Manipulation in Money Markets*

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Interest rate derivatives are among the most actively traded financial instruments in the main currency areas. With values of positions reacting immediately to the underlying index of daily interbank rates, manipulation has become an increasing challenge for the operational implementation of monetary policy. To address this issue, we study a microstructure model in which a commercial bank may have strategic recourse to central bank standing facilities. We characterise an equilibrium in which market rates will be manipulated with strictly positive probability. Our findings have an immediate bearing on recent developments in the Sterling and the Euro money markets.

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1. Introduction

Central banks around the globe increasingly focus on steering some short-term money market interest rate in their implementation of the monetary policy stance. For example, this is the case of the Federal Reserve in the U.S., the European Central Bank (ECB) in the Euro area, and the Bank of England in the UK. More broadly, central banks seem to increasingly attach greater value to stable day-to-day and even intra-day money market conditions. With this aim, so-called corridor systems have been adopted in several currency areas, for example, in Australia, Canada, the Euro area, and New Zealand. More recently, the Bank of England has also adopted such a system (see Bank of England [4]).

This paper wishes to contribute to the ongoing discussion on the appropriate design of corridor systems by showing that manipulation is a potential issue in such money markets. Specifically, a commercial bank might have been speculating on, say, a rise in policy rates, so that it would look as an attractive perspective if market rates would increase somewhat. To create temporarily higher rates, this bank may then take up loans from the in-

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1 In a corridor system (see, e.g., Woodford [37]), the central bank stands ready to provide overnight liquidity in unlimited amounts, generally against collateral, at a rate somewhat above market rates, and stands ready to absorb liquidity overnight in unlimited amounts at a rate somewhat below market rates. By setting a corridor around the central bank target or policy rate, the range of variation of overnight interest rates will be bounded, on a day-to-day basis, by the rates on the standing lending and deposit facilities, allowing short-term market interest rates to be steered with limited volatility around the desired level. The Federal Reserve has a semi-corridor system following the introduction of its primary credit facility, one percentage point above the Fed Funds Target, with zero being the standard lower bound.

2 Furine [17] shows with a search model that the actual recourse of a lending facility may be less than suggested by the statistics of individual refinancing costs when the market attaches a stigma to its use, but also that the availability of a lending facility might reduce incentives for active participation in the interbank market. Pérez Quirós and Rodríguez Mendizábal [30] conclude that the introduction of a deposit facility may lead to a stabilisation of market rates. This is because the deposit facility reduces the costs of running into a “lock-in” situation, in which reserve requirements are satisfied before the last day of the reserve maintenance period.

3 Manipulation in financial markets has attracted significant academic attention during the last two decades or so. Besides the contributions cited below, see for instance Allen and Gale [1], Bagnoli and Lipman [2], Benabou and Laroque [5], Gerard and Nanda [18], and Vila [36].
terbank market and deposit the funds subsequently with the central bank. Under certain conditions, this will cause a rise in the market rate, adding value to the manipulator’s net position. We will discuss under which conditions this and similar strategies are profitable, and which incentive effects are created by this possibility. We will also discuss some of the means at the disposal of the central bank to eliminate this kind of behaviour.

In the Euro area, variations of this type of manipulative strategy may have occurred on at least two occasions since the start of Stage Three of the Economic and Monetary Union in January 1999.

**Manipulation episode at the end of the maintenance period**

**24 May – 23 June 2000** In this maintenance period, the ECB raised key policy rates from 3.75% to 4.25%. Ahead of the policy decision, market participants were betting on whether the rate increase was going to happen, and also about the likely scale (25 basis points vs. 50 basis points). Indeed, second-quarter turnover in interest swaps more than doubled in 2000. Already on the first day of the maintenance period the money market index EONIA was at 4.06%, reflecting market expectations (cf. Figure 1). On Monday, June 19, 2000, there was a EUR 4.999 bn total recourse to the deposit facility. This recourse occurred before the last main refinancing operation, and consequently did not affect market rates. However, at the close of trading on the next day, the allotment day of the last main refinancing operation in the maintenance period, there was another EUR 11.207 bn total recourse to the deposit facility. This recourse changed the liquidity conditions ahead of the crucial part of the maintenance period; thus the EONIA increased immediately. Made distinctly before the end of the reserve maintenance period, the recourse was indeed quite unusual. It is not unlikely that an individual commercial bank had strong incentives to attempt manipulation. The Eurosystem launched a fine-tuning operation and provided EUR 7.000 bn at overnight maturity on Wednesday, June 21, achieving a temporary relief to market conditions. However, on the penultimate day there was another large...

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For explanations of technical terms in the context of the implementation framework of the Eurosystem, we refer the reader to Section 2.
composite recourse to the deposit facility of a total size comparable to the fine-tuning operation. The Eurosystem did not perform an additional fine-tuning on the last day of the maintenance period. Overall the maintenance period ended tight with a net recourse to the marginal lending facility and EONIA 26 basis points above the new policy rate.

**Manipulation episode at the end of the maintenance period 24 April – 23 May 2003** At the start of the maintenance period the EONIA index was at 2.55%, and the minimum bid rate at the main refinancing operation was 2.50%. During this maintenance period, expectations were formed about a policy rate cut by the ECB in the subsequent period. The ECB indeed cut key policy rates by 50 basis points on 6 June 2003. There has been an unusually strong activity in the swap market during May 2003. On Tuesday, 20 May, on the allotment day of the last main refinancing operation in the maintenance period, there were several (non-strategic) recourses to the deposit facility adding to a total of EUR 1.462 bn. Even though this is not a large total recourse it seems that it had an impact on liquidity conditions because the EONIA was 23 bp above the minimum bid rate on the next day. The movement of the market may have triggered the response that followed. On the next day, there was an active request by an individual market participant for lending from the Eurosystem of EUR 9 bn (cf. Figure 2). This recourse apparently changed or even reversed liquidity conditions ahead of the crucial part of the maintenance period. Thus the EONIA decreased immediately. On the following Thursday, there was another recourse to the lending facility of EUR 1.8 bn by the same market participant. Again, the timing of the recourses was unusual. Apparently, there had been an attempt to imitate the manipulation strategy employed in 2000, now in the context of expectations of decreasing policy rates. The ECB launched a liquidity absorbing fine-tuning operation on Friday, the last day of the maintenance period, drawing EUR 3.850 bn from the market. Overall the maintenance period ended slightly loose with a net recourse to the deposit facility and EONIA 29 bp below the policy rate.

For a central bank, manipulation is undesirable mainly for two reasons.
First, from an operational perspective, manipulation has the potential to add volatility to the overnight rate, and to complicate the liquidity management of both commercial banks and the central bank. Second, manipulation may affect the market’s confidence in a smooth implementation of monetary policy, which may have an impact on longer-term refinancing conditions and therefore on the effectiveness of monetary policy.

To address these issues, we consider a model in which a strategic trader with private information may trade in a swap market first and may then manipulate the market rate. The potential manipulator faces a trade-off between the costs of taking control of the market rates and the additional value for her derivatives position. It turns out that the trade-off will sometimes, but not always, induce the trader to leverage her derivatives position, and to subsequently manipulate the money market. With several informed traders, a public good problem reduces individual incentives for manipulation, yet does not eliminate the problem. We then discuss policy measures such as fine-tuning and the narrowing of the corridor set by the standing facilities. The discussion covers both elements of the new operational design by Bank of England and recent experiences of the Eurosystem.

Technically, our analysis follows the microstructure literature on informed trade that is associated with the seminal work of A. Kyle [22]. In this literature, an individual trader with private information may cause a price effect because the market extracts the information contained in the aggregate order flow. In an important contribution to this literature, Kumar and Seppi [21] (henceforth K&S) have studied manipulation in futures markets with cash settlement. The present paper adapts the assumptions underlying the K&S model to reflect the institutional backdrop of a corridor system. The main adaptation concerns the way in which prices are moved in the market for the underlying (see also Section 6). In K&S, there is asymmetric information in the spot market. Because market participants may mistake uninformed trading for informed trading, the order flow created by the uninformed manipulator moves the price of the underlying asset. In contrast, there is no informed spot trading in our model. Indeed, while we do not deny the existence of private information in the money market, the evidence discussed
above suggests that private information was not necessarily the central element in its mechanics. In our model, it is assumed that the manipulator gains temporary control of the market rate by using the standing facilities.\textsuperscript{5}

The difference in the way in which prices are moved in the market for the underlying asset causes a qualitative change in the predictions of the analysis. In our model, a recourse to one of the standing facilities incurs an immediate loss in net interest earnings equivalent to the difference between the corridor rate and the current market rate. To cause the market rate to change marginally, this spread of typically more than 100 basis points has to be paid on the absolute amount of the recourse, which may be unprofitable. Indeed, the strategy does not pay off unless the involved positions exceed a certain size. This is why in our set-up, the probability and the extent of equilibrium manipulation depends on the design of the corridor system, which enables us to discuss alternatives for policy makers within our formal framework.\textsuperscript{6}

The rest of the paper is structured as follows. Section 2 provides some background on the Euro money market. This section can be skipped by those that are already acquainted with the institutional details. Section 3 sets up the basic model. In Section 4, we study the decision problem of the informed trader. Section 5 analyses the strategic game between manipulator and market makers. In Section 6, we consider welfare consequences, policy options, and the extension to several manipulators. Section 7 concludes. All proofs can be found in the Appendix.\textsuperscript{7}

\textsuperscript{5}In the past, market rates have also reacted to insufficient demand in central bank operations. It is not clear, however, that these so-called underbidding episodes have been deliberate attempts of manipulation (cf. Ewerhart [14] and Nyborg, Bindseil, and Strebulaev [26]).

\textsuperscript{6}Another difference is that in K&S, the manipulator’s order in the futures market does not convey any information. In our model, the private information of the manipulator causes the swap rate to exhibit a reaction to the order flow, and implies an endogenously finite market order.

\textsuperscript{7}Note that strategic recourses, while similar in nature, differ from short squeezes (cf. Nyborg and Strebulaev [27, 28]). In contrast to a short squeeze, a strategic recourse does not presuppose a temporary monopoly situation. Strategic recourses are also not directly related to bidding behaviour in central bank operations.
2. Institutional background on the Euro money market

When a customer of a commercial bank A instructs this bank to transfer money into another party’s account at another commercial bank B, then by purely mechanical consideration, bank A’s holdings of central bank money will diminish, and bank B’s holdings will increase. This and similar types of transactions may, when accumulating over the business day, re-allocate significant amounts of liquidity between individual credit institutions located in the Euro area, which is a motive for them to trade secured and unsecured short-term credit in the Euro money market.

A central bank that has chosen to implement monetary policy by steering short-term interest rates may do so by seeking control of aggregate liquidity conditions in the money market and by using additional instruments to stabilise interest rates further. This is the approach favoured by many modern central banks (cf. Bindseil [7] or Borio [8]). In the case of the Eurosystem, the control of liquidity conditions is gained by the combination of open market operations, standing facilities, and reserve requirements.

The ECB provides the necessary refinancing of the banking system through its open market operations, where the bulk of interbank liquidity is offered in the weekly regular operations, the so-called main refinancing operations (MROs). The maturity of these operations used to be two weeks until March 2004, and has been one week since then. Other operations include the monthly longer-term refinancing operations (LTROs) with a maturity of three months, and so-called fine-tuning operations (FTOs). The latter can be used in a very flexible way, yet only vis-à-vis a subpopulation of the counterparties of the Eurosystem.

The interest rate corridor in the Euro area is constituted by the Eurosystem’s standing facilities, the marginal lending facility and the deposit facility. There is no administrative procedure. That is, a recourse to either the marginal lending facility or the deposit facility can be requested by any eligible counterparty to the Eurosystem, where intraday debit positions on the counterparty’s settlement account with the national central bank are automatically considered as a request for recourse to the lending facility. The
use of the facilities occurs after the close of the market (at 6:30 p.m.). By 9:15 a.m. on the subsequent trading day, the market is informed through Reuters page ECB40 about the aggregate recourses to each of the two standing facilities. Significant recourses are typically observed only on the last one or two days of the maintenance period, when demand and supply in the money market become increasingly inelastic.

*Reserve requirements* for credit institutions are expressed in terms of an average balance to be held over a so-called reserve maintenance period (usually about a month) on the counterparty’s settlement account with the respective national central bank. Non-compliance with minimum reserve obligations implies sanctions. In contrast to the U.S., required reserves are remunerated in the Euro area at a rate close to funding costs.

The combination of the above instruments makes market conditions in the Euro money market usually a very stable signal of the current monetary stance. Nevertheless, both the average level and the volatility of market rates may vary over time. Especially after the last main refinancing operation in the maintenance period, market rates may differ visibly from the mid of the corridor. Deviations of the EONIA from the mid of the interest rate corridor occur in response to liquidity flows, so-called autonomous factors, which are beyond the direct control of the central bank’s liquidity management, and which affect the aggregate liquidity position of the banking system. These factors include treasury accounts with some national banks, banknotes that are paid out or collected at counters of commercial banks, and changes to consolidated net foreign assets held by the Eurosystem. Movements of the market rates occur also at certain calendar dates such as the end of the quarter and the end of the year, when commercial banks manage their balance sheets more carefully, and in connection with events that are perceived by the market to have a potential effect on financial stability. Further deviations of the market index from the mid of the corridor have been observable occasionally.

The *derivates market* allows to either hedge the risks of a change in short-term interest rates, or to speculate on them. Among the most actively traded instruments in this market is the overnight interest rate swap (OIS)
of various maturities, ranging from one week to two years. For instance, an institutional investor might speculate on the timing of an expected increase in policy rates using a swap contract with a maturity of one month. In contrast, a commercial bank that wishes to freeze refinancing conditions in the interbank market until the next main refinancing operation may prefer a swap with a maturity of only a week. In terms of payments streams, the OIS is an instrument which exchanges a fixed interest rate against an index of daily interbank rates (almost always EONIA). The OIS differs from the plain vanilla interest rate swap (cf. Bicksler and Chen [6]) which is used for longer maturities and with reference to the Euribor. Also, for plain vanilla interest rate swaps, the floating rate is determined at one settlement date and paid at the next. In contrast, the floating rate leg of an OIS is determined and paid at maturity. Overnight interest rate swaps have been known in the U.S. for quite some time as call money swaps.

For many market participants, it is much easier to realise a short-term interest rate position with swaps than with transactions in the deposit market (see, e.g., Pelham [29], or Elliott [10]). The swap is the more liquid instrument, and involves less credit risk. As a consequence, the swap curve has emerged as one of the main benchmark yield curves for the Euro area. In June 2005, Euribor FBE (the European Banking Federation) and Euribor ACI (the Financial Markets Association) launched the EONIA Swap Index, which is published by Reuters over Telerate for maturities ranging from 1 week to 12 months. There has been a continued strong expansion of the EONIA swap market over the last few years.

The OIS market is a highly competitive, high volume OTC market, with dominant players featuring in the main European financial centers. The market organisation is highly concentrated, with a handful of dealers accounting for about half of the trading activity. Among the most active dealers are commercial banks that are headquartered in the Euro area. Dealers contract both with other dealers and with customers. The range of institutions participating in the OIS market as customers is very broad, covering both the financial sector (credit institutions, insurance companies, pension funds, hedge funds, money market funds, etc.) and the non-financial sector (gov-
ernments). Leveraged funds are especially active in this market. For further
details on the Euro money market and the overnight swap market, the reader
is referred to descriptive studies by Remolona and Wooldridge [32], Santillán,
Bayle, and Thygesen [33], Hartmann, Manna, and Manzaranes [19], and to
the ECB’s annual Euro money market study [12].

3. Formal set-up

Our market environment is an adapted version of Kumar and Seppi [21], as
discussed in the Introduction. We envisage a developed money market with
reserve requirements and averaging provision, embedded into a symmetric
corridor system. Our analysis will focus on the last two days of the reserve
maintenance period, when the regular (weekly) refinancing operations by the
central bank have already established “neutral” conditions, and the market
is essentially left on its own. Following the conventional terminology used in
fixed income and money markets, prices are replaced by interest rates. The
set-up is then as follows.

Three assets are traded in the money market: first, a riskless bond (“net
interest”), which serves as a numeraire; second, a standardised overnight de-
posit contract (“liquidity”) with endogenous interest rate $r$; and third, an
overnight interest rate swap (“OIS”) on the deposit contract, traded at the
swap rate $r^*$. Sign conventions for fixed-for-floating swaps tend to be ambigui-
ous in general and depend on whether the hedging or the speculation motive
is stressed. Throughout the present paper we will follow the accounting con-
vention that the receive-floating party is long the swap, so that the position
in her portfolio obtains a positive sign. This convention has the consequence
that increasing market rates are desirable for the holder of a long position in
the OIS, and conversely for a short position.

The sequence of events is summarised in Figure 3. A swap market on
date 1, organised in the early afternoon of the trading day, is followed by
a spot market for the underlying deposit contract on date 2, organised at a
similar time of the day. A liquidity shock hits the market shortly before the
end of date 2. Following the shock, but still before the close of the market

Figure 3
at the end of date 2, there is last-minute trading in the deposit contract. All net interest payments are settled at date 3. At any date before date 3, market participants may have recourse to the standing facilities provided by the central bank.

Altogether five types of traders participate in these markets: An informed trader, two independent groups of nondiscretionary traders in the swap and deposit markets, swap dealers, and money market specialists. Risk-neutrality is assumed throughout for both the informed trader and the market makers. The informed trader obtains a random initial endowment $X_0$ in the swap before date 1. This initial position may be the result of OTC trading with non-bank customers, and is assumed to be private information.\(^8\) The game between the traders has then the following structure.

At date 1, the informed trader submits a market order $X_1$ to the swap dealers, where $X_1 > 0$ when paying the fixed rate. The group of nondiscretionary swap traders, composed of non-financial firms, submits an additional order volume of $Y$, which is distributed independently from $X_0$ with mean $E_Y[Y] = 0$. Aggregate order volume is then given by $Z = X_1 + Y$. The dealers observe the aggregate order flow $Z$ in the swap market. In addition, dealers will be understood to be informed, about whether the informed trader submits a non-zero market order.\(^9\) Formally, let $b = 1$ if $X_1 \neq 0$, and $b = 0$ otherwise, and assume that swap dealers observe $b$. The risk-neutral swap dealers are then willing to clear the swap market at the competitive rate $r^*(Z,b)$. Since only the case $b = 1$ is interesting, we will henceforth drop the second argument and write simply $r^*(Z)$ for $r^*(Z,1)$.

The liquidity effect. With the close of the market at the end of date 1, the informed trader may have recourse $S$ to the standing central bank facilities,

\(^8\)In practice, the initial position $X_0$ may not necessarily be stochastic. A commercial bank could actively manage short-term interest rate positions vis-à-vis non-bank customers in a specific way, e.g., by asking customers to swap variable interest income into fixed interest income. That would make the initial position endogenous. However, a commercial bank may be able to do this once, but not several times, because it would openly steel profits from its customers.

\(^9\)Non-anonymity significantly simplifies the exposition without affecting our main results. For a model without this assumption, we refer the interested reader to our working paper [16].
where $S > 0$ corresponds to a recourse to the lending facility, and $S < 0$ to a recourse to the deposit facility. At date 2, the recourse $S$ is observable by all market participants. The informed trader and the group of discretionary money market traders, composed of commercial banks, may submit market orders at date 2. However, as the order flow is not informative, the market specialists set the conditions for deposit contracts at date 2 to some rate $r(S)$ that depends only on $S$. Shortly before the end of date 2, an autonomous factor shock changes the aggregate liquidity situation of the banking system, where $V > 0$ stands for an absorption of liquidity. The dispersion of the shock among market participants does not affect the price and is therefore not explicitly modelled.

The liquidity shock $V$ is distributed independently from $X_0$ and $Y$, according to the cumulative distribution function $\Phi_V(S) = \text{pr}\{V \leq S\}$. We assume that the support of the distribution of $V$ is a (non-degenerate) interval $I_V$, that $\Phi_V(.)$ is continuous everywhere as well as differentiable in the interior of $I_V$ with density $\Phi'_V(.) > 0$, and that $\Phi'_V(S)$ is nondecreasing for $S < 0$, and nonincreasing for $S > 0$. These assumptions are satisfied for many common distributions of interest including the normal and uniform distributions. It is finally assumed that the central bank implements monetary policy in a neutral way, so that the median of the distribution of $V$ is zero.

Let $r^L$ denote the interest rate paid by commercial banks on credit received from the central bank through the lending facility, and by $r^D < r^L$ the rate paid by the central bank on deposits made by commercial banks. All market participants try to cover their positions at the end of date 2, so that the price in the last-minute trading is either $r^L$ (when $S - V < 0$) or $r^D$ (when $S - V > 0$). Under these conditions, the liquidity effect from strategic recourses is determined exclusively by the change in the relative probabilities of a tight or a loose end of the maintenance period. Thus, the market for the overnight contract appears in the reduced form which has become standard in the literature since Poole [31].

**Poole’s Lemma.** The market rate at date 2 for the deposit contract after a
net recourse of $S$ is given by a nonincreasing function

$$r(S) = \Phi_V(S)r^D + (1 - \Phi_V(S))r^L.$$  \hfill (1)

In particular, for $S = 0$, the market rate $r(0)$ corresponds to the midpoint $r^0 = (r^D + r^L)/2$ of the corridor.

In the context of money markets, the price change caused by an inflow or outflow of liquidity into or out of the banking system, as captured by Poole’s Lemma, is known as the liquidity effect. For the Euro area, the existence of the liquidity effect after the last main refinancing operation in a reserve maintenance period is undisputed. That is to say, after the last regular operation, market rates are generally expected to move in response to the release of public information about flows of liquidity, e.g., when a recourse to a standing facility of the central bank occurs.\(^{10}\) As we will discuss now, the liquidity effect gives a possibility to abuse those standing facilities.

4. Sporadic manipulation

A market participant who intends to take temporary control of the market rate will be aware of the costs and benefits of such a strategy. There are costs because the use of standing facilities is bound to interest rate levels which almost always differ significantly from market conditions. There are benefits because short-term interest rate positions may gain in value. In this section, we analyse under which conditions a strategic recourse is profitable.

Formally, net interest income $\pi$ for the informed trader is the sum of three components, as suggested by equation (2) below. First, there is the net return on the initial position $X_0$ in the swap. The initial position is valued with the interest rate $r(S)$ realised at date 2, while funding costs for this position are already sunk at date 1, and can be normalised to $r^0$. Next, there is the net return on the market order $X_1$. Here as well, the position is valued using the interest rate $r(S)$ realised at date 2. Funding costs are

\(^{10}\)The empirical evidence for the U.S. is less conclusive. See in particular Hamilton [20], Thornton [34], and Carpenter and Demiralp [9].
given by the swap rate at date 1, which will be denoted by $r^*$. The third income component is the net interest paid for the strategic recourse $S$ to the standing facilities. This component is generally negative. E.g., a recourse to the credit facility costs $r^L$, but yields only $r(S) \leq r^L$. Summing up, the informed trader obtains a net interest income

$$\pi(X_0, X_1, S) = X_0(r(S) - r^0) + X_1(r(S) - r^*) + S(r(S) - r^{L/D}(S)), \quad (2)$$

where we write

$$r^{L/D}(S) = \begin{cases} r^L & S > 0 \\ r^0 & S = 0 \\ r^D & S < 0 \end{cases}$$

for the interest rate that the informed trader either pays for having recourse to the marginal lending facility or receives for depositing money with the central bank. The terms $r^0$ and $r^*$ correspond to the fixed leg of the swap positions $X_0$ and $X_1$, respectively. The variable interest rate $r(S)$ is received or paid on all three positions.

**Controlling the market rate.** The starting point for the analysis is to note that a strategic recourse to one of the standing facilities is not always optimal. Denote by $X = X_0 + X_1$ the total swap position. For concreteness, assume a long position, i.e., $X > 0$. The reader will note that it is never optimal in this situation to have recourse to the marginal lending facility. Indeed, to increase the value of the position, the market rate must go up and liquidity must become scarcer, so the informed trader will have recourse to the deposit facility ($S < 0$). We differentiate the informed trader’s objective function (2) with respect to $S$. Then the necessary first-order condition governing the informed trader’s decision about $S$ at date 1 becomes

$$-r'(S)(X_0 + X_1 + S) = r(S) - r^D. \quad (3)$$

As captured by the left-hand side of equation (3), the marginal benefit of manipulating the interest rate upwards is the increase in the market value of the aggregate net position $X + S$. The marginal cost, on the other hand, is the interest rate differential between a deposit in the market and a deposit with the central bank. The resulting trade-off may be one-sided, however.
Specifically, it turns out that if $X$ is small enough in absolute terms, then the benefit will always be smaller than the cost, so that manipulation does not pay off. A similar consideration can be made for the case of a negative $X$. Thus, as depicted in Figure 4, the optimal recourse to the standing facilities, drawn as a function of the informed trader’s position in short-term interest rate instruments, is zero for small absolute values of $X$.

**Proposition 1.** The optimal strategic use of standing facilities $S^*(X)$ at the end of date 1 involves no recourse for $|X| \leq \Delta/\rho$, where $\Delta = r^L - r^0$ is the half-width of the corridor, and $\rho = |r'(0)|$ is the liquidity effect. Moreover, when $|X| > \Delta/\rho$, then $S^*(X) \neq 0$, and $X$ and $S^*(X)$ are of opposite sign.

Thus, only a market participant with a sufficiently large exposure has an incentive to attempt manipulation. Such a trader may have built up a sufficiently large long position in the swap market ($X > \Delta/\rho$) and will have recourse to the deposit facility ($S < 0$) to cause prices to rise. In this case, it cannot be optimal to use the lending facility, i.e., to manipulate the market rate downwards, because the alternative choice $S = 0$ avoids the non-trivial costs of the lending facility, and does not lower the value of the swap position. In the other scenario, a trader with a sufficiently large short position ($X < -\Delta/\rho$) will have recourse to the marginal lending facility ($S > 0$), and profit from the softening of the market.

**Some back-of-the-envelope calculations.** Which size of position makes manipulation profitable? We have estimated elsewhere (see Ewerhart, Cassola, Ejerskov, and Valla [15]) that for the Euro area under the system in use before March 2004, $\rho \approx 0.09\%$/bn EUR. The figure captures the response to a publicly observable, one-day liquidity shock of 1 bn EUR, which occurs immediately after the last main refinancing operation, and which is not corrected for by later fine-tuning. With this estimate, we can perform the following crude calibration. The corridor half-width being $\Delta = 1\%$, Proposition 1 predicts that manipulation is the consequence of profit maximisation for positions with a notional of at least

$$\frac{\Delta}{\rho} \approx \frac{1\%}{0.09 \%$/bn EUR} = \text{EUR 11.1 bn}.$$
The reader will note that this figure does not take account of expectations about potential central bank interventions after an attempted manipulation (these are difficult to quantify), and may therefore understate the actual threshold.

How much money could have been made with manipulation? The costs of an individual strategic recourse can be calculated relatively precisely because it corresponds essentially to the 100 basis points penalty for just one day. For instance, the recourse at the close of the market on 22 May 2003 (cf. Table 1) led to an approximate cost of

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S(r(S) - r^L) \approx \text{EUR } 1.8 \text{ bn} \cdot \frac{(2.21\% - 3.50\%)}{360} = -\text{EUR } 64'500.00.
\]

The gains are more difficult to estimate. In general, they depend on the unobservable position of the informed trader, and on the number of days for which the effect on market rates holds on. Pelham [29] reports of individual deals in May 2003 reaching the dimension of EUR 25 bn. An individual contract of that dimension would increase in value by one to several hundred thousand Euros per diem in response to the movements of the EONIA in the manipulation episodes. For instance, only on 23 May 2003, the receiver of the fixed rate of that contract gained

\[
X_0(r(S) - r^0) \approx -\text{EUR } 25 \text{ bn} \cdot \frac{(2.21\% - 2.50\%)}{360} = +\text{EUR } 201'388.88,
\]

compared to a situation where the EONIA equals the policy rate. However, overall, net profits for the individual manipulator should have been much higher than this lower bound suggests, because the exposures of the main players in the manipulation episodes have very likely been much larger than the single contract. These considerations suggest a relatively strong incentive for manipulation.

5. Trading in the swap market

Once a market participant considers manipulation as a profitable strategy, she will seek to improve the effectiveness of this strategy by leveraging her
initial position in short-term interest-rate instruments. In the model, this possibility is reflected by the informed trader’s endogenous choice of the swap market order $X_1$. As mentioned in Section 2, an additional position taking could also be accomplished by satisfying reserve requirements unevenly over time. Which of these and possible other instruments is used by the individual treasurer is ultimately an empirical question. To keep the model tractable, we will focus in the sequel on the case of leverage using the swap market.

While position taking is essentially costless in the liquid OIS market, it cannot be accomplished at zero cost. In an equilibrium with rational expectations, swap dealers will anticipate the possibility of manipulation and will extract the information contained in the order flow. E.g., a large positive market order, unless compensated by nondiscretionary trading, indicates to the dealers that the informed trader has already a relatively large initial long position $X_0$, making a recourse to the deposit facility more likely. Competition between swap dealers will force those dealers to set the swap rate close to market expectations about the deposit rate at date 2. That is, conditional on the observable order volume $Z = X_1 + Y$, the swap rate will be set to

$$r^*(Z) = E_{X_0,Y}[r(S^*(X_0 + X_1)|Z].$$

(4)

Here, the swap rate $r^*(Z)$ will typically be increasing in $Z$. Thus, as a consequence of the dealers’ rational anticipation, creating significant leverage may be costly for the informed trader.

But when large trades can affect market expectations, it will typically be easier for the informed trader to leverage the existing position than to hedge it. Indeed, as our next result shows, neither hedging nor a change of the market side can be optimal in a symmetric market environment.

**Proposition 2.** Assume that $X_0$, $Y$, and $V$ are symmetrically distributed with mean zero, and that $E_Y[r^*(X_1 + Y)]$ is increasing in $X_1$. Then the informed trader’s optimal market order satisfies $X^*_1(X_0) \geq 0$ for $X_0 > 0$ and $X^*_1(X_0) \leq 0$ for $X_0 < 0$.

While hedging is never optimal in a symmetric environment, we will see below that under general conditions, and in extension of Proposition 1, the informed
trader will abstain from leveraging her initial position with strictly positive probability. That is to say, if the initial position $X_0$ is relatively small in absolute value, then the informed trader does not prefer being active in the swap market, and does not prefer either manipulating the deposit market. The intuitive reason for this result is that the informed trader needs to build up a sizable position to make a strategic recourse ex post optimal. But then, there would be already too much information contained in the size of the deal $X_1$, which would make the whole plan unprofitable.

These considerations are reflected in our description of the equilibrium, which will be provided below. By an equilibrium, we mean functions for the market order $X_1^*(.)$ and for the strategic recourse $S^*(.)$, and a pricing rule $r^*(.)$ such that (i) for any $X_0$ in the support of the initial distribution, the informed trader maximises expected net income from interest $E_Y[\pi]$ by choice of $X_1 = X_1^*(X_0)$ and $S = S^*(X_0 + X_1)$, and (2) for any $Z = X_1 + Y$ in the support of the equilibrium distribution, the swap dealers set the swap rate $r^* = r^*(Z)$ competitively as captured by (4).\textsuperscript{11}

The “sporadic” nature of manipulation, while valid much more generally, precludes the possibility of a tractable equilibrium with normally distributed random parameters. The point to note is that, as manipulation occurs only for sufficiently large initial positions, the conditional distribution of market orders is determined by the tails of the distribution of the initial position. Tail distributions, however, of normal distributions are not normal. Thus, despite the theoretical desirability of the normal distribution that has been pointed out by Nöldeke and Tröger [24, 25] and Bagnoli, Viswanathan, and Holden [3], it is preferable in our situation to consider a set-up with uniform distributions, just because the tail distribution of a uniform distribution is again uniform. We will assume therefore in the sequel that the random variables $X_0$, $Y$, and $V$ are uniformly distributed on intervals $[-\delta_X, \delta_X]$, $[-\delta_Y, \delta_Y]$, and $I_V = [-\delta_V, \delta_V]$, respectively, where $\delta_X, \delta_Y, \delta_V > 0$.

In the uniform set-up, boundary conditions must be considered explicitly.\textsuperscript{11}

\textsuperscript{11}In principle, the market makers should form expectations also off the equilibrium distribution of net aggregate orders. However, in our model, these expectations, if rational, would amplify the price effect in the swap market, making a deviation by the informed trader even less attractive.
Two restrictions on the parameter values have to be imposed. First, we
assume that the liquidity shock is sufficiently dispersed, as captured by
\[ \delta_V > \frac{\delta_X + \delta_Y}{3}. \] (5)
This restriction will ensure that the manipulated market rate does not reach
the boundary of the corridor, which is a useful simplification. Further, to
focus on the interesting case of manipulation, we will assume that the initial
position of the informed trader is sufficiently large with positive probability,
i.e,
\[ \delta_X > \delta_V. \] (6)
When these two conditions are satisfied, we find an explicit equilibrium with
the following characteristics:

**Proposition 3.** Under conditions (6) and (5), there is an equilibrium in the
manipulation game. For \( |X_0| < \delta_V \), there is no manipulation, i.e., \( X_1^*(X_0) = S^*(X_0) = 0 \). For \( |X_0| \geq \delta_V \), however, the informed trader will leverage her
position and subsequently manipulate the deposit rate.

In practice, the decision to manipulate by a commercial bank will depend not
only on the initial endowment but also on (i) the overall trading and collateral
capacities of the bank, (ii) the internal allocation of the bank’s risk budget
between markets, and (iii) its general readiness to take strategic measures in
the search of profit opportunities, including the involved daringness vis-à-vis
the monetary authority and potentially other regulatory institutions. For
these reasons, we would expect that even in a large currency area, only few
commercial banks may be prepared for manipulative actions such as those
described in this paper. Depending on the central bank’s stance on this issue,
it may also be difficult for an individual institution to repeat an unwanted
manipulative strategy. Still, a central bank will have to formulate a credible
response to such strategies.
6. Welfare consequences, policy measures, and further discussion

6.1 Social cost of manipulation

As mentioned in the introduction, manipulation is not welcomed by a central bank because it might negatively affect the reputation of the monetary authority and may also add volatility to money market conditions. We will use three different measures for the welfare loss. First, the probability of manipulation

\[ \text{pr}\{S^* \neq 0\} > 0. \]

Second, we consider the expected extent of manipulation, conditional on manipulation

\[ E[|S^*| |S^* > 0] > 0. \]

Finally, the volatility of the market rate at date 2, measured by the unconditional standard deviation

\[ \sigma_M = (E[(r(S) - r^0)^2])^{1/2} > 0. \]

The first two measures are related to central bank reputation, while the volatility measure is related to implementation issues.

6.2 Preventing manipulation

What mechanisms could prevent the need of money markets to accommodate volatility caused by strategic recourses?

The width of the corridor. In the early discussion of the problem (cf. Vergara [35]), it had been suggested that a wider interest rate corridor should effectively defuse the risk of manipulation. Indeed, it is not implausible to conjecture that a larger average spread between the EONIA rate and the respective facility rate should increase the cost of affecting the market rate sufficiently to make manipulation unattractive. However, as our next result shows, this intuition is incorrect. Formally, the corridor can be widened by raising the marginal lending rate from \( r^L = r^0 + \Delta \) to \( r^0 + \Delta' \), where \( \Delta' > \Delta \).
is the half-width of the widened corridor, and by simultaneously lowering the deposit rate from \( r^D = r^0 - \Delta \) to \( r^0 - \Delta' \). These changes are then implemented consistently over the whole two-day maintenance period.

**Proposition 4.** **Widening or narrowing the interest rate corridor has no effect either on the probability or on the extent of manipulation.**

To see why the proposition holds in the case considered in Proposition 3, recall that by Poole’s Lemma, the size of the liquidity effect is proportional to the width of the corridor, i.e.,

\[
\rho = |r'(0)| = |\Phi_V(0)|(r^L - r^D) = 2|\Phi_V(0)|\Delta.
\]

Replacing \( \rho \) in the equilibrium conditions shows that \( \Delta \) cancels out in all expressions, so that both the probability and the extent of manipulation remain unaffected by the size of the width.

Proposition 4 is much more general and does not depend on distributional assumptions. Intuitively, the interest rate corridor has two roles as an instrument in the implementation of monetary policy. On the one hand, the standing facilities impose an effective boundary to money market conditions. On the other, however, the width of the corridor is a linear scaling factor for the size of the liquidity effect. Once this double role of the corridor is taken into account, the above result should be ultimately straightforward: While a wider corridor makes strategic recourses more costly for the commercial bank, the gains are scaled up as well.

The volatility caused by manipulation could be lowered by having a tighter corridor. However, in practice, a corridor that is too tight would create incentives for exclusive trading with the central bank, and would consequently dry out the interbank market. This would constitute a problem because considerations of credit risk imply a certain dispersion of the interest rates that are applied to bilateral transactions in the money market. The optimal size of the corridor should reflect the central bank’s trade-off between smoothing implementation and setting prudential incentives.

**Fine-tuning.** The model can be extended in a straightforward way to incorporate the possibility of central bank intervention. Let \( \alpha_0 \in [0; 1] \) denote
the probability that the central bank intends fine-tuning at the end of date 2.\footnote{In a more descriptive set-up, the probability of fine-tuning would be correlated with the size of the liquidity imbalance on the last day of the reserve maintenance period. In this case, the manipulator may choose a lower $S^*$ to avoid the fine-tuning. While the corresponding equilibrium would be untractable, we conjecture that the qualitative features of predictions would be essentially the same.} The parameter values $\alpha_0 = 0$ and $\alpha_0 = 1$ correspond to no intervention and regular fine-tuning, respectively. In the case of the Eurosystem, we would assume that $\alpha_0$ traditionally has been close to zero. Indeed, before 11 May 2004, the ECB had generally been quite reluctant to use additional operations to correct for end-of-period imbalances, apparently because there had been no good reason for an intervention, and also because some volatility seems to be desirable to provide incentives for bidding in the main refinancing operations (cf. ECB [11]). However, following the initial experiences with the new operational framework, the ECB gradually increased its willingness to intervene on the last day, with fine-tuning after February 2005 occurring almost regularly at the end of the maintenance period. Thus, nowadays, with quasi-regular fine-tuning at the end of each maintenance period, $\alpha_0$ should be much closer to one.

Fine-tuning operations may not always lead to the desired result. This indeed happened in the Euro area when market participants found the condition in a liquidity-draining fine-tuning operation not sufficiently attractive to participate (cf. ECB [13]). Formally, we assume a conditional probability $\pi > 0$ that a given fine-tuning operation does not lead to the desired outcome. The probability of successful fine-tuning is then given by $\alpha = \alpha_0(1 - \pi)$. We assume that an unsuccessful operation fails completely, while conditional on a successful fine-tuning operation, the market rate at the end of date 2 is $r^0$. The market rate after manipulation will then be $r(S)$ with probability $1 - \alpha$, and $r^0$ with probability $\alpha$. In expected terms, a recourse of $S$ in the context of a central bank reaction captured by $\alpha$ implies a market rate at date 2 of

$$r(S, \alpha) = \alpha r^0 + (1 - \alpha)r(S).$$

(7)

In particular, with fine-tuning, the deviation of the market rate from $r^0$ at date 2 is bounded by $(1 - \alpha)\Delta$. Thus, in a sense, fine-tuning attenuates
the liquidity effect more effectively than a more dispersed liquidity shock. Adapting Proposition 3, we arrive at the following result.

**Proposition 5.** Assume that the probability $\pi$ of an operational failure is smaller than one. Then a higher probability of fine-tuning $\alpha_0$ lowers both the probability and the extent of manipulation, as well as the volatility of money market conditions.

Thus, deviations of the money market rate caused by strategic recourses can be effectively reduced by an appropriate and immediate reaction of the central bank, and should therefore be expected to be a transient phenomenon in practice. An immediate reaction is needed, because if the recourse is not compensated immediately in the morning of the subsequent day, the market rate could have moved, and a gain for the manipulator would result.

**The BoE design.** Our analysis may throw some light on the innovative design of the standing facilities in the new operational framework of the Bank of England (see Macgorain [23]). The final design involves having a corridor half-width of 1 percent, as in the case of the Eurosystem, but having the corridor narrowed down to 0.25 percent on the final day of the reserve maintenance period. This design element effectively drives a wedge between the cost of the strategic recourse on the right-hand side of equation (3), which remain high, and the benefit from the interest rate movement on the left-hand side of (3), which is significantly reduced. Formally, for $t = 1, 2$, denote by $r_t^L$ and $r_t^D$ the facility rates at date $t$, and by $\Delta_t = (r_t^D - r_t^L)/2$ the corridor half-width at date $t$. To avoid unnecessary complications, we assume $\Delta_2 \leq \Delta_1$.

**Proposition 6.** Narrowing of the corridor only on date 2 by some factor $\beta = \Delta_1/\Delta_2 > 1$ is equivalent to successful fine-tuning with probability $\alpha = 1 - 1/\beta$.

The narrowing of the corridor on the last day of the reserve maintenance period may therefore become a complement or substitute for fine-tuning, for instance, when the probability $\pi$ of an operational failure is not negligible.
The Bank of England has combined narrowing of the corridor with a regular fine-tuning policy and flexible reserve requirements. Preliminary evidence from the Stirling money market suggests that this combination of policy measures is indeed quite powerful.\footnote{Information policy does not appear to us as a useful instrument to combat manipulation. While in principle, the central bank could withhold information about recourses, the manipulator has an incentive to actively disseminate this information. Moreover, the resulting ambiguity might lead to even more gaming.}

6.3 Several manipulators

Intuitively, the possibility of profitable manipulation should provoke imitation or climbing on the bandwagon by other major players in the interbank market. To study this possibility in formal terms, we generalise our model to the case of $N \geq 2$ informed traders $i = 1, \ldots, N$. Consider an informed trader $i$ with an initial position $X_0^i$ and a submitted market order $X_1^i$. There is an interaction with the other informed traders in particular because the value of $i$’s position depends not only on her own recourse $S^i$, but also on the aggregate net recourse

$$S^{-i} = \sum_{j \neq i} S_j$$

of the other informed traders. Formally, this interdependence is reflected in the net income from interest for trader $i$, which is given by

$$\pi^i(X_0^i, X_1^i, S^i, S^{-i}) = X_0^i (r(S^i + S^{-i}) - r^0) + X_1^i (r(S^i + S^{-i}) - r^*) + S_i (r(S^i + S^{-i}) - r^{L/D}(S^i)),$$

in straightforward generalisation of (2). To keep the model tractable, we focus on the second stage of the manipulation game. Formally, we will disallow swap trading at date 1, and assume that initial swap positions $X_0^i$ are perfectly correlated.

**Proposition 7.** There exists an equilibrium in the second stage of the manipulation game with $N \geq 2$ informed traders. Similar to the case $N = 1$, there
is no recourse provided that $|X^i| \leq \delta_V$. If, however, $\delta_V < |X^i| < (2+1/N)\delta_V$, then informed trader $i$’s equilibrium recourse $S_{i,*}$ is given by

$$S_{i,*} = -\text{sign}(X^i)\frac{|X^i| - \delta_V}{N + 1}.$$ 

Thus, with $N \geq 2$ informed traders, there is a public good problem between the informed traders because all informed traders will benefit from an individual trader’s strategic recourse. However, it must be conjectured that competition among potential manipulators alone will not preclude the possibility of manipulation.

7. Concluding remarks

In this paper, we have pointed out that in money markets that are embedded in a corridor system, composed of central bank lending and deposit facilities, there is the potential for manipulative action that abuses these facilities. Anecdotal evidence for the Euro area suggests that this strategy may be perceived by the market as more than just a theoretical possibility. We have used a microstructure model to show that manipulation can be profitable for a commercial bank with suitable ex-ante characteristics. Manipulation remains a feature of the equilibrium even if dealers in the derivatives market form rational expectations about potential manipulation. A widening or narrowing of the corridor over the whole maintenance period is not helpful. Instead, regular fine-tuning fights manipulation effectively, or alternatively narrowing the corridor on the last day of the maintenance period, because the costs of manipulation remain high, while the benefits decrease. The discussion supports the common perception that the monetary authority has powerful instruments to combat manipulation, but also that further vigilance in these operational matters appears recommendable.

Appendix: Proofs

Proof of Poole’s Lemma. The distribution of $V$ having no mass points, the probability that the maintenance period ends with ample liquidity amounts

25
to
\[
\Pr\{S - V > 0\} = \Pr\{V < S\} = \Pr\{V \leq S\} = \Phi_V(S).
\]
Similarly, the probability that \(S - V < 0\) is given by \(1 - \Phi_V(S)\). This proves (1). The monotonicity of \(r(S)\) follows from
\[
r(S) = r^L - \Phi_V(S)(r^L - r^D).
\]
As the median of the distribution of \(V\) is zero, we have \(\Phi_V(0) = 1/2\), which proves the Lemma. \(\Box\)

**Proof of Proposition 1.** Assume first that \(X \geq 0\). Then any \(S > 0\) is strictly inferior to no recourse, i.e., to \(S = 0\). Thus, \(S^\ast(X) \leq 0\). Using (3) and Poole’s Lemma, we find the necessary first-order condition
\[
X = -S + \frac{1 - \Phi_V(S)}{\Phi'_V(S)},
\]
where \(S < 0\) and such that \(\Phi'_V(S) > 0\). It is easy to check that the right-hand side of equation (8) is strictly decreasing in \(S < 0\), and approaches \(\Delta/\rho\) for \(S \to 0\). Hence, equation (8) has a unique solution \(S^\ast(X) < 0\) for any \(X > \Delta/\rho\). Clearly, this is the global optimum when \(I_V\) is unbounded from below. Assume now a finite lower boundary \(V < 0\) of \(I_V\). Then clearly, any \(S < V\) is inferior to \(S = V\), so that also in this case, the global optimum is determined by (8). For \(0 \leq X \leq \Delta/\rho\), an interior solution is not feasible. Therefore, \(S^\ast(X) = 0\) when \(I_V\) is unbounded from below. When \(I_V\) is bounded from below, then \(V \leq -\Delta/\rho\) because \(\Phi_V(S)\) is convex for \(S < 0\). But then,
\[
\pi(X_0, X_1, V) - \pi(X_0, X_1, 0) = X(r^L - r^0) + V(r^L - r^D) < 0.
\]
Thus, also when \(I_V\) is bounded from below, \(S^\ast(X) = 0\) for \(0 \leq X \leq \Delta/\rho\).
The case \(X < 0\) can be treated in an analogous way. Hence the assertion. \(\Box\)

**Proof of Proposition 2.** Without loss of generality, assume \(X_0 > 0\) (the other case follows by symmetry). Consider first a change in the market side, i.e., a market order \(X_1 < 0\) such that \(X_0 + X_1 < 0\). We claim that submitting this market order is suboptimal, even if followed by \(S = S^\ast(X_0 + X_1)\). As an
alternative plan of action, consider $\hat{X}_1 = -2X_0 - X_1$, followed by $\hat{S} = -S$. Indeed, in this case $X_0 + \hat{X}_1 = -(X_0 + X_1)$, so that in a symmetric market environment,

$$\pi(X_0, \hat{X}_1, \hat{S}) - \pi(X_0, X_1, S) = X_1(E_Y[r^*(X_1 + Z)] - r^0) - \hat{X}_1(E_Y[r^*(\hat{X}_1 + Z)] - r^0)$$

Clearly, $|X_1| > |\hat{X}_1|$ and consequently also

$$|E_Y[r^*(X_1 + Z)] - r^0| > |E_Y[r^*(\hat{X}_1 + Z)] - r^0|.$$

Thus, (9) is positive, proving our claim. Consider now the case of hedging, i.e., $-X_0 \leq X_1 < 0$. Then $X_0 + X_1 \geq 0$ and therefore $S^*(X_0 + X_1) \leq 0$ by Proposition 1. We claim that a deviation to $\hat{X}_1 = 0$ without changing $S = S^*(X_0 + X_1)$ is already a better trading strategy. To see why, note that $r(S) \geq r^0$ and that $r^0 > E_Y[r^*(X_1 + Z)]$. But then,

$$\pi(X_0, \hat{X}_1, \hat{S}) - \pi(X_0, X_1, S) = -X_1(r(S) - E_Y[r^*(X_1 + Z)]) > 0.$$

Thus, also hedging cannot be optimal.

**Proof of Proposition 3.** This result follows immediately from Lemma A.1 below for $\alpha = 0$ (i.e., no fine-tuning).

**Proof of Proposition 4.** Start from an equilibrium in the manipulation game. Assume first that the half-width of the corridor is scaled up from $\Delta > 0$ to some $\Delta' > 0$, where $\Delta' < r^0$. Let $\gamma = \Delta'/\Delta > 1$. Then, by Poole’s Lemma, the liquidity effect $r(S) - r^0$ is scaled up by the factor $\gamma$. Consider now, as an equilibrium candidate in the model with corridor $\Delta'$, a competitive swap spread $r^* - r^0$ that is scaled up by the factor $\gamma$. It is then straightforward to check that the objective function (2) of the manipulator is multiplied by $\gamma$. The optimal strategy of the informed trader concerning the choice of $X_1$ and $S$ as a function of $X_0$ remains unchanged. Thus, neither the distribution of aggregate market orders $Z$ arriving at the dealer’s desk, nor
the dealer’s posterior belief on $S$ given his observation of $Z$ is affected. From equation (4), we get that the scaled down pricing function in the swap market is indeed competitive. A similar argument can be made for a narrowing of the corridor. Hence the assertion.¶

Proof of Proposition 5. This result follows immediately from Lemma A.1 below.¶

Lemma A.1. For $\alpha < 1$, let $\delta_{\bar{V}} = \delta_{V}/(1 - \alpha)$. Assume

$$\delta_X > \delta_{\bar{V}} \text{ and } \delta_X + \delta_{\bar{V}} < 2\delta_{\bar{V}} + \delta_{V}.$$  \hfill (10)

Then the following is an equilibrium in the manipulation game with fine-tuning. For $|X_0| < \delta_{\bar{V}}$, there is no manipulation, i.e.,

$$X^*_1(X_0, \alpha) = S^*(X_0 + X^*_1(X_0, \alpha), \alpha) = 0.$$  \hfill (11)

For $|X_0| \geq \delta_{\bar{V}}$, however, the informed trader will submit a market order

$$X^*_1(X_0, \alpha) = \theta \text{sign}(X_0)(|X_0| - \delta_{\bar{V}}),$$  \hfill (12)

and will have recourse to the standing facilities

$$S^*(X_0 + X^*_1(X_0, \alpha), \alpha) = -\frac{1 + \theta}{2}\text{sign}(X_0)(|X_0| - \delta_{\bar{V}}) \hfill (12)$$

at the end of date 1. Here, $\theta = \delta_{\bar{V}}/(\delta_X - \delta_{\bar{V}}) > 0$ is a measure for the informational advantage of the informed trader. The swap dealers set the competitive rate to

$$r^*(Z, \alpha) = r^0 + \frac{\rho}{4}\frac{1 + \theta}{\theta}Z.$$  \hfill (13)

The probability of manipulation, the extent of manipulation, and the volatility of the market rate at date 2 are respectively given by

$$\text{pr}\{S^* \neq 0\} = \frac{\delta_X - \delta_{\bar{V}}}{\delta_X},$$  \hfill (14)

$$E[|S^*| | S^* \neq 0] = \frac{\delta_X - \delta_{\bar{V}} + \delta_{\bar{V}}}{4}, \text{ and} \hfill (15)$$

$$\sigma_M = \frac{\delta_X - \delta_{\bar{V}} + \delta_{\bar{V}}}{2\delta_{\bar{V}}}\sqrt{\frac{\delta_X - \delta_{\bar{V}}}{3\delta_X}}(r^L - r^0).$$  \hfill (16)
Proof. We have to show that conditions (i) and (ii) in the definition of the equilibrium are satisfied. First, we consider the decision problem of the informed trader. Let $X_0 \in [-\delta_X; \delta_X]$. Assume that the swap dealers apply the linear pricing rule

$$
\begin{equation}
    r^*(X_1 + Y) = r^0 + \lambda(X_1 + Y),
\end{equation}
$$

where $\lambda > 0$ is a constant. We will show at a later stage of the proof that

$$
\begin{equation}
    \lambda > \frac{\rho}{2} \frac{\delta_Y}{\delta_Y + \delta_V - \delta_X}.
\end{equation}
$$

But then, by Lemma A.2 below, the informed trader does not participate in the swap market for $|X_0| < \hat{\delta}_V$, and submits the bid

$$
\begin{equation}
    X_1^*(X_0, \alpha) = \frac{\rho}{4\lambda - \rho} (X_0 - \hat{\delta}_V \text{sign}(X_0))
\end{equation}
$$

for $|X_0| \geq \hat{\delta}_V$. From (19), the distribution of $X_1$, conditional on $b = 1$, is uniform on the interval $[-\delta_1; \delta_1]$, where

$$
\begin{equation}
    \delta_1 = \frac{\rho}{4\lambda - \rho} (\delta_X - \hat{\delta}_V)
\end{equation}
$$

is the maximum market order of the informed trader. By Theorem 3.1 in Bagnoli, Viswanathan, and Holden [3], a linear equilibrium requires $\delta_Y = \delta_1$. This proves (11). Solving (20) for $\lambda$, and subsequently using (17) proves (13). Further, inequality (18) is equivalent to

$$
\begin{equation}
    \frac{4\lambda}{\rho} = 1 + \frac{\delta_X - \hat{\delta}_V}{\delta_Y} > 1 + \frac{\delta_X - \hat{\delta}_V}{2\delta_Y + \hat{\delta}_V - \delta_X}
\end{equation}
$$

Subtracting one on both sides and invoking (10) shows that (18) is indeed satisfied. Finally, Lemma A.3 below and (11) deliver (12). Checking the expressions (14), (15), and (16) is a straightforward exercise. This completes the proof of Lemma A.1. □

Lemma A.2. Assume (10) and (18). Then the informed trader does not participate in the swap market for $|X_0| < \hat{\delta}_V$, and submits the bid (19) whenever $|X_0| \geq \hat{\delta}_V$.\[29\]
Proof. When the central bank fine-tunes successfully with probability $\alpha$, then the net interest for the informed trader amounts to

$$\pi(X_0, X_1, S, \alpha) = X_0(r(S, \alpha) - r^0) + X_1(r(S, \alpha) - r^*) + S(r(S, \alpha) - r^{L/D}(S)).$$

Assuming (17), the expected profit for the informed trader is given by

$$E_Y[\pi] = -\lambda X_1^2 + (X_0 + X_1 + S)(r(S, \alpha) - r^0) + S(r^0 - r^{L/D}(S)).$$

Using Lemma A.3, the informed trader’s objective function is given by

$$h(X_1) = E_Y[\pi(X_0, X_1, S^*(X_0 + X_1), \alpha)] =
\begin{cases}
-\lambda X_1^2 & \text{if } |X_1| < \hat{\delta}_V \\
-\lambda X_1^2 + \frac{\rho}{4}(|X_1| - \hat{\delta}_V)^2 & \text{if } \hat{\delta}_V \leq |X_1| < \hat{\delta}_V + 2\delta_V \\
-\lambda X_1^2 + \rho\delta_V(|X_1| - \delta_V - \hat{\delta}_V) & \text{if } |X_1| \geq \hat{\delta}_V + 2\delta_V.
\end{cases}$$

From (10) and (18), we obtain $4\lambda > \rho$. Under this condition, the objective function $h(X_1)$ is continuously differentiable and strictly concave on $\mathbb{R}$. The necessary and sufficient condition for the optimum is therefore $h'(X_1) = 0$. Note that the third case $|X_0 + X_1^*| \geq \hat{\delta}_V + 2\delta_V$ is not possible. This is because in this case the first-order condition would imply $|X_1^*| = \rho\delta_V/(2\lambda)$, but then, using (18), we obtain $|X| \leq |X_0| + |X_1^*| < \hat{\delta}_V + 2\delta_V$, a contradiction. Assume now $|X_0| \geq \hat{\delta}_V$. By straightforward extension of Proposition 2, we have $\text{sign}(X_1^*) = \text{sign}(X_0)$ for $X_0 \neq 0$. But then clearly $|X| < \hat{\delta}_V$ is impossible, which yields (19). Consider now $|X_0| < \hat{\delta}_V$. Formula (19) would imply a reversed sign for $X_1$, so this is clearly not feasible. Hence $X_1^* = 0$ in this case. This proves the assertion.\]

Lemma A.3. In the uniform model with fine-tuning, let $\hat{\delta}_V = \delta_V/(1-\alpha)$, as before. Then $S^*(X, \alpha) = 0$ for $|X| < \hat{\delta}_V$, while $S^*(X, \alpha) = -\text{sign}(X)(|X| - \hat{\delta}_V)/2$ for $\hat{\delta}_V \leq |X| < \hat{\delta}_V + 2\delta_V$, and $S^*(X, \alpha) = -\text{sign}(X)\delta_V$ for $|X| \geq \hat{\delta}_V + 2\delta_V$. 

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Proof. Consider first the case $X > 0$. Without fine-tuning, Poole’s Lemma implies $r(S) = r^0 - S\Delta/\delta_V$ for $|S| \leq \delta_V$. Using (7) yields $r(S, \alpha) = r^0 - S\Delta/\delta_V$ for $|S| \leq \delta_V$. Clearly, $S^*(X, \alpha) \leq 0$. The necessary first-order condition for an interior solution reads $X = \hat{\delta}_V - 2S$. Thus, an interior solution of the informed trader’s problem at the end of date 1 exists and is given by $S^*(X, \alpha) = -\hat{\delta}_V - 2\delta_V$. Otherwise, there is a boundary solution. From

\[ \pi(X_0, X_1, -\delta_V, \alpha) - \pi(X_0, X_1, 0, \alpha) = (1 - \alpha)(X - 2\hat{\delta}_V)\Delta \]

it is obvious that $S^*(X, \alpha) = 0$ for $|X| \leq \hat{\delta}_V$, and $S^*(X, \alpha) = -\delta_V$ for $X \geq \hat{\delta}_V + 2\delta_V$. An analogous consideration can be made for the case $X < 0$. Hence, the assertion. □

Proof of Proposition 6. Write $r(S, \alpha, \Delta)$ for the market rate in a corridor system with half-width $\Delta$ at date 2 around $r^0$, after a recourse of $S$, and with a probability $\alpha$ of successful fine-tuning. Using (7) and Poole’s Lemma yields

\[ r(S, \alpha, \Delta) = \alpha r^0 + (1 - \alpha)\Phi_V(S)r^D_2 + (1 - \Phi_V(S))r^L_2 \]

\[ = \Phi_V(S)(\alpha r^0 + (1 - \alpha)r^D_2) + (1 - \Phi_V(S))(\alpha r^0 + (1 - \alpha)r^L_2) \]

\[ = r(S, 0, (1 - \alpha)\Delta). \]

Thus, fine-tuning with probability $\alpha = 1 - 1/\beta$ is equivalent to narrowing the interest rate corridor on date 2 from $\Delta_1$ to $\Delta_2 = (1 - \alpha)\Delta_1 = \Delta_1/\beta$. □

Proof of Proposition 7. Under the assumptions made, trader $i$’s objective function reads

\[ \pi'(X_i^i, S^i, S^{-i}) = X_i^i(r(S^i + S^{-i}) - r^0) + S^i(r(S^i + S^{-i}) - r^{L/D}(S^i)), \]

where $X_i = X_0^i$. Consider first the case $X_i^i \geq 0$. Then, for any $S^{-i}$, choosing $S^i > 0$ is always (weakly) inferior for trader $i$ than $S^i = 0$ because

\[ \pi'(X_i^i, S^i, S^{-i}) - \pi'(X_i^i, 0, S^{-i}) \leq 0. \]

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Thus, $S^i \leq 0$. Moreover, in the uniform case, choosing $S^i$ such that $S^i + S^{-i} < -\delta_V$ is inferior to choosing $S^i$ equal to $-\delta_V - S^{-i}$ because

$$\pi^i(X^i, S^i, S^{-i}) - \pi^i(X^i, -\delta_V - S^{-i}, S^{-i}) = (S^i + S^{-i} + \delta_V)(r^U - r^D) < 0.$$  

Thus, if $S^{-i} < -\delta_V$ then an optimal recourse is given by $S^i,(X^i, S^{-i}) = 0$. Moreover, if $S^{-i} \geq -\delta_V$ then $S^i,(X^i, S^{-i}) \in J^- = [-\delta_V - S^{-i}; 0]$. For values $S^i \in J^-$, trader i’s objective function is differentiable and strictly concave with respect to $S^i$. The interior solution $S^i,(X^i) = (\delta_V - X^i - S^{-i})/2$ stays within $J^-$ provided that $\delta_V - S^{-i} \leq X^i \leq 3\delta_V + S^{-i}$. In a symmetric equilibrium, $S^{-i} = (N - 1)S^i$. Thus $S^i,(X^i) = (\delta_V - X^i)/(N + 1)$ for $\delta_V \leq X^i \leq 2 + \delta_V/N$, and we have established an equilibrium. The case of $X^i \leq 0$ can be treated in an analogous way. ¶
References


Figure 1. Daily recourses to standing facilities in the Eurosystem, EONIA and key policy rates (24 May - 23 June 2000).
Figure 2. Daily recourses to standing facilities in the Eurosystem, EONIA and key policy rates (24 April - 23 May 2003).

Note: The ECB reduced key policy rates by 50 bp on 6 June 2003

The ECB conducts a liquidity absorbing fine-tuning operation EUR 3.9 bn on the last day of the maintenance period

Recourse to ML on the settlement day of the last MRO in the reserve maintenance period
Figure 3. Time structure of the model.

- Initial position $X_0$
- Date 1: Swap trading $X_1, Y, r^*(Z)$
- Standing facilities $S$
- Date 2: Deposit trading $r(S)$
- Liquidity shock $V$
- Date 3: Settlement of interest $\pi$
Recourse to credit facility lowers overnight rate to the benefit of the fixed leg receiver.

Recourse to deposit facility raises overnight rate to the benefit of the floating leg receiver.

Figure 4. Net usage of standing facilities as a function of the informed trader’s swap position.
Table 1. EONIA and use of standing facilities on the last week of the reserve maintenance periods 24 May through 23 June, 2000, and 24 April through 23 May, 2003.

<table>
<thead>
<tr>
<th>Day</th>
<th>Notes</th>
<th>EONIA (%)</th>
<th>Marginal lending (EUR bn)</th>
<th>Deposit facility (EUR bn)</th>
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<tr>
<td>June 2000</td>
<td><strong>Monday 19</strong></td>
<td>4.21</td>
<td>0.3</td>
<td>5.0</td>
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<td></td>
<td><strong>Tuesday 20</strong></td>
<td>4.22</td>
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<td></td>
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<td></td>
<td>(Fixed rate tender 4.25%)</td>
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<td></td>
<td><strong>Wednesday 21</strong></td>
<td>4.38</td>
<td>0.2</td>
<td>1.5</td>
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<td>Settlement day MRO;</td>
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<td>maturity; EUR 7 bn</td>
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<td><strong>Thursday 22</strong></td>
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<td>0.1</td>
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<td><strong>Friday 23</strong></td>
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<td>3.1</td>
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<td>May 2003</td>
<td><strong>Monday 19</strong></td>
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<td><strong>Wednesday 21</strong></td>
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<td><strong>Thursday 22</strong></td>
<td>2.57</td>
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<td><strong>Friday 23</strong></td>
<td>2.21</td>
<td>0.1</td>
<td>0.7</td>
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<td>Last day of RMP; ECB drains EUR 3.9 bn via FRT at 2.5%</td>
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