On Market Liquidity and Liquid Balances

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Motivation:

How does the economy’s liquidity affect the stock market’s liquidity?

- We use the term “liquidity” to denote (at least) two distinct concepts.

  A. A *market* – for anything – is said to be liquid if it is easy to trade; buyers and sellers can readily be found – at similar prices – to accommodate any need to trade.

  B. An *agent* – a firm, an individual, a country – is said to be liquid when a large fraction of its assets are held in cash, or other securities readily exchangeable for goods and services.

- Is it a linguistic accident that we use the same word to describe two such different things?

- Why care?

  - To understand/control the risks posed by market illiquidity, in particular, *financial fragility*. 
Motivation – continued

- In times of stress, central banks often signal their willingness to “provide liquidity” – meaning cash – to the economy in the hope of averting financial melt-downs.

- What they are trying to affect, however, is the other type of liquidity.
  - They are trying to ensure that markets can accommodate needed reallocations.
    - Goal is not to artificially prop up prices in the face of bad news.
  - If markets are illiquid, the result may be disequilibrium prices, leading to distressed liquidations (“fire sales”), leading to inefficiencies, and perhaps a spiral of further sales.
  - If markets could always absorb trade demand, there would be no danger.

- Moreover, there is academic (as well as anecdotal) evidence that this “liquidity substitution” works.
  - Chordia, Sarkar, Subrahmanyam (2004); Fujimoto (2004).

- Understanding the mechanism may shed light on both the real function of central banks and the real causes of liquidity and liquidity risk.
Outline

1. Motivation.

2. The conventional view. Related literature.

3. This paper’s contribution.

4. The model: An economy with time-varying liquid balances.

5. Results: How/why does level of liquid balances affect stock market liquidity?

6. Conclusions.
Related Literature

- Conventional understanding of the role of financial liquidity in affecting market conditions:
  
  (A) Market liquidity is determined by (segmented) intermediaries whose ability to make orderly markets is limited due to **financial constraints**.
  (B) Authorities can (exogenously) loosen those constraints.


- Closely related literature: “Limits to Arbitrage”.

- See also Holmstrom and Tirole (1998, 2001); Attari and Mello (2001); Vayanos (2004); Morris and Shin (2004); Caballero and Krishnamurthy (2005); Brunnermeier and Pedersen (2005); …

- This line of reasoning requires some or all of the following:
  - Segmented markets.
  - Asymmetric information.
  - Incomplete contracts.
  - Credit channel for monetary policy.
This Paper:

1. Delineates a different, frictionless, equilibrium channel for the same effect.

   When there is more cash in the economy, the propensity to consume out of available goods is lower; in that state, a marginal perturbation in an agent’s asset holdings will have less impact on his IMRS (discount rates) hence less impact on his asset valuations; this means he would be more willing to accommodate such trades (“supply liquidity”) than he would when cash is low.

2. Illustrates magnitudes and covariances with plausible calibrations.

3. Highlights distinct implications.

   The fact of a linkage between the two types of liquidity need not imply that the channel is through segmented, constrained intermediaries.
A Model with Time-Varying Liquid Balances

- What do we mean *in real terms* when we say the economy is “liquid”?
  - What does it mean to say e.g. “the central bank supplied liquidity”?

- This paper takes the view that the defining feature of cash *in this context* is that it can be readily exchanged for goods and services *regardless of what is going on in financial markets*.

- In other words, “cash” is a real asset that can be technologically transformed into consumption.

- For other assets, claims to them may be sold for goods (market conditions permitting). But the physical capital stock itself may not be readily converted to consumption.
  - Extreme case: irreversibility.

- The assumption here, then, is that “cash” is essentially the same as a stored consumption goods. No fiat money in the model.

- Note that the net sum of all agents’ cash balances is equal to the reserves of the banking system, i.e. the monetary base. Hence the monetary interpretation.
The Model – continued

- So consider a discrete-time, representative-agent, single-good economy with CRRA preferences, having two assets:

  1. Cash balances (or stored consumption goods);
  2. A dividend stream.

- First asset will be taken to be riskless – but this is not its defining feature! Key assumption is that it is freely consumable.
  - No explicit borrowing constraint.
  - In cases considered, agents will always keep positive balances.

- Second asset is non-transformable; a lognormal dividend flow.

- Could add a separate labor endowment – to generate “liquidity shocks”. But this is not necessary for the development.

- Note that this is a standard “buffer-stock savings” model (Deaton, 1991; Carroll, 1997) although applied at an aggregate level.
### The Model: Notation

#### Quantity

<table>
<thead>
<tr>
<th>Time-t dividend $D_t$</th>
<th>Dividend growth $\tilde{R}_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividend growth $\tilde{R}_{t+1}$</td>
<td>Earnings growth $R$</td>
</tr>
<tr>
<td>Cash growth $G_t$</td>
<td>Beginning of period balances $B_t = G_t - D_t$</td>
</tr>
<tr>
<td>Total available goods $G_t$</td>
<td>Consumption $C_t$</td>
</tr>
<tr>
<td>Law of motion $G_{t+1} = R (G_t - C_t) + \tilde{R}_{t+1} D_t$</td>
<td></td>
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</tbody>
</table>

#### Parameter

<table>
<thead>
<tr>
<th>Coefficient of relative risk aversion $\gamma$</th>
<th>Subjective discount rate $\phi = -\log \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective discount rate $\phi = -\log \beta$</td>
<td>Return to cash $r = \log R$</td>
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<tr>
<td>Return to cash $r = \log R$</td>
<td>Dividend growth rate $\mu$</td>
</tr>
<tr>
<td>Dividend growth rate $\mu$</td>
<td>Dividend volatility $\sigma$</td>
</tr>
<tr>
<td>Dividend volatility $\sigma$</td>
<td>Time interval $\Delta t = 1$</td>
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</tbody>
</table>
The Model : Basic Properties

- Model can be characterized by ratio \( v = \frac{D}{G} \) taking values in (0,1).
  - Interpret this as degree of (real) illiquidity.
  - It varies endogenously based on agents’ consumption choices.
  - Under some restrictions, it is even stationary. (Model doesn’t assume this.)

- Intuitively, when \( v \to 0 \), agents will consume relatively less of their available goods because this amounts to eating the capital stock.
  - However they will consume relatively more of their income, or even dissave, since they expect to have higher income in the future (thanks to geometric dividend growth).

- Optimal consumption can be expressed as \( C_t = G_t \ h(v_t) \).
  - Can show \( h' > 0, h'' < 0 \) very generally.
  - Explicit solution for \( h() \) only available numerically.
The Model: Consumption Function

The dark line is the optimal consumption function $h(v) \equiv C/G$ plotted against the ratio of dividends to total goods-on-hand $v \equiv D/G$. Also shown is income as a fraction of $G$ plotted as a dashed line.
The Model: Consumption Moments

The left panel plots the conditional mean of log consumption growth, and the right hand panel plots the conditional standard deviation. The horizontal axis is $v$, the liquid balances ratio.
The Model – continued

- Summarizing consumption dynamics, even though the external environment is \textit{i.i.d.}, the degree of liquidity endogenously becomes a (stationary) state-variable.

- High-liquidity (low $v$) states feel very different from low-liquidity ones.

- When cash balances are high, consumption growth and volatility are low.

- Hence marginal utility is more volatile when liquidity is low.

- Moreover, when liquidity is low, dividends make up a larger fraction of consumption. Hence, mechanically, they then covary more (negatively) with marginal utility.

- Both effects imply a \textbf{time-varying risk premium}.
  - When the economy is more liquid, risk premia are lower.
The Model: Asset Pricing Functions

The left panel plots the ratio $g \equiv P/D$, and the right hand panel plots $P/G$. The horizontal axis is $v$, the dividends-to-cash ratio.
The Model: Asset Return Moments

The left panel plots the conditional mean of continuously compounded excess returns, and the right hand panel plots their conditional standard deviation. The horizontal axis is \(v\), the dividend-cash ratio.
The Economy: Summary

- A frictionless economy in which the level of liquid balances – or the real liquidity – varies endogenously.

- It looks like periods with low liquidity are periods of high stress: the stock market is low, volatility is high, expected returns are high.

- An intervention in such a state – an increase in the “supply of liquidity” – would seem to improve things.
  - N.B. Proposition 2.2 establishes that value-neutral interventions (i.e. open market operations) leave the consumption, price, and value functions unchanged.
  - Hence analysis of their effects is dynamically consistent.

What does the model say about market liquidity?
A Definition of Illiquidity in Frictionless Markets

- Even though there are no exogenous costs of securities trading in the model, the demand curves of the representative agent are not flat (Johnson, 2005).
  - Any agent with the same preferences and holdings as the representative agent, in fact, would NOT trade unlimited quantities of risky assets at the (marginal) price.
  - Asking him to alter his optimal holdings will perturb the dynamics of his consumption stream, altering his discount rates, hence marginal valuations.

- The implied price elasticity, $I$, defines a meaningful measure of illiquidity.
  - Formally, the directional (log) derivative of price of an asset with respect to quantity of that asset, when the quantity of some other asset (the medium of exchange) is varied so as to keep the value function constant.
  - The incremental change in price, $P^{(1)}$, of “asset one” following a value-neutral trade of $\Delta X^{(1)}$ units of it for $\Delta X^{(0)} = (P^{(1)}/P^{(0)})\Delta X^{(1)}$ units of “asset zero”.

Definition of Market Illiquidity – continued

- $\mathcal{I}$ measures how fast the price moves away from someone (an informationless trader) who wishes to alter her portfolio for exogenous reasons.
  - It captures the price-impact function she faces in “demanding liquidity” from the representative agent.
  - Equivalent to the limiting bid/ask spread that (representative) agents would quote if required to make markets competitively.

- Note that we can compute this quantity whether or not any trades actually occur in the equilibrium, just as we can for prices themselves.
Market Liquidity: Two Simple Examples

- Consider a claim to an asset with exactly one cashflow: $D_T$. Then, as usual,
  $$P_t^{(1)} = E_t \left[ \beta^{T-t} u'(C_T) D_T^{(1)} / u'(C_t) \right].$$

- Explicitly keep track of shares outstanding: $D_T \equiv D_T^{(1)} X^{(1)}$.

- If the asset is the only source of time-$T$ consumption: $C_T = D_T$, and if the unit of exchange is any other claim not paying off at $T$ (or $t$), then,
  $$\frac{dP_t^{(1)}}{dX^{(1)}} = \frac{1}{u'(C_t)} E_t \left[ \beta^{T-t} u''(C_T)(D_T^{(1)})^2 \right] = -\gamma E_t \left[ \beta^{T-t} \frac{u'(C_T)}{u'(C_t)} \frac{D_T^{(1)}}{X^{(1)}} \right] = -\gamma \frac{P_t^{(1)}}{X^{(1)}}$$

- Or $\mathcal{I} = \gamma$.

- The effect of asking the agent to substitute away from time-$T$ consumption causes him to raise his marginal valuation of such consumption by the percentage $\gamma$.
  - This is an intertemporal substitution effect, not a risk bearing effect. No assumption is made about whether the net effect of the exchange raises or lowers overall consumption risk.
Market Liquidity : Two Examples – continued

- Now consider how the calculation changes when the unit of exchange (asset zero) is (a claim to) current consumption.

- Implies an extra term in \( dP^{(1)}/dX^{(1)} \) from varying current marginal utility.

\[
E_t \left[ \beta^{T-t} u'(C_T) D_T \right] \frac{d}{dX^{(1)}} \left( \frac{1}{u'(C_t)} \right) = -P^{(1)} \frac{u''(C_t)}{u'(C_t)} \frac{dC_t}{dX^{(1)}} = \gamma \frac{P^{(1)}}{C_t} \frac{dC_t}{dX^{(1)}}.
\]

- Value neutrality implies \( dC_t/dX^{(1)} = -P^{(1)} \) so that \( I \) equals

\[
\gamma \left[ 1 + \frac{P^{(1)}X^{(1)}}{C_t} \right].
\]

- The future consumption effect is amplified by a (non-negative) term equal to the percentage impact of the exchange on current consumption.

- Intuition:
  - Marginally reducing current consumption (in exchange for shares) raises current marginal utility.
  - So, if the representative agent is required to purchase \( \Delta X^{(1)} \) shares and forego consumption of \( P^{(1)} \Delta X^{(1)} \), his discount rate rises (he wants to borrow).
  - Hence and he would pay \textit{less} than \( P^{(1)} \) for the next \( \Delta X^{(1)} \) shares offered.
Market Liquidity in the Model

- In the model of Section 2, the obvious unit of exchange is “cash” – i.e. (claims to) units of the consumable asset.
  - Clearly $P^{(0)} = 1$ always.

- We want to investigate the illiquidity of the other asset.

- Computing $\mathcal{I}$ is straightforward, but not directly expressible in terms of primitives.

  $$\mathcal{I} = -v(1 + vg(v)) \frac{g'(v)}{g(v)} = (1 - v \frac{f'(v)}{f(v)})(1 + f(v)).$$

- Easy to evaluate, but what do they mean?
Market Liquidity in the Model  – continued

- The behavior of \( I \) in this model can be understood in terms of the effects in the two simple examples.
  
  - When the agent exchanges \( \Delta X^{(1)} \) shares for \( \Delta G = P^{(1)} \Delta X^{(1)} \) in cash, some of the payment is consumed and some is saved.
  
  - When relatively more is saved, the impact on future marginal utility will be lower because some of this comes from interest income.

  - When relatively more is saved, the impact on current marginal utility will be lower because the impact on current consumption is lower.

  (See the text for an explicit computation in the two-period case.)

- Hence \( I \) is higher when more of available wealth \( (G_t) \) is consumed and less saved.

- And we already showed that more of available wealth is consumed when \( \nu \) is higher, so....

  \textbf{Market illiquidity rises as the economy becomes more illiquid.}
Market Liquidity: $I$

A couple of examples. (Parameters in Tables 1 and 2.)

- The configuration on the right is non-stationary
Market Liquidity: Summary

- Empirical consensus for actual stock market price impact functions is around unit order of magnitude, i.e., a 1% price decline for a sale of 1% of shares outstanding.
  - So magnitude of model effects is economically significant.

- Variation in market liquidity in the model can be dramatic, particularly in nonstationary cases.
  - Note that nonstationary cases can be made stationary with a repeated intervention policy, and that solutions are dynamically consistent with this feature.

- Markets are illiquid when:
  1. Stocks are cheap;
  2. Expected returns are high;
  3. Volatility is high;

- But none of these relations is causal, in either direction!
Implications

Q: Can the story here be distinguished empirically from a constrained intermediary story?

A: Not clear, but...

- My story says that monetary liquidity and market liquidity are *always* linked, not just in crises, i.e. when constraints would bind.

- Controlling for *economy’s* liquid balances (real money supply) there should be no special role for *intermediary’s* financial position in determining prices, liquidity, volatility.

Caveat: Obviously highly simplified model, even within frictionless representative agent class.

- One exogenous state variable, power utility, etc.

Yet: At least it delivers quantitative predictions, given primitives.

- I know of no financial constraints type model that has been fully developed enough to do this yet.
Summary / Conclusions

- Understanding causes of dynamic variation in market liquidity is important for **Practitioners**: Anyone who may need to implement a dynamic strategy is exposed to liquidity risk;

  **Regulators**: Financial fragility is really another term for market illiquidity.

- I study a limiting characterization of market illiquidity in a simple model with two assets, one of which is money-like.

  *When there is less cash around, people are less willing to accommodate others’ trade demands.*

- The reason: the proceeds of the trade in such a state have a bigger impact on current consumption, hence discount rates.

- While the mechanism here is not inconsistent with a financial constraints channel, it does, at least, imply that the observation of a linkage between the two kinds of liquidity cannot be taken as evidence for (the importance of) such a channel.

  - More aggressively, the model suggests that “liquidity crises” are not necessarily evidence of market failure or inefficiency.