Analyzing Credit Risk in Default Swap Transaction Data: Is Fixed-Income Markets’ Information Sufficient to Evaluate Credit Risk?

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Abstract

We investigate the influence of various variables on credit default swap transaction data. Credit derivatives are arguably a superior proxy to credit risk than bond spreads. The variables considered include fixed-income as well as equity markets data. We thus provide an international analysis of corporate credit risk, and some results on sovereign risk. Simple models explain a significant portion of the cross-sectional variation in our sample (with R^2 reaching 80%). We uncover important behavioral differences between high rated and low rated, US and non US, corporate and sovereign underlyings. We also find evidence of momentum. Contrarily to earlier results, equity market information matters for both high and low ratings, albeit in different ways. We implement a spread-based reduced-form model. Equity information explains close to 50% of the errors obtained. Overall, strong results show the importance of considering equity market information beyond ratings and even bond spreads when pricing credit risk.

JEL Classification: G12, G13, G15

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1 Introduction

Credit risk has received much attention in the academic literature. The bulk of the work has focused on theoretical valuation issues. There is far less research on the empirical side. Nearly all of the empirical work investigating credit risk has focused on the bond market. The main approach was to explain the determinants and the dynamics of the credit spread, hence the difference between the yield on a bond of a risky counterparty and a government bond. Government and corporate bonds differ in a variety of ways, which makes the credit spread an imperfect proxy for credit risk. Some of the issues are addressed in Duffee (1998). Financial innovation has led to the emergence of a new kind of derivative written directly on a credit risk, credit derivatives. The Credit Default Swap (CDS) is the most used Credit Derivative. A Credit Default Swap is an instrument that provides its buyer with a lump sum payment made by the seller in the case of default (or other "credit event") of an underlying reference entity against the periodic payment made by the buyer. This periodic payment expressed as a function of its notional value is the CDS rate. No academic study that we know of has investigated the empirical behavior of credit default swap rates (while one recent study fits reduced from models of credit default swap rates and investigate the modelization of the instrument, see Vorst and Houweling (2002)). Such a study has strong implications for our understanding of credit risk behavior. It represents an opportunity to study credit risk from another instrument than the fixed income instruments (bonds, swaps) analyzed previously.

We test for the influence of the theoretical factors predicted by the reduced and the structural form literature. Moreover, we test for the stability of the influences by using a cross-section of credit default swap rates on a variety of underlyings.

Our study differs from all existing studies done on factors influencing credit risk in some respect. Credit derivatives have been around for some years but they have only in the recent past begun to be used widely in the market. Credit default swaps are supposed to allow the transfer of pure credit risk from one counterparty to another. They can be designed to provide protection against consequences of default in a variety of ways. In the purest form they provide a pre-specified payment in the event of default. If this is a fixed payment then the value of the credit default swap is only influenced by the occurrence of default. The payment can be specified in terms of an otherwise risk free bond, which makes it dependent on the term structure of interest rates. In this case the following relationship has to hold

\[ \text{Credit risky bond} + \text{Credit default swap} = \text{Risk free bond} \]
Due to this fundamental relationship we can use the prices of credit default swaps, respectively their swap rate, in order to analyze indirectly the factors influencing the credit risk of the underlying parties of the credit default swaps. This approach has many advantages over using bond spreads to analyze credit risk. It is not subject to some of the flaws of previous studies investigating credit risk as measured by the spread of the risky bond yield relative to the risk free rate. As the contracts are written directly on a credit event they are not subject to the distortion of call features and other covenants that distort bond spreads (see Duffee (1998)). Furthermore, credit default swaps are not interest rate based instruments (while other credit risk instruments such as bonds and swaps are). They allow for a direct analysis of credit risk (and the influence of interest rates thereupon), rather than for an indirect analysis of credit risk as embedded in an interest-rate based security. Finally, they present a somewhat standardized instrument to study and compare credit risk in different countries, as well as credit risk coming from corporates versus sovereigns. However they are also subject to some disadvantages which are linked to their nature as OTC contracts. The main disadvantage is their lack of liquidity and of a secondary market. Because of this problem, we do not rely on quotes, which often do not reflect in exotic markets at what price the transaction would truly take place. We have the advantage of using actual transaction prices\(^1\).

We proceed with a brief literature review of empirical studies of credit risk before addressing how classical theoretical models would price credit default swaps. We then describe our data and the variables we considered based on the theoretical and empirical literature. The empirical investigation that follows provides results on the impact of these variables as well as on international differences and other underlying differences. We then check in a more sophisticated way whether fixed income information is sufficient by implementing a reduced-form model of default swaps. This reduced-form model is based on stochastic interest rates and bond spread and thus integrates most fixed-income markets information. We investigate the factors driving the model’s errors.

2 Literature Review

The literature on such a recent instrument as the Credit Default Swap is by necessity scarce. Nonetheless, several papers have addressed the theoretical pricing of credit derivatives during the last few years. Longstaff and Schwartz (1995) present the pricing of credit spread options based on an

\(^1\) It should also be mentioned that most academic studies that analyze bond spread rely on one database only (the so-called Warga database). Having an alternative data set to work from is also important to understand credit risk.
exogenous mean-reverting process for credit spreads. Das (1995) uses a structural-form compound option model to price credit derivatives in a stochastic interest environment. The structure used corresponds more to credit spread options though than to credit default swaps. Duffie (1999) presents a simple argumentation for the replication of Credit Default Swaps as well as a simple reduced form model of the instrument.

Hull and White (2000 a and b) develop a reduced-form type pricing model, with an extension to several underlyings and non perfectly correlated default. They calibrate their model based on the traded bonds of the underlying on a time series of credit default swap prices on one underlying.

Das and Sundaram (2000) produce a reduced form model of credit derivatives in general relying on a Heath Jarrow Morton structure for both risk free interest rates and credit spreads. Their modelisation allows for a very general pricing and fitting of credit derivatives. We come back to this model later when we implement it.

A recent working paper, Vorst and Houweling (2002), investigates the implementation of a reduced form model "à la Duffie-Singleton" on credit default swap quotes. They illustrate the difficulty of implementing this type of model but show that they dominate simple practitioners use of bond spreads. They also show that the risk-free rate used (government bonds, versus repo rates, versus swap rates) influence the quality of the fit obtained in major ways.

While the literature on Credit Default Swaps is scarce and no complete empirical analysis has been produced yet that we know of, there is a more significant empirical literature on credit risk in general.

Some papers have concentrated on a direct analysis of credit ratings as provided by the big rating agencies. These ratings are important as they are used extensively in practice as a proxy for credit risk. Some theoretical models also rely on ratings and rating transitions like Jarrow et al (1997). Moon and Stotsky (1993) evaluate the determinants of ratings by each rating agency in a systematic econometric analysis. Hite and Warga (1997) analyze changes in ratings during the life of a bond and find some information content for down grades at announcement, and little or none for upgrades. The earlier studies by Katz (1974), Hettenhouse and Sartoris (1976), Weinstein (1977) and Pinches and Singleton (1978) all concluded that there is a lag between the arrival of new information and rating changes. Hence ratings do not necessarily provide much new information, except for small not very frequently traded firms. Further evidence on the changing of ratings through time is provided by Lucas and Lonski (1993). They find a trend towards lower rate debt issues combined with a higher rating volatility in the bond market during 1970-1990. Some more evidence on the quality of ratings is provided by Cantor et al. (1997) by analyzing the influence of split rating on pricing. Finally Nickel et al. (2000) analyze the
stability of rating transitions. The work of Altman and Kishore (1996a) and Izvorski (1997) provide some more evidence on default and recovery rates. Altman and Kishore (1996a) find that history of default and the resulting default rates are sensitive to some specific issues and that ratings have no explanatory power on recovery rates once seniority is taken into account. Izvorski (1997) concludes that maturity, seniority and the state of the economy are the main determinants of the value of firm specific contracts. The influence of maturity is a debated issue as there is no general consensus among the different modelling approaches on its influence on credit risk pricing particularly for below investment grade issues.

There are only a few studies investigating the determinants of credit spreads. Duffee (1998) finds that the credit spread is negatively related to the level of interest rates and the term spread. He also finds that the sensitivity to changes in the term structure is more pronounced for lower rated bonds. He observes further that changes in bond values might be due to the influence of the call feature present in a bond. Allessandrini (1998) confirms these findings and concludes further that the business cycle effect is mainly captured by the changes in long-term interest rates. The study by Friedson and Garman (1998) differs from the previous by using new issues of high-yield bonds to analyze the factors that influence the pricing. They find that changes in the risk free rate, credit spreads and the slope of the yield curve influence the pricing.

The more recent study by Collin-Dufresne et al. (2001) uses time series of quoted bond prices to analyze the influence of various financial variables that should in theory influence changes in yield spreads. They find that these variables have only limited explanatory power. Moreover the residuals of the regressions are highly cross-correlated pointing to the influence of an unobserved common factor. The authors remain short of determining this common factor but find little support for structural form variables explanatory power of credit spread changes. While our results seem to differ strongly from theirs (as we document the importance of equity market information for the credit risk level of both high and low rated underlyings), Collin-Dufresne et al. truly address changes in bond spreads using time series (which can be linked to other elements than credit risk and has a strong time dimension) while we address levels of credit default swap rates on cross-section data. Some of our results seem to confirm segmentation issues in fixed income markets first established by these authors. This stresses the importance of the credit derivatives market to understand the empirics of credit risk (and underlines possible problems with bond spreads use for credit risk investigation). We also provide information on international evidence for corporates and sovereigns. Some further evidence for co-movements of credit spreads is provided by Batten et al. (1999) in their study on Australian Eurobonds. Finally, Yu
(2002) examines the effect of accounting transparency on credit spread term structures.

Some studies have analyzed credit risk from other instruments, notably on swaps like Sun and Sundaresan (1993) on swap quotes and Cossin and Pirotte (1997) on swap transaction data. Their results tend to be less advanced on credit risk issues than the previously quoted studies on bond spreads.

There is a different group of studies that focus on sovereign debt. Analogous to the literature on corporate ratings, sovereign ratings were subject to close examination. Cantor and Packer (1996) find that the public information is contained in ratings and that rating changes do have a significant effect on the prices of outstanding debt. Classens and Penacchi (1996) construct a model that takes into account a number of issuer specific factors. They calibrate their model to the observed prices of Mexican Brady bonds. Kamin and Kleist (1999) analyze the determinants and the evolution of emerging market spreads during the 1990s. They find a strong relationship of credit ratings, maturity and currency denomination with emerging market instruments spreads. They also find evidence for a changing risk premium during the time span under consideration. Cumby and Evans (1997) and Dungey et al. (1999) consider credit quality to be some unobservable random variable. Dungey et al. (1999) provide a decomposition of international spread changes in a Kalman filter framework. Their main result is that for the Commonwealth countries the first of their three factors (the common factor) displays long-swings that explains most of the changes in credit spreads. For other countries some country specific influences seem to be more important. A significant improvement within those studies is presented by Eichengreen and Mody (1998). They consider the determinants of emerging market debt values by taking into account a possible selectivity bias as some low rated countries might have been unable to issues bonds following the emerging market turbulences. Their findings confirm that higher quality translates into a higher probability of issue and a lower spread. However fundamental information can only explain a fraction of the overall variation in their sample.

There has not been any advanced study we are aware of that bear on Credit Risk as reflected in Credit Derivatives. We next analyze how different models would price Credit Default Swaps in order to understand what variables are important to consider in our empirical analysis and how those variables will affect Credit Default Swap rates. In the first part of our empirical analysis, we do not fit a specific model to our data (as we want to separate model testing from empirical analysis). The role of the next section is to uncover stylized facts about different variables in order to make our empirical analysis more clear and pertinent.
3 The Factors Influencing Credit Risk

3.1 The Pricing of a Credit Default Swap

In this section, proposed mostly for heuristic reasons, we derive the general structure of the pricing of a credit default swap and illustrate the differences in pricing among various possible theoretical approaches. Two strands dominate the theoretical literature on credit risk today: the structural and the reduced form ones. We will work out the explicit pricing for a structural form model and for a default intensity-based reduced form model in order to show the influence of the various parameters analyzed in the subsequent sections. The pricing is done from the buyers point of view. The buyer will have to make periodic payments as long as there is no default of the underlying party. On the other hand he will receive some payment in the case of default. This payment can be specified in a variety of ways. The payment could be defined as a fixed amount in the simplest case. It can be the difference between the pre-default value and the recovery value of a bond. The most common definition is to define it as the difference between the face value of the bond and the recovery value after default. The payout is sometimes corrected for accrued interest and implied interest payments. The value of a default swap to the buyer of protection (if we omit the second order effect of counterparty risk studied by Hull and White (2000)) is thus given by

\[
\text{Value of a credit default swap} = \sum_{i=1}^{N} \exp \left( - \int_{0}^{t_i} r(u) \, du \right) \cdot \Pr(\text{no default until time } t_i) \cdot \text{Swap rate} \\
\cdot \text{Notional} + \exp \left( - \int_{0}^{\tau} r(u) \, du \right) \Pr(\text{default at time } \tau) \cdot \text{Payment} \tag{1}
\]

where \(i\) is the index of the payments, \(N\) is the number of payments until maturity and \(r(u)\) is the interest rate. The expectations operator in the above equation is needed as the interest rate could be stochastic and correlated with the variables influencing the probability of default. Various models will differ on how they determine the probabilities and the payment at default.

3.2 A Simple Structural Analysis of Credit Default Swaps

In a first step we will develop the pricing for a structural model, with non stochastic interest rates. It is clear that this model is only applicable directly for corporate underlyings. Some evolution of the model could be used for sovereigns in the spirit of Classens and Penacchi (1996). Following the basic Merton
(1974) framework we assume that the firm value $V$ follows a geometric Brownian motion given by

$$\frac{dV}{V} = (r - d) \, dt + \sigma_v \, dz$$

(2)

where $r$ is the instantaneous risk-free rate, $d$ is the continuous dividend yield and $\sigma_v$ is the volatility of the firm value process. The firm will default if this value breaches some pre-specified default level denoted $H$. We will assume that this default level is an exogenously fixed constant. It could be a deterministic function of time. Black and Cox (1977) use a deterministic exponential barrier to model the effect of bond indenture provisions on the value of risky debt. We could use a similar functional form to model the default boundary. Another possible extension would be in the spirit of Leland (1994) and Leland and Toft(1996). In their model the default boundary is determined endogenously by maximizing the value of equity respectively to the value of the firm. The value of the credit default swap in our case is comparable to the sum of a number of barrier options on the firm value. The methodology and notation we use for the pricing of the barrier options follows the work of Rich (1994). The probability of no default until time $i$ is obtained as

$$\text{Prob}(\text{no default until time } i) = \text{Prob} \left( \inf_{t \in [0, t_i]} V_t < H \right)$$

(3)

where

- $V_t = \text{firm value process}$
- $H = \text{default boundary (possibly the face value of debt)}$
- $\mu = r - d - \frac{\sigma^2}{2}$ for $t \in [0, t_i]$

With these probabilities we can price one leg of the credit default swap. The value of the periodic payments is obtained as

$$Part1 = \text{Swap rate} \cdot \text{Notional} \cdot \sum_{i=1}^{N} \exp(-r_{t_i}) \left( N \left( \frac{-\ln(H/V_0) + \mu t_i}{\sigma \sqrt{t_i}} \right) - \exp \left( \frac{2 \ln(H/V_0) \mu}{\sigma^2} \right) N \left( \frac{\ln(H/V_0) + \mu t_i}{\sigma \sqrt{t_i}} \right) \right)$$

(4)

Now we proceed to the pricing of the second leg of the default swap, which is the payment in the case of default. We assume that the recovery value is just a discount on the face value. This is not an unreasonable assumption if the recovery rate can be estimated ex-ante with some degree of certainty. We could make the recovery rate a function of time. The valuation could always be done by solving numerically the integral given in equation number 1. On the other hand the recovery rate could be a
function of the value of the assets at time of default. However this value is known to us as default is triggered when the firm value crosses the level $H$ as long as default occurs at any time before maturity. Therefore the assumption that the recovery rate is a function of the asset value at the time of default, would just make it dependent on $H$ contrary to Merton (1974) where a European type setting is used. If the underlying firm value process is a jump-diffusion process, the modelling of a recovery rate that depends on the asset values of the firm at default would be more complicated. Under the above assumptions the value of the second part is given by

$$Part2 = E_0 \left[ \exp (-r\tau) \cdot \Pr(\text{default at time } \tau) \cdot \text{Payment} \right]$$

$$= \int_0^T \text{Payment} (H, \tau) \exp (-r\tau) h(\tau) \, d\tau$$

where:

$$h(\tau) = \text{first passage time density of the process } v \text{ at the level } H$$

$$h(\tau) = \frac{-\ln \left( \frac{H}{V_0} \right) - \ln \left( \frac{H}{V_0} \right) + \mu \tau}{\sigma \sqrt{\tau}}$$

Under the assumption that the recovery rate is a function of $H$ or the face value we can simplify the above integral to

$$Part2 = \text{Payment} \cdot \left( \left( \frac{H}{V_0} \right)^{a+m} N (w) + \left( \frac{H}{V_0} \right)^{a-m} N \left( w - 2m\sigma \sqrt{T} \right) \right)$$

where:

$$a = \frac{\mu}{\sigma^2}$$

$$m = \frac{\sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2}$$

$$w = \frac{\ln \left( \frac{H}{V_0} \right) + m\sigma^2 T}{\sigma \sqrt{T}}$$

The valuation equation would be more complicated if the interest rate were considered stochastic and correlated with the firm value process. We might not be able to obtain a closed form solution for this case depending on the choice of the process for the stochastic interest rate.

Because the value of the firm follows a diffusion process the probability of default goes to zero as the maturity of the contract goes to zero. Therefore the credit spread on a risky bond implied by such a model goes to zero as well. Empirically however default spreads do not go to zero with decreasing maturity but they remain positive. One possibility to take into account that firm value can drop suddenly is to model the firm value process as a jump-diffusion process. Zhou (1997) follows this path and obtains a closed form solution for the value of a bond under some restrictions and proposes the use
of the Monte Carlo methodology for the valuation in the general case.

The factors that influence the value of the credit default swap in a classical structural model such as the one proposed here are the distance from the default boundary, the value of the assets of the company, the volatility of the value of the company, the level of the interest rate and the time to maturity of the credit default swap. In the earliest structural model, the Merton model, it was assumed, that the default boundary was simply the face value of debt. In later models the default boundary was given exogenously or determined endogenously as in Leland(1994), Leland and Toft (1996) and Anderson and Sundaresan (1996). Even if we don’t observe the exact value of this boundary we can identify the influence of various factors on the distance from this boundary. A decrease in the stock price will lower the distance to this boundary, the same is true for an increase in leverage. An increase in volatility would also increase the probability of default as the likelihood increases that the firm value process crosses the default boundary.

3.3 A Simple Reduced Form Approach and Implications

We turn now to a reduced form approach. In the reduced form approach, the probability of default is governed by a hazard rate. This is common to all the models like Jarrow and Turnbull (1995), Duffie and Singleton (1997) and Lando (1994/97). The hazard rate denoted h(t) could be a function of a various other variables. An alternative modelization (see for example Das and Sundaram (2000)) relies on a direct modelization of the bond spreads. We come back to this alternative later when we implement it.

Lando (1994/97) derives some simple representations for the valuation of credit derivatives. The value of a contingent claim that pays some fixed amount if default has not occurred before that time is obtained as

$$IE^Q \left[ \exp \left( -\int_t^\Gamma r(s) \, ds \right) X \cdot I(\Gamma > T) \right] =$$

$$I(\Gamma > t) \left[ \exp \left( -\int_t^\Gamma r(s) + \lambda(s) \, ds \right) X \right]$$

where \( \Gamma \) is the time of default and \( \lambda \) is the intensity of the Poisson process governing the default probability. This expression is exactly the value of one of the payments of the credit default swap rate times the notional conditional on no default up to that point. The other leg of the default swap would be valued with the expression for a credit derivative that pays off \( Z(\Gamma) \) if the underlying defaults at
time $\Gamma$ and zero otherwise. This expression is given by

$$E^Q_t \left[ \exp \left( - \int_t^\Gamma r(s) \, ds \right) Z(\Gamma) \right] =$$

$$\mathbb{1}(\Gamma > t) E^Q_t \left[ \int_t^T Z(s) \cdot \lambda(s) \exp \left( - \int_t^s r(u) + \lambda(u) \, du \right) ds \right]$$

where $\Gamma = \text{is the time of default}$

The payment $Z(s)$ would in our case be defined as

$$Z(s) = \text{Recovery value}$$

where $\text{FV} = \text{face value}$

The recovery value can be defined in a variety of ways. Jarrow and Turnbull (1995) define it as an ex-ante known value. Another possibility is to treat it as being dependent on a number of state variables. Das and Tufano (1996) for instance assume, that the recovery rate is correlated with the default free spot rate. Duffie and Singleton (1998) specify the recovery as a fraction of the pre-default value of the bond (Recovery of Market Value). This solution has some advantages on the modelling side while recovery of Face Value (where the creditor receives a fraction of the promised face value) or Recovery of Treasury (where the creditor receives a fraction of an identical but default free bond) are more common (see Jarrow and Turnbull (1995)).

A variety of authors have suggested that the default rate could depend on some state variables reflecting the economic environment and some firm specific information. Lando (1994/1997) assumes that the hazard rate depends on a number of state variables. Jarrow and Turnbull (2000) use the same kind of process and assume that the hazard rate is a function of the random elements in the evolution of a stock price index and the short term interest rate. Their hazard rate function is given by

$$\lambda(t) = a_0(t) + a_1 r(t) + \beta \sigma_1 W_1(t)$$

where $a_0, a, \beta$ are constants; $r$ is the risk free rate and $W_1$ is a Brownian motion governing the unexpected part of the returns of a stock price index. Their model can be calibrated to fit some observed structure of default spreads. The constant in their model could depend as well on some parameters related to the firm value like the rating, the leverage or the stock price volatility. In the
reduced form framework the valuation formula would be given by

\[ Value \ of \ a \ credit \ default \ swap = \]

\[ -I(\Gamma > t) \cdot \sum_{i=1}^{N} E_{t}^{Q} \left[ \exp \left( -\int_{t}^{T_{i}} r(s) + \lambda(s) \ ds \right) X \right] \]

\[ + I(\Gamma > t) \cdot E_{t}^{Q} \left[ \int_{t}^{T} Z(s) \cdot \lambda(s) \ exp \left( -\int_{t}^{s} r(u) + \lambda(u) \ du \right) \ ds \right] \]

The reduced and the structural form approaches differ significantly in the way they model the default probabilities. On the other hand the economical differences become much smaller if we use a structural model with a jump-diffusion process for the firm value, thus allowing for sudden drops in the value of the firm's assets, or if we introduce a hazard rate that depends on economic and firm specific factors. Therefore the factors influencing the prices of default swaps are basically the same, but the weighting of their influence is different. Based on these models we identify a number of factors that should affect the prices of credit default swaps and credit risk in general. These factors are primarily in the case of reduced form models the default process, the interest rate and the recovery rule as well as the maturity of the instrument.

4 The Credit Default Swap Data and the Use of Transaction Data

The credit default swap data is obtained from a major London interdealer broker. The data consist of several thousand one-way and two-way quotes and 392 realized trades. In this study we focus on the realized trades. A one way quote in an OTC market is just the request to sell or to buy a specific instrument for some specific price. It is not cleared market data with a well defined bid ask spread. Practitioners know that quotes obtained on screens may actually not be obtained in reality. Quotes, while they may be instructive to analyze in themselves, do not provide for a clear market price as the latter could empirically differ widely from the quotes, even two way quotes (for example depending on the counterparties considered, the amount asked for, or simplicity the willingness to actually do a deal). The traded data on the other hand is market cleared data, hence it represents the market consensus on the fair value of the credit default swap at transaction time. This is particularly important to consider for a young, rather exotic market where pricing is difficult to assess. Therefore we will restrict ourselves to the 392 observations of traded contracts. These trades took place during the period from January 1998 to February 2000, with observations of all qualities well-spread over the period. In
the sample we have 69 sovereign and 323 corporate underlyings. The underlyings come from a variety of countries. The domiciles of the underlyings are:

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<td>Spain</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Sweden</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Thailand</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Tunisia</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>19</td>
<td>0</td>
</tr>
<tr>
<td>United States</td>
<td>165</td>
<td>0</td>
</tr>
</tbody>
</table>

The majority of underlyings are from the United States followed by European countries. The corporations underlying the contracts tend to be large, with a minimum market capitalization for the US ones of $500 millions and an average of $40 billions. Most of the contracts are denoted in US dollars. The remaining ones are in Euro or Yen. The prices of the credit default swaps are denoted in basis points per annum. The notional amounts of the contracts range from 1 to 20 million US$. The payment at default is defined as the difference between the notional of the credit default swap and the recovery value of a bond on this underlying with the same notional. Deviations from the general structure outlined above are possible. For some of the contracts more detailed procedures in case of default are specified directly between the counterparties without the involvement of the broker. These could include for instance the payment of the value of the bond during some time before the default and other refinements. Table 1 summarizes the basic statistics of the data. Rates on sovereigns are in general higher than on corporates but their ratings tend to be lower. While US and Non-US corporates seem to have similar rates and ratings, actual distributions vary strongly, with Non-US corporates having much higher skewness and kurtosis. Overall, the data requires much deeper analysis to check whether the relationships and distributions differ from sample to sample. We proceed to this after gaining a bit more understanding of how the market is constituted.
5 The Credit Default Swap Market

We hereby provide some information on how the credit default market is structured, as it appears from our database (i.e., as originated by an interdealer broker).

Regarding the underlyings themselves, while some names are seen more frequently than others, it is not a very concentrated market as seen in Figure 1.

There is some concentration as far as industries are concerned though, as 34% of our transactions occurred with financial intermediaries as underlying risk (Table 2).

More of a concentration occurs at the sovereign level, with China occurring in 24% of our sovereign transactions (Figure 2).

The market players themselves are rather concentrated, with a small number of banks taking a large part of the buying or selling side of the transactions considered. Interestingly, the "league tables" of our sample do not vary widely on the seller and on the buyer side. Not only are the top players the same ones but the average rating of the buyers is very similar to the one of the sellers. Notice though the exception of Credit Anstalt, that seems to have been a big buyer of protection of the period (Table 3). Also, there is much higher concentration on the sellers’ side (C10 of 74%) than on the buyers’side (C10 of 57%). Selling protection is thus a more concentrated market, as reflected in our database, than buying protection.

6 The Determinants of Credit Risk: Variables We Consider and Our Sources

In this section we outline the factors to be analyzed in the subsequent econometric analysis. As shown earlier, the choice of those factors is justified by the existing theoretical literature on the pricing of credit sensitive contracts. Overall, structural models stress the influence of the value of the assets of the company, its volatility, the distance from the default boundary (as influenced notably by the leverage), the level of interest rate and the maturity. Reduced form models use also the latter two but exogeneize the default process and the recovery rule.

1. Credit ratings

Credit ratings are the most widely observed measure of credit quality of a specific debt issue or the issuing entity in general and remain the most commonly used information for the default process and the hazard rate $\lambda$. All the reduced form models rely in one way or another on the estimation of a default
probability. In practice default probabilities are estimated very often by rating classes. Some models like Jarrow et al. (1997) are directly based on the estimation of the rating migration matrix. Numerous studies on credit ratings have shown that often changes in ratings are anticipated by the market. Thus we expect that ratings have only a limited explanatory power for price changes. However other studies particularly on sovereign ratings (for example Cantor and Packer (1996)) have shown that ratings subsume efficiently all the fundamental information. Moreover they seem to provide some additional information beyond the fundamentals used in their study. Our research uses a cross-section of initial prices of credit default swaps. We are investigating the various factors that influence the level of credit default swap rates, not the changes. Therefore credit ratings should have a significant explanatory power in our regressions. The main critique concerning ratings is their infrequent revisions.

The credit ratings we use are the ratings of the underlying company for its long term debt. They range from AAA to C in Standard and Poor’s rating system and from AAA to B3 in Moody’s notation. We have used the Standard and Poor’s rating whenever possible. If only the Moody rating was available we used it instead of the Standard and Poor’s rating. This choice was made as we had more ratings available from Standard and Poor’s than from Moody’s. We use the ratings in two ways in our regressions. We introduce dummy variables that represent each rating and thus let us analyze the impact of each rating with no assumption on its relationship to the other ratings. We also have translated the alpha numeric rating classes into a numerical scale ranging from 1 for the highest to 17 for the lowest credit rating. This procedure, while being common in the literature, might introduce a bias because we implicitly assume that the influence of a rating change is the same between AA and A or BB and B. It is clear that the rating change can have a more dramatic influence for lower quality underlyings than for higher quality underlyings and we will indeed investigate this point. However working with a number of dummy variables is not well suited for some subsamples (as we observe only very few observations for some of the rating classes). We thus use both methodologies to confirm our results, using dummy variables in some regressions and the numeric-translation in some alternatives. We also use extensively an intermediate approach and allow for behavioral differences among high and low ratings by using either a single dummy variable or subsamples. The numerical values that we assigned to the different rating classes can be found in the following table:
<table>
<thead>
<tr>
<th>Rating</th>
<th>S&amp;P</th>
<th>No. of Transactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AAA</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>AA+</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>AA</td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td>AA-</td>
<td>33</td>
</tr>
<tr>
<td>5</td>
<td>A+</td>
<td>35</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>52</td>
</tr>
<tr>
<td>7</td>
<td>A-</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>BBB+</td>
<td>42</td>
</tr>
<tr>
<td>9</td>
<td>BBB</td>
<td>56</td>
</tr>
<tr>
<td>10</td>
<td>BBB-</td>
<td>36</td>
</tr>
<tr>
<td>11</td>
<td>BB+</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>BB</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>BB-</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>B+</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>B</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>B-</td>
<td>4</td>
</tr>
<tr>
<td>17</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>NA</td>
<td></td>
<td>26</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>393</td>
</tr>
</tbody>
</table>

It is interesting to notice that our data offer a wide spread of ratings rare in empirical academic studies, from AAA to B with half of the underlyings being BBB-rated or less. Most empirical credit risk work up-to-now has not been able to truly investigate in one sample both very high and very low credit risk.

2. Interest rate

It is interesting to note that most of the current credit risk management models as used by practitioners, whether based on ratings or on structural variables (such as leverage and variance) do not include stochastic interest rates. On the other hand, the spot rate is a factor that appears in all of the current academic credit risk pricing models. In general the spot rate is negatively correlated with the credit spread. This impact is confirmed in empirical studies (see Duffee (1998) and others). We expect to find a negative relationship between the US spot rate and the observed credit default swap rates.

We use the 3 month treasury constant maturity rate series from the database of the federal bank of St.Louis (FRED) as the proxy for the short term risk free rate. We are working with monthly observations and chose the latest observation before the trading date of the credit default swap. The choice of the US rate is certainly a viable choice for the US corporate underlyings as well as a usual choice for the sovereign underlyings as the US government is regarded normally as the highest grade counterparty in the world. In order to examine the influence of this choice we use additionally series of benchmark yields from datastream for the following countries: Australia, Germany, Japan and the UK.

3. Slope of the yield curve

The slope of the yield curve does not appear in most of the structural models directly, but we would still expect it to have a significant impact via its influence on the expected short rate in the
future and due to the fact that it is related to future business conditions. Some interest rate models like Brennan and Schwartz (1979) model explicitly the short and the long rate, while others take it into account implicitly by assuming that the short rate is mean reverting around the long rate level. Das (1995) is a model which uses the whole risk-free term structure. We interpret the economic influence of the yield curve as conveying information on future spot rates and economic conditions. Generally a steeper slope of the term structure is considered to be an indicator of improving economic activity in the future. Harvey (1988) finds that the slope of the yield curve has a positive relationship with future consumption. Estrella and Hardouvelis (1991) confirm that a positively sloped yield curve is associated with an increase in real economic activity as measured by consumption, consumer durables and investment. Finally Estrella and Mishkin (1995) test the predictive power of various financial variables in probit models for the prediction of recessions. They find that among all the variables examined the slope of the yield curve has the highest power, with a decrease in the slope being associated with an increase in the probability of a recession.

Therefore we introduce slopes of the yield curves of some other countries as the economies in the US, Europe and Japan are not at the same stages in the business cycle.

In order to measure the slope of the yield curve we use the difference between the long term and the short term interest rate series from the federal bank of St.Louis (FRED). We include again the series for Australia, Germany, Japan and the UK. In this case we measure the slope of the yield curve as the difference between the benchmark yield over 10 years and the benchmark yields below five years. The series are taken from Datastream.

4. Time-to-maturity

We expect that time to maturity should have an influence on observed Credit Default Swap rates. However there is no consensus in the literature as to the shape of the term structure of credit spreads. Most of the structural models predict an upward sloping term structure for investment grade and a downward sloping term structure for speculative grade debt. But the expected term structures can be more complicated than that as illustrated by Merton’s (1974) hump-shaped or Das’ (1995) ”N-shaped” term structures. Collin-Dufresne (2001) points out that some shape effect is mainly due to these models using implicitly declining leverage ratios. We use the time-to-maturity reported in the database.

The time to maturity is reported in the database either directly as time to maturity reported in weeks, months or years or as a specific ending data. We translate all the times to maturity in a notation of weeks. In some cases we round the resulting number of weeks as the contract might have been initiated any day of the week The error introduced by using weeks instead of days is very small as most
of the observations are expressed in years anyway and the maturities are rather long ranging anywhere from some months up to 10 years.

5. Stock prices

Stock prices contain information on the underlying companies. Negative information on the firm is reflected faster in the stock price than in the rating. In all the structural form models like Merton (1974) or the various extensions like Shimko et al (1993), Longstaff and Schwartz (1995), Leland (1994), Leland and Toft (1997) default is triggered by the firm value process. Leland (1994) shows that it is possible to reformulate the Merton model in terms of the stock price instead of \( V \) (the value of the firms’ assets). Based on the structural models Kealhofer, McQuown and Vasicek (KMV) have developed and marketed a model for the pricing and management of risky debt. They use stock prices to back out expected default probabilities.

In the context of a reduced form model Jarrow and Turnbull (2000) use a default rate which is dependent on the random element of a stock price index. Following their approach we could make the default rate dependent on the evolution of a stock price.

The influence of stock price changes are twofold. The stock price might reflect business conditions ahead of time. On the other hand a drop in the stock price induces a higher leverage ratio if one assumes that the level and value of debt fluctuates less strongly than the value of equity. We will adjust the stock returns for returns in the associated index in order to control for systematic stock movements. Moreover we will construct a dynamic measure of leverage in order to isolate the leverage effect.

The stock price data is collected from Reuters with a weekly frequency. We use the data to estimate changes in the stock price 4 weeks, 12 weeks, 26 weeks and 1 year before the trading date of the credit default swap. We use the absolute as well as the real percentage changes in the stock price as an explanatory variable. Due to the fact that we use the changes from a month up to a year, the choice of the weekly frequency instead of daily does not seem to matter much. The prices are Friday’s closing prices as reported by Reuters. We use the data from the last Friday before the trade of the credit default swap.

6. Variance or volatility of the firms’ assets

All the structural models contain as an input the volatility of the assets of the firm. The credit spread is expected to increase with a higher volatility. Ronn and Verma (1986) show how to link the volatility of the firm value with the volatility of the stock price. As a proxy for the variability in the firms assets we use the historical annualized variance of the stock returns based on the weekly quotes reported by Reuters. We use a running window of 52 weeks to estimate the variance for
every trading week. The weekly variance is annualized by using the scaling implied by a geometric Brownian motion for the underlying stock price. We use the historical variance because there are no liquid options for a lot of the underlyings in our sample which makes it impossible to use the implied volatility from traded option prices.

7. Leverage

All of the structural models agree that the level of leverage has a significant impact on credit risk. Either they are directly based on the ratio of firm value to debt value or they depend on the distance of the firm value process from some default triggering level (also called the distance from default). Collin-Dufresne and Goldstein (2001) note that most of the structural form models use implicitly declining expected leverage ratios. This fact explains why these models predict a declining term structure of yield spreads for speculative grade debt. Collin-Dufresne and Goldstein develop a model which yields stationary leverage ratios. In the context of reduced form models, the leverage ratio has only an indirect influence via the hazard rate. In all the models higher leverage is associated with an increase of credit risk. As a proxy for leverage we will use a variable defined as book value of long-term debt divided by market value of equity. As the market value of equity changes at the same speed as the stock price we use this variable to control for leverage effects when we include stock price changes as an explanatory variable.

In order to control for changes in leverage we construct a dynamic proxy for leverage using the debt reported in the Equities 3000 database from Reuters and the market values from Datastream. We use as a proxy for leverage the ratio of the total liabilities and the market value of the firms’ equity. The use of leverage differs significantly between the various countries. Therefore we restrict the use of the leverage proxy to the US sub-sample.

8. Index returns

In order to control for factors affecting all the securities in some markets we verify the robustness of our results regarding the influence of stock prices by using market index adjusted returns as well as index returns in combination with stock price changes. The indices include the following:

<table>
<thead>
<tr>
<th>Australia</th>
<th>All Ordinaries</th>
<th>Italy</th>
<th>Mibtel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>ATX</td>
<td>Korea</td>
<td>KS 11</td>
</tr>
<tr>
<td>Belgium</td>
<td>Bel 20</td>
<td>Netherlands</td>
<td>AEX</td>
</tr>
<tr>
<td>Canada</td>
<td>TSE 30</td>
<td>Spain</td>
<td>MSI</td>
</tr>
<tr>
<td>France</td>
<td>CAC 40</td>
<td>Switzerland</td>
<td>SMI</td>
</tr>
<tr>
<td>Germany</td>
<td>Xetra DAX</td>
<td>UK</td>
<td>FTSE 100</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>Hang Seng</td>
<td>US</td>
<td>S&amp;P 500</td>
</tr>
<tr>
<td>India</td>
<td>BSEN</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
9. Idiosyncratic factors

The default probability and thus the credit default swap rates might also depend on a number of idiosyncratic factors. Nickel et al. (2000) investigate the stability of rating transition matrices. They find that rating transitions vary significantly between US and non-US obligors, between industries and the stage of the business cycle. Following the same lines we want to test for differences among various sub-samples. Our sample is composed of sovereign and corporate debt. Moreover it encompasses underlyings from a variety of countries. In practice different hazard rates are used for different industries. We will differentiate between sovereign and corporate and US and non US corporate underlyings by using dummy variables and sub-samples. We will not differentiate among the various industries as we do not have enough observations of the same industry in the different subgroups.

We are also dealing with an exotic product where liquidity effects may matter (notably when traders rely on replication for pricing). We use market capitalization as a proxy for liquidity and investigate its impact as well.

7 Estimations and Results

7.1 The Credit Rating and Time to Maturity

In a first step we want to analyze the influence of the rating on the credit default swap rates. Despite all their deficiencies, ratings are still considered the most important single source of information on the credit quality of a borrower. Therefore we expect a strong connection between ratings and credit default swap prices. The influence of the rating does not need to have the same influence on lower grade and high grade underlyings. We will investigate this question with a set of dummy regressions. The second variable that we will look at is the time to maturity. Although time to maturity is a natural variable to consider in any derivative contract, the theoretical influence of the time to maturity on credit default swap rates and credit risk in general is ambiguous. Many structural form models predict for example that we could observe a hump shaped term structure of credit spreads for low rated underlyings and a decreasing one for very low rated underlyings. Some predict even more complex term structures while reduced form models can accommodate many shapes.

We estimate all the equations as simple linear regressions on the level of default swap rates and as a semilog model on the logarithm of credit default swap rates. This second set of regressions represents a crude attempt at checking for some non linearities and confirming or infirming results.
obtained from linear regressions. It should be clear that the relationships we are looking for are most probably non linear. This work should be thought of as looking for the impact of variables on credit default swap rates via a linear approximation and a semi log approximation (we have also tested for a full log specification with very similar results, confirming the strength of the results beyond the issue of specification). Obviously, a better attempt would be to test for a more precise shape for the relationships, as we do in a later part of the paper. To be well done though for the general analysis, this would have to rely on the direct testing of a model. Unfortunately, such a methodology would have the double task of testing the model and testing the results, as no model has faced a consensus in the literature yet. This double testing would limit the analysis of the results by itself. Vorst and Houweling (2002) also proceed with an attempt to fit a reduced form model and show the difficulties of fitting such a model in a satisfactory way.

Nonetheless, we can check via two ways whether we capture the right shape by using either linear or semi-log modelization: we can fit a power function that has more degrees of freedom on its shape than either linear or semi-log or we can check with rating dummies (either for classes of ratings or for each rating) that leave the shape free.

In the power test, we fit for the curvature of the relationship while we impose it for both linear and semi-log regressions. In other words, we check for a relationship of the type:

\[ CDSR = \alpha Rating^\beta \]  

where \( \beta \) captures the shape of the relationship. This can written equivalently

\[ \ln(CDSR) = \ln \alpha + \beta \ln(Rating) \]  

Interestingly, results in Table 4 (represented graphically in Figure 3) show that semilog tends to perform better and that the power test does not provide for the best fit. Constraining on the shape of the impact of ratings may not be as high a hurdle as one could expect intuitively. We thus proceed with a general analysis using both linear and semi-log relationships (as power tests cannot be extended directly to multiple independent variables).

Next, we introduce a dummy for ratings below BBB. The choice of the BBB rating was done out of statistical considerations in order to have a balanced sample size for all the subgroups. This dummy allows for a different sensitivity of the credit default swap rate with respect to the rating for highly rated underlyings and lower rated underlyings. We consider the following regressions
\[ CDSR = const + \alpha_0 \cdot dummy_1 + \alpha_1 \cdot rating \]
\[ + \alpha_2 \cdot dummy_1 \cdot rating + \beta_2 \cdot time + \varepsilon \]  

\[ \text{Log}(CDSR) = const + \alpha_0 \cdot dummy_1 + \alpha_1 \cdot rating \]
\[ + \alpha_2 \cdot dummy_1 \cdot rating + \beta_2 \cdot time + \varepsilon \]

where

<table>
<thead>
<tr>
<th>CDSR</th>
<th>Credit default Swap rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>Credit Rating</td>
</tr>
<tr>
<td>Time</td>
<td>Time to maturity in weeks</td>
</tr>
<tr>
<td>dummy_1</td>
<td>dummy variable for underlyings rated below A</td>
</tr>
<tr>
<td>\varepsilon</td>
<td>Error term</td>
</tr>
</tbody>
</table>

Previous empirical studies have found evidence that the market distinguishes between sovereign and corporate underlyings as well as between underlyings from various geographical regions. In order to gain more evidence, we run the above regression on the following sub-samples:

- Corporate underlyings
- US Corporate underlyings
- Non US Corporate underlyings
- Sovereign underlyings

In order to test for the statistical significance we estimated a pooled regression with unrestricted intercepts and coefficients. Tables 5 and 6 show the result of the individual regressions and the coefficient tests from the pooled regression. We have corrected for heteroskedasticity when necessary (using the White test and procedure).

The first obvious result (Table 5) lies in the striking significance of the different regressions, with Fisher tests never under 20. This significance is notably linked to the significance of the ratings variable that remains strongly significant whatever the specification (linear or semi-log) and the sample considered (sovereign, US or non-US corporates). The strongest impact economically of ratings happens with Sovereign credit default swap rates (with the largest coefficient in both specifications). This confirms the result that Sovereign and Corporate ratings are not interchangeable and that the Sovereign rating differences are larger (from a pricing point of view) than the Corporate ones.

The use of the dummy variable multiplied by the rating is a way of testing whether low rated underlyings, whether Sovereign or Corporates, have a different price-rating relationship than highly
rated underlyings. The coefficient is only significant for the US Corporate sub-sample (and for the Sovereign sub-sample in the linear regression). The value of the coefficient of the rating variable multiplied by the dummy is quite larger than the value of the coefficient of the rating variable on its own in the linear specification, thus indicating a potentially very strong threshold impact of ratings. This indeed indicates that the influence of the rating on the level of the credit default swap rate varies significantly for low rated underlyings and high rated underlyings. This can be considered in line with some theoretical results from the structural form literature, which predict that low rated riskier debt behaves significantly differently from high rated debt. We find the same evidence in the sovereign sub-sample with the same magnitude of impact of a rating change for low rated debt in the linear specification. This variable is not significant for the rest of the sub-samples. It may be a first hint that US Corporates ratings may behave or be considered differently from the non US ones in the Credit Risk pricing market. This is due to the fact that non US Corporates include a wide range of underlyings for which credit risk pricing does not react in the same way to changes in their rating. Ratings do differ as far as their pricing impact on credit risk in the US versus non US Corporates. One should thus be wary of importing models fitted in one sample for use in the other sample.

The time to maturity is nearly never significant. It has a positive coefficient in the case where it is significant (whole sample in semi-log configuration), meaning that a longer time to maturity leads to a higher CDS rate (or higher credit risk). The reason for the overall lack of significance might be coming from our sample. About half of our observations have a time to maturity of five years or a value very close to 5 years. We have also tested for the impact of a variable constituted of the dummy variable multiplied by the time variable. The idea is to test whether the maturity has a different impact for low rated underlyings and for high rated underlyings, as would be expected from structural form models. We did not obtain significant results, which may point once more to a sample problem as far as maturity is concerned. Finally we have tested the possibility that the influence of time is more complicated than assumed by a simple regression. We have introduced dummies for various time bands. However the more complicated structure did not yield any better results. Nearly all of the coefficients were insignificant. The reason for this could be the same as mentioned above.

Our next regressions investigate more precisely the presence of non linear effects in ratings. We first regress our CDS spreads on credit rating dummies representing rating classes (whenever we had enough observations we have done the same test at the one-notch difference instead of the one-class difference with similar results). We have each time omitted the worst rating class so that the constant in the regression represents the coefficient to that class. Table 6 gives the results. On the whole
sample, each rating class appears significant except for one. High ratings have a negative coefficient as expected (lower spread than the worst class) but non linearities appear (the top 3 classes do not differ much from each other while the lower classes clearly differ from the higher ones). Nonetheless some incongruities (such as the fact that the BB class requires a higher spread than the B class) may be linked to the mix of truly different data such as sovereign, US and non US corporates. Regressions on each sample separately show strongly different behaviors for each subsample. In US corporates, there is a clear distinction (that we will use further) between high (AAA to A) and low (BBB and lower) ratings, with seemingly no strong difference between the high ratings (except for the AAA class that stands somewhat out) and no strong difference between the low ratings (although these non significant differences are ordered in means as expected). This further reveals the strong threshold effect that exists in US corporate ratings.

Ratings’ impact on non-US corporates spreads is somewhat more erratic and bears less explanatory power, confirming results already found in the linear expression. Sovereign underlyings produce an almost picture perfect regression of what would expect from ratings’ impact, with a significant impact of each class, close to linear differences from one class to the other (except for the closeness of the two top classes) and a strong overall explanatory power. Ratings do matter for sovereign underlyings, their impact is consistent with common expectations and threshold effects do not seem as important as for US corporates.

Overall, it is remarkable that with ratings only we are able to explain up to 68 percent of the variation in the US corporate subsample and close to this for sovereign. It is also noticeable that our subsamples vary widely in behaviors, US corporates presenting a clear threshold effect in ratings, sovereign being influenced quasi linearly and non US corporates presenting the least clear relationship to ratings. And it becomes very clear that large variations in the sample, and notably in the corporate samples, are not explained by the rating. Investigating how successful other variables will be at approximating Credit Default Swap rates remains thus important.

7.2 The US Interest Rate, the Slope of the Yield Curve and the Credit Spread

Most of the empirical papers predict an increase in the credit spread if the level of interest rate decreases. Other interest rate variables can be considered for which interpretation may be more complex. The influence of the slope of the yield curve can be seen as a proxy for the state of the economy. A steeper term structure of interest rates is associated with an improvement of the business
climate while a flatter term structure would be associated with a decrease in the economic activity. We have also considered the spread between long term AAA corporate bonds and long term government bonds which is a direct measure for the riskiness of this rating class in the US (and thus a measure of minimal credit risk).

Table 7 depicts the correlation of the three interest rate variables for all the groups.

Due to the fact that all these three variables are quite strongly correlated we might face the problem of multicollinearity. As is typical in such a case the three variables seem to be insignificant based on individual t-tests, but the whole group is highly significant. In order to obtain more reliable estimates of the influence of any of the three interest rate variables, we estimate an additional set of regressions with only one of them at a time. Table 8 shows the result for the whole sample as well as US corporate, non US corporate and sovereign sub-samples.

The whole group of interest rate variables is highly significant as shown by a Fisher test as well as a Wald test, but the individual variables appear to be insignificant when included at the same time. However all the interest rate variables are highly significant when used one by one in the regressions. This is probably linked to the high multicollinearity exposed in Table 7.

The short rate is negatively related to the CDS rate. Higher interest rates lead to lower CDS rates, or to lower credit risk. An interpretation for this could be that the interest rate is correlated to positive macroeconomic prospects. Interestingly though, the impact of the short rate on the CDS rate of US corporates is much higher for highly rated corporates than for low rated corporates where it becomes insignificant. This may be linked to the fact that low rated corporates are very sensitive to their financing costs which increase significantly as rates increase. It shows that interest rates’ impact on credit risk matters overall but is rather complex.

The AAA spread has a positive sign as expected. Therefore we confirm the findings of previous empirical research for the US sub-sample. The adjusted $R^2$ of the regressions increases significantly due to the inclusion of the interest rate variables. Therefore a pricing model of credit default swaps should include at least some information related to the rating and the interest rate environment. However all the three interest rate variables have similar explanatory power for the US sub-sample.

Only the US short rate is significant when we use all the three variables in the same regression. The whole group on the other hand has a significant influence in all the sub-samples. The individual regressions show a different picture. For the whole sample all of the interest rate variables are significant except the US slope in the linear regressions. For the non US corporate and the sovereign underlyings only the US short rate remains consistently significant. The US slope and the AAA spread
lose their significance. This result is interesting as it indicates that for non US underlyings the US rate matters in a sense as the world’s risk free rate, but the slope and the spread, which we interpreted as proxies for the state of the US economy and the US credit risk, have no explanatory power. This keeps confirming the differences in behavior between US and non US Corporates.

7.3 The Influence of Local Interest Rate Factors

We have found evidence that the slope of the US yield curve is significant for the US sub-sample. However it is not significant for non US underlyings. We want to test for the influence of non US interest rates. We have tested for the influence of the levels and the slopes of the yield curves of the following countries: Australian rates for Australian companies, Japanese rates for Asian companies, German rates for countries of continental Europe and British Pound rates for underlyings in the UK. The rates and the slopes of the individual country rates are highly correlated and we encounter the same kind of multicollinearity problems as with the US interest rate. Therefore we restrict ourselves to use only the level of the US short rate and the slopes of the individual country yield curves. The results of the regressions are shown in table 9.

In general we find that the level of the short term US rate remains significant throughout all the sub-samples. All the additional yield curve slopes are significant except the Japanese slope. The Japanese slope is significant for the corporate sub-sample including or excluding US corporations. It is however not significant for the sovereign and the overall sample. The reason for the insignificance in the sovereign sub-sample might be due to the fact that there are only five observations of credit default swaps on the Japanese government. The signs of the coefficient are as expected for the Australian and the German slopes. They are positive and thus support the view that a steeper yield curve is associated with improving business conditions, and thus associated with lower credit risk. The coefficient of the Japanese slope changes sign but is insignificant and thus we can not draw any additional conclusions. The sign of the UK slope is negative, while we observed an inverted term structure for the whole sample period in the UK.

We find evidence that the US interest rate has a strong influence on credit default swap rates even after controlling for the effects of the local term structure. The slope of the local term structure adds additional information. We have interpreted the slope of the yield curve as an indicator of future economic conditions. As the US and the rest of the world are not at the same stage in the business cycle, the economic outlook for the various economies is different. The finding that the slope of the local interest rates matter is consistent with this interpretation.
7.4 The Variance

Although we do not test for any specific models in this part of the paper, it is interesting to investigate further stylized facts expected from theoretical models. Most of the structural form models predict that an increase in the variability of the firm’s asset value leads to higher credit risk. We use the historical variance as a proxy for the variability of the firm value. All the variables used so far were not firm specific. Even if the rating encompasses theoretically all the relevant information contained in the stock price, it is a very sluggish measure of credit quality. The information contained in the stock price gets updated almost instantly on the arrival of new information relevant to the underlying firm. Therefore we expect that the variance and the other stock price related variables add information because they are firm specific and up to date. Variance and other stock related variables are at the center of the structural form models of credit risk (and can be incorporated in reduced form models). Table 10 depicts the results of the regressions.

The influence of the variance of the underlying stock price is positive and significant for all the sub-samples, whether US or non US Corporates, in the linear or semi log specification. The coefficient is higher for the non US corporates. The contribution to the overall explanatory power of the regression is quite high on an already remarkable level. The result is a clear indication that firm specific information matters and the prediction of the structural form models is verified in our sample. Firm volatility adds information beyond what is captured in ratings for the pricing of Credit Default Swaps. We have also tested for different shapes for the relationship. For example, we have tested the influence of the volatility with and without the variance. If the volatility is included instead of the variance it is significant and has a positive coefficient. In combination with the variance both are significant. However the volatility has a negative coefficient. We have tested for the influence of the variance raised to a higher power. However these terms are not significant beyond the variance.

7.5 Stock Price Changes

The variance measures the overall variability of the past stock prices. It contains no information if the future prospects of the firm have improved or worsened. A booming firm can have the same variance as a firm on the verge of bankruptcy. The influence of past stock prices can be seen from several perspectives. A decrease in the stock price will lead to a smaller equity and without a reduction in the amount of outstanding debt to an increase in leverage. On the other hand a decline in the stock price can indicate a worsening in the prospects of a firm. The third possible influence is from the demand
side. A decline of the stock price might induce bondholders to seek protection from credit risk. A stronger decline might lead to more demand and thus raise the price, e.g. the credit default swap rate.

In order to test for the information contained in the returns on the stock prior to the trading date of the credit default swap, we include the market adjusted change in the stock price during the year before the trade. We adjust the change in the stock price for the change in the associated market index.

We have tested different time intervals to calculate the return on the stock. In general the longer the calculation period the more significant the influence. We adjust the returns on the stock for the return on the market in order to control for market wide changes in the economy. The results obtained with or without the market adjustment do not vary significantly. Moreover the changes in the stock price remain significant and of the same sign. Market adjusted results are somewhat more significative though, showing that underperformance related to the market may drive credit risk somewhat more that absolute underperformance. Table 12 shows the results of the estimation using market adjusted changes.

The change in the stock price is significant for all the subgroups. The 1 year change has a negative sign pointing to the fact that an increase in the stock price is associated with a lower credit risk while a decrease leads to a higher risk.

### 7.6 Leverage

One of the interpretations of why past stock returns may have explanatory power for the credit default swap rate could be linked to their effect on leverage. Most of the structural form models use the ratio of market value of equity and debt value as an indicator of default probability (distance to default). We restrict ourselves to the US sub-sample for the following estimation for two reasons. Leverage is strongly dependent on the country and the industry, for accounting, tax and agency reasons. A country with large conglomerates including a house bank may view a higher leverage as more acceptable than an economy with independent banks for agency reasons. Secondly as the debt value is almost always produced as an accounting number it depends strongly on the accounting principles used in every country. In order to have a somewhat reliable number we restrict the following estimation to the US sub-sample hoping the US accounting principle (US GAAP) provides a reliable estimate of the true leverage.

The results (table 13) show that leverage has a significant influence on the credit default swap rates in our sample beyond the rating, interest rate and volatility information. More surprisingly we note that the influence of the stock price changes remains significant even after controlling for the
leverage effects. Overall, including the different variables, interest rate variables as well as the firm specific variables such as variance of the value of the firm’s assets, leverage and past stock price evolution improves the adjusted \( R^2 \) to a high level of 78\% in the linear form. This result confirms that the variables that were provided by theoretical models do explain most of the variation in credit risk pricing.

Nonetheless, the significance of past stock price changes is puzzling. It seems to point to the presence of behavioral, momentum like issues in credit risk markets. (One rational markets explanation would be that our past price changes variable captures changes in volatility that our historic volatility measure cannot capture. We have tried to correct for this with some companies for which implied volatilities or some form of correction were available but have not resolved the problem.)

7.7 The Impact of Liquidity as Proxied by Market Capitalization

As Credit Defaults Swaps constitute an exotic market where pricing is often attempted by replication and thus affected by the existence and the liquidity of the considered securities, we try to capture liquidity effects by a market capitalization proxy. Tables 14 and 15 show the results of the regressions.

The proxy is significant for high rated US corporates but not for the low rated ones. This could be a sign that liquidity effects are leading differences in high rated US corporates. Nonetheless, interpretation is not secure as significance drops in multivariate regressions, notably when they include leverage. Overall, liquidity effects as measured by this proxy do not seem to impact prices significantly in this market.

7.8 The Overall Impact of Structural Variables versus Ratings

We further explore how structural variables (variance, leverage, time, and interest rate) fare in predicting CDS rates versus ratings (and time). Ratings are the most used source of information for credit risk and remain overall the single most useful source of explanatory power in our regressions indeed.

When considering the overall sample of US Corporates (table 16), ratings provide for a \( R^2 \) of 47\% in the linear form while structural variables provide for 31\%. These structural variables are not subsumed in ratings as combining them with ratings increases the explanatory power significantly to 65\%.

Nonetheless, when considering subsamples, the picture changes and the importance of considering structural variables becomes even stronger. Looking at high rated US corporates, ratings have very low
explanatory power (4%) and this explanatory power is subsumed in size differences as discussed above. On the other hand, structural variables (notably with the leverage variable) keep a high explanatory power of 48%. Regarding low rated corporates on the other hand, ratings do matter significantly while structural variables (notably the variance variable) help boost the explanatory power, thus further illustrating the strong threshold effect that affects US corporates.

Notice that contrarily to previous studies (Collin-Dufresne et al. (2001) and others), we find that equity market information matters for both high and low rated companies. It adds information to other variables (and notably ratings) in both cases, although its impact differs in both situations.

We have still a different picture for non US corporates, illustrating once more the non homogeneity of credit risk and its complexity. Ratings in non US corporates matter and provide most of the explanatory power for the high rated corporates while most of the explanatory power is provided by structural variables (variance) for low rated corporates. Overall though, this confirms the importance of considering structural variables (or equity market information) in determining credit risk spreads.

Reduced form models, which have many advantages compared to structural models, notably as far as implementation is concerned, should thus incorporate structural variables in some way or another (e.g. Jarrow and Turnbull (2000)). Structural models, on the other hand, can keep instructing us on the theoretical shape of the relationship between structural variables and credit risk.

8 A Reduced-Form Model Implementation and Consequences

A more sophisticated way of checking whether fixed income information is sufficient to price credit risk is to implement a model using all available information on interest rate and bond spreads. While the simple econometric analysis provided above allows for many results, it is indeed instructive to fit an actual model of credit default swaps and analyze the errors provided. While no current model dominate all others, the best fitting ability seems to be obtained by reduced form models. While these can be made more complex to accommodate for equity markets information, we hereby proceed to what they are first designed to do, i.e. insure compatibility of information in fixed income markets, notably between bond spreads, risk free rates and credit default swap rates.

We hereby implement a reduced-form model that relies on fixed income market information. We use a classical reduced form model that relies on dynamics for credit spreads rather than on a default process, the Das and Sundaram (2000) model. Houweling and Vorst (2002) fit a default-based reduced
form model on quotes and refine their technology in order to obtain the best possible fit (by choosing a different risk-free term structure methodology depending on the credit quality). Our goal is not as much to fit the model as to check whether errors obtained from an advanced fitting from fixed-income market data can be explained by equity market data.

The idea of the Das-Sundaram model is to construct a tree that provides for the evolution of the riskfree interest rate as well as the evolution of the credit spread. We assume a stochastic process for the forward spread curve as well as for the forward interest rate. Therefore, this is a tree model with four branches emanating from each node. At each node we have information about the forward rate, interest rate, spread, risk neutral probability of default and recovery rate\(^2\), all the necessary elements to compute our CDS price. In what follows, we describe the inputs necessary for this model.

### 8.1 Inputs of the Model

#### 8.1.1 Interest Rate

The first thing we have to model is the evolution of the riskfree interest rate. Das-Sundaram allows to use the Heath, Jarrow and Morton (1992) model for the riskfree term structure. There are obvious advantages of using this type of model. The model matches the initial term structure by construction and does not require any assumption on the preferences. It is also important to notice that, in HJM models, the evolution of forward interest rates of all maturities is simultaneously and exogenously specified, while in spot interest rate models, only the evolution of the spot interest rate is directly modeled.

The forward rate process in discrete time is described by:

\[
    f(t + h, T) = f(t, T) + \alpha(t, T)h + \sigma(t, T)X_1 \sqrt{h}, \forall T \geq t
\]

where \( X_1 \) is a random variable, \( h \) is the discrete time interval and \( f(t, T) \) is the one period forward rate at time \( t \) starting at time \( T \), \( \alpha(t, T) \) is the drift of the process and \( \sigma(t, T) \) its volatility. The instantaneous spot rate therefore is:

\[
    r(t) = f(t, t) = f(0, t) + \sum_{j=0}^{\frac{t}{h}-1} [\alpha(jh, t)h + \sigma(jh, t)X_1 \sqrt{h}]
\]

From the HJM model we know that the risk-neutral drift term, \( \alpha(. \) ), is completely determined by

\(^2\) It is assumed that the Recovery Market Value hypothesis holds, i.e. in default, a zero coupon risky bond is worth a fraction of its market value.
the volatility terms at each date.

\[ T/h = \sum_{k=t/h+1}^{T/h-1} \alpha(t, kh) = \frac{1}{h^2} \left( E^t \left[ \exp \left\{ - \sum_{k=t/h+1}^{T/h-1} \sigma(t, kh) X_1 h^{3/2} \right\} \right] \right) \] (17)

Hence, in order to fit this model to reality, the main concern lies in finding the volatility parameter. There are several ways of modeling this volatility. To simplify the implementation, we consider \( \sigma(.) \) as a constant, which implies the Ho & Lee (1986) model. We estimate this constant volatility using past data. The choice of past data is not arbitrary: if we were going to estimate the volatility implicitly, we would have to use very liquid derivatives, deep in the money. These instruments would probably present noise that should be avoided through cross section averaging in order to have coherent estimators. Unfortunately, these instruments are not available and we have to follow the common practice of using past data.

Our information consists of daily Treasury bond data for the five years prior to the date of the contract we want to price, extracted from the Research Department at BNP Paribas. We also obtain from the mentioned institution the daily yield curve through maturities of 6, 12, 18, 24...60 and 66 months. When these maturities are not available in the bond data, bootstrap methodology is used to compute the desired rate at the selected maturity.

Once we have the yield curve for every day, we compute the forward rates. The daily changes in forward rates for the previous five years to the date of the CDS are used to calculate the volatility.

At the same time we also compute the initial forward rate curve for all the dates of the contracts.

### 8.1.2 Forward Spread Curve

Let \( \varphi(t, T) \) be the forward rate on the risky bond implied from the current yield curve for risky bonds. The forward spread, \( s(t, T) \) on the risky bonds is defined as:

\[ s(t, T) = \varphi(t, T) - f(t, T) \] (18)

As in Das-Sundaram, we assume that the forward spread curve, \( s(t, T) \), follows the discrete stochastic process:

\[ s(t + h, T) = s(t, T) + \beta(t, T) h + \eta(t, T) X_2 \sqrt{h} \] (19)

where \( X_2 \) is a random variable which can be correlated with the interest rate, \( \beta(t, T) \) is the drift term and \( \eta(t, T) \) the volatility. The instantaneous spot spread can be written as:
The drift term is fully determined by the volatility as shown in the equation

\[
s(t, t) = s(0, t) + \sum_{j=0}^{T/h-1} [\beta(jh, t)h + \eta(jh, t)X_2 \sqrt{h}]
\]

(20)

Like before, we need to estimate the volatility of this process and the initial forward spread curve. Obviously, we have to use past data. It is more convenient to model the spread rather than modelling the yield of the defaultable security and take the difference with respect to the yield of the riskless bond. Two main reasons can be argued: We can avoid carrying the errors from the two models (for the risky and for the risk-free yields) and if we fit the model with observed spreads, these can take into account the liquidity effect.

In theory, we should compute this forward spread for every particular firm. Unfortunately, we cannot do this in practice because firms do not issue enough bonds to construct a yield or a forward rate curve. A possible solution is to regroup firms with similar characteristics in order to complete the lack of information. We choose this approach. We will base our criteria to choose similar firms in their rating and in their sector. We select the rating because it is a major indicator of the probability of default. The sector is chosen because Altman & Kishore (1996) and Altman et al. (2000) show that the recovery rate is strongly influenced by the sector the firm belongs to. In general, industry seems to have a strong impact on spread behavior.

We obtain from the Research Department at BNP Paribas the daily rate curves for the groups we are interested in. The way of constructing the curve is similar to the one described for the riskless curves. With the daily yield curves we calculate the implicit forward rates for the risky bonds. To compute the credit spreads, we subtract the riskless forward rates. We obtain in this way a daily risky yield curve for every rating and sector we are interested in. Unfortunately, we did not obtain long time history for these curves but one year history. Therefore, in order to compute the volatility factor of the credit spread curve, we have to use in-sample estimation: we compute the volatility of the forward rates at the same time the CDS takes place.

8.1.3 Probability of Default

In order to obtain the probability of default, we rely on the ratings of the firm that issues the underlying bond. In order to keep our model simple, we use the data provided by the rating agencies, Standard &
Poor’s and Moody’s. They compute cumulative probabilities of default, averaged over the last twenty years.

For example, we have the following table of cumulative probabilities of default in 1998 (Source: Moody’s)

<table>
<thead>
<tr>
<th>Rating</th>
<th>1 year</th>
<th>2 years</th>
<th>3 years</th>
<th>4 years</th>
<th>5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.04%</td>
<td>0.14%</td>
</tr>
<tr>
<td>Aa</td>
<td>0.03%</td>
<td>0.05%</td>
<td>0.10%</td>
<td>0.25%</td>
<td>0.39%</td>
</tr>
<tr>
<td>A</td>
<td>0.01%</td>
<td>0.06%</td>
<td>0.21%</td>
<td>0.37%</td>
<td>0.54%</td>
</tr>
<tr>
<td>Baa</td>
<td>0.12%</td>
<td>0.39%</td>
<td>0.75%</td>
<td>1.26%</td>
<td>1.70%</td>
</tr>
<tr>
<td>Ba</td>
<td>1.34%</td>
<td>3.71%</td>
<td>6.21%</td>
<td>8.77%</td>
<td>11.44%</td>
</tr>
<tr>
<td>B</td>
<td>6.78%</td>
<td>13.19%</td>
<td>19.13%</td>
<td>24.11%</td>
<td>28.59%</td>
</tr>
</tbody>
</table>

These probabilities of default are based on historical defaults. However, since our model is set up in a risk-neutral world, we cannot use directly the probability of default estimated from historical data.

Das and Sundaram (2000) show that if we assume a non-stochastic recovery rate, we have the following relationship between the risk neutral probability of default \( \lambda(t) \) and the actual probability of default \( \lambda^p(t) \):

\[
\lambda(t) = \lambda^p(t) \left[ \frac{1 - \exp(-s(t,t)h)}{1 - \exp(-[s(t,t) - \xi(t)]h)} \right]
\] (22)

with
- \( \lambda^p(t) \equiv \) actual probability of default.
- \( \lambda(t) \equiv \) risk-neutral probability.
- \( \xi(t) \equiv \) premium for bearing default risk at time \( t \).
- \( s(t,t) \equiv \) spread at time \( t \).

The recovery rate can also be obtained as:

\[
\phi(t) = \frac{1}{\lambda^p(t)} \left\{ \exp(-[s(t,t) - \xi(t)]h) - 1 + \lambda^p(t) \right\}
\] (23)

Since the recovery rate is determined by the model, we avoid its parametrization. This is appreciated because there is a considerable lack of empirical studies on recovery rates.

Das and Sundaram (2000) assume that \( \xi(t) = \pi s(t,t) \) where \( \pi \) is a constant and \( s(t,t) \) is the instantaneous spot spread. This is analogous to the specification in the Cox-Ingersoll-Ross model of the term structure where the risk-premium is proportional to the short rate. We also follow this methodology. It may be worth noting that the risk premium \( \xi(t) \) can be also estimated from the relation between actual probability and the recovery rate, which we derived in (23). To follow this last
method, we should model the recovery rate.

8.2 Implementing the Model

Das and Sundaram (2000) assume that the random variables \( X_1 \) and \( X_2 \) follow a binomial distribution with values 1 and -1. Each variable can take these values with probability \( \frac{1}{2} \). We assume that the correlation between the variables is \( \rho \). Therefore, the joint distribution of \((X_1, X_2)\) is:

\[
(X_1, X_2) = \begin{cases} 
(1, 1), \text{w.p. } \frac{1+\rho}{4} \\
(1, -1), \text{w.p. } \frac{1-\rho}{4} \\
(-1, 1), \text{w.p. } \frac{1-\rho}{4} \\
(-1, -1), \text{w.p. } \frac{1+\rho}{4}
\end{cases}
\]

At each node, we have information about the interest rate, the spread rate and the risk neutral probability of default. Hence, we have enough information to price our CDS since the recovery rate can be found as a function of the above mentioned variables.

Note that we will have as many trees as CDS to price. What we have to remark is that:

- The interest rate described in the trees will be different depending on the day. This is because every tree uses a different initial forward rate curve, unless the CDS have same quoted date.
- The spread between trees will be different depending on the rating and the sector of the underlying.

If default does not happen, the buyer of the CDS pays their swap rate every six months. We consider this a good time step for our tree model. Lower time step could increase significantly the timing of computing the price. The algorithm used to compute the price is exactly the same as described in Das and Sundaram (2000), with the difference in how to model the inputs used in the tree. This is a recursive algorithm which is a function of itself at each step. Recursion is available since the lattice satisfies no-arbitrage conditions at each node. Hence, each subtree on the lattice may be treated separately. Knowledge of the values \( F(\tau), S(\tau), \lambda \) and \( \phi \) allows a direct extension into the next period’s node, where \( F(\tau) = (f(\tau, \cdot)) \) represents the forward interest curve and \( S(\tau) = (s(\tau, \cdot)) \) represents the forward spread curve.

At the initial node are current values of the forward rate curve and spread curve \((F(t) \text{ and } S(t))\). From the initial node, four branches emanate (see figure 1). At the end of each of these branches is another set of \( F(t+h) \) and \( S(t+h) \), and so on until the terminal nodes are reached. The recursive scheme contains two components i) the recursive call and ii) the terminating conditions. The recursive function calls itself by passing the next periods values \( F(t+h, T) \) and \( S(t+h, T) \) to itself. In between, the forward values are calculated ensuring that the conditions of no-arbitrage are met, i.e. the
function solves for the risk-neutral drift parameters ($\alpha(.)$ and $\beta(.)$). At each node, there exists the probability of ending the contract (default) or continuing to maturity.

Information generated at each node

8.3 The Data

To have compact data, we select only CDS with US underlyings and with maturity time of 5 years. We then restrict ourselves to companies that we can group and obtain spread information on. Because of this, we consider groups homogenous in rating and in industry. Because of data requirements (notably spreads for industry groupings), we actually fit the model on 75 credit default swaps of different industries and different ratings.

We have formed five groups:

<table>
<thead>
<tr>
<th>Industry</th>
<th>Rating</th>
<th># observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto</td>
<td>A</td>
<td>21</td>
</tr>
<tr>
<td>Bank and Finance</td>
<td>AA</td>
<td>10</td>
</tr>
<tr>
<td>Bank and Finance</td>
<td>A</td>
<td>31</td>
</tr>
<tr>
<td>Industrial</td>
<td>AA</td>
<td>7</td>
</tr>
<tr>
<td>Tobacco</td>
<td>A</td>
<td>6</td>
</tr>
</tbody>
</table>

We present below the descriptive statistics for actual credit default swap rates for the groups we have selected:

We observe that the higher prices are those with a bond from a tobacco company as underlying, followed by the auto industry. We can also observe that the highest prices are related to a lower rating. These are some of the characteristics that our model should capture.
8.4 Results

As we presented before, to find our theoretical prices we need to find the term structure of the riskless forward rates, the term structure of the credit spreads and the correlation between them. The first two were obtained from the Research Department at BNP Paribas. We obtained the daily yield curves and the corresponding values for each of the above mentioned groups (except for the Bank & Finance rated A). We then construct the forward curves for the Government bonds and for the corporate bonds. The forward credit spread is just constructed as the difference between them.

We provide for correlation coefficients per industry/group rating between spreads and interest rates (table 18). Duffee (1998) finds that for the period 1985-95 the correlation is around -0.2, whereas Altman and Kishore (1996) report a correlation of -0.07 for the period 1978-1996. Except for the case of banks and finance sector, our correlation is within the range of both papers.

Table 19 shows us that our theoretical prices are able to capture, in part, the tobacco effect, i.e. the highest prices are also in the case where the underlying is a bond from a tobacco company. The second highest is the auto industry. However, we can notice a change of order. In the real prices, the mean of the CDS with a bond from bank-finance A sector was higher than that from industry AA sector. In the theoretical model, this order is reversed. This could be due to the high credit spreads for the industrial sector as well as for a not very accurate fit of the credit spread for the bank-finance A group. We notice that real prices are in general bigger than theoretical ones.

We should remark here that the theoretical prices were very robust to changes in parameters such as π, volatility of the forward rate, volatility of the forward credit spread and probability of default (if it was kept in a rational level). However, they were quite sensitive to changes in the forward credit spread input.

8.5 Explaining the Model’s Errors

We next proceed into explaining the model’s errors. The errors’ descriptive statistics are provided in table 20.

We then regress these errors on the fundamental variables that we have analyzed the impact of during the first part of this paper (table 21). Because of data constraints, we have simplified ratings and have not considered notch differences. We check that this rating simplification explains for part of the errors. Deepening on the specification, we obtain between 26% and 35% explanatory power of more precise rating considerations.
We also check that interest rate dynamics have been well integrated in the model. Indeed, short term US rates, long term rates or the slope of the yield curve have no explanatory power per se on the errors of the model (results not reproduced here but available from the authors). The sophisticated modelization of the interest rate dynamics has thus been successful at capturing the impact of interest rates on credit default swap rates.

Finally, we check for the impact of equity markets information on our model’s errors. Results are significant both statistically and economically, especially for variance and one year changes. Equity market information provides for close to 50% explanation of the model’s errors. Leverage does not have much explanatory power here (while it has on a stand-alone basis). Strong correlations in our sample between leverage and variance, as documented in the correlation table, may explain this partly. Interestingly, bond spreads, even modelled through a sophisticated approach, thus need to be completed with equity market information in order to price credit default swaps well. This may well be a confirmation of market segmentation problems in fixed income markets as suggested in Collin-Dufresne and alii (2001). This result confirms the need to go beyond bond spreads to study credit risk. It also confirms the importance of the credit derivatives market to understand credit risk empirics.

9 Conclusions

We have investigated the influence of various factors on Credit Default Swap rates and therefore on credit risk as reflected in a recent credit derivative market. Credit Default Swap rates can arguably be considered the best pricing information we have on credit risk, at least when transaction rates are considered (as quotes may vary from actual transactions, especially in an exotic market). They do not suffer from the well-documented limitations of bond spreads as a measure of credit risk. This paper reflects the complexity of credit risk, a risk that is not homogenous amongst underlyings (whether sovereigns or corporates, US and non US based).

Starting from theoretical models, we have identified some factors that should influence the CDS rates. We have compared econometrically the influence of those factors on various subgroups in our sample. We find that all of the theoretical factors have a significant influence and that taken together these factors drive much of the variation in the pricing of Credit Default Swaps (up to 82% despite our probably misspecified linear form).

The rating is the most important single source of information on credit risk overall, although all the
other factors add significant information to ratings and matter more than ratings for some subgroups. This latter fact stresses the importance of alternative theories such as structural form credit risk theories (and the need to incorporate such variables in reduced form theories). Ratings themselves can have strong threshold effects as we find that the sensitivity of the level of credit default swap rates to ratings is different for high rated debt and for low rated debt. Hence a difference in rating for high quality US corporate underlyings can have close to no explanatory power while structural variables such as market value leverage do have explanatory power. In general such structural variables (variance notably for low rated underlyings) have explanatory power and are not subsumed in ratings, and this for both high rated and low rated underlyings. Sovereign and Corporates have different sensitivities to rating information and US and non US corporates show different behaviors in relationship to ratings (with threshold effects seemingly not as significant for non US corporates as for US corporates). US interest rates influence the credit default rates of all the subgroups, e.g. US rates do matter for credit default swap rates of underlyings from other countries. For US companies the level, the slope of the yield curve and the spread matter. For non US underlyings the level of the US rates is important, however the local slope of the yield curve matters more than the US slope. Considering the slope of the yield curve as an indicator of future economic activity this points to the fact that default is linked to the performance of the local economy, as would be expected. We thus have further evidence that international credit risk markets are not homogenous and that national markets may differ both in behavioral structure (thresholds) as well as in the variables to consider (local term structures). As predicted by structural models the variance of the stock price is positively linked with the default swap rates. It adds specific information beyond what is contained in ratings and thus should be considered when estimating credit risk. Declines in the stock price are positively associated with the observed default swap rates. We find evidence that the size and the direction of the change in stock prices matter, which seems to point out to the presence of behavioral issues in credit risk markets. Further in line with structural form models we find that leverage has a significant influence on the default swap rates. Even when we control for leverage and market index changes the influence of past stock price changes remains significant. Liquidity effects as measured by market capitalization do not seem to matter. Overall, considering stochastic interest rates and what we call structural variables remains a necessity beyond the use of ratings for a good approximation of credit risk, and that whether high or low rated companies are considered. For high rated companies, ratings explain very little and structural form variables seem the only good source of information while for low rated companies, both ratings and structural form variables have strong and separate explanatory power.
By fitting a reduced-form model, we go even further into investigating the impact of the different variables on credit default swap rates. We notably look for what could drive the errors of the model. There again, we find that equity markets information, such as volatility, provides for a large component of the answer. Introducing straightforward equity markets’ information provides for close to 50% explanation of the errors of the model. This seems to confirm the presence of market segmentation in fixed income markets and stresses the importance of considering equity information when assessing credit risk.
10 References


