Stock Price, Earnings, and Book Value in Managerial Performance Measures

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Abstract

This paper examines the role of stock price and accounting data in providing managerial incentives. We develop a multiperiod agency model in which a manager must be given incentives to undertake investments and to exert personally costly effort. Investments are “soft” and therefore entail measurement error for the accounting system as it seeks to separate investment- from operating expenditures. This separation is of no concern to the stock market which draws on its own information about the firm’s future cash returns resulting from past investments. Stock price, however, is also an imperfect performance indicator because it includes all value relevant information, parts of which are not incentive relevant. We demonstrate that an optimal performance measure can be expressed as a weighted average of market value added and residual income and we relate the relative weights placed on these two components to the softness of investments and the precision of the market’s information.
1 Introduction

In order to provide top level managers with desired incentives, firms rely on a range of performance indicators based on accounting information and external market information. Managerial compensation packages frequently comprise multiple ingredients including cash bonuses based on earnings or return on assets, stock grants and stock options.\(^1\) Survey evidence also indicates that there is considerable variation in the structure of managerial compensation schemes not only across different industries but also across firms in the same industry. An observation commonly made in connection with high-technology firms is that, in comparison to traditional industries, high-technology firms tend to rely more heavily on stock based compensation.\(^2\)

This paper develops a multiperiod principal-agent model in which a firm’s stock price, earnings and book value evolve endogenously in response to the incentive scheme chosen for the firm’s manager. Our main objective is to examine the relative weights placed on accounting earnings and stock price in optimal performance measures. We relate these weights to the riskiness of the firm’s investment projects and to the “softness” of investments, i.e., the ability of the accounting system to separate investment expenditures from operating expenditures.\(^3\) In addition, we identify the precision of the market’s forecast of future cash flows as a determinant of the optimal relative weights.

Our multiperiod model is predicated on the notion that accounting information lags behind the stock market in its ability to reflect value creating decisions. Specifically, the firm’s market price is assumed to incorporate future economic benefits of current investment decisions. This forward looking ability of the stock market makes stock price a potentially useful incentive tool for aligning the long-term objectives of managers and owners. In the basic version of our model, the market provides a perfect assessment of future cash flows at each point in time.\(^4\) In doing so, however, the market must include all value relevant information regardless of whether this information is incentive relevant. In particular, the

\(^1\) See, for instance, Jensen and Murphy (1990), Rappaport (1999) and Bushman and Smith (2000).

\(^2\) See Ittner, Lambert, and Larcker (2001) for a review of the related empirical literature.

\(^3\) This notion is related to the work of Smith and Watts (1992) who examine the link between a firm’s investment opportunity set and the use of stock based incentive plans.

\(^4\) In contrast, in the one-period models of Paul (1992), Bushman and Indjejikian (1993), Kim and Suh (1993) and Feltham and Wu (2000), the market’s valuation of the firm is afflicted by exogenous random disturbances.
firm’s stock price must reflect any random components of future cash flows which are beyond the manager’s control.

The main role of the accounting system in our model is to separate investment expenditures from current operating expenses, but this separation is assumed to involve measurement errors. For example, investment expenditures for product development, personnel training or process improvement can frequently not be traced directly, but instead are measured according to internal allocation rules which rely on select allocation bases. By capitalizing the measured investment expenditures, accounting income shields managers, at least partially, from the negative cash flow effect of current investment expenditures, and thereby provides investment incentives. At the same time, however, the accounting measurement errors introduce risk factors of their own which ultimately impose an agency cost.

Aside from relying on current stock price and accounting income, the principal could generate the desired investment incentives by rewarding the delivery of cash flows in future periods when the investment returns materialize. Clearly however, there is no need to await future returns if the market has perfect foresight of those returns. In such a setting it will be optimal to generate the desired investment incentives by relying on current stock price and current accounting income. At the same time, the principal wants to use lagged stock price as an instrument for eliminating the variability of the cash returns associated with previous investments. In that sense, lagged stock price serves as an instrument of “intertemporal relative performance evaluation”.

The weight on current market price relative to the weight on current income in an optimal performance measure reflects the need to balance two risk factors: measurement error in the identification of investment expenditures and variability of future cash flows reflected in the firm’s current market price. To optimize this tradeoff, the principal adopts a conservative asset valuation rule, i.e., a dollar of measured investment expenditures is capitalized at a rate of less than one, even though accounting measurements are unbiased on average. Furthermore, the optimal capitalization rate is decreasing in the amount of accounting measurement error. In the limit, it becomes desirable to adopt a policy of full expensing.

Earlier work on agency problems with multiple performance indicators has shown that for the purpose of linear aggregation the relative weight attached to any two signals should depend on their “signal-to-noise” ratio.\textsuperscript{5} Despite several complicating features of our model,\textsuperscript{5}

\textsuperscript{5}See, for instance, Banker and Datar (1988), Lambert and Larcker (1987), Feltham and Xie (1994) and
our findings are fully consistent with the earlier results. In particular, the ratio between the optimal weight on market price (relative to accounting income) and the optimal capitalization rate is equal to the ratio of each signal’s sensitivity multiplied by its precision.

In the earlier one-period models of Bushman and Indjejikian (1993) and Kim and Suh (1993) the firm’s market price aggregates signals received by the market and the firm’s accounting system. Each of these signals reflects the manager’s unobservable effort plus some random noise. Ideally, the manager’s rewards would be based on the firm’s terminal cash flow, yet because this cash flow is assumed to be unavailable for contracting purposes, it becomes essential to rely on both the accounting signal and current stock price. In contrast, our multiperiod model with investment choices allows for the possibility of basing managerial compensation exclusively on the stream of delivered cash flows. To do so, however, would unduly constrain the creation of intertemporal incentives, thus making stock price and accounting numbers valuable performance indicators beyond the actual cash flows.

While the basic version of our model gives the market perfect ability to project the firm’s future cash flows, we also consider the more realistic scenario in which the market receives imperfect information, i.e., its aggregate signal of future cash flows resulting from current investment is subject to an (unbiased) error term. One might expect that as market information becomes less precise, an optimal performance measure puts less weight on current stock price and instead relies more heavily on accounting measurements and future cash flows. We find, however, that this intuition is generally not correct due to a countervailing effect: with less precise information, the market’s valuation of the firm relies less on its own noisy signal and more on the manager’s equilibrium investment choice. As a consequence, the sensitivity of the signal provided by current stock price decreases, and this may make it desirable for the principal to put a nominally larger weight on stock price.

To begin with, our analysis considers a dividend policy under which the entire current cash flow (net of compensation payments) is fully paid out to the firm’s owners. We then demonstrate that our findings on the relative use of accounting income and market price can be adapted without substantive changes to arbitrary dividend policies. To replace market price by a measure that is invariant to past dividend payments, the principal can resort to

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The second half of Bushman and Indjejikian’s (1993) paper considers a setting in which the manager’s actions are two-dimensional, yet income only reflects one of those actions.
market value added, i.e., cum-dividend price less the compounded beginning of the period price. Similarly, residual income (or economic value added) is invariant to current and past dividend payments, yet it preserves the information content of accounting income. We find that a weighted average of market value added and residual income is an optimal performance measure irrespective of the firm’s dividend policy. In fact, a linear combination of these two value added measures becomes essentially the unique optimal performance measure if the principal seeks memoryless compensation schemes, for which the compensation parameters are not conditioned on past information variables.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 presents our basic results in a setting in which the optimal performance measure must balance the two risk factors related to noisy accounting measurements and variability of future investment returns. Imperfect market information is introduced in Section 4, while Section 5 examines alternative dividend policies. We conclude in Section 6.

2 Model Description

We consider a principal-agent relationship which extends over $T$ periods. In each period, the agent (manager) contributes to current operating cash flow through personally costly effort. In addition, the agent makes investment decisions. These investments require an initial cash outflow in a given period and result in uncertain cash inflows in the next period.\footnote{It would be straightforward to extend our analysis so as to allow for the possibility that investment returns are received over several periods.} For any $1 < t \leq T$, the firm’s total cash flow is:

$$\tilde{c}_t = a_t + \tilde{\varepsilon}_t + (m_{t-1}(b_{t-1}) + \tilde{\mu}_{t-1}) - b_t .$$

Here, $a_t$ represents the agent’s effort and $\tilde{\varepsilon}_t$ is a random variable which reflects operating noise. The cash investment in period $t$ (which starts at date $t-1$ and ends at date $t$) is denoted by $b_t$, with $b_t \in [\underline{b}, \bar{b}]$. The function $m_{t-1}(\cdot)$ represents the expected gross return from investment undertaken in period $t-1$; this expected return function is assumed to be concave such that $m'_{t-1}(b_{t-1}) \to \infty$ as $b_{t-1} \to \underline{b}$, and $m'_{t-1}(b_{t-1}) \to 1$ as $b_{t-1} \to \bar{b}$. The actual investment returns are uncertain as reflected in the noise term $\tilde{\mu}_{t-1}$. For simplicity, the amount of risk is assumed to be independent of the scale of investment for $b_t \in [\underline{b}, \bar{b}]$. The realization of $\tilde{\mu}_{t-1}$ occurs at the end of period $t-1$. 

7It would be straightforward to extend our analysis so as to allow for the possibility that investment returns are received over several periods.
Initially, we assume that all parties know the actual value of $f_t \equiv m_t(b_t) + \mu_t$ at date $t$. However, the signal $f_t$ is assumed to be non-contractible in period $t$, and therefore, the compensation payment at date $t$ cannot be tied directly to $f_t$ even though this cash flow will be realized in the next period. One interpretation of this assumption is that knowledge of $f_t$ reflects the "collective wisdom" of the market and is based on a variety of information sources in the economy, e.g., analysts’ reports. In accordance with this interpretation, $f_t$ is reflected in the firm’s market price at date $P_t$.

The firm’s accounting system is assumed to generate a verifiable and contractible signal $y_t$ of the current investment expenditure $b_t$. Our model captures the notion that for "softer" investments, like those in product development, process improvement and personnel training, the accounting system is imperfect in its ability to separate investment expenditures from current operating expenditures. As argued in the Introduction, such separation of expenditures is prone to involve measurement errors when the different expenditures are not directly traceable but instead must be derived through some allocation procedure. To capture this measurement error, we assume that the signal $y_t$ is equal to the true expenditures plus a random measurement error term. Thus, the accounting system generates the following verifiable measure of investment:

$$ \tilde{y}_t = b_t + \tilde{\eta}_t , $$

where $\tilde{\eta}_t$ is a zero-mean normally distributed random variable which reflects measurement errors. A high variance of the random variable $\tilde{\eta}_t$ effectively amounts to a setting in which operating expenditures cannot be separated from investment expenditures.\footnote{In the analysis of Kanodia, Singh and Spero (2000), the accounting system also generates an imperfect measure of investment. In the absence of agency considerations, these authors examine the impact of measurement error on the investment decisions of an owner/manager who wishes to maximize the value of his firm. Along similar lines, Kanodia and Mukherji (1996) demonstrate the importance of separating investment expenditures from operating expenditures.}

The signal $y_t$ can be used to record an asset value:

$$ A_t = k_t \cdot y_t , $$

where $k_t$ is a design variable for the principal which reflects the rate at which the available signal of current investment expenditures is capitalized. Without loss of generality, the
representation in (2) presumes that investments are fully amortized at the end of their useful lives, i.e., after one period.  

It should be noted that, in contrast to the earlier work of Bushman and Indjejikian (1993), Kim and Suh (1993) and Sloan (1993), we do not assume that the market is intrinsically noisy. In the basic version of our model the market delivers a noiseless and unbiased valuation of the firm. In doing so, the market price must include all value relevant information including the realization of the noise term \( \mu_t \). We also note that, in contrast to the above mentioned studies, there is no need for the market to rely on the available accounting information in our settings. Given knowledge of \( f_t \), the market does not learn anything from \( A_t \) about the future benefits of current investments. Furthermore, the market does not care about current cash flow if it is fully paid out in dividends to shareholders. Obviously, this feature will change with alternative dividend policies as examined in Section 5 below.

\[
\begin{align*}
&\text{Figure 1: Events in Period } t \\
&t-1 \\
&\quad (a_t, b_t) \quad (\epsilon_t, \mu_t, \eta_t) \quad P_t \quad s_t \\
&\quad \text{chosen} \quad \text{realized} \quad \text{realized} \quad \text{paid}
\end{align*}
\]

To keep our multiperiod agency model analytically tractable, we adopt a so called LEN framework with linear contracts, exponential utility and normally distributed noise terms. Specifically, the noise terms \( \tilde{\mu}_t \), \( \tilde{\epsilon}_t \), and \( \tilde{\eta}_t \) are assumed to be independent and normally distributed such that \( \tilde{\epsilon}_t \sim N(0, \sigma_{\epsilon}^2) \), \( \tilde{\mu}_t \sim N(0, \sigma_{\mu}^2) \) and \( \tilde{\eta}_t \sim N(0, \sigma_{\eta}^2) \).

The principal is risk-neutral and therefore seeks to maximize the present value of future expected cash flows net of compensation payments (as a consequence the principal also

\[\text{If one were to extend our model so as to allow for investment returns to be delivered over multiple periods, the choice of depreciation method would not be essentially indeterminate in the current framework. The reason is that in our setting the principal knows the profitability of investments, but she cannot verify the actual investment expenditure. In contrast, Rogerson (1997) and Reichelstein (2000) consider settings in which investments are observable and the role of the depreciation schedule is to motivate a better informed manager to accept all positive NPV projects, and only those.}\]
maximizes the cum-dividend price of the firm). The manager is assumed to be risk-averse and his preferences at date $t$ can be described by an additivity separable exponential utility function of the form:

$$U_t = - \sum_{i=t+1}^{\infty} \gamma^{i-t} \cdot \exp\{-\hat{\rho} \cdot (\phi_i - e_i(a_i))\}. \quad (4)$$

In each period, the agent’s current utility is given by consumption of money, $\phi_i$, less the cost of effort $e_i(a_i)$. We assume that $a_i \in [0, \overline{a}]$, and that $e_i(\cdot)$ is convex with $e'_i(0) = 0$ and $\lim e'_i(a_i) = \infty$ as $a_i \to \overline{a}$. The coefficient $\hat{\rho}$ represents the agent’s degree of risk aversion.

Consistent with earlier literature on repeated agency models, the agent is assumed to have access to third party banking. Specifically, the agent can borrow and lend in each period at the interest rate $r$. Denoting the compensation payment in period $t$ by $s_t$ and the agent’s savings at date $t$ by $W_t$, consumption is given by

$$\phi_t = s_t + (1 + r) \cdot W_{t-1} - W_t. \quad (5)$$

Access to third party banking for the agent ensures that the choice of incentive scheme does not need to be concerned with smoothing the agent’s consumption over time. In combination with additivity separable exponential utility, these two assumptions imply that in any period $t$ the agent’s preferences over alternative incentive schemes are independent of his current wealth $W_{t-1}$. We therefore set $W_0 = 0$, without loss of generality. Upon leaving the firm, the agent is assumed to be able to earn a net-wage of zero, i.e., he can earn a fixed wage $\hat{s}_t$ by exerting effort $\hat{a}_t$, such that $\hat{s}_t - e_t(\hat{a}_t) = 0$.

The new contractible information variables in period $t$ are $c_t, A_t$ and $P_t$. The entire information available to both parties at the beginning of period $t+1$ can therefore be represented as:

$$I_t \equiv (I_{t-1}, (c_t, A_t, P_t)) \quad (6)$$

We consider linear long-term compensation schemes of the form:

$$\{s_t = \alpha_t + \beta_t \cdot \pi_t\}_{t=1}^{T}, \quad (7)$$

where $\pi_t$ is the manager’s performance which can be based on any linear combination of the variables in $I_t$. The parameters $\alpha_t$ and $\beta_t$ of the compensation scheme can then, without
loss of generality, be chosen independently of the history\textsuperscript{10}.

Given long-term contracts and the agent’s access to credit, there is no need for the principal to condition current compensation on past information variables which have no bearing on current or future realizations. In particular, there is no loss of generality in restricting attention to performance measure of the form:

$$\pi_t = c_t + A_t + w_t \cdot A_{t-1} + u_t \cdot P_t + v_t \cdot P_{t-1},$$

(8)

where $w_t$, $u_t$, and $v_t$ are coefficients to be chosen by the principal in conjunction with the compensation parameters $\alpha_t$ and $\beta_t$.\textsuperscript{11} Without loss of generality, we have normalized the coefficient on cash flow $c_t$ to one. There is also no loss of generality in setting the coefficient on asset value $A_t$ equal to one because the asset valuation rule, i.e., the capitalization rate $k_t$, is chosen endogenously by the principal.

In each period, the stock market values the firm at its expected value of future cash flows. Initially, we consider a full-payout dividend policy, i.e., dividends are equal to total cash flow less the managerial compensation payment in each period.\textsuperscript{12} The firm’s ex-dividend fair market value then becomes:

$$P_t = \sum_{i=t+1}^{T} \gamma^{i-t} \cdot E[\tilde{c}_i - \tilde{s}_i | I_t^- , f_t],$$

(9)

where $\gamma \equiv (1+r)^{-1}$ denotes the discount factor based on the interest rate $r$, and $I_t^-$ denotes the informational variables in $I_t$ except for $P_t$. We note from expressions (7)-(8) that the manager’s future compensation depends on current market price, which in turn depends on future compensation. In particular, the market anticipates that for every dollar of current market value the manager will be paid $v_{t+1}$ in the following period. Given linear contracts,

\textsuperscript{10}One immediate drawback of our focus on linear compensation schemes is that it excludes the use of stock options which are issued either “at the money” or at a strike price sufficiently large so that managers prefer not to exercise the option with some “significant probability”.

\textsuperscript{11}As explained further in Section 3, the beginning of the period stock price, $P_{t-1}$, needs to be included in the performance measure only if the principal must meet the agent’s participation constraint in each period. In contrast, if the agent can be “locked into” the long-term contract and therefore the participation constraints only hold on an ex-ante basis, then it would be sufficient to consider performance measures based on $c_t$, $A_t$, and $P_t$ only.

\textsuperscript{12}Alternatively, we could have employed the convention that the market price also includes the current managerial compensation payment.
however, it is readily verified that the equilibrium price is:

\[
P_t = \lambda_t \cdot f_t + \omega_t \cdot A_t + K_t
\]  

where

\[
\lambda_t \equiv \frac{\gamma \cdot (1 - \beta_{t+1})}{1 + \gamma \cdot \beta_{t+1} \cdot v_{t+1}}
\]

\[
\omega_t \equiv -\frac{\gamma \cdot \beta_{t+1} \cdot w_{t+1}}{1 + \gamma \cdot \beta_{t+1} \cdot v_{t+1}}
\]

and the constant \( K_t \) depends on the compensation parameters of future periods.\(^\text{13}\)

### 3 Optimal Performance Measures

To characterize optimal performance measures and the weights on current and lagged stock price, we first specify the exact nature of the contracting commitments made by the two parties. While our analysis supposes that the principal commits to an incentive scheme over the entire planning horizon, we assume initially that the agent cannot commit himself to stay beyond the current period. This specification seems descriptive of most managerial contracting arrangements. In order for the agent to stay with the firm at any given point in time, the stream of discounted future payoffs must exceed his market alternative. We then say that the compensation scheme satisfies the *interim participation* constraints. This contracting scenario will later be contrasted with one in which the agent can be “locked” into the relationship and therefore only the initial participation constraint has to be satisfied.

An immediate implication of the sequential rationality requirement imposed by the interim participation constraints is that the agent’s rewards (and penalties) can effectively not be deferred to later periods. To see this, we recall that the realizations of the noise terms \( \mu_t \) and \( \eta_t \) are known to all parties at date \( t \). Since both of these random terms have unbounded supports, the interim participation constraints can be satisfied only if the performance measure in period \( t + 1 \) and thereafter is independent of \( \mu_t \) and \( \eta_t \). In particular,

\[
c_{t+1} + w_{t+1} \cdot A_t + v_{t+1} \cdot P_t \equiv a_{t+1} + \epsilon_{t+1} + (m_t(b_t) + \mu_t) - b_{t+1} + w_{t+1} \cdot k_t \cdot (b_t + \eta_t) + v_{t+1} \cdot P_t
\]

\(^\text{13}\)See the proof of Lemma 1 in the Appendix for details.
must be independent of $\mu_t$ and $\eta_t$. Therefore, we obtain: $v_{t+1} = -(1+r)$ and $w_{t+1} = 0$. Substituting these values into (10), the expression for the market price $P_t$ simplifies to:

$$P_t = \gamma \cdot f_t + K_t$$  \hspace{1cm} (11)

The sequential rationality requirement embedded in the interim participation constraints implies that lagged stock price must exactly offset the future cash returns from current investments. In that sense, lagged stock price is effectively used as an instrument of “inter-temporal relative performance evaluation”. As a consequence, the future cash returns do not generate any investment incentives, but instead such incentives must be generated via the current stock price or current accounting information.

The following result shows that the agent’s preferences take a particularly simple form in the multiperiod LEN model with interim participation constraints.\textsuperscript{14}

**Lemma 1**: Suppose an incentive scheme of the form in (7)-(8) satisfies the interim participation constraints. Then $v_t = -(1+r)$, $w_t = 0$, and the certainty equivalent of the agent’s expected utility is given by:

$$CE_t(I_t) = W_t + \sum_{i=t+1}^{T} \gamma^{i-t} \{ E[s_i(\tilde{\pi}_i)|I_t] - e_i(a_i) - \rho \cdot \beta_i^2 \cdot (\sigma_{\varepsilon}^2 + k_i^2 \cdot \sigma_{\eta}^2 + \gamma^2 \cdot u_i^2 \cdot \sigma_{\mu}^2) \}$$  \hspace{1cm} (12)

where $\rho \equiv \frac{1}{2} \cdot \hat{\rho} \cdot (1 - \gamma)$.

Lemma 1 shows that the principal’s problem is intertemporally separable in the present setting, i.e., the effort level and investment amount for period $t$ can be chosen in a myopically optimal fashion. In particular, the risk-neutral principal’s objective for period $t$ is to choose $(a_t^*, b_t^*)$ so as to:

$$\max_{(a_t, b_t)} \{ a_t + (\gamma \cdot m_t(b_t) - b_t) - e_t(a_t) - \rho \cdot \beta_t^2 \cdot [\sigma_{\varepsilon}^2 + k_t^2 \cdot \sigma_{\eta}^2 + u_t^2 \cdot \gamma^2 \cdot \sigma_{\mu}^2] \}$$  \hspace{1cm} (13)

subject to the incentive compatibility constraint:

$$\left( a_t^*, b_t^* \right) \in \arg\max_{(a_t, b_t)} \{ \alpha_t + \beta_t \cdot E[a_t + \tilde{\varepsilon}_t - b_t + k_t \cdot (b_t + \tilde{\eta}_t) + u_t \cdot \gamma \cdot (m_t(b_t) + \tilde{\mu}_t)] - e_t(a_t) \}$$  \hspace{1cm} (14)

\textsuperscript{14}All proofs are provided in the Appendix.
In the current LEN model, these incentive compatibility constraints are equivalent to the two first-order conditions:

\[ e'_t(a_t) = \beta_t \]  \hspace{1cm} (15)

and

\[ u_t \cdot \gamma \cdot m'_t(b_t) = 1 - k_t \]  \hspace{1cm} (16)

The above optimization problem reveals that the principal faces an “induced” agency problem with respect to the investment choice \( b_t \). Even though investments do not impose a direct personal cost on the manager, there is an indirect cost due to the moral hazard problem. In order to provide the agent with effort incentives, the principal must reward the delivery of cash flows in each period, i.e., \( \beta_t \) must be positive. However, since every dollar of investment reduces current cash flow by one dollar, the agent needs a countervailing investment incentive. Rewarding the manager for increases in the current stock price does provide such incentives, yet the cost of this instrument is that the manager becomes exposed to the undesirable risk inherent in the uncertain investment returns. The variance term in (13), combined with the incentive constraints in (15) and (16), reflects this interaction. As a benchmark, we denote the first-best level of investment by \( b^*_t \), i.e.,

\[ b^*_t \in \arg\max_{b_t} \{ \gamma \cdot m_t(b_t) - b_t \}. \]

Even though accounting measurements are presumed to be imperfect, such measurements are nonetheless effective in generating investment investments. For instance, the manager would be completely shielded from the investment expense if the accounting system were to fully capitalize the signal \( y_t \), i.e., if \( k_t \) were set equal to one. The principal could then instruct the agent to invest the first-best amount \( b^*_t \), and this would indeed be optimal in the special case where the accounting measurement is perfect, i.e., \( \sigma^2_{\eta} = 0 \). However, when the separation between operating and investment expenditures is subject to measurement errors, i.e., \( \sigma^2_{\eta} > 0 \), reliance on the accounting signal entails a new source of risk for the agent. Since the corresponding risk premium is increasing in the capitalization rate \( k_t \), one would expect that the principal relies on both the accounting signal and the firm’s stock price in order to induce investments.

To facilitate the interpretation of our results in this section, it will be convenient to express the performance measure as a linear combination of accounting income as well as
current and lagged stock price, i.e.,

$$\pi_t = Inc_t + u_t \cdot P_t + v_t \cdot P_{t-1},$$

where $Inc_t = c_t + A_t - A_{t-1}$. This normalization, which sets the coefficient on $A_{t-1}$ equal to one, is without loss of generality if the fixed payments $\alpha_t$ is allowed to be a (linear) function of $A_{t-1}$.$^{15}$

**Proposition 1** The optimal investment $b_t^*$ is below the first-best level $b_t^\circ$. The weight on market price (relative to the weight on income) in an optimal performance is equal to:

$$u_t^* = \frac{1 - k_t^*}{\gamma \cdot m_t'(b_t^*)}$$

with the optimal capitalization rate $k_t^*$ given by:

$$k_t^* = \frac{\sigma^2}{\sigma^2 + (m_t(b_t^*))^2 \cdot \sigma^2_\eta}.$$  \hspace{1cm} (18)

The optimal weight on the market price, $u_t^*$, and the optimal capitalization rate, $k_t^*$, in Proposition 1 reflect that the principal seeks to minimize the overall variance in the manager’s performance measure subject to the two interacting incentive compatibility constraints. The expression for the optimal $k_t^*$ in (18) reveals that accounting measurement error calls for conservative capitalization in the sense that $0 < k_t^* < 1$. Thus, the observed amount $y_t$ is neither fully capitalized nor fully expensed. The intuition for this result is that with full capitalization the manager would be completely shielded from the investment expenditure, and hence there would be no further need to include current stock price in the performance measure. Yet, the principal could create identical investment incentives more cheaply by putting a small weight on current stock price. This follows from the fact that the objective function in (13) is quadratic in $u_t^*$, and hence the resulting amount of risk is negligible for small values of $u_t^*$.

Our finding in Proposition 1 can be used to address the following question. If the nature of the investment is such that GAAP accounting calls for direct expensing, what would be the

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$^{15}$In Proposition 3 below, we focus on memoryless compensation schemes for which $\alpha_t$ and $\beta_t$ are independent of the history.
resulting economic loss, if the firm also were to follow an accounting policy of direct expensing for the purpose of performance evaluation in order to maintain conformity with GAAP? The above result shows that if the principal were to restrict himself to GAAP accounting income (which would be equal to cash flow) and stock price as performance indicators, there would be more severe underinvestment, i.e., the principal would induce a level of investment below \( b_t^* \). In addition, the manager would be exposed to unnecessary risk, and the principal would have to bear the attendant cost of a higher risk premium. The cost associated with these distortions can be evaluated explicitly in the current LEN framework.

For “hard” investments, like those in property, plant, and equipment, one would expect only small measurement errors by the accounting system. Indeed expression (18) shows that as the variance \( \sigma_\eta^2 \) becomes small, optimality calls for full capitalization and no reliance on the firm’s stock price. Conversely, the internal accounting measurements receive almost no weight and the desired investment incentives are generated exclusively by the firm’s market price when \( \sigma_\eta^2 \) becomes large. Such situations may arise in connection with “soft” investments, like those in customer satisfaction or process improvements, for which it is difficult to separate operating from investment expenditures in current cash flows.

Without going to extreme values, it is natural to ask whether the optimal \( k_t^* \) and \( u_t^* \) vary monotonically with the variances of the noise factors \( \tilde{\eta}_t \) and \( \tilde{\mu}_t \). Equations (17) and (18) show that, for a given investment level \( b_t^* \), the optimal capitalization rate indeed increases monotonically in \( \sigma_\mu^2 \) and decreases monotonically in \( \sigma_\eta^2 \). This reasoning is incomplete, however, since both the optimal \( a_t^* \) and \( b_t^* \) are also functions of the respective noise terms. To simplify the following comparative statics result, we hold the bonus parameter \( \beta_t \) exogenously fixed at some \( 0 < \beta_t^* < 1 \). This specification may reflect that the marginal cost of effort \( e'_t(\cdot) \) is equal to \( \beta_t^* \) for \( a_t \leq a_t^* \), while \( e'_t(\cdot) \) is prohibitively large for \( a_t > a_t^* \). Despite changes in the underlying parameters of the problem, the principal will then continue to set \( \beta_t = \beta_t^* \).

---

16 The literature on accounting for intangibles has frequently suggested that there may be substantial economic costs associated with a policy of expensing investments in intangible assets; see, for example, Lev and Sougiannis (1996). In a pure valuation context, these costs appear to be more difficult to capture than in the current management control setting.
Corollary to Proposition 1: For fixed bonus parameters $\beta^*_t$:

(i) the optimal weight on the market price, $u^*_t$, is decreasing in $\sigma^2_\mu$.

(ii) the optimal capitalization rate, $k^*_t$, is decreasing in $\sigma^2_\eta$.

It is useful to compare the above expression for the relative weights on current stock price and current accounting information with the results in Banker and Datar (1989). The main insight from the analysis of Banker and Datar (1989) is that when multiple signals are to be linearly aggregated, the relative weight on a signal should be given by its sensitivity times its precision. In our context, the signals $P_t$ and $y_t$ are included in the manager’s performance measure to provide investment incentives (rather than to provide effort incentives). Specifically, the sensitivity of the signal $P_t$ is given by:

$$
\frac{\partial E[\tilde{P}_t]}{\partial b_t} = \gamma \cdot m'_t(b^*_t),
$$

while the sensitivity of the signal $y_t$ is given by:

$$
\frac{\partial E[\tilde{y}_t]}{\partial b_t} = 1.
$$

Since $Var(\tilde{P}_t) = \gamma^2 \cdot \sigma^2_\mu$, $Var(\tilde{y}_t) = \sigma^2_\eta$ and the precision of a signal is defined to be the inverse of its variance, we find that the relative weight on each signal is indeed proportional to its sensitivity times its precision, that is$^{17}$, 

$$
\frac{u^*_t}{k^*_t} = \frac{m'_t(b^*_t) \cdot \sigma^2_\eta}{\gamma \cdot \sigma^2_\mu}.
$$

It is also instructive to relate our findings to the multi-action congruity literature and to examine how the relative weight placed on stock price varies with changes in the sensitivity of investments. For the purpose of this comparative statics analysis, we fix the investment choice at some arbitrary level $\hat{b} \in [\underline{b}, \overline{b}]$ and allow $m'_t \equiv m'_t(\hat{b})$ to vary. In order to induce the investment choice $\hat{b}$, one might expect that the relative weight on the stock price would be a decreasing function of $m'_t$. Differentiating (17) with respect to $m'_t$, however, yields:

$$
\frac{\partial u^*_t}{\partial m'_t} = \frac{\sigma^2_\eta \cdot [\sigma^2_\mu - (m'_t)^2 \cdot \sigma^2_\eta]}{\sigma^2_\mu + (m'_t)^2 \cdot \sigma^2_\eta^2},
$$

which reveals that $u^*_t$ is non-monotonic in $m'_t$. This observation is consistent with the findings of Datar, Kulp, and Lambert (2001) who show that in multi-action agency models, the weight placed on a signal can be a non-monotonic function of its sensitivity to the action being motivated.
To conclude this section, we examine the significance of the interim participation constraints. As explained above, in order for the agent to have an incentive to stay with the firm at the beginning of each period, the beginning of the period stock price must be used to eliminate the variability in the impending investment returns. As a consequence, the investment incentives in each period must be generated entirely by current stock price and the current asset value. If the agent can commit himself to stay with the firm for \( T \) periods, the coefficients on the beginning of the period stock price, \( v_t \), and the coefficient on the beginning of the period asset value, \( w_t \), become unconstrained. We recall that for any linear incentive scheme the firm’s stock price is given by

\[
P_t = \lambda_t \cdot f_t + \omega_t \cdot A_t + K_t,
\]

with \( \lambda_t \) and \( \omega_t \) as given by in connection with expression (10).

Accordingly, the variance term in the agent’s certainty equivalent in (12) is replaced by

\[
\beta_t^2 \cdot \sigma_z^2 + \Gamma_t^2 \cdot \sigma_{\mu}^2 + \Lambda_t^2 \cdot \sigma_{\eta}^2
\]

where

\[
\Gamma_t \equiv \beta_t \cdot u_t \cdot \lambda_t + \gamma \cdot \beta_{t+1} \cdot (1 + v_{t+1} \cdot \lambda_t)
\]

and

\[
\Lambda_t \equiv \beta_t \cdot k_t + \gamma \cdot \beta_{t+1} \cdot v_{t+1} \cdot \omega_t \cdot k_t
\]

represent the aggregate coefficients on \( P_t \) and \( A_t \), respectively, across the two periods \( t \) and \( t+1 \). We note in particular that \( \Gamma_t \) reduces to \( \beta_t \cdot u_t \cdot \gamma \) if \( v_{t+1} = -(1+r) \), while \( \Lambda_t \) reduces to \( \beta_t \cdot k_t \) when \( w_{t+1} = 0 \) and thus \( \omega_t = 0 \).

When the manager can commit to stay with the firm, the principal gains intertemporal flexibility in providing investment incentives. For any given value of \( \beta_t \), identical incentives can be generated by rewarding the agent either in the current or in the next period for increases in \( P_t \) and \( A_t \). Thus, the optimal values of \( u_t, k_t, v_t \) and \( w_t \) are indeterminate, though the aggregate values of \( \Gamma_t \) and \( \Lambda_t \) are determined uniquely by the incentive compatibility conditions.

It can be shown that from the principal’s perspective it does not matter whether the agent can credibly commit to stay with the firm for the entire planning horizon. If the compensation scheme is constrained to satisfy the sequential rationality requirement of voluntary participation at each point in time, the principal loses some degrees of freedom in the design
of optimal performance measures. Yet, her overall expected net-payoff is the same as in a setting with full commitment in which only the initial participation constraint applies.\footnote{See Dutta and Reichelstein (2002) for a formal demonstration in a related model.}

It can also be shown that with full commitment by both parties stock price remains an essential variable in optimal performance measures. To eliminate $P_t$ entirely from the agent’s performance measure, the principal would need to set both $u_t$ and $v_{t+1}$ equal to zero. At the same time, optimality requires that the incentive compatibility conditions be satisfied, and furthermore, the performance measure have minimal variance. These conditions cannot be satisfied simultaneously if $u_t = v_{t+1} = 0$. For the same reasons it would not be optimal to rely exclusively on the stream of delivered cash flows. First, a cash flow based performance measure would ignore the valuable signal provided by the accounting information. Secondly, even if accounting measurements were entirely unreliable (i.e., $\sigma^2_\eta$ gets large and therefore $A_t$ could be ignored), the stock price variable $P_t$ remains essential in order for the principal to generate the desired effort and investment incentives with minimal risk exposure. A compensation scheme based exclusively on cash flows would become overly constrained, and therefore suboptimal.
4 Imperfect Market Information

We have thus far assumed that the market has perfect foresight about the future payoffs from the firm’s current investments. This section examines the more realistic scenario in which the market’s aggregate information about the investment payoffs is imperfect. To model this, we represent the market’s information by the realization of a noisy signal about the future cash flows. In particular, the market is assumed to observe the realization of the random variable:

\[ \tilde{f}_t = m_t(b_t) + \mu_t + \tilde{\delta}_t. \]  

where \( \tilde{\delta}_t \) is normally distributed with mean zero and variance \( \sigma^2_\delta \). The magnitude of the variance \( \sigma^2_\delta \) then becomes a convenient measure of the market’s residual uncertainty since the market faces greater uncertainty about the future investment returns as \( \sigma^2_\delta \) increases.

Unlike the perfect market setting in the previous section, the market no longer directly observes the returns from investments, \( m_t(b_t) + \mu_t \). In forming expectations regarding next period’s cash flows, the market must rely on not only on its signal \( f_t \) but also on its conjecture about the agent’s investment choice in the current period. In fact, since the noise term, \( \tilde{\delta}_t \), by itself is not value relevant, the market pays attention to \( f_t \) only because it is informative about the realization of \( \mu_t \). Let \( \hat{b}_t \) denote the market’s conjecture about the agent’s investment choice in period \( t \). The market rationally anticipates the agent’s investment choice, and therefore \( \hat{b}_t = b^*_t \), in equilibrium. As before, the market’s valuation of the firm is not affected by the asset value \( A_t \). Since \( \hat{\mu}_t \) and \( \tilde{\delta}_t \) are normally distributed, the conditional expectation for the future cash inflow is:

\[ E[m_t(\hat{b}_t) + \hat{\mu}_t | \tilde{f}_t = f_t] = (1 - h) \cdot m_t(\hat{b}_t) + h \cdot f_t, \]  

where

\[ h \equiv \frac{\sigma^2_\mu}{\sigma^2_\mu + \sigma^2_\delta}. \]

\[ \text{We are invoking here the formula for conditional expectations of normally distributed random variables:} \]

\[ E[\tilde{X} | \tilde{Y} = Y] = E[\tilde{X}] + \frac{\text{Cov}(\tilde{X}, \tilde{Y})}{\text{Var}(\tilde{Y})} \cdot (Y - E[\tilde{Y}]). \]
Expression (23) shows that the market’s expectation for the future cash flows is a weighted average of its noisy signal $f_t$ and its conjecture about the expected investment payoffs $m_t(\hat{b}_t)$. The market puts more weight on its signal as this signal becomes more precise, i.e., $h$ increases in the precision of the market information, $\frac{1}{\sigma^2}$. Given (23) and a contract of the form in (7)-(8), the firm’s market value at date $t$ becomes:

$$P_t = \lambda_t \cdot [(1 - h) \cdot m_t(\hat{b}_t) + h \cdot f_t] + \omega_t \cdot A_t + K_t,$$

where the constant $K_t$ depends on the compensation parameters of future periods, and $\lambda_t$ and $\omega_t$ are identical to the ones derived in connection with expression (10).

The following result extends Lemma 1 to the imperfect market setting:

**Lemma 2:** If a compensation scheme of the form in (7)-(8) satisfies the interim participation constraints, then $v_t = -(1 + r)$, $w_t = 0$, and the certainty equivalent of the agent’s expected utility at date $t$ becomes:

$$CE_t(I_t) = W_t + \sum_{i=t+1}^{T} \gamma^{i-t} \cdot \{E[s_i(\bar{\pi}_i)|I_t] - c_i(a_i) - \rho \cdot \beta^2 \cdot [\sigma^2 + \kappa^2(1-h) + \nu^2 \cdot \gamma^2 \cdot h \cdot \sigma^2 + (1-h) \cdot \sigma^2]\}.$$  

The arguments underlying the above result are almost identical to those used in proving Lemma 1, and we therefore omit the proof of Lemma 2. As before, the certainty equivalent of the agent’s expected utility in (25) takes a mean-variance form. Direct comparison shows that the variance term in (25) differs from the corresponding expression in the perfect market setting in two respects. First, the variance term contains an additional component, namely $(1-h) \cdot \sigma^2$, which reflects the agent’s residual uncertainty regarding the payoffs in the current period from the investment made in the previous period. Second, the variance of the current market price differs from the one in the perfect market setting. With noisy market information, $Var[\hat{P}_t] = \gamma^2 \cdot h^2 \cdot (\sigma^2 + \sigma^2) = \gamma^2 \cdot h \cdot \sigma^2$.

---

When the market had perfect information, it did not matter that the manager was not paid at date $T+1$, since $f_t$ perfectly anticipated the future return. This argument no longer applies with imperfect market information, and it will therefore be convenient to suppose that there is no investment decision in the last period. Thus, $b_T = 0$ and, as a consequence, $P_T = 0$. Alternatively, one could extend the model by allowing for compensation payments at date $T+1$.

To see this, we note that $Var[\bar{\mu}_{t-1}|f_{t-1}] = (1-h) \cdot \sigma^2$. 

18
As in the perfect market setting, the interim participation constraints require that the manager’s expected compensation be independent of \( f_t \) and \( \eta_t \) in period \( t+1 \) and thereafter. This again implies \( v_t = -(1 + r) \) and \( w_t = 0 \), and therefore the firm’s market value at date \( t \) simplifies to:

\[
P_t = \gamma \cdot [(1 - h) \cdot m_t(\hat{b}_t) + h \cdot f_t] + K_t.
\]  

From the certainty equivalent expression in (25), it follows that the manager will make his investment choice so as to maximize \( E[\tilde{s}_t + \gamma \cdot \tilde{s}_{t+1} | I_{t-1}] \). Substituting expression (26) and collecting the relevant terms yields the following necessary condition for an interior level of investment, \( b_t \):

\[
\beta_t \cdot [-1 + k_t + u_t \cdot \gamma \cdot m'_t(b_t) \cdot h] + \beta_{t+1} \cdot \gamma \cdot [m'_t(b_t) - h \cdot m'_t(b_t)] = 0
\]  

Since the agent’s preferences are of the mean-variance form in (25), the principal again seeks to maximize the stream of future expected cash flows less the cost of effort and the attendant risk premium:

\[
\sum_{t=1}^{T} \gamma^t \cdot \{ a_t + \gamma \cdot m_t(b_t) - b_t - e_t(a_t) - \rho \cdot \beta_t^2 \cdot [\sigma^2 + k_t^2 \cdot \sigma^2 + u_t^2 \cdot \gamma^2 \cdot h \cdot \sigma^2 + (1 - h) \cdot \sigma^2] \},
\]  

subject to the investment incentive compatibility constraint in (27) and the effort incentive compatibility condition \( \beta_t^* = e_t'(a_t) \). In solving the above optimization problem, the principal chooses the induced level of effort \( a_t \), the induced level of investment \( b_t \), the weight on the market price \( u_t \), and the capitalization rate \( k_t \). We note that the principal’s optimization problem is no longer intertemporally separable since the incentive compatibility condition in (27) involves the bonus parameter in the subsequent period.
Proposition 2: With imperfect market information, the weight on the market price (relative to the weight on income) in an optimal performance is equal to:

\[ u^*_t = z_t \cdot \frac{1 - k^*_t}{\gamma \cdot h \cdot m_t(b^*_t)} \]  (29)

with the optimal capitalization rate \( k^*_t \) given by:

\[ k^*_t = z_t \cdot \frac{\sigma^2_{\mu}}{\sigma^2_{\mu} + h \cdot (m'_t(b^*_t))^2 \cdot \sigma^2_{\eta}} \]  (30)

and

\[ z_t \equiv \left[ 1 - \gamma \cdot \frac{b^*_t}{\beta^*_t} \cdot (1 - h) \cdot m'_t(b^*_t) \right]. \]

The expressions for the optimal weight on current stock price and the optimal capitalization rate are essentially the same as in the perfect market setting, except for the common scaling factor \( z_t < 1 \). To provide intuition for our finding in Proposition 2, we recall that in the perfect market setting it was optimal to generate the investment incentives exclusively by relying on income and current stock price. With interim participation constraints, the future cash returns from current investment had no incentive effect because they were neutralized by the inclusion of lagged stock price. This “clean” separation of tasks no longer applies when the market receives an imperfect signal of future cash flows.

With imperfect market information, it is no longer possible to shield the manager entirely from the investment payoffs, \( m_{t-1}(b_{t-1}) + \mu_{t-1} \), in period \( t \).\(^{22}\) One desirable consequence of this less precise intertemporal relative performance evaluation is that the rewards tied to future cash flow generate some investment incentives. Specifically, we recall from expression (26) that the market price is a weighted average of its signal \( f_t \), which depends on the manager’s actual investment choice, and \( m_t(\hat{b}_t) \), which depends on the market’s conjecture \( \hat{b}_t \) of

\[ f_{t-1} = m_{t-1}(b_{t-1}) + \mu_{t-1} + \delta_{t-1} \]

If the market’s noisy signal \( f_{t-1} = m_{t-1}(b_{t-1}) + \mu_{t-1} + \delta_{t-1} \) were directly available for contracting purposes, the manager’s risk exposure would be minimized by putting weight \( -h \equiv -\frac{\sigma^2_{\mu}}{\sigma^2_{\mu} + \sigma^2_{\delta}} \) on the signal \( f_{t-1} \). To see this, note that the optimal weight to filter out the unwarranted noise is given by

\[ -\frac{\text{Cov}[\hat{f}_{t-1}, \hat{\mu}_{t-1}]}{\text{Var}[f_{t-1}]} = -h. \]

Since the market discounts its signal \( f_{t-1} \) precisely by the factor \( h \) in pricing the firm, the optimal weight on the lagged stock price remains unchanged from that in the perfect market setting, i.e., \(-1 + r\). Thus, the lagged stock price must receive the same weight as required to satisfy the manager’s interim participation constraint. This reflects that in order for the interim participation constraints to hold, the manager’s current performance measure must be invariant to \( f_{t-1} \).
the manager’s investment choice. From the manager’s viewpoint, the expected contribution to his performance measure in the next period equals \((1 - h) \cdot [m_t(b_t) - m_t(\hat{b}_t)]\). Since the manager treats the market’s conjecture \(\hat{b}_t\) as fixed, his investment choice \(b_t\) increases next period’s compensation at the rate of \(\beta_t \cdot (1 - h) \cdot m_t'(b_t)\). To induce \(b_t^*\), the principal can thus correspondingly reduce the weights on current stock price and the accounting signal.

Proposition 2 shows that the variance term \(\sigma^2\), which captures the degree of market imperfection, affects the optimal choice of coefficients \(u_t^*\) and \(k_t^*\) directly as well as indirectly via the induced action and investment choices. In order to focus on the direct effect, we consider a specialized setting in which the optimal action and investment choices are invariant to the parameters of the underlying agency problems. This will be the case if these choices are not interior but always the maximal values, i.e., \(a_t = \bar{a}\) and \(b_t = \bar{b}\). The proof of the following result (in the Appendix) provides sufficient conditions for the principal to prefer these boundary values. For notational convenience, we define \(e_t' \equiv e_t'(\bar{a})\), \(e_t'' \equiv e_t''(\bar{a})\), and \(m_t' \equiv m_t'(\bar{b})\).

**Proposition 2’**: Suppose it is optimal to induce \(a_t = \bar{a}\) and \(b_t = \bar{b}\) in each period. Then the weight on market price (relative to the weight on income) in an optimal performance is equal to:

\[
u_t^* = \max \{0, z_t\} \cdot \frac{1 - k_t^*}{\gamma \cdot h \cdot m_t}
\]

with the optimal capitalization rate \(k_t^*\) given by:

\[
k_t^* = \max \{0, z_t\} \cdot \frac{\sigma^2}{\sigma^2 + h \cdot (m_t')^2 \cdot \sigma^2_{\eta}}
\]

where

\[
z_t \equiv \left[1 - \gamma \cdot \frac{e_t'+1}{e_t'} \cdot (1 - h) \cdot m_t'ight].
\]

This result shows that with effort and investment choices on the boundary, it may be optimal to set both \(u_t^*\) and \(k_t^*\) equal to zero. In contrast, Proposition 1 established that an optimal performance measure always puts positive weights on both the accounting information and the market price. The reason for this difference is that with imperfect market

\[23\text{Comparison of the objective functions in (13) and (28) immediately reveals that the optimal } b_t^* \text{ in the perfect market setting generally differs from the optimal } b_t^* \text{ in the imperfect market setting.}\]
information an optimal performance measure effectively returns \((1-h) \cdot m_{t-1}(b_{t-1})\) in period \(t\) for the investment expenditure \(b_{t-1}\) made in period \(t-1\). Consequently, if the market information is sufficiently imprecise (i.e., \((1-h)\) is sufficiently large) and the bonus parameters across two consecutive periods are properly aligned, it will not be necessary to rely on current stock price or accounting information to induce the optimal investment decision.

**Corollary to Proposition 2’**

(i) The optimal weight on the market price, \(u_t^*\), is decreasing (increasing) in the precision of the market information, if \(\frac{e_{t+1}}{e_t}\) is below (above) a certain threshold level.

(ii) Relative to the optimal weight on the market price, the optimal capitalization rate, \(k_t^*\), remains constant in the precision of the market information.

As market information becomes less precise, the principal wants to rely more, rather than less, on the market price. At first glance, this result appears counterintuitive. It must be kept in mind, however, that lower precision of the market’s information has two opposing effects on the optimal coefficient \(u_t^*\). First, the market relies more on its equilibrium beliefs about the agent’s investment choice and less on its information about the actual investment choice, as market information becomes less precise. As a consequence, stock price becomes a less powerful instrument for inducing investment incentives, and therefore the principal needs to increase \(u_t^*\) in order to induce a given investment choice. Secondly, however, next period’s performance measure becomes a more powerful incentive device as the market information becomes less precise. This effect will tend to lower the principal’s reliance on the current market price in providing investment incentives. The expression for \(z_t\) shows that the magnitude of this second effect depends on the relative magnitudes of the bonus coefficients in periods \(t\) and \(t+1\). When \(\frac{e_{t+1}}{e_t}\) is relatively small, the first effect dominates in the sense that \(u_t^*\) increases as the market information becomes less precise. On the other hand, when \(\frac{e_{t+1}}{e_t}\) is relatively large, the second effect dominates, and the optimal weight on the market price decreases as the precision of the market information declines.

The result that the ratio \(\frac{k_t^*}{u_t^*}\) remains constant in \(\sigma_\delta^2\) also appears counterintuitive at first sight. One might have expected that the principal would rely more on the accounting
information as market price becomes a less precise indicator of future cash flows. We recall, however, that the investment incentive is determined by the effective weight on the market signal \( f_t \), i.e., \( u_t^* \cdot h \cdot \gamma \), rather than by the nominal weight on the market price, i.e., \( u_t^* \).

While the ratio \( \frac{k^*_t}{u^*_t} \) remains constant, the ratio of the effective weights \( \frac{k^*_t}{u^*_t \cdot h \cdot \gamma} \) increases as the market information becomes less precise. Put differently, the sensitivity of the market price, as given by \( \gamma \cdot h \cdot m_t' \), declines as the market information becomes less precise. At the same time, however, the precision of the market price, as given by \( (\gamma^2 \cdot \sigma^2 \cdot h)^{-1} \), improves because the market relies less on its own signal in pricing the firm. These two effects completely offset each other, and the relative weights on the two signals remain unchanged.

We finally note that the preceding analysis has assumed throughout that investment related cash inflows are uncertain, i.e. \( \sigma^2 > 0 \). In the limit case of a deterministic return relationship (i.e., \( \sigma^2 = 0 \)), the market can perfectly predict the agent’s “equilibrium” investment choice without relying on its noisy signal \( f_t = m_t(b_t) + \delta_t \). Consequently, the market signal becomes irrelevant for valuation purposes even though it is incrementally informative about the agent’s investment choice.\(^{24}\) Since the firm’s market price only depends on the agent’s conjectured but not his actual investment choice, market price becomes useless for incentive purposes and therefore the optimal weight \( u_t^* \) will be indeterminate. However, with some uncertainty, no matter how small, in the investment return relationship, the market signal becomes useful for forecasting future cash flows and the optimal \( u_t^* \) is determined uniquely by (29).

5 Alternative Dividend Policies

In this section, we extend the above analysis by allowing for the possibility that past cash flows are partly or fully retained in the business. For simplicity, we first focus on a setting in which the market receives perfect information, i.e., \( f_t = m_t(b_t) + \mu_t \).\(^{25}\) Let \( d_t \) denote the amount of dividend paid in period \( t \) and \( C_t \) denote the firm’s cash balance at the end of

\(^{24}\)A similar result is obtained in Kanodia, Singh and Spero (2000), where the market ignores a noisy measure of investment in pricing the firm. Bagwell (1995) obtains a related conclusion in a leader-follower oligopoly setting.

\(^{25}\)As discussed at the end of this section, the case of imperfect market information requires only minor extensions.
period $t$. This balance is invested at the interest rate $r$, and therefore satisfies the relation:

$$C_t = (1 + r) \cdot C_{t-1} + c_t - d_t,$$

where the cash flow $c_t$ is given by (1). The firm’s book value then becomes $BV_t = C_t + A_t$, and comprehensive income measurement requires that:

$$\text{Inc}_t = d_t + BV_t - BV_{t-1} = (c_t + A_t - A_{t-1}) + r \cdot C_{t-1}.$$

Accounting income therefore is the sum of operating income and financing (or interest) income. Like in our previous settings, the market does not obtain new information from the accruals, i.e., the variable $A_t$ in our setting. However, when cash flow are no longer paid out as dividends, the firm’s market price obviously must reflect the firm’s cash balance in addition to expected future cash flows. Specifically, the firm’s market price is now given by:

$$P_t = C_t + \gamma \cdot f_t + K_t,$$

where $K_t$ again is a constant determined by the agent’s compensation scheme in future periods. To examine the impact of alternative dividend policies on optimal performance measures, suppose dividend payments follow a rule of the form:

$$d_t = L_t(I_{t-1}) + \theta_1 \cdot c_t + \theta_2 \cdot A_t,$$

for some linear function $L_t(\cdot)$ and coefficients $\theta_1$ and $\theta_2$. This class of dividend policies includes commonly used dividend arrangements such as fixed payout ratio based on accounting income.

The literature on equity valuation has long argued that expected future dividends are not an operational criterion for valuing a firm since dividend payments reflect value distribution rather than value creation. Consistent with this view, Ohlson (1999) examines a multiperiod agency model (in which there is no external capital market) and concludes that optimal contracts do not need to depend on dividend payments. Our multiperiod LEN framework allows us to draw the sharper conclusion that optimal performance measures must be invariant to the firm’s dividend policy.

26For consistency with our earlier analysis, we continue to use income before compensation. The interpretation is that the firm’s current shareholders pay the agent’s compensation from their dividend. We note, however, that our results would remain unchanged if we were to use income after compensation.
While accounting income is not affected by current dividends, it does depend on the firm’s past dividends through the interest income \( r \cdot C_t \). On the other hand, it is well known that residual income is independent of both current and past dividend payments. In our model residual income amounts to:

\[
RI_t = Inc_t - r \cdot BV_{t-1} = a_t + \varepsilon_t - b_t + f_{t-1} + A_t - A_{t-1}(1 + r).
\]

Thus, residual income by itself is not an optimal performance measure since it does not reflect the future investment return \( f_t \). In contrast, consider the following measure of market value added\(^{27}\):

\[
MV A_t \equiv P_t + d_t - (1 + r) \cdot P_{t-1}.
\]

This measure expresses the value added for shareholders in period \( t \). It is equal to the stock’s abnormal return times the beginning of the period stock price. Clearly, \( MV A_t \) is also invariant to the firm’s dividend policy. Specifically, we have:

\[
MV A_t = a_t + \varepsilon_t - b_t + \gamma \cdot f_t + K_t - (1 + r) \cdot K_{t-1}.
\] (34)

While market value added does reflect the future investment return \( f_t \), it does not capture the accounting information, \( A_t \). On the other hand, Proposition 1 has shown that an optimal performance measure must include both \( f_t \) and \( A_t \). This suggests that a properly weighted average of residual income (or economic value added) and market value added does constitute an optimal performance measure for arbitrary dividend policies.

In deriving the following result, we focus on compensation schemes which do not involve memory of past events and transactions. Formally, the compensation schemes \( \{s_t\}_{t=1}^T \) are said to be memoryless if the parameters \( \alpha_t \) and \( \beta_t \) do not depend on \( I_{t-1} \), the information that is available for contracting at the beginning of period \( t \). All memory requirements are embedded in the performance measures \( \pi_t \). In particular, \( \pi_t \) can be based on any linear combination of the variables \( (c_\tau, P_\tau, A_\tau, d_\tau, C_\tau) \) for \( \tau = t \) and \( \tau = t - 1 \). As argued in Section 2, there is no need for further memory in performance measures provided the agent has access to the credit market and therefore can borrow and lend at the principal’s cost of capital \( r \).

\(^{27}\)Our definition of market value added differs from that in the Value Based Management literature, where market value added is frequently defined as stock price minus "capital invested"; see, for instance, Stewart (1994).
In order for any incentive scheme to meet the interim participation constraints at the beginning of period $t$, compensation in that period must be independent of $P_{t-1}$, $A_{t-1}$ and $f_{t-1}$. Market value satisfies this requirement, yet residual income does not. For this reason, we focus on “calibrated” residual income, defined as residual income less the compounded difference between beginning of the period market and book value. Straightforward algebra shows that:

$$RI^c_t = RI_t - (1 + r) \cdot (P_{t-1} - BV_{t-1}) = a_t + \varepsilon_t - b_t + A_t + (1 + r) \cdot K_{t-1}.$$ 

Thus, calibrated residual income is indeed independent of $f_{t-1}$ and $A_{t-1}$. The following result shows that a linear combination of calibrated residual income and market value added is essentially the unique optimal performance measure if one insists on memoryless compensation schemes. To state this claim formally, two performance measures will be called equivalent if one is obtained from the other through an affine transformation, i.e., through the addition and multiplication of constants. Obviously, any claim about uniqueness can only apply to the entire class of equivalent measures.

**Proposition 3**: Suppose $\{s_t = \alpha_t + \beta_t \cdot \pi_t\}_{t=1}^T$ is an optimal memoryless compensation scheme for some dividend policy of the from in (33). Then $\pi_t$ is equivalent to:

$$\pi_t^* = (1 - u_t^*) \cdot RI^c_t + u_t^* \cdot MVA_t$$

with the capitalization rate, $k_t^{**}$, given by $k_t^{**} = k_t^*/(1 - u_t^*)$, and $k_t^*$ and $u_t^*$ as in (17) and (18).

The above result does not imply that the coefficient on current dividends must be zero in an optimal performance measure. It would be possible, for example, to have current dividends proportional to current cash flow and to assign a positive coefficient to dividends. To preserve optimality, however, the other coefficients would then have to be aligned so that the resulting performance measure is equivalent to the convex combination of $MVA_t$ and $RI^c_t$ given in (35).

The uniqueness of $\pi_t^*$ derives from the fact that with memoryless compensation schemes an optimal performance measure must satisfy:

$$E[\pi_t|f_{t-1}] = a_t - b_t \cdot m_t(b_t) + k_t^* \cdot b_t,$$
with \( u_t^* \) and \( k_t^* \) as given in Proposition 1. As one might expect, the optimal \( \pi_t^* \) puts its entire weight on the accounting component, \( RIR_t^* \), if there is no accounting measurement error. In that case, \( k_t^* = k_t^{**} = 1 \) and \( u_t^* = 0 \). Conversely, if there is no uncertainty in the future investment returns, we find that \( k_t^* = k_t^{**} = 0 \) and furthermore \( u_t^* = 1 \), since in this extreme case \( b_t^* = b_t^* \) and \( \gamma \cdot m_t'(b_t^*) = 1 \). For intermediate values of \( k_t^* \) and \( u_t^* \), it is readily verified that the resulting capitalization rate \( k_t^{**} \) is between zero and one, thus preserving our finding of conservative accounting.

We note that even when accounting measurements are very imprecise, \( MVA_t \) is not an optimal performance measure. By attaching a relative coefficient of one to current stock price, \( MVA_t \) will induce first best investments. Yet, as demonstrated above, the principal prefers underinvestment due to the attendant risk premium. On the other hand, it follows from (13) that the desired amount of underinvestment decreases with the size of the bonus parameter \( \beta_t \). In that sense, the loss associated with the \( MVA_t \) performance measure becomes small if accounting measurement errors are large and, at the same time, the optimal bonus parameters \( \beta_t \) are small.

To conclude this section, we note that Proposition 3 remains essentially unchanged when the market’s signal of future investment returns, \( f_t \), is subject to random disturbances. The coefficients \( u_t^* \) and \( k_t^* \) then have to be chosen as in Proposition 2. Furthermore, as explained in Section 4, it can no longer be guaranteed that the new coefficients \( u_t^*, k_t^* \) and \( k_t^{**} \) are between zero and one.

6 Conclusion

We have analyzed a multiperiod agency setting to examine the role of stock price and accounting information in managerial performance measures. From an incentive perspective, stock price and accounting data provide fundamentally different types of information. While stock price is “forward-looking”, accounting information is historical or “backward looking”. Since the firm’s current stock price reflects the market’s assessment of future cash flows, it is an effective instrument for providing managers with investment incentives. The drawback of stock price as a performance indicator is that it must reflect all value relevant factors even if some of those factors are beyond the manager’s control. If investment expenditures, as measured by the accounting system, are properly capitalized, accounting income can also...
be used to generate the desired investment incentives. Accounting income effectively shields the manager’s current performance measure from investments by ‘deferring’ the associated expenditures to future periods. To the extent that the separation of investment and operating expenditures is subject to measurement errors, however, accounting income is also an imperfect indicator. Our analysis establishes that an optimal performance measure will generally need to strike a balance and rely on both accounting information and stock price.

Our performance evaluation framework generates a number of unintuitive comparative statics results on the relative use of market- and accounting information in optimal performance measures. In particular, we argue that the relative weight on stock price may increase as the market receives less precise information about the firm’s future cash flows. Furthermore, our analysis shows that when the market information is relatively imprecise, the principal may find it optimal to rely less, rather than more, on accounting information. These findings may prove useful in future empirical work which seeks to link the use of stock price in managerial compensation schemes to the characteristics of different investments, firms and industries.

Our results were derived in a framework in which the rules for income measurement are chosen optimally from a control perspective. When the accounting rules are constrained to be conservative and therefore to expense all “soft” investments, possibly in order to maintain conformity with GAAP, our results show that the optimal investment policy would be less efficient and they also provide a measure of the resulting economic losses. Finally, we find that market value added (MVA) and economic value added (EVA) may be useful measures in order to calibrate stock price and accounting income to the relevant history, in particular to the firm’s dividend policy.
Appendix

Proof of Lemma 1:

Step 1: We first derive the expression for the equilibrium market price, given a linear compensation scheme of the form in (7)-(8). We note that $P_T = \gamma \cdot f_T$. Date $T-1$ market price is given by:

$$P_{T-1} = \gamma \cdot E[\hat{\pi}_T | I_{T-1}, f_{T-1}]$$

Substituting $\pi_T = c_T + w_T \cdot A_T + u_T \cdot P_T + v_T \cdot P_{T-1}$ and solving for $P_{T-1}$ yields

$$P_{T-1} = \lambda_{T-1} \cdot f_{T-1} + \omega_{T-1} \cdot A_{T-1} + K_{T-1}$$

where $\lambda_{T-1}$ and $\omega_{T-1}$ are as defined in connection with (10), and $K_{T-1}$ is a constant depending on $\alpha_T$, $\beta_T$ and $v_T$. For arbitrary $t$, expression (10) follows by induction.

Step 2: We next argue that if $P_t(f_t) = \lambda_t \cdot f_t + \omega_t \cdot A_t + K_t$, for arbitrary coefficients $K_t$, $\lambda_t$ and $\omega_t$, and the agent’s linear compensation scheme is of the form $s_t = \alpha_t + \beta_t[c_t + A_t + w_t \cdot A_{t-1} + u_t \cdot P_t + v_t \cdot P_{t-1}]$, then the certainty equivalent of the agent’s expected utility at date $t$ is given by:

$$CE_t(I_t) = W_t + \sum_{i=t+1}^{T} \gamma^{i-t} \cdot \{E[s_i(\tilde{\pi}_i) | I_i] - e_i(a_i) - \rho \cdot RP_i$$

with the risk premium, $RP_i$ given by:

$$RP_i \equiv \beta_i^2 \cdot \sigma_z^2 + \left[\beta_i \cdot u_i \cdot \lambda_i + \beta_{i+1} \cdot \gamma (1 + \lambda_i \cdot v_i) \right]^2 \cdot \sigma_{\mu}^2 + \left[\beta_i \cdot (1 + u_i \cdot \omega_i) + \gamma \cdot \beta_{i+1} \cdot w_{i+1} \right]^2 \cdot k_i^2 \cdot \sigma_{\eta}^2$$

Proof of Step 2: Given the realization of $f_t$ in period $t$, the agent chooses his savings at date $t$ optimally. With exponential utility and normally distributed noise terms, the agent’s expected utility at date $t$ can be expressed as:

$$EU_t = -\frac{1}{r} \cdot exp\{-\rho \cdot r \cdot CE_t\}$$

with the certainty equivalent $CE_t$ determined by:

$$CE_t(I_t) = W_t + \gamma \{E[\hat{s}_{t+1} + CE_{t+1}(\hat{I}_{t+1}) | I_t] - e_t(a_t) - \rho \cdot Var[\hat{s}_{t+1} + CE_{t+1}(\hat{I}_{t+1}) | I_t]\}$$
For details the reader is referred to Dutta and Reichelstein (1999, Lemma 1). The claim then follows immediately by induction after noting that:

\[ \text{Var}[\tilde{s}_{i+1} + CE_{i+1}(\tilde{I}_{i+1}) | I_i] = RP_i. \]

**Step 3:**
In order for the agent to be willing to stay with the firm at the beginning of period \( t + 1 \), his performance measure in period \( t + 1 \) must be independent of the noise terms \( \mu_t \) and \( \eta_t \). This is equivalent to the condition that

\[ c_{t+1} + w_{t+1} \cdot A_t + v_{t+1} \cdot P_t \]

be constant with regard to \( f_t \) and \( A_t \). Since \( c_{t+1} = a_{t+1} + \varepsilon_{t+1} + f_t - b_t \), it follows that \( \lambda_t \cdot v_{t+1} = -1 \), or \( v_{t+1} = -(1 + r) \). Similarly, the requirement that the period \( t \) performance measure be constant in \( A_t \) implies that \( w_{t+1} = 0 \). Expression in Lemma 1 then follows by substituting \( v_t = -(1 + r) \) and \( w_t = 0 \) in the certainty equivalent expression in Step 2. \( \square \)

**Proof of Proposition 1:** In order to implement any given \( a_t^* \) and \( b_t^* \), the bonus coefficient \( \beta_t \) must be chosen so that \( \beta_t = e'(a_t^*) \) and the coefficients to \( k_t \) and \( u_t \) must satisfy the incentive compatibility constraint:

\[ u_t \cdot \gamma \cdot m_t'(b_t^*) + k_t = 1. \]

The principal seeks to minimize the total variance:

\[ k_t^2 \cdot \sigma^2 \mu + \gamma^2 \cdot u_t^2 \cdot \sigma^2 \mu. \quad (A1) \]

Substituting the incentive compatibility condition for \( b_t^* \), into the objective function in (A1), one obtains the first-order condition for the optimal \( u_t^* \):

\[ u_t^* = \frac{\sigma^2 \mu \cdot m_t'(b_t^*)}{\gamma \sigma^2 \mu + (m_t(b_t^*))^2 \cdot \sigma^2 \eta}. \]

Since

\[ u_t^* = \frac{1 - k_t^*}{\gamma \cdot m_t'(b_t^*)}, \]

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it follows that:

\[ k_t^* = \frac{\sigma^2}{\sigma^2 + (m_t'(b_t^*))^2 \cdot \sigma^2_0}. \]

To demonstrate that \( b_t^* < b_t^c \), we note that

\[ b_t^c \in \text{argmax}_{b_t} \{ \gamma \cdot m_t(b_t) - b_t \}, \]

while

\[ b_t^* \in \text{argmax}_{b_t} \{ \gamma \cdot m_t(b_t) - b_t - \rho \cdot \beta^2_t \cdot V_t(b_t) \} \]

where

\[ V_t(b_t) \equiv \left[ (k_t^*(b_t))^2 \cdot \sigma^2_0 + \gamma^2 \cdot (u_t^*(b_t))^2 \cdot \sigma^2_0 \right] = \frac{\sigma^2_0 \cdot \sigma^2_0}{\sigma^2_0 + (m_t'(b_t))^2 \cdot \sigma^2_0}. \]

Since \( V_t(\cdot) \) is increasing in \( b_t \), it follows that \( b_t^* < b_t^c \).

To complete the proof, it remains to show that for any choice of \((\beta_1, ..., \beta_T), (k_1, ..., k_T), \) and \((u_1, ..., u_T)\) the principal can choose the fixed payments \((\alpha_1, ..., \alpha_T)\) so as to satisfy the interim participation constraints. Given Lemma 1, the interim participation constraint at date \( t \) is equivalent to the condition that \( CE_t \geq 0 \).

Direct substitution show that

\[ P_T = \gamma \cdot f_T - \gamma \cdot \alpha + \gamma \cdot \hat{a}_T(\beta'_T), \]

where \( \hat{a}_T(\beta'_T) \) is the effort level induced by \( \beta'_T \), i.e. \( \beta'_T \equiv e'_T(\hat{a}_T(\beta'_T)) \). Therefore:

\[ E[\hat{s}_T \mid f_{T-1}] = \alpha + \beta_T \cdot (a_T + f_{T-1} + v_T \cdot P_{T-1}(f_{T-1})) \]

\[ = \alpha + \beta_T \cdot \left[ a_T + \frac{1}{1 - \beta_T} \cdot \alpha_T - \hat{a}_T(\beta'_T) \right]. \]

For any \( \beta_T \) the principal can thus choose \( \alpha_T \) so that the agent’s participation constraint in period \( T \) is satisfied with equality.

For period \( T - 1 \), the beginning of the period price is given by:

\[ P_{T-1} = \gamma \cdot f_{T-1} - \frac{\gamma}{1 - \beta_{T-1}} \cdot \alpha_{T-1} + K_{T-1}', \]

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where $K'_{T-1}$ is a constant depending on $\beta_{T-1}$, $u_{T-1}$, $k_{T-1}$, $\alpha_T$ and $\beta_T$. Thus,

$$E[\tilde{s}_{T-1} \mid f_{T-2}] = \alpha_{T-1} + \beta_{T-1} \cdot [a_{T-1} - b_{T-1} \cdot (1 - k_{t-1})] + u_{T-1} \cdot \gamma \cdot m_{T-1}(b_{T-1}) + \frac{1}{1 - \beta_{T-1}} \alpha_{T-1} - \frac{1}{\gamma} K'_{T-1}].$$

We conclude that for any $\beta_{T-1}$, $u_{T-1}$, $k_{T-1}$, $\beta_T$ and $\alpha_T$, the principal can choose $\alpha_{T-1}$ to satisfy the agent’s participation constraint with equality in period $T-1$. Proceeding inductively, it follows that the fixed payments $\alpha_t$ can be chosen so as to satisfy the participation constraint in each period with equality.

**Proof of Corollary to Proposition 1:** The incentive compatibility constraint for any investment, $b_t$, is $u_t \cdot \gamma \cdot m'_t(b_t) = (1 - k_t)$. We can therefore restate the principal’s objective as:

$$B(u_t, k_t) - \rho \cdot \beta_t^2 \cdot [u_t^2 \cdot \gamma^2 \cdot \sigma^2_\mu + k_t^2 \cdot \sigma^2_\eta],$$

where $B(u_t, k_t)$ denotes the present value of cash flows corresponding to the level of $b_t$ induced by $u_t$ and $k_t$. Thus the marginal cost of $u_t$ is increasing in $\sigma^2_\mu$ and the marginal cost of $k_t$ is increasing in $\sigma^2_\eta$. A revealed preference argument then shows that $k_t^*$ is decreasing in $\sigma^2_\eta$, while $u_t^*$ is decreasing in $\sigma^2_\mu$. \(\square\)

**Proof of Proposition 2:** In order to induce $a_t^*$, the bonus parameter has to be chosen such that $\beta_t^* = e'_t(a_t^*)$ for each $t$. For a given choice of $a_t^*$, $a_{t+1}^*$, and $b_t^*$, the principal chooses the coefficients $u_t$ and $k_t$ to minimize the variance:

$$k_t^2 \cdot \sigma^2_\eta + u_t^2 \cdot \gamma^2 \cdot h \cdot \sigma^2_\mu \quad (A2)$$

subject to the incentive compatibility constraint in (27). We therefore substitute (27) into the principal’s objective function in (A2). Expression (30) for the optimal $k_t^*$ then follows from the principal’s first-order condition. Expression (29) for the optimal $u_t^*$ follows from the incentive compatibility constraint in (27). Like in Proposition 1, the fixed payments $\{\alpha_t\}$ can be so chosen as to meet the manager’s interim participation constraints, i.e, the manager would not want to quit at the beginning of period $t+1$ provided he has invested $b_t^*$. Furthermore, the fixed payments could be ”backloaded” so that the manager never would have an incentive to underinvest at date $t$, and subsequently quit his job. \(\square\)
Proof of Proposition 2': The principal chooses the optimal effort $a_t$ and the optimal investment $b_t$ to maximize:

$$\gamma \cdot m_t(b_t) - b_t + a_t - e_t(a_t) - \rho \cdot e'_t(a_t) \cdot [\sigma^2 + (k^*_t)^2 \cdot \sigma^2 + (u^*_t)^2 \cdot \gamma^2 \cdot h \cdot \sigma^2 + (1 - h) \cdot \sigma^2]$$

where $u^*_t$ and $k^*_t$ are as given in Proposition 3. We first provide sufficient conditions to ensure that the principal always finds it optimal to induce the maximum effort $a_t = \bar{a}$ and the maximum investment $b_t = \bar{b}$. For notational convenience, we define $e'_t \equiv e'_t(\bar{a})$, $e''_t \equiv e''_t(\bar{a})$, and $m'_t \equiv m'_t(\bar{b})$. In order to induce $a_t = \bar{a}$, the principal will set $\beta^*_t = e'_t$ in each period. It will be optimal to induce $a_t = \bar{a}$ and $b_t = \bar{b}$ in each period provided $m''_t(\cdot)$ is decreasing, $e'''_t(\cdot)$ is increasing, and the following conditions hold:

$$\gamma \cdot m'_t - 1 - 2 \cdot \rho \cdot (e'_t)^2 \cdot \left[ \frac{\sigma^2 \cdot \sigma^2 + 3 \cdot (m'_t)^2 \cdot \sigma^2}{m_t \cdot \sigma^2} \right] > 0 \quad (A3)$$

$$1 - e'_t - 2 \cdot \rho \cdot e'_t \cdot e''_t \cdot (\sigma^2 + \sigma^2 + \sigma^2) > 0 \quad (A4)$$

To see this, we differentiate the above objective with respect to $a_t$. If $e''_t(\cdot)$ is increasing and condition (A4) holds, the above objective function is strictly increasing in $a_t$. Similarly it can be verified that the principal’s objective function is increasing in $b_t$ provided that condition (A3) holds and $m''_t(\cdot) \leq 0$. This shows that conditions (A3) and (A4) are sufficient to ensure that the optimal action and investment choices are $a_t = \bar{a}$ and $b_t = \bar{b}$, respectively.

In order to induce $a_t = \bar{a}$, the bonus parameters have to satisfy

$$\beta^*_t = e'_t. \quad (A5)$$

The manager will choose $b_t = \bar{b}$ provided that:

$$u_t \geq z_t \cdot \frac{1 - k_t}{\gamma \cdot h \cdot m'_t} \quad (A6)$$

where

$$z_t \equiv 1 - \gamma \cdot (1 - h) \cdot m'_t \cdot \frac{e'_{t+1}}{e'_t}.$$  

The principal chooses the coefficients $u_t$ and $k_t$ to minimize (A4) subject to the incentive compatibility constraints in (A5) and (A6). Note that if $u_t < 0$ satisfies the incentive compatibility constraint in (A6), then $u_t = 0$ will also satisfy it. Consequently, $u_t < 0$
cannot be optimal. Furthermore, (A6) will optimally hold as an equality whenever \( u_t > 0 \).

As a result,

\[
  u_t^* = \max \left\{ 0, z_t \cdot \frac{1 - k_t^*}{\gamma \cdot h \cdot m_t^*} \right\}.
\]

(A7)

Substituting (A7) into (A4) and optimizing with respect to \( k_t^* \) yields:

\[
  k_t^* = \max \{0, z_t\} \cdot \frac{\sigma^2}{\sigma^2 + h \cdot (m_t^*)^2 \cdot \sigma^2}.\]

(A8)

Substitution of (A8) into (A7) gives:

\[
  u_t^* = \max \{0, z_t\} \cdot \frac{m_t^* \cdot \sigma^2}{\gamma \cdot [\sigma^2 + h \cdot (m_t^*)^2 \cdot \sigma^2]}.
\]

(A9)

Proof of Corollary to Proposition 2:

When \( u_t^* > 0 \), we differentiate (A9) with respect to \( \sigma_\delta^2 \) and, after simplifying, obtain:

\[
  \text{Sign} \left[ \frac{\partial u_t^*}{\partial \sigma_\delta^2} \right] = \text{Sign} \left[ \theta - \frac{e_{t+1}^*}{e_t^*} \right] \]

where

\[
  \theta \equiv \frac{1}{\gamma \cdot m_t} \cdot \frac{(m_t^*)^2 \cdot \sigma^2}{\sigma^2 + h \cdot (m_t^*)^2 \cdot \sigma^2} \in (0, 1).
\]

This proves part (i). Combining expressions (A8) and (A9) yields:

\[
  k_t^* = \frac{\gamma \cdot \sigma^2}{m_t^* \cdot \sigma^2} \cdot u_t^*,
\]

which proves that the ratio \( \frac{k_t^*}{u_t^*} \) is independent of \( \sigma_\delta^2 \).

Proof of Proposition 3. We first demonstrate the sufficiency part, i.e., the performance, \( \pi_t^* \), is indeed optimal for a memoryless compensation scheme. As argued in the text:

\[
  MVA_t = a_t + \epsilon_t - b_t + \gamma \cdot f_t + K_t - (1 + r) \cdot K_t - 1
\]
and
\[ RIF^c_t = a_t + \epsilon_t - b_t + k^{**}_t \cdot y_t + K_{t-1} \]

Thus, if \( k^{**}_t = \frac{k_t^*}{(1-u_t^*)} \), we obtain:
\[
\pi_t^* = (1 - u_t^*) \cdot RI^c_t + u_t^* \cdot MVA_t
\]
\[
= a_t + \epsilon_t - b_t + k^*_t \cdot y_t + u_t^* \cdot \gamma \cdot f_t + \Delta.
\]

where \( \Delta \) denotes a constant depending on \( u_t^*, K_t \) and \( K_{t-1} \). Proposition 1 now shows that for suitably chosen constants \( \alpha_t \) and \( \beta_t \),
\[
\{ s_t = \alpha_t + \beta_t \cdot \pi_t^* \}_{t=1}^T
\]
is an optimal incentive scheme which satisfies the interim participation constraints.

To demonstrate necessity, we note that any performance measure \( \pi_t \), which can be written as a linear combination of \( \{ c_\tau, P_\tau, A_\tau, d_\tau, C_\tau \} \) for \( \tau = t \) and \( \tau = t - 1 \) can be rewritten as:
\[
\pi_t = x^c_t \cdot (a_t + \epsilon_t - b_t) + x^f_t \cdot \gamma \cdot f_t + x^y_t \cdot k_t \cdot y_t +
\]
\[
x^c_{t-1} \cdot (a_{t-1} + \epsilon_{t-1} - b_{t-1}) + x^f_{t-1} \cdot \gamma \cdot f_{t-1} + x^y_{t-1} \cdot k_{t-1} \cdot y_{t-1} +
\]
\[
... + x^c_1 \cdot (a_1 + \epsilon_1 - b_1) + x^f_1 \cdot \gamma \cdot f_1 + x^y_1 \cdot k_1 \cdot y_1.
\]

The above expression reflects that each of the variables \( c_t, P_t, A_t, d_t \) can be expressed as linear combinations of the “carrier” variables \( (a_\tau + \epsilon_\tau - b_\tau), f_\tau \) and \( k_\tau \cdot y_\tau \) for \( 1 \leq \tau \leq t \), with the coefficients \( x^c_\tau, x^f_\tau \) and \( x^y_\tau \) representing the aggregate coefficients on these carrier variables.

In order to satisfy the interim participation constraints with a memoryless compensation scheme, all coefficients \( x^c_\tau, x^f_\tau \) and \( x^y_\tau \) must be equal to zero for \( \tau \leq t - 1 \). To induce the optimal \( b_t^* \) and minimize the manager’s risk exposure, an optimal performance measure must be equivalent to:
\[
(a_t + \epsilon_t - b_t) + u_t^* \cdot \gamma \cdot f_t + k^*_t \cdot y_t.
\]

Therefore \( \frac{x^f_t}{x^c_t} = u_t^* \) and \( \frac{x^y_t \cdot k_t}{x^c_t} = k^*_t \), proving our claim. \( \square \)
References


