the case of private ownership, the insider sells his shares at date 1 if there is a liquidity shock. The public case differs from the private case due to the transparency of earnings. Earnings \(x_1\) can be observed by all investors.

2.3.1. Selling behavior at date 1

The analysis of the equilibrium is similar to the private ownership case. As before, the insider chooses project 2 (exploration) with probability \(\sigma \in [0, 1]\). Regardless of the project chosen, the expected market value of the firm when \(x_1 = 0\) is \(p\). In this case, there is no information asymmetry between the insider and the market. Earnings transparency means that the market always knows when \(x_1 = 0\). The market also knows that project 1 is always chosen after \(x_1 = 0\). Although the market does not know which project was chosen at date 0, that knowledge is not value relevant when \(x_1 = 0\). Thus, shares are always fairly valued when \(x_1 = 0\) and the insider gains nothing by selling shares. We can assume that the insider sells or keeps his shares when \(x_1 = 0\); the equilibrium payoffs are not affected by this choice.

The insider may however choose to sell shares after \(x_1 = S\). Although the market knows that \(x_1 = S\) has occurred, it does not know which project was chosen at date 0. If project 1 was chosen, the expected value of the firm is \(1 + p\). If project 2 was chosen, the expected value of the firm is \(1 + \theta p\). Thus, the insider is always weakly better off when the market believes that project 2 was initially chosen. That creates a value-relevant information asymmetry, which may distort the incentives of the insider when making project choice decisions.

The next two lemmas characterize the behavior of a late-consumer insider after \(x_1 = S\).

**Lemma 3** In the public ownership case, in equilibrium a late-consumer insider never sells shares after \(x_1 = S\) if project 2 was chosen.

Therefore, the insider never sells voluntarily at date 1 after exploration. The intuition is
that, if project 2 was chosen, after \( x_1 = S \) the firm is always sold with a discount because the market can never be certain that project 2 was chosen.

**Lemma 4** *In the public ownership case, in equilibrium a late-consumer insider weakly prefers to sell shares with after \( x_1 = S \) if project 1 was chosen.*

If the insider chooses project 1 (exploitation) and obtains a success, the insider trades with probability one (to simplify the exposition, we assume that the insider sells when he is indifferent). Selling after \( x_1 = S \) if project 1 was chosen is always profitable as long as the market assigns some non-negative probability to project 2. Given Lemmas 3 and 4, without the possibility of a liquidity shock, trading after \( x_1 = S \) would always reveal the insider’s choice of project. However, liquidity shocks allow an insider who chooses project 1 to trade after \( x_1 = S \) without revealing his choice of project. In equilibrium, a late consumer insider may pool with an early consumer insider.

### 2.3.2. Project choice in date 0

In equilibrium, the market must have correct beliefs and thus must assign probability \( \sigma \) to project 2 being chosen. When the market observes a success and the insider sells shares, the market assigns probability \( s \) that project 2 has been chosen. The difference between \( \sigma \) and \( s \) is that \( \sigma \) is the unconditional probability of choosing project 2 while \( s \) is the probability of project 2 being chosen given that the insider sells shares and the market observes \( x_1 = S \):

\[
s \equiv \Pr(\text{project 2} \mid \text{sell}, x_1 = S) = \frac{\Pr(\text{sell}, x_1 = S \mid \text{project 2}) \Pr(\text{project 2})}{\Pr(\text{sell}, x_1 = S)} \tag{30}
\]

The inputs for this formula are as follow. From Lemma 3, the insider only sells after choosing project 2 and \( x_1 = S \) if he suffers a liquidity shock:

\[
\Pr(\text{sell}, x_1 = S \mid \text{project 2}) = \mu \delta p. \tag{31}
\]
From Lemmas 3 and 4, the probability of selling and \( x_1 = S \) is

\[
\Pr(sell, x_1 = S) = (1 - \sigma) p + \sigma \delta p \mu. \tag{32}
\]

Finally, in equilibrium the unconditional probability of project 2 is \( \sigma \). Thus, we have that equilibrium beliefs must be

\[
s(\sigma) = \frac{\sigma \mu \delta}{(1 - \sigma) + \sigma \mu \delta}. \tag{33}
\]

Given such beliefs, the market value of shares sold after a success is

\[
V(\sigma) = 1 + s(\sigma) \theta p + (1 - s(\sigma)) p = 1 + p + s(\sigma) p (\theta - 1). \tag{34}
\]

We now calculate the expected gains for the insider from choosing either project 1 or project 2. As before, the expected value of one share if the insider chooses project 1 is given by

\[
q_1(\sigma) = p V(\sigma) + (1 - p) p. \tag{35}
\]

If the insider chooses project 1, the probability of success is \( p \). In the case of a success, the insider sells and obtains \( V(\sigma) \). If there is a failure, the market value of the firm becomes \( p \) as the best project to choose in date 2 is project 1, again with probability \( p \) of success.

The expected gain per share for the insider from choosing project 2 (exploration) is

\[
q_2(\sigma) = \delta p [\mu V(\sigma) + (1 - \mu)(1 + \theta p)] + (1 - \delta p) p. \tag{36}
\]

At date 1, the probability of success is \( \delta p \). In the case of a success, the insider only sells if there is a liquidity shock, which happens with probability \( \mu \). If the insider is not forced to sell (no liquidity shock), he keeps his shares until date 2 and selects project 2 again, now with probability \( \theta p \) of success.

The next proposition fully characterizes the unique equilibrium.
Proposition 3 In the public ownership case, there is a unique equilibrium \( \sigma^* \in [0, 1] \), which is given by

\[
\sigma^* = \frac{s^*}{\mu \delta + s^* (1 - \mu \delta)},
\]

where

\[
s^* = \max \left\{ \frac{\delta [1 + p(\theta - 1)] - \delta \mu p(\theta - 1) - 1}{p(\theta - 1) (1 - \delta \mu)}, 0 \right\}.
\]

One particular implication of this proposition is that an equilibrium with full innovation, \( \sigma = 1 \), is never possible if \( \mu < 0 \). The intuition for this result is as follows. If the market expects exploration with probability 1, then choosing exploitation becomes a dominant strategy. The insider increases the probability of success by exploiting and then chooses to sell his shares in date 1. The strategy \( \sigma = 1 \) can only be an equilibrium when there is no liquidity shock, \( \mu = 0 \). In this case, the market knows that there is a sale in case of success only if the insider exploited. The insider cannot disguise exploitation and chooses to explore if \( \mu = 0 \).

The following corollary facilitates the comparison with the private case.

Corollary 1 The unique equilibrium is such that

1. If \( v_1 - v_2 \geq -\delta \mu (\theta - 1) \), then \( \sigma^* = 0 \)

2. If \( v_1 - v_2 < -\delta \mu (\theta - 1) \), then \( \sigma^* \in (0, 1) \).

A necessary condition for \( s^* > 0 \), and so \( \sigma^* > 0 \), is \( v_2 > v_1 \). Exploration must be efficient in order for the insider to choose it with positive probability. But \( v_2 > v_1 \) is not sufficient. We need the difference between \( v_2 \) and \( v_1 \) to be sufficiently high for \( \sigma > 0 \) to be possible. Thus, it is harder to make the insider explore when the firm is public.

The results in this section show that public ownership creates a bias against innovation. But it always induces the efficient choice of project when \( v_1 - v_2 > 0 \).
2.3.3. Comparative statics

To perform comparative statics with respect to the fundamental parameters, first notice that \( \sigma^* \) is strictly increasing in \( s^* \) when the solution is interior. Assuming an interior solution for simplicity, we have

\[
\frac{\partial \sigma^*}{\partial \theta} = \frac{1 - \delta}{p(1 - \delta \mu)(\theta - 1)^2} \frac{1}{\partial s^*} \frac{\partial \sigma^*}{\partial \theta} > 0.
\] \hspace{1cm} (39)

Intuitively, an increase in \( \theta \) makes innovation more valuable and increases the amount of innovation.

For \( \delta \), we have a similar effect:

\[
\frac{\partial \sigma^*}{\partial \delta} = \frac{p(\theta - 1)(1 - \mu) + (1 - \mu) \frac{\partial \sigma^*}{\partial \delta}}{p(\theta - 1)(1 - \delta \mu)^2} \frac{\partial \sigma^*}{\partial \delta} > 0.
\] \hspace{1cm} (40)

An increase in \( \delta \) makes exploration less risky because it increases the expected probability of success. As for the case of \( \theta \), an increase in \( \delta \) increases the amount of innovation because it makes innovation more valuable.

Finally, we have that

\[
\frac{\partial \sigma^*}{\partial \mu} = -\delta (1 - \delta) \frac{1 + (\theta - 1)p}{p(\theta - 1)(1 - \delta \mu)^2} \frac{\partial \sigma^*}{\partial \mu} < 0.
\] \hspace{1cm} (41)

If liquidity shocks occur very often, it is easier for the insider to hide the choice of project 1. Frequent liquidity shocks make the market believe that the insider is selling because of a liquidity shock, and not because of success under exploitation. As it is easier to hide the choice of exploitation, the incentives to choose innovation decrease.

In sum, exploiting the old method is better when \( \theta \) or \( \delta \) are low, or when \( \mu \) is high.
2.3.4. The value of being public

Now we compute the expected value of the firm to the insider, taking into account the value of shares initially sold to public investors. As before, we have

\[ \alpha^* = 1 - \frac{I}{c_2[\sigma^* v_2 + (1 - \sigma^*) v_1]}. \]

(42)

Thus, the ex ante value of the firm to the insider is given by

\[ W_{public} = \alpha^* [\sigma^* q_2(\sigma^*) + (1 - \sigma^*) q_1(\sigma^*)] = \]

\[ = \sigma^* v_2 + (1 - \sigma^*) v_1 - \frac{I}{c_2}. \]

(43)

(44)

In the public ownership case, inefficiencies may arise because there is too little innovation in equilibrium (i.e. \( \sigma^* < 1 \) when \( v_2 > v_1 \)) and because raising funds through equity offerings is costly (\( c_2 < 1 \)).

3. The decision to go public or private

Now we analyze the decision to go public or private. The insider chooses to go private or public depending on whether \( W_{private} \) is greater than \( W_{public} \) or not. To differentiate between the two cases, let \( \sigma_{private} \) denote the private-ownership equilibrium and \( \sigma_{public} \) denote the public-ownership equilibrium. To simplify notation, define the relative cost advantage of public offerings compared to private offerings as

\[ a = \frac{c_1 - c_2}{c_1 c_2}. \]

(45)

If private offerings are cheaper than public ones (\( c_1 > c_2 \)), then \( a > 0 \), and vice versa.

The following proposition follows immediately from the comparison of \( W_{private} \) with \( W_{public} \).
Proposition 4 The private ownership structure is preferable to the public ownership structure if and only if

\[ v_1 + (\sigma_{\text{private}} - \sigma_{\text{public}})(v_2 - v_1) \geq aI \]  

(46)

In our model, the choice between public and private is driven by two considerations. The first one is the main novelty of our model: the choice between public versus private depends on the relative efficiency of innovative projects. The second one is the relative cost of capital advantage of public offerings compared to private offerings.

Notice that \( \sigma_{\text{private}} - \sigma_{\text{public}} \geq 0 \) for any given constellation of parameters. Thus, going private is more attractive than going public when innovation is efficient \( (v_2 - v_1 > 0) \). In fact, if we shut down the other effect by setting \( a = 0 \), whether innovation is efficient or not is the only consideration for the choice of ownership structure, as shown in the next corollary.

Corollary 2 Let \( a = 0 \).

1. If innovation is efficient \( (v_2 > v_1) \), the insider chooses to go private.

2. If the conventional project is efficient \( (v_2 < v_1) \), the insider strictly prefers to go public if \( \frac{v_1 - v_2}{(1-\delta)} < T(0) \) and is indifferent between going public or private if \( \frac{v_1 - v_2}{(1-\delta)} \geq T(0) \).

3. If both projects are equivalent \( (v_2 = v_1) \), the insider is indifferent between going public or private.

4. Robustness and extensions

4.1. Illiquid private securities

As we discuss in subsection 2.1.4, private securities are probably more difficult to unload quickly after a liquidity shock because they are not traded in organized markets. To capture the relatively illiquidity of private securities, we now assume that for each dollar sold in shares at date 1 if the firm is private, the insider only pockets \( k < 1 \).
Most of the analysis of the private case remains unchanged with this modification. In particular, Lemma 1 is not affected and its proof implies that the case \( k < 1 \) has no effect for the behavior of a late-consumer insider when \( x_1 = S \).

The optimal choice of \( b \) however is slightly different. To analyze this case, note that a necessary condition for selling shares with positive probability is

\[ kV(\sigma, b) \geq p. \]  

(47)

Because \( V(\sigma, b) > p \), we have that \( kV(\sigma, b) > p \) for \( k \) close enough to 1. Therefore, the insider sells for sure after a failure if the market is liquid enough. That is, as \( k \) approaches 1, eventually we get \( b = 1 \). If the market for firm securities in date 1 is very illiquid, that is, \( k \) is close to zero, then in equilibrium a late-consumer insider never sells shares, i.e. \( b = 0 \).

For intermediate values of \( k \), the equilibrium is in strictly mixed strategies with \( b \in (0, 1) \) with \( b \) increasing in \( k \). The next Lemma formalizes these results.

**Lemma 5** In the private ownership case with \( k \in (0, 1) \), a late-consumer insider sells shares with equilibrium probability \( b(\sigma) \) at date 1 after observing \( x_1 = 0 \), where

\[
b(\sigma) = \begin{cases} 
1 & \text{if } k \geq k_1(\sigma) \\
\frac{k[v_1 + \sigma(v_2 - v_1)] - p}{(1 - \sigma)(1 - p)[1 - p + \sigma p(1 - \delta)]} & \text{if } k_2(\sigma) < k < k_1(\sigma) \\
0 & \text{if } k \leq k_2(\sigma)
\end{cases}
\]

where

\[
k_1(\sigma) \equiv \frac{\mu p + (1 - \mu)[1 - p + \sigma p(1 - \delta)]p}{\mu [v_1 + \sigma(v_2 - v_1)] + (1 - \mu)[1 - p + \sigma p(1 - \delta)]p},
\]

\[
k_2(\sigma) \equiv \frac{p}{v_1 + \sigma(v_2 - v_1)}.
\]

The threshold values \( k_1(\sigma) \) and \( k_2(\sigma) \) define three regions for the behavior of the insider. If the market is liquid enough, \( k \geq k_1(\sigma) \), then the insider always sells after having a failure.
Figure 4. Late-consumer insider’s probability of selling shares after $x_1 = 0$.

We call this Region 1 in Figure 1. In Region 2 the insider plays a strictly mixed strategy considering to sell shares, and in Region 3 a late-consumer insider never sell shares.

We use Figure 1 to illustrate the effect of a change in the probability of a liquidity shock on the equilibrium strategy of the insider. If $\mu$ increases, $k_1 (\sigma)$ decreases: a late-consumer insider sells shares with probability 1 for a larger set of values for $k$. Intuitively, if $\mu$ increases, then it becomes easier for the insider to disguise a failure behind a liquidity shock.

We redefine $T$ (the intrinsic value of the option to exit early for a late-consumer insider) as

$$T (\sigma) = \max \{kV (\sigma) - p, 0\}. \quad (48)$$

Notice now that this option has zero value if the underlying $kV (\sigma)$ is low, which may happen either because the market is very illiquid (low $k$) or because the market is "cold," i.e. the market believes that when an insider sells shares, $x_1 = 0$ is very likely ($\mu$ is low). In terms
of the regions in Figure 4, we find that $T(\sigma)$ is strictly positive in Region 1, while zero in Regions 2 and 3.

The next proposition generalizes our previous results to the case where $k \leq 1$.

**Proposition 5** *In the private ownership case with $k \leq 1$, an equilibrium always exists and is given by:

1. If $v_1 - v_2 < 0$, then $\sigma^* = 1$.

2. If $v_1 - v_2 > 0$, then $\sigma^*$ is uniquely given by

$$
\sigma^* = \begin{cases} 
1 & \text{if } \frac{v_1 - v_2}{1 - \delta} \leq T(1) \\
T^{-1}\left(\frac{v_1 - v_2}{p(1 - \delta)}\right) & \text{if } \frac{v_1 - v_2}{(1 - \delta)} \in (T(1), T(0)) \\
0 & \text{if } \frac{v_1 - v_2}{(1 - \delta)} \geq T(0) 
\end{cases}
$$

3. If $v_1 - v_2 = 0$, then $\sigma^* \in \arg\min_{\sigma \in [0,1]} T(\sigma)$.

The key difference between our benchmark case of $k = 1$ and the case in which $k < 1$ is the possibility of an equilibrium as in case 3 of the proposition above. In such a case, an equilibrium $\sigma^*$ is not unique. This case only happens when $v_1 = v_2$ and is illustrated in Figure 5.

Finally, the ex ante value of the firm to the insider is given by

$$
W_{\text{private}} = \alpha^* [\sigma^* q_2 (\sigma^*) + (1 - \sigma^*) q_1 (\sigma^*)] = \sigma^* v_2 + (1 - \sigma^*) v_1 - \frac{I}{c_1} - L
$$

where

$$
L \equiv \alpha^* \{\mu + (1 - \mu) [\sigma^* (1 - \delta p) + (1 - \sigma^*) (1 - p)]\} V (\sigma^*) (1 - k).
$$

The new term in this equation represents the expected illiquidity costs associated with the insider selling his shares at date 1. This represents another source of deadweight costs
Figure 5. Equilibrium $\sigma^*$ when $v_1 - v_2 = 0$ and $\sigma_L < 1$.

associated with private ownership: the insider’s trading of shares due to liquidity shocks or privileged information is costly because private securities are illiquid.

The choice between public and private is then modified to include this cost. Now the private ownership structure is preferable to the public ownership structure if and only if

$$v_1 + (\sigma_{private} - \sigma_{public}) (v_2 - v_1) \geq aI + L. \quad (52)$$

We conclude that the case of $k < 1$ enriches the model but does not change any of the qualitative results.

4.2. Debt

We have assumed that the firm can only issue one type of securities: straight share contracts. We now consider the case in which debt securities are also available.

Our goal is not to characterize fully the optimal capital structure, but to understand the extent to which the availability of debt securities makes the choice of ownership irrelevant.
Thus, we focus only on the public case when $v_2 - v_1 > 0$. In this case, debt is likely to have an impact on incentives to innovate. The asset substitution effect (Jensen and Meckling, 1976) makes risky projects more attractive when there is leverage. Thus, this effect may offset the public ownership bias against innovation. Here we investigate whether this conjecture is true.

Suppose that the firm finances its investment fully with debt with face value $D$, a zero-coupon long-term bond, to be paid in the end of period 2.\footnote{As it will become clear, financing the initial investment fully with debt is the optimal financing choice unless the cost of debt capital is higher than the cost of equity capital.} If debt is not paid in full, bondholders seize the company’s cash flows. It is trivial to show that nothing changes if $D \leq S$, thus here we focus on the interesting case in which $D \in (S, 2S)$. In this case, default occurs unless the firm observes to successes in a row.

Suppose that the insider chooses project 2 with probability $\sigma'$. Given that in equilibrium the market’s belief that the insider has chosen 2 must be $\sigma'$, the insider will not trade voluntarily after choosing project 2. However, the insider will trade with probability 1 if used project 1 and $x_1 = S$. When the market observes $x_1 = S$ and shares being sold, market prices in equilibrium are

\[ V (\sigma') = s (\sigma') \theta p (2S - D) + (1 - s (\sigma')) p (2S - D). \quad (53) \]

Simplifying,

\[ V (\sigma') = [1 + s (\sigma') (\theta - 1)] p (2S - D) \quad (54) \]

Thus, the per share expected utility from choosing project 1 is

\[ q_1 (\sigma') = p V (\sigma'), \quad (55) \]
while the per share expected utility from project 2 is

\[ q_2(\sigma') = \mu \delta p V(\sigma') + (1 - \mu) \delta \theta p^2 (2S - D). \tag{56} \]

For the insider to be willing to randomize between 1 and 2, we need

\[ s(\sigma') = \frac{(1 - \mu) \delta \theta - (1 - \mu \delta)}{(\theta - 1)(1 - \mu \delta)} > 0. \]

Crucially, we have that \( s(\sigma') < 1 \), implying that first best \( (s = 1) \) cannot be implemented in this case.

To find out the effect of debt on the probability of innovation, we need to compare \( s(\sigma') \) with

\[ s^* = \min \left\{ \frac{\delta [1 + p(\theta - 1)(1 - \mu)] - 1}{p(\theta - 1)(1 - \mu \delta)}, 0 \right\}. \tag{57} \]

Algebra shows that

\[ s(\sigma') - s^* = \max \left\{ \frac{(1 - \delta)(1 - p)}{p(\theta - 1)(1 - \delta \mu)}, 0 \right\} > 0, \tag{58} \]

which implies that debt increases the amount of innovation in the public case when \( v_2 - v_1 > 0 \). Thus, if firms want to innovate more but remain public, it is optimal to lever up. However, we also find that \( s(\sigma') < 1 \), so the public ownership structure with debt is still inferior to the private ownership case when \( v_2 - v_1 > 0 \). In sum, the capital structure is not a (perfect) substitute for the ownership structure in incentivizing innovation.

5. Conclusions

Our results suggest that public and private firms invest in fundamentally different ways. Private firms take more risks, invest more in new products and technologies, and pursue more radical innovations. Private firms are more likely to choose projects that are complex,
difficult to describe, and untested. Organizational change is also more likely under private ownership. Mergers and acquisitions, divestitures, and changes in organizational structure and management practices are more easily motivated under private ownership.

On the other hand, public firms choose more conventional projects. Their managers appear short-sighted; they care too much about current earnings. They find it difficult to pursue "complex" projects that the market does not appear to understand well. Public firms go private after bad shocks, when it is clear that their business models are no longer working.

Anecdotal and systematic evidence corroborates the link between private ownership and innovative change. Firms that go private pursue more influential innovations (Lerner, Sorensen, and Stromberg, 2008) and engage more in organizational change (Davis et al., 2009). There is also some evidence that private equity owned firms introduce innovations in management practices (Bloom, Sadun, and Van Reneen, 2008).

Moon (2006) describes the acquisition by Morgan Stanley Capital Partners of an oil and gas subsidiary of a utility that was undergoing a restructuring. The company had good long term-prospects according to independent analysts, but faced several years of negative cash flows due to the restructuring efforts. Although finding strategic buyers for the company seemed the most logical solution, none of the public firms in the industry appeared to be willing to deal with the complexity of the business and with its negative cash flows. Private equity investors, on the other hand, were keen to deal with this uncertainty and with the prospect of negative cash flows in the short run.

There are still some untested implications of our model. Our model predicts that cash-flow volatility should be higher in private than public firms. Private firms should be more profitable during technological revolutions, while public firms should be more valuable in mature but growing industries.

Our model also has implications for the decision to go public or private. Firms are likely to go public after a technological breakthrough, that is, when it makes sense to exploit a newly discovered technology. Firms are likely to go private after suffering permanent negative
productivity shocks, that is, when their existing technologies or business models become permanently unprofitable. Chemmanur, He, and Nandy (2007) find that firms go public at the peak of their productivity and then performance declines after going public. This is consistent with firms going public only after perfecting a new technology; they become public in the "harvesting" period. Our model also explains the other side of this story: companies go private when performance is particularly poor.

6. Appendix—Proofs

Lemma 1.

Proof. Define

\[ h \equiv \Pr(x_1 = 0 \mid sell) \tag{59} \]

\[ s \equiv \Pr(\text{project 2} \mid sell, x_1 = S). \tag{60} \]

A rational market should value each share sold by the insider at date 1 at

\[ hp + (1 - h) [s (1 + \theta p) + (1 - s) (1 + p)]. \tag{61} \]

Furthermore, as a minimum rationality requirement, the market must believe that the insider is (weakly) more likely to sell after after a failure than after a success. This is because, for any given price of shares sold at date 1, if the insider chooses to sell when \( x_1 = S \), then he should also sell if \( x_1 = 0 \). Thus, if the market observes shares being sold, the lowest possible weakly rational value for \( h \) is \( 1 - p \) (that is, the market believes that both types sell shares with the same probability and project one was chosen).

We now need to consider two cases.

Project 1 was chosen at date 0. A late-consumer insider would only sell after
observing $x_1 = S$ if

$$hp + (1 - h) \left[ s (1 + \theta p) + (1 - s) (1 + p) \right] \geq 1 + p. \quad (62)$$

This condition is easier to satisfy when the market believes that project 2 was chosen when $x_1 = S$, i.e. $s = 1$. Setting $s = 1$ implies

$$1 + \theta p - h [1 + p (\theta - 1)] \geq 1 + p \quad (63)$$

This condition is easier to satisfy when $h$ is low. The lowest possible rational $h$ occurs when $h = 1 - p$ in which case the condition becomes

$$\theta p - (1 - p) [1 + p (\theta - 1)] \geq p. \quad (64)$$

This expression is easier to satisfy when $\theta$ is high. For any given $p$, the maximum $\theta$ is $1/p$. Thus, a necessary condition for equation 62 to hold is

$$p^2 - 2p + 1 \leq 0, \quad (65)$$

which implies $p = 1$, which is ruled out by assumption. Thus, condition 62 can never be satisfied.

**Project 2 was chosen at date 0.** After a success, the best option is to stick with project 2. The condition for selling is thus

$$hp + (1 - h) \left[ s (1 + \theta p) + (1 - s) (1 + p) \right] \geq 1 + \theta p. \quad (66)$$

This condition never holds, since the right-hand side is decreasing in $h$ and increasing in $s$, and

$$(1 - p) p + p (1 + \theta p) < 1 + \theta p.$$
Lemma 2.

Proof. For $b = 1$ to be an equilibrium strategy for the insider we need that $V(\sigma, 1) \geq p$, or

$$m(\sigma, 1) [\sigma v_2 + (1 - \sigma) v_1] + [1 - m(\sigma, 1)] p \geq p$$

which holds trivially for any $\sigma$ because $v_1 > p$ and $v_2 > p$. For $b < 1$ to be an equilibrium, we need $m(\sigma, b) = 0$, which can never happen since $\mu > 0$. ■

Proposition 1.

Proof. The insider chooses the project after selling $1 - \alpha$ shares of the company. Thus, his goal is to maximize the value of his equity stake $\alpha q(\sigma)$, where $q(\sigma)$ is the value of the firm given that the market expects $\sigma$. His incentive compatibility constraint for choosing project 2 when the market expects project 2 to be chosen is

$$\alpha q_2(\sigma) \geq \alpha q_1(\sigma),$$

which implies

$$(1 - \delta p)(p + T(\sigma)) + \delta p (1 + \theta p) \geq (1 - p)(p + T(\sigma)) + p (1 + p),$$

which simplifies to condition 18. Reversing the inequalities proves part 2. ■

Proposition 2.

Proof. To prove each case it is sufficient to find out the set of $\sigma$’s that satisfy the incentive compatibility constraints.

Case 1. Because $T(\sigma) > 0$, then the IC for project 1 cannot be satisfied. The IC for project 2 is trivially satisfied, thus $\sigma^* = 1$ is the only equilibrium.
Case 2. Suppose that \( v_1 - v_2 > 0 \). Notice that

\[
V'(\sigma) = \frac{\partial m(\sigma, 1)}{\partial \sigma} [v_1 + \sigma (v_2 - v_1) - p] + (v_2 - v_1) m(\sigma, 1)
\]

which is strictly negative when \( v_1 - v_2 > 0 \) because

\[
\frac{\partial m(\sigma, 1)}{\partial \sigma} < 0 \text{ and } v_1 + \sigma (v_2 - v_1) - p > 0.
\]

Thus, the highest possible value for the option to exit is

\[
T(0) = \frac{\mu p}{\mu + (1 - \mu) (1 - p)}
\]

Thus, if

\[
v_1 - v_2 \geq p (1 - \delta) T(0)
\]

then the unique equilibrium occurs when \( \sigma = 0 \).

Now, note that \( T(\sigma) \) is minimized at \( \sigma = 1 \), so

\[
T(1) = \frac{\mu v_2 + (1 - \mu) (1 - \delta) p}{\mu + (1 - \mu) (1 - \delta)} - p
\]

\[
T(1) = \frac{\mu p \delta [1 + p (\theta - 1)]}{\mu + (1 - \mu) (1 - \delta)}.
\]

Note that because \( T(\sigma) \) is strictly decreasing for \( \sigma \in [0, 1] \), its inverse \( T^{-1} \) is well defined in that domain. If

\[
v_1 - v_2 < p (1 - \delta) T(0) \text{ and } v_1 - v_2 > p (1 - \delta) T(1)
\]

then there exists a unique \( \sigma^* \in [0, 1] \) such that

\[
\sigma^* = T^{-1} \left( \frac{v_1 - v_2}{p (1 - \delta)} \right).
\]
Lemma 3.

Proof. Because rational market beliefs imply that shares sold after $x_1 = S$ can be valued at most at $1 + \theta p$, the late-consumer insider strictly prefers to keep his shares unless the market believes that $\sigma = 1$. But if the market believes that $\sigma = 1$, if the insider chooses $\sigma = 0$ his expected payoff is

$$p (1 + \theta p) + (1 - p) p > \delta p (1 + \theta p) + (1 - \delta p) p$$

(78)

Thus $\sigma = 1$ cannot be an equilibrium. ■

Lemma 4.

Proof. Because rational market beliefs imply that shares sold after $x_1 = S$ can be valued at least at $1 + p$, the late-consumer insider strictly prefers to sell his shares unless the market believes that $\sigma = 0$, in which case he is indifferent between selling or not selling. ■

Proposition 3.

Proof. For the insider to be willing to randomize between projects 1 and 2, we must have equal expected gains from both projects, that is

$$pV (\sigma) + (1 - p) p = \delta p [\mu V (\sigma) + (1 - \mu) (1 + \theta p)] + (1 - \delta p) p.$$  

(79)

Solving for $s$,

$$s^* = \frac{\delta [1 + p (\theta - 1)] - \delta \mu p (\theta - 1) - 1}{p(\theta - 1) (1 - \delta \mu)},$$

(80)

as long as the numerator is positive. If negative, the equilibrium $s^*$ is zero, because project 1 always gives higher payoffs than project 2. Notice that $s$ is always strictly lower than 1 (the difference between the numerator and the denominator is $- (1 - \delta) [1 + p (\theta - 1)]$).

Using (33), $\sigma^* = s / [\mu \delta + s (1 - \mu \delta)]$ when $s > 0$, and $\sigma = 0$ when $s = 0$: there is a one-to-one mapping between $\sigma$ and $s$. ■

Lemma 5.
Proof. $V(b, \sigma)$ can be rewritten as

$$V(b, \sigma) = \frac{\mu [v_1 + \sigma (v_2 - v_1)] + (1 - \mu) [1 - p + \sigma p (1 - \delta)] bp}{\mu + (1 - \mu) [1 - p + \sigma p (1 - \delta)] b}$$

The proof has three parts.

(1) For $b = 1$ to be an equilibrium strategy for the insider we need that $kV(\sigma, 1) \geq p$.

$$k \frac{\mu [v_1 + \sigma (v_2 - v_1)] + (1 - \mu) [1 - p + \sigma p (1 - \delta)] p}{\mu + (1 - \mu) [1 - p + \sigma p (1 - \delta)]} \geq p \quad (81)$$

If $b = 1$, then the condition for selling is

$$k \geq \frac{\mu p + (1 - \mu) [1 - p + \sigma p (1 - \delta)] p}{\mu [v_1 + \sigma (v_2 - v_1)] + (1 - \mu) [1 - p + \sigma p (1 - \delta)] p} \equiv k_1(\sigma) \quad (82)$$

Because $v_1 + \sigma (v_2 - v_1) > p$, then $k_1(\sigma)$ is strictly lower than 1. Thus, if $k > k_1(\sigma)$, $b = 1$ is an equilibrium strategy.

(2) For $b = 0$ to be an equilibrium strategy for the insider we need that $kV(\sigma, 0) \leq p$.

Similar algebra reveals that this condition is equivalent to

$$k \leq \frac{p}{v_1 + \sigma (v_2 - v_1)} \equiv k_2(\sigma).$$

Algebra shows that $0 < k_2(\sigma) < k_1(\sigma)$.

(3) If $k \in (k_2(\sigma), k_1(\sigma))$, an equilibrium must be in strictly mixed strategies. Imposing the condition $kV(\sigma, b(\sigma)) = p$ leads to

$$b(\sigma) = \mu \frac{k [v_1 + \sigma (v_2 - v_1)] - p}{(1 - k) (1 - \mu) [1 - p + \sigma p (1 - \delta)] p} \quad (83)$$

Simple substitution shows that hat $b(\sigma) = 0$ if $k = k_2(\sigma)$ and $b(\sigma) = 1$ if $k = k_1(\sigma)$.  

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Furthermore, \( b(\sigma) \) is strictly increasing in \( k \)

\[
\frac{\partial b}{\partial k} = \mu \frac{v_1 + \sigma (v_2 - v_2) - p}{p (1 - \mu) (1 - k)^2 (1 - p + p\sigma (1 - \delta))} > 0
\]

Thus, \( b(k) \in (0, 1) \) for \( k \in (k_2, k_1) \).

**Proposition 5.**

**Proof.** To prove each case it is sufficient to find out the set of \( \sigma \)'s that satisfy the incentive compatibility constraints.

**Case 1.** Because \( T(\sigma) \geq 0 \), then the IC for project 1 cannot be satisfied. The IC for project 2 is trivially satisfied, thus \( \sigma^* = 1 \) is the only equilibrium.

**Case 2.** Suppose that \( v_1 - v_2 > 0 \). Suppose there is an equilibrium \( \sigma > 0 \). From the incentive compatibility constraints, it must be that \( T(\sigma) > 0 \). That implies \( b(\sigma) = 1 \). Thus, for \( \sigma > 0 \) we have

\[
V(\sigma) = m(\sigma, 1) [v_1 + \sigma (v_2 - v_1)] + [1 - m(\sigma, 1)] p
\]

(85)

Notice that

\[
V'(\sigma) = \frac{\partial m(\sigma, 1)}{\partial \sigma} [v_1 + \sigma (v_2 - v_1) - p] + (v_2 - v_1) m(\sigma, 1)
\]

(86)

which is strictly negative when \( v_1 - v_2 \geq 0 \) because

\[
\frac{\partial m(\sigma, 1)}{\partial \sigma} < 0, v_1 + \sigma (v_2 - v_1) - p > 0.
\]

(87)

Thus, the highest possible value for the option to exit is \( T(0) \). Thus, if

\[
v_1 - v_2 \geq p (1 - \delta) T(0)
\]

(88)

then the unique equilibrium occurs when \( \sigma = 0 \).
To analyze the other cases, first we define $\sigma_L$ as

$$\sigma_L = \min \left\{ V^{-1} \left( \frac{p}{k} \right), 1 \right\}. \quad (89)$$

That is, $\sigma_L$ is the lowest value of $\sigma \in [0, 1]$ that minimizes $T(\sigma)$. Note that because $T(\sigma)$ is strictly decreasing for $\sigma \in [0, \sigma_L]$, its inverse $T^{-1}$ is well defined in that domain.

If

$$v_1 - v_2 < p (1 - \delta) T(0) \text{ and } v_1 - v_2 > p (1 - \delta) T(1) \quad (90)$$

then there exists a unique $\sigma^* \in [0, \sigma_L]$ such that

$$\sigma^* = T^{-1} \left( \frac{v_1 - v_2}{p (1 - \delta)} \right). \quad (91)$$

Finally, if

$$v_1 - v_2 \leq p (1 - \delta) T(1) \quad (92)$$

then only equilibrium with $\sigma^* = 1$ can happen.

**Case 3**: $v_1 - v_2 = 0$. If $\sigma_L = 1$, then for any $\sigma < 1$ we have $T(\sigma) > 1$, thus the only equilibrium occurs with $\sigma^* = 1$. If $\sigma_L < 0$, then $T(\sigma) = 0$ for any $\sigma \in [\sigma_L, 1]$, proving the result.

**References**


Bloom, Nicholas, Sadun, Rafaella, and John Van Reneen, 2008, Do private equity owned firms have better management practices? working, Stanford and LSE.


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