Working Paper
Series

FINRISK
National Centre of Competence in Research
Financial Valuation and Risk Management

National Centre of Competence in Research
Financial Valuation and Risk Management

Working Paper No. 14

The Impact of Clearing on the Credit Risk of a Derivatives Portfolio

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First version: June 2002
Current version: September 2002

This research has been carried out within the NCCR FINRISK project on "Conceptual Issues in Financial Risk Management".

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Abstract

Most derivative contracts are traded over-the-counter, i.e., bilaterally between two counterparties. Recently, clearing services have become available that allow to transfer over-the-counter derivatives to a central counterparty (clearing house). We develop a framework to determine the effects of clearing on the credit risk of derivatives portfolio. To facilitate our analysis, we compare the cost of credit risk of a portfolio of over-the-counter derivatives and a portfolio of similar but cleared derivatives. We show that, under certain verifiable assumptions, the cost of credit risk of a portfolio of cleared derivatives is always lower or equal than the cost of credit risk of a portfolio of un-cleared derivatives. We further show that, under the same assumptions, clearing enhances the value of a portfolio. The theoretical analysis is applied in a numerical example on a portfolio of swap contracts.

JEL classification: G13, G19, G21
Keywords: Derivative contracts, default risk, portfolio credit risk, cost of credit risk, clearing, central counterparty

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1 Introduction

Today, derivative instruments are traded in two rather dissimilar ways. Either they are traded over-the-counter (OTC), i.e., directly between counterparties. Or they are traded on a regulated exchange. According to a recent study by the Bank of International Settlements, about 80% of derivatives are currently traded OTC whereas less than 20% are traded on exchanges.[1]
Derivative instruments traded OTC are executed with arbitrary counterparties in the market. As these counterparties might default during the life of the derivative contracts, these derivatives inherently bear (counterparty) credit risk, or default risk.[2] [3]
Regulated banks are required to support the default risk borne by a derivatives portfolio by capital. Obviously, the provision of capital is costly. These costs, related to the credit risk of a portfolio of OTC-traded derivatives, need to be considered and are at the center of this paper.[4]

The most significant difference between OTC- and exchange-traded deriva-

[1]The percentage figures are based on notional amounts. Cf. Bank for International Settlements (2002).

[2]In what follows, we use the terms (counterparty) credit risk and default risk interchangeably. Credit risk in often defined in a broader sense including lending risk and issuer credit risk. As derivative portfolios typically do not bear lending or issuer credit risk, credit risk and default risk in this case are the same. Cf. Picoult (2001).

[3]Credit risk, or default risk, can be split up into settlement risk and pre-settlement risk. Settlement risk refers to the risk of a counterparty failing to meet contractual obligations *at* settlement. Pre-settlement risk refers to the risk of a counterparty failing to meet contractual obligations *before* settlement. Settlement risk arises especially in relation to contracts that include an exchange of the underlying at maturity, which is often the case with foreign exchange or commodity derivatives. The more important of the two components, however, is pre-settlement risk. Pre-settlement risk is sometime called replacement cost risk. Cf. Bank for International Settlements (1998).

[4]Cf. Picoult (2001).

tives is that the latter are cleared by a central counterparty (clearing house). In other words, derivative contracts traded on an exchange are executed with a single counterparty, the clearing house, which processes all transactions and guarantees performance.[5]

Historically, clearing has almost exclusively been available for exchange-traded contracts. Recently, however, some clearing houses started to offer clearing services for derivatives that are traded OTC.[6] This means that a derivative can be transferred to a clearing house after it has been traded by two counterparties. It is then processed similar to an exchange-traded instrument. Clearing houses offering such services claim that they significantly reduce credit risk of derivative portfolios, thus reducing economic capital requirements and costs.

Clearing services, however, do not come for free. They trigger costs in the form of clearing fees and charges for collateral which has to be provided to the clearing house. A market participant therefore faces the question whether clearing of OTC-traded contracts is economically beneficial or not.

The aim of this paper is to analyze the impact of clearing on the cost of a derivatives portfolio. More specifically, we analyze how clearing affects the cost of credit risk of a given portfolio of derivatives. We also consider the

[5]Central counterparties are either part of a derivatives exchange or independent entities. Examples of the former are the CME clearing division and Eurex Clearing; an example of the latter is the London Clearing House.

[6]At the time of writing, such services were offered by the London Clearing House for swap and repo contracts, as well as for certain commodity contracts; services for other contracts were in preparation. Several organizations offered or were preparing to offer services for electricity and natural gas contracts, including the Board of Trade Clearing Corporation, the London Clearing House, Clearing Bank Hannover, European Energy Exchange, Nordpool/NOS, UKPX (OM).

effect clearing has on the value of such a portfolio, (partially) offsetting the costs of clearing. This last point is particularly important. Clearing does have an effect on the value of a derivative instrument due to the change in counterparties.

We start our analysis by briefly reviewing current approaches to the valuation of derivatives subject to default risk (Section 2). We introduce a model that reflects the credit qualities of counterparties and allows us to calculate the effect of changes in counterparty credit quality on the value of a derivative instrument. Having determined the value of a single derivative instrument, we determine the value of a portfolio of derivative instruments with an arbitrary number of counterparties. The value of the portfolio is the basis for further analysis.

In Section 3 we define the notion of credit risk of a portfolio of derivative instruments. There are many definitions of credit risk floating around in the literature, so we consider it necessary to "produce" our own. We recall the concept of coherent risk measures by Artzner, Delbaen, Eber, and Heath (1999) and utilize the notion of Expected Shortfall.

Based on our measure of credit risk, we introduce the notion of cost of credit risk, which determines the cost of carrying a certain amount of credit risk.

The concepts of Sections 2 and 3 provide a framework for the description of clearing. This is the subject of Section 4.

Section 5 contains the main part of the analysis. Equipped with the concepts of the previous sections, we investigate the effect of clearing on the value of a portfolio of derivatives, its credit exposure, its credit risk, and its cost of risk. We show that, under realistic assumptions, clearing increases the value of a portfolio of OTC-traded derivatives, reduces its credit exposure, reduces its credit risk, and reduces the cost of credit risk. Obviously, these benefits need to be compared to the costs of clearing in order to determine the value of clearing.

We point out at this stage already that a crucial parameter in the evaluation of clearing is the credit quality of the clearing house (reflected by the probability of default). The difference between the credit quality of the clearing house and the credit qualities of the other market participants drives a lot of the results. While we assume that the credit quality of the clearing house is higher than the credit qualities of the market participants during large parts of our analysis, we make some qualifying remarks on this assumption at the end of the section.[7]

Our analysis to this point is theoretical in nature. While developing the theory, we make several assumptions for ease of exposition. In Section 6, we provide a numerical example. We apply our framework to a portfolio of OTC-traded swap contracts. The example resembles clearing services currently available in the market. Section 7 concludes.

[7] With our assumption we are in line with the market. Some CCPs are rated, e.g., the Board of Trade Clearing Corporation in Chicago, which has a 'AAA' rating. Additional comfort is usually gained from the fact that in most cases CCPs are highly regulated by national authorities. On the other hand, a CCP should not be assumed to be default-free.

2 Valuing derivative securities subject to default risk

Valuation models for financial instruments subject to default risk are usually categorized into *firm-value* (*structual*) models and *reduced-form* (*intensity*) models. Our approach is based on the latter class of models, where default is modelled by a bankruptcy process. This process is usually a one-jump process where a jump indicates default. The jump process is driven by a jump intensity, usually called default intensity. One of the first models was proposed by Jarrow and Turnbull (1995). Extensions were developed in Jarrow, Lando, and Turnbull (1997), Das and Tufano (1996), Duffie, Schroder, and Skiades (1996), Duffie and Singleton (1997), Duffie and Singleton (1999), Madan and Unal (1998), and others. A review of the most popular models is given in Bielecki and Rutkowski (2002).

Starting with Johnson and Stulz (1987), some authors have proposed approaches to pricing derivative instruments subject to credit risk, including Hull (1989a) and Hull (1989b). Ammann (2001) gives a comprehensive overview.

More recently, the issue of market incompleteness has been taken into account. It is unlikely that markets are complete in the presence of default risk. In other words, it is unlikely that default risk of any firm can be perfectly hedged by an investment in one or more tradeable securities. It is equally unlikely that the market can easily be completed. Thus, the market is incomplete and the valuation models mentioned above no longer apply, at least not in a straightforward fashion.[8] See Schachermayer (2001) for a recent survey of the related literature. Collin-Dufresne and Hugonnier (2001) propose an approach to the valuation of derivatives subject to default risk, explicitly taking into account the issue of market incompleteness.

Almost all of the approaches proposed so far focus on valuing single derivative securities. The portfolio models in the credit risk literature that have been suggested concentrate on portfolios of loans or bonds rather than on derivative securities, i.e., they do not take into account the specific nature of derivatives. We introduce a model that addresses the specific issues of valuing a portfolio of derivative securities.

2.1 General set-up

We assume a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, endowed with some filtration $\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq T}$, where $T > 0$ denotes the fixed time horizon. $\mathbb{P}$ denotes the physical probability measure, i.e., it is assumed to represent real-world probabilities. $\mathbf{Z}(t) = (Z_1(t), \ldots, Z_d(t))'$ is a d-dimensional vector of of risk factors.[9] All risk factors are assumed to be $\mathcal{F}_t$-adapted.

Our analysis will be conducted from the perspective of a representative agent with some utility function U. The agent aims at maximizing expected utility of her terminal wealth, i.e.

$$\mathbf{E}\left[U\left(e + \int_0^T H_u dS_u\right)\right] \longrightarrow \max!$$

[8] In more technical terms, if the payoff of a contingent claim cannot be spanned by tradeable securities, an equivalent martingale measure, if it exists, might not be unique.

[9] $\mathbf{Z}(t)$ might include any type of risk factors including stock prices, interest rates, macroeconomic variables, etc.

where e is the agent's initial endowment, H is an admissible trading strategy, and S is the vector of price processes of the securities.

The set of market participants, or counterparties, is given by $\{1, \dots, N\}, N \in \mathbb{N}$. The credit quality of a counterparty n at time t is expressed by the default intensity of the counterparty, denoted by $\lambda_n(t)$. The meaning of $\lambda_n(t)$ is described in Sections 2.2 and 2.3.

The value of a derivative security $i \in 1, \dots I, I \in \mathbb{N}$, is modelled as a function of time and $\mathbf{Z}(t)$. Its payoff function is denoted by $\xi(t, \mathbf{Z}(t))$ for some function $f : \mathbb{R}_+ \times \mathbb{R}^d \longrightarrow \mathbb{R}^0_+$, and its value at time t is denoted by $v^d_{in}(t)$. Derivative securities are assumed to be subject to default risk, which is indicated by superscript d. The value of a derivative security $v^d_{in}(t)$ depends on the credit quality of the counterparty, λ_n. A similar default-free security is denoted by $v_{in}(t)$. Note that the values of two similar derivative securities traded with different counterparties are typically not the same.

The (unit) holdings of a counterparty in the different derivative securities at time t are given by an $(I \times N)$-dimensional vector $\theta(t)$. The ith column vector of $\theta(t)$ is given by $\theta_i(t)$, and the nth row vector is given by $\theta_n(t)$. Thus, $\theta_{in}(t)$ denotes the holding in security i traded with counterparty n, where $\theta_{in}(t) \in \mathbb{R}$. $\theta_{in}(t) > 0$ indicates a long position, whereas $\theta_{in}(t) < 0$ indicates a short position. Let $\Theta(t) = \sum_n \sum_i \theta_{in}(t)$. Then

$$\sum_{n=1}^{N} \sum_{i=1}^{I} \frac{\theta_{in}(t)}{\Theta(t)} = 1 \tag{2.1}$$

i.e., the portfolio weights have to sum up to one. The portfolio of a counterparty can be characterized by $\theta(t)$.

The value of the portfolio of a counterparty at time t is given by

$$V(\theta(t)) = \sum_{n=1}^{N} \sum_{i=1}^{I} \theta_{in}(t) v_{in}^{d}(t).$$

Throughout our analysis we ignore transaction costs such as operational expenses, etc. We do include clearing fees, though, as we assume that clearing fees are charged as a compensation for risk transferred to the clearing house.

2.2 Modelling default

In order to determine the value of a defaultable derivative, we need to model the default process relevant to the respective security. We model default by a digital process with one jump in $t \in [0, \tau]$. The jump process is therefore given by the indicator function, i.e.,

$$N(t) = \mathbb{1}_{\{\tau \leq t\}} \tag{2.2}$$

where τ denotes the default event. τ is a stopping time adapted to $\mathcal{F}_t$. $N(t)$ is equal to one if default has occurred by time t, and zero otherwise.[10]

The process $N(t)$ is driven by a compensating intensity, or hazard rate, $\lambda(t)$ such that

$$N(t) - \int_0^t \lambda(s) ds \tag{2.3}$$

is a martingale. The intensity of default, $\lambda(t)$, measures the likelihood of default per unit of time. Typically, $\lambda \in (0, 1)$. $N(t)$ can be shown to be a

[10]The model therefore does not provide for partial defaults.

semimartingale.[11] If $\lambda(t)$ is constant, $N(t)$ is a standard Poisson process; if $\lambda(t)$ varies deterministically over time, then $N(t)$ is a time-inhomogeneous Poisson process; and if $\lambda(t)$ is a random variable, $N(t)$ becomes a doubly-stochastic process, also called a Cox process.[12]

We are interested in the first jump during the time interval $[0, T]$. From the properties of Poisson processes[13], we know that

$$\mathbf{P}(N(u) - N(t) = 0) = \exp\left(\int_t^u \lambda(s)ds\right) \tag{2.4}$$

for $u, t \in [0, T]$ such that $u > t$. Equation 2.4 reflects the probability that default will not have occurred at time u. It is usually called *survival probability.* In the remainder, we set $N(t) = 0$, i.e., we assume that default has not occurred by time t when the derivative security is valued. This condition could easily be relaxed by conditioning on $\tau > t$. However, we do not explicitly state this condition in order to keep notation simple.

2.3 Pricing a defaultable security

Let $\xi^d(T, \mathbf{Z}(T))$ denote the *actual* payoff function of a derivative security subject to default risk expiring at time T and dependent on the state variables $\mathbf{Z}(t)$. $\xi(T, \mathbf{Z}(T))$ denotes the *promised* payoff dependent on some state variables $\mathbf{Z}(t)$.[14] The payoff of the default-risky security can then be stated

[11] See Brémaud (1981).

[12] Rating transitions are modelled by a change of λ.

[13] Cf. Brémaud (1981).

[14] We assume $\mathbf{Z}(t)$ to be market risk factors such as the value of a stock, an interest rate, etc.

as follows:[15]

$$\begin{aligned}
\xi^d(T, \mathbf{Z}(T)) &= \xi(T, \mathbf{Z}(T))\mathbb{1}_{\{T<\tau\}} + \xi(T, \mathbf{Z}(T))\delta(T, \mathbf{Z}(T))\mathbb{1}_{\{\tau \le T\}} \\
&= \xi(T, \mathbf{Z}(T))(1 - N(T, \mathbf{Z}(T))) \qquad (2.5) \\
&\quad + \xi(T, \mathbf{Z}(T))\delta(T, \mathbf{Z}(T))N(T, \mathbf{Z}(T)) \\
&= \xi(T, \mathbf{Z}(T))(1 - N(T, \mathbf{Z}(T)))\,(1 - \delta(T, \mathbf{Z}(T)))\,.
\end{aligned}$$

where $\delta(t, \mathbf{Z})$ is the time-varying recovery rate, $\delta(t, \mathbf{Z}) \in [0, 1]$. As $\delta(t, \mathbf{Z})$ depends on $\mathbf{Z}(t)$, it is a random variable. If $T < \tau$ almost surely, default is certain not to occur before time T, the maturity of the security. In this case $\mathbb{1}_{\{\tau \le T\}} = 0$ and the payoff of the security simplifies to $\xi^d(T, \mathbf{Z}(T)) = \xi(T, \mathbf{Z}(T))$. To simplify notation, we write $\xi(T)$ for $\xi(T, \mathbf{Z}(T))$, $\delta(T)$ for $\delta(T, \mathbf{Z}(T))$, and $N(T)$ for $N(T, \mathbf{Z}(T))$.

We now suppose that there exists a non-empty set of martingale measures $\mathcal{Q}$ such that every element of $\mathcal{Q}$ is equivalent to $\mathbb{P}$. The existence of an equivalent martingale measure implies that there is no arbitrage. However, as mentioned previously, there are typically multiple equivalent martingale measures. The agent therefore has to choose one equivalent martingale measure for valuation. Schachermayer (2001) describes various approaches to the choice of a pricing measure. For ease of exposition, we assume that the agent chooses an equivalent martingale measure $\mathbb{Q} \in \mathcal{Q}$ such that valuation under $\mathbb{Q}$ is consistent with the agent's goal of utility maximization.

The (arbitrage-free) value of the derivative security to the agent at time $t < T$

[15] Cf. Ammann (2001)

is then given by

$$v^d(t) = B(t)\mathbf{E}_{\mathbb{Q}}[B^{-1}(T)\xi^d(T)|\mathcal{F}_t] \tag{2.6}$$

where $B(t)$ denotes the savings account. Substituting for $\xi^d(T)$, we get

$$\begin{aligned} v^d(t) &= B(t)\mathbf{E}_{\mathbb{Q}}[B^{-1}(T)(\xi(T)\mathbb{1}_{\{T<\tau\}} + \xi(T)\delta(T)\mathbb{1}_{\{\tau\leq T\}})|\mathcal{F}_t] \\ &= B(t)\mathbf{E}_{\mathbb{Q}}[B^{-1}(T)\xi(T)\mathbb{1}_{\{T<\tau\}}|\mathcal{F}_t] \\ &\quad + B(t)\mathbf{E}_{\mathbb{Q}}[B^{-1}(T)\xi(T)\delta(T)\mathbb{1}_{\{\tau\leq T\}}|\mathcal{F}_t] \end{aligned} \tag{2.7}$$

According to Equation 2.5, we can also write

$$\begin{aligned} v^d(t) &= B(t)\mathbf{E}_{\mathbb{Q}}[B^{-1}(T)\xi(T)(1-N(T))(1-\delta(T))|\mathcal{F}_t] \\ &= v(t) - B(t)\mathbf{E}_{\mathbb{Q}}[B^{-1}(T)\xi(T)N(T)(1-\delta(T))|\mathcal{F}_t] \end{aligned} \tag{2.8}$$

where $v(t)$ denotes the value of a similar default-free security. Equation 2.8 implies that the value of a security subject to default risk can be expressed as the difference between the value of a similar default-free claim and the expected loss given default of the security.

A crucial point in the evaluation of Equation 2.8 is the specification of $\lambda(t)$, which drives the default process $N(t)$, of the recovery rate $\delta(t)$, and the state variables $\mathbf{Z}(t)$.

Assuming that $\lambda(t)$ is independent of $\mathbf{Z}(t)$ and $\delta(t)$, Equation 2.8 can be restated as

$$v^d(t) = v(t) - (1-\mathcal{S}(t,T))B(t)\mathbf{E}_{\mathbb{Q}}[B^{-1}(T)\xi(T)(1-\delta(T))|\mathcal{F}_t] \tag{2.9}$$

where $\mathcal{S}(t,T)$ is defined as

$$\mathcal{S}(t,T) = \mathbb{Q}(\tau > T | \tau > t) = \exp\left(-\int_t^T \lambda(s)ds\right) \tag{2.10}$$

Assuming independence of $\delta(t)$ and $\mathbf{Z}(t)$, Equation 2.9 can now be restated as

$$v^d(t) = v(t)\left(1 - (1 - \mathcal{S}(t,T))(1 - \mathbf{E}_{\mathbb{Q}}[\delta(T)|\mathcal{F}_t]\right) \tag{2.11}$$

If we further assume that $\lambda(t) = \lambda$ and $\delta(T) = \delta$ are exogenous constants, we get

$$v^d(t) = v(t)\left(e^{-\lambda(T-t)} + \delta\left(1 - e^{-\lambda(T-t)}\right)\right) \tag{2.12}$$

This is equal to the result proposed by Jarrow and Turnbull (1995). We made the rather strong assumption that $\lambda(t)$, $\delta(t)$, and $\mathbf{Z}(t)$ are mutually independent. This means, in economic terms, that neither default process nor recovery rate are influenced by the state variables $\mathbf{Z}(t)$.[16] We further assumed that λ and δ are exogenous constants.

Generalizations are apparent but make the expressions much more complex, and in many cases do not yield closed-form solutions. Equation 2.12 is good enough for our purposes.

One advantage of the expression in Equation 2.12 is its analytical tractability. In fact, it nicely shows the effect of default risk on the value of the derivative. We can easily derive the effect of a change in the default intensity, i.e., in the credit quality of the counterparty, on the value of the derivative. More

[16] This means, in heuristic terms, that market risk and credit risk are independent.

specifically,

$$\frac{\partial v^d(t)}{\partial \lambda} = -v(t)(1-\delta)e^{-\lambda(T-t)}(T-t), \tag{2.13}$$

still assuming independence of $v(t)$ and λ.

Expression 2.12 models the value of a defaultable security as a function of the intensity λ, i.e., of the credit quality of the counterparty. This means that the value of the security varies with changing credit qualities. In order to differentiate between different counterparties, we denote by v^d_{in} the value of the ith security dealt with counterparty n.

3 Credit risk of derivative positions

The results presented in the previous section allow us to determine the value of a portfolio of derivative securities which are subject to default risk. We now turn to the issue of determining the credit risk associated with such a portfolio. As mentioned previously, credit risk, in heuristic terms, is the risk of loss from a counterparty default.

We first define a credit risk measure. In the second step, we show how to determine the value of this measure as well as its cost.

3.1 Preliminaries

The total value at time t of a portfolio of derivative securities traded with counterparty n is given by

$$V(\theta_n(t)) = \sum_{i=1}^{I} \theta_{in}(t) v_{in}^d(t) \tag{3.1}$$

where $\theta_{in}(t)$ denotes the unit holding in security i at time t, $\theta_{in}(t) \in \mathbb{R}_+^0$.
Assuming a total of N counterparties we get:

$$V(\theta(t)) = \sum_{n=1}^{N} \sum_{i=1}^{I} \theta_{in}(t) v_{in}^d(t) \tag{3.2}$$

Each of the $v_{in}^d(t)$ depends on one or more state variables $\mathbf{Z} = (Z_1, \ldots, Z_d)'$. $V(\theta(t))$ is therefore the value of the portfolio at time t, whereby $V(\theta(t))$ is itself a $\mathcal{F}_t$-measurable random variable.
Before we proceed, we introduce the notion of credit exposure.

Definition 3.1. *The* current credit exposure *to a counterparty n at time t is given by*

$$CE_c(\theta_n(t)) := \max\left(\sum_{i=1}^{I} \theta_{in}(t) v_{in}^d(t), 0\right) \tag{3.3}$$

where $i \in \{1, \ldots, I\}, \theta_{in}(t) \in \mathbb{R}_+^0$.

In other words, the current credit exposure is positive only if the market value of the portfolio w.r.t. counterparty n is positive at time t. If it is negative, credit exposure is zero.
Current credit exposure refers to the credit exposure at time t. In order to take potential future fluctuations into account, we define future credit

exposure.

Definition 3.2. *The* future credit exposure *to a counterparty n over time period Δt at time t is given by*

$$CE_f(\theta_n(t+\Delta t)) := \max\left(\sum_{i=1}^{I}\theta_{in}(t)B^{-1}(t+\Delta t)v_{in}^{d}(t+\Delta t), 0\right) \tag{3.4}$$

where $B^{-1}(t+\Delta t)$ denotes the discount factor.

It is necessary to discount $v_{in}^{d}(t+\Delta t)$ back to time t in order for current and future credit exposure to be comparable.
Very often, market participants require collateral from their counterparties to reduce their credit exposure. This is analyzed in more detail in Section 4.2. In case collateral is held, Equation 3.3 changes to

$$CE_c(\theta_n(t)) := \max\left(\sum_{i=1}^{I}\theta_{in}(t)v_{in}^{d}(t) - M_{On}(t), 0\right) \tag{3.5}$$

where $M_{On}(t)$ denotes the amount of collateral held from counterparty n. The same obviously applies to 3.4.
We are interested in the credit loss in relation to the portfolio during the time period Δt, whereby Δt is some discrete time interval. In credit risk management, Δt is usually a year or longer.

Definition 3.3. *The* credit loss *w.r.t counterparty n over time period Δt is given by*

$$L_n(t, t+\Delta t) := CE_f(\theta_n(t+\Delta t))(1-\delta_n(t+\Delta t))\mathbb{1}_{\{\tau_n \leq (t+\Delta t)\}} \tag{3.6}$$

where $\delta_n(t+\Delta t)$ *is the recovery rate and* τ_n *is the time of default of counterparty* n*, as defined previously.* $\mathbb{1}_{\{\tau_n \leq (t+\Delta t)\}}$ *is the indicator function assuming value* 1 *if default occurs between* t *and* $t+\Delta t$*, and* 0 *if not (cf. Equation 2.2).*

Note that credit loss is expressed in terms of values at time t. $L(t, t+\Delta t)$ is assumed to be observable at time $t+\Delta t$, but it is typically random at time t. The distribution of $L(t, t+\Delta t)$ is called the loss distribution. We typically write

$$L(t+\Delta t) := L(t, t+\Delta t) \tag{3.7}$$

3.2 Credit risk defined

Given our definition of credit loss in Equation 3.3, we now define the notion of credit risk. The are numerous approaches to defining and measuring credit risk in the literature. The most popular at present is probably still Value-at-Risk. Unfortunately, it has several shortcomings that make it dangerous to use. We therefore propose an alternative risk measure, Expected Shortfall.

Before we state a definition of credit risk, we formally define the concept of risk measure.

Definition 3.4. *Let* $(\Omega, \mathcal{F}, \mathbb{P})$ *be a probability space and* $\mathbf{Z}$ *a non-empty set of* $\mathcal{F}$*-measurable real-valued random variables. Then any mapping* $\rho : Z \longrightarrow \mathbb{R} \cup \{\infty\}$ *is called a* risk measure*.*

Artzner, Delbaen, Eber, and Heath (1999) introduce the concept of *coherent risk measures*. They suggest a set of properties, namely monotonicity, positive homogeneity, transaltion invariance, and subadditivity. Any risk measure that has these four properties is called coherent. Artzner, Delbaen, Eber, and Heath (1999) show that Value-at-Risk is not coherent, as it does not generally have the subadditivity property.[17]

In the following, we specify a notion of credit risk using the concept of Expected Shortfall.[18] As a prerequisite we introduce the notion of a quantile.

Definition 3.5. *Let* $\alpha \in (0,1]$ *be fixed and let* L *be a real-valued random variable on a probability space* $(\Omega, \mathcal{F}, \mathbb{P})$*. Define* $\inf \emptyset = \infty$*. We then call*

$$q_\alpha(L) = \inf\{l \in \mathbb{R} : \mathrm{P}[L \leq l] \geq \alpha\}$$

the α*-quantile of* L*.*

α is called the confidence level. Based on Definition 3.5, we are now ready to define Expected Shortfall.

Definition 3.6. *Let* $\alpha \in (0,1]$ *be fixed and let* L *be a real-valued random variable on a probability space* $(\Omega, \mathcal{F}, \mathbb{P})$ *with* $\mathbf{E}[\max(0, L)] < \infty$*. Define* $q_\alpha(L)$ *as in Definition 3.5. We then call*

$$\mathrm{ES}_\alpha(L) = -(1-\alpha)^{-1}\left(\mathbf{E}[L\mathbb{1}_{\{L \geq q_\alpha(L)\}}] + q_\alpha(L)\big\{\alpha - \mathrm{P}[L < q_\alpha(L)]\big\}\right) \quad (3.8)$$

Expected Shortfall (ES) *at level* α *of* L*.*

Thus, ES is the expected value of the loss L of a portfolio over some time interval at level α. See Tasche (2002) for a more detailed description and

[17] In other words, Value-at-Risk fails to reward diversification. Furthermore, Value-at-Risk is law-invariant and fails to recognize concentration of risks. For more details see Artzner, Delbaen, Eber, and Heath (1999) and Tasche (2002).

[18] Cf. Tasche (2002).

properties of Expected Shortfall.

We use the notion of ES of Definition 3.6 to formalize the notion of credit risk.

Definition 3.7. *Let $\theta(t)$ be a portfolio with value $V(t)$. Given the credit loss of this portfolio over the time period $t + \Delta t$, $L(t + \Delta t)$, as defined in Definition 3.3, and its distribution function, the* credit risk *at level α over time period $t + \Delta t$ is given by*

$$\rho_\alpha^C(t + \Delta t) = \mathrm{ES}_\alpha(L(t + \Delta t)) \tag{3.9}$$

where $\mathrm{ES}_\alpha(L(t + \Delta t))$ is defined as in Definition 3.6.

The confidence level α might be set set by the market participant itself or by a regulator.

We now have a coherent risk measure to work with. In heuristic terms, ρ_α^C gives us the amount of capital necessary to make a certain risk L acceptable. If ρ_α^C is negative, capital may be withdrawn. In other words, ρ_α^C gives us the amount of (economic) capital necessary to support a certain business activity, such as a derivatives position.

3.3 Cost of risk

We assume that a given derivatives position or portfolio needs to be supported by a certain amount of economic capital. We further assume that the amount of capital is determined by the credit risk of the position, $\rho_\alpha^C(\theta(t))$. This leads to the following definition:

Definition 3.8. *The* cost of credit risk *of a portfolio $\theta(t)$ with credit risk $\rho_\alpha^C(\theta(t))$ is given by*

$$C_E(\rho_\alpha^C(\theta(t))) = c(\rho_\alpha^C(\theta(t))) \tag{3.10}$$

where $c(\rho_\alpha^C(\theta(t)))$ is some function $c : \rho_\alpha^C(\theta(t)) \longrightarrow \mathbb{R}_+$.

The function $c(\rho_\alpha^C(\theta(t)))$ is determined by the cost of capital of the firm. For the purpose of this paper, we assume $c(\rho_\alpha^C(\theta(t)))$ to be linear in $\rho_\alpha^C(\theta(t))$. In addition, we assume that the amount of economic capital required to support a certain amount of risk equals the size of the risk. While this assumption is fine in theory, in practice it is often not the case. The mapping of risk into some amount of economic capital takes arbitrary forms, and it often depends on jurisdiction, type of instrument, etc.

We have defined a model to determine the value of a portfolio of derivative securities. Furthermore, we have introduced a measure of credit risk. Taking into account the notion of cost of risk in Section 3.3, we are able to compute the cost of risk of a portfolio of derivatives.

4 Clearing

In this section we describe the process of clearing and its impact on a portfolio of derivatives.

4.1 Novation and multi-lateral netting

When a derivative transaction is transferred to and accepted by a clearing house, the clearing house becomes the counterparty to the transaction. In other words, it becomes the buyer to the seller and the seller to the buyer.[19] All transactions transferred to the clearing house are thus executed with the clearing house as the (central) counterparty.

Assume a market participant has a portfolio $\theta(t)$ of non-cleared derivatives. The value of this portfolio is given by Equation 3.2, i.e.,

$$V(\theta(t)) = \sum_{n=1}^{N}\sum_{i=1}^{I}\theta_{in}v_{in}^{d}$$

Assuming all transactions are transferred to a clearing house, the value of the portfolio changes to:

$$V(\theta_C(t)) = \sum_{i=1}^{I}\sum_{n=1}^{N}\theta_{in}v_{iC}^{d} \tag{4.1}$$

$V(\theta_C(t))$ thus reflects the value of a cleared portfolio. Note that v_{in}^{d} changes to v_{iC}^{d}, where v_{iC}^{d} is the value of security i after is has been transferred to the clearing house. This will be discussed in more detail in Section 5.2.

The credit exposure of a non-cleared portfolio, as defined in Equation 3.3, is given by

$$CE_c(\theta(t)) = \sum_{n=1}^{N}\max\left(\sum_{i=1}^{I}\theta_{in}v_{in}^{d} - M_{On}(t), 0\right), \tag{4.2}$$

[19]This process, where the clearing house is substituted for the initial counterparty, is called *novation*. The transfer and acceptance process differs among the different clearing houses.

whereas the exposure of a similar cleared portfolio is given by

$$CE_c(\theta_C(t)) = \max\left(\sum_{i=1}^{I}\sum_{n=1}^{N}\theta_{in}v_{in}^d - M_V(\theta_C(t)), 0\right), \tag{4.3}$$

where $M_V(\theta_C(t))$ is the amount of collateral the market participant receives from the clearing house (see Section 4.2.)

It can be shown that $CE_c(\theta_C(t)) \leq CE_c(\theta(t))$ (see Section 5.1). Equation 4.3 reflects multilateral netting. In other words, a central counterparty enables a market participant to offset exposures in certain instruments across counterparties. This is different to the case of bilateral netting, where exposures can only be offset with respect to a single counterparty.

4.2 Margining

Derivative transactions are usually covered by collateral. This means that counterparties post a certain amount of collateral to each other to cover credit risk. In the OTC market the calculation of the amount of collateral is often done in a rather unsophisticated way. Large, highly-rated counterparties often do not post any collateral at all.

For exchange transactions, which are cleared, collateralization is highly formalized. The process is usually called *margining*. Once a transaction is transferred to a clearing house, the market participant has to post different types of margins, which are usually calculated based on composition and size of the portfolio.

Definition 4.1. *Let $\theta(t)$ be a portfolio of derivatives. A mapping $M(\theta(t)) : \theta(t) \to \mathbb{R}$ is called the* margin requirement *for portfolio $\theta(t)$.*

For OTC-traded derivatives the margin requirement is usually set quite arbitrarily. Often, the size of $M(\theta(t))$ depends on the credit quality of the counterparty. In the case of cleared derivatives, though, the margin requirement is calculated according to some highly formalized rules. Often, it does not depend on the credit quality of the counterparty. The margin requirement set by a clearing house is usually split up into two types: *initial* margin and *variation* margin.

Definition 4.2. *Let $\theta_C(t)$ be a portfolio of derivatives, and let $\rho_\alpha^C(\theta_C(t))$, the credit risk of this portfolio, be defined as in Definition 3.7. The* initial margin *for this portfolio is given by a mapping $M_I(\theta_C(t)) : \theta_C(t) \longrightarrow \mathbb{R}$ for some $\alpha \in [0,1]$.*[20]

$M_I(\theta_C(t))$ is supposed to cover the potential loss, at some "confidence level" α, arising in the liquidation of the portfolio $\theta_C(t)$.[21] In other words, it is supposed to cover the cost of a liquidation of the position in case the market participant defaults and the clearing house has to unwind the portfolio in the market.[22]

[20]The calculation of initial margin is usually based on the whole portfolio that a counterparty holds with the clearing house. Therefore, when evaluating the effect of a new position on the initial margin of a derivatives portfolio, one has to focus on marginal effects. A new position in a derivative instrument might result either in an increase or in a decrease of initial margin of a portfolio, depending on the structure of the existing portfolio. In our analysis, we implicitly assume that the portfolio we consider is a 'stand-alone' portfolio.

[21]Most clearing houses use stress testing or concepts that are based on the loss distribution to calculate initial margin. The most prominent system is SPAN, which was developed by the Chicago Mercantile Exchange. For a detailed description of SPAN see Chicago Mercantile Exchange (1995). See Bylund (2002) for an analysis of SPAN and a comparison with other margining methodologies.

[22]The cost of unwinding a derivatives position is usually called *replacement cost*.

The second type of margin is called variation margin.

Definition 4.3. *Let $\theta_C(t)$ be a portfolio of derivatives, and let $V(\theta_C(t))$ be its value at time t. The* variation margin *of this portfolio is given by*

$$M_V(\theta_C(t)) = V(\theta_C(t)) - V(\theta_C(t - \Delta t)) \tag{4.4}$$

for some time interval Δt.

$M_V(\theta_C(t))$ offsets the daily movements of the value of the portfolio. It is settled daily, i.e., it is calculated and processed at the end of each business day. Variation margin is typically settled in cash.[23]

Definition 4.4. *The margin requirement for a cleared portfolio $\theta_C(t)$ at time t is given by*

$$M_C(\theta_C(t)) = M_I(\theta_C(t)) + M_V(\theta_C(t)) \tag{4.5}$$

where $M_I(\theta_C(t))$ and $M_V(\theta_C(t))$ are defined as in Definitions 4.2 and 4.3, respectively.

The margin requirement has to be covered by collateral. Whereas variation margin usually has to be covered by cash, a wider range of collateral is accepted for initial margin including letters of credit and high-quality securities.[24]

[23]Variation margin changes the cash flow pattern of a derivatives contract, and thus has an impact on its value. See e.g. Murawski (2002) for results in this direction as well as an overview of the relevant literature.

[24]Collateral that bears market risk is typically subject to a hair-cut. A hair-cut is a form of discount applied on the market value of collateral to account for possible negative movements in the value of collateral.

As mentioned above, OTC transactions can be collateralized as well. We denote the margin requirement for a portfolio $\theta(t)$ of OTC-traded derivatives by $M_O(\theta(t))$.

Assuming that the collateral used to cover margin requirements is of high quality[25], margins reduce the credit exposure of a portfolio of derivatives.[26] Of particular interest in this regard is variation margin. It offsets the daily changes in portfolio value in cash. In other words, it completely offsets the credit exposure of a portfolio "accumulated" up to the present time. The remaining exposure of this portfolio is the potential future exposure.[27]

4.3 Costs of clearing

There are two types of costs that arise in connection with clearing: fees and cost of capital in relation to collateral.[28]

Every transaction cleared by a clearing house is usually subject to a fee. This fee might be calculated on a per-transaction basis, on a volume basis, or it might even be a fixed fee per time period.

The more significant component, however, is the cost of capital in relation to collateral. As mentioned in the previous section, a position with the clearing house has to be supported by a certain amount of collateral. Therefore, a

[25]By high quality we mean that the collateral bears little market and credit risk, like cash, letters of credit, T-bonds, etc.

[26]For this reason, margins are sometimes called *exposure modifiers*.

[27]For a more detailed analysis see Murawski (2002).

[28]Market participants might be obliged to contribute to a capital pool, usually called *default fund*. Such a capital pool is maintained by most clearing houses to (mutually) cover losses in case a counterparty defaults and margins held are not sufficient to cover the cost of unwinding the position. For ease of exposition, we ignore the (capital) cost arising from such an obligation.

market participant incurs opportunity cost since he cannot utilize the collateral otherwise. Furthermore, processing of collateral is usually subject to a fee.

Definition 4.5. *The* cost of clearing *of a position $\theta(t)$ is given by*

$$C_K(\theta_C(t)) = C_M(M_C(\theta_C(t))) + C_F(\theta_C(t)) \tag{4.6}$$

where $M_C(\theta_C(t))$ is given by Definition 4.4. $C_M(M_C(\theta_C(t)))$ is some mapping $C_M : M_C(\theta_C) \longrightarrow \mathbb{R}$ and represents the cost of collateral; and $C_F : \theta_C(t) \longrightarrow \mathbb{R}_+$ denotes the clearing fee.

4.4 Clearing house credit quality

A very important parameter in the analysis of clearing is the credit quality of the clearing house, expressed by λ_C. Clearing houses are usually assumed to be of very high credit quality, which is justified by their – often high – ratings.[29] In this section, we make some observations of the special nature of clearing houses' credit exposures.

As a first observation it should be stated that the portfolio of a clearing house is always balanced with regards to market risk. This is a consequence of the fact that a clearing house always assumes both sides of a transaction. Therefore, the portfolio of a clearing house does not have any market risk exposure.[30]

[29] Cf. page 5.

[30] Obviously, this is no longer the case once one of the participants defaults and the

Remark 4.6. *The market risk of a clearing house's portfolio is zero.*

Our second observation concerns diversification effects of a clearing house's portfolio. Assume a set of counterparties $\{1, \ldots, N\}$. Let $CE_c(\theta_n(t))$ denote the current credit exposure of the clearing house to counterparty n with position θ_n. Additionally, let $\rho_\alpha^C(\theta_n(t))$ denote the credit risk of that position. It can be shown that $\sum_n CE_c(\theta_n(t)) \geq CE_c(\sum_n \theta_n(t))$. Furthermore, $\sum_n \rho_\alpha^C(CE_c(\theta_n(t))) \geq \rho_\alpha^C(\sum_n CE_c(\theta_n(t)))$ (follows from subadditivity property of credit risk). This means that the credit risk of the clearing house's portfolio is lower or equal to the sum of the credit risks of the individual counterparties.

Our third observation is a consequence of margining. Assuming a recovery rate of zero, the potential loss of a clearing house due to a credit event is given by the credit exposure to the respective counterparty. The credit exposure is significantly reduced by the margin posted by the counterparty. As stated previously, variation margin offsets the changes in portfolio value up to time t. The current credit exposure, i.e. the exposure "accumulated" between 0 and t, is zero. Thus, the clearing house's credit exposure only depends on potential future changes of $V(\theta(t))$. This is fundamentally different from the uncleared case where transactions are only settled at maturity. In this case, a market participant is exposed to both, current credit exposure "accumulated" between time 0 and t, and future credit exposure between t and maturity.

clearing house has to close out the participant's position in the market. In this case, the clearing house is exposed to (i) changes in the market value of the position, and (ii) changes in the value of the collateral posted by this participant.

Despite the points mentioned above, a clearing house should not be assumed to be default-free. Its credit exposure is still (strictly) positive. Furthermore, a clearing house is exposed to liquidity risk, operational risk, as well as model risk.

We assume that the default intensity of the clearing house, $\lambda_C(t)$, is exogenously given. We also assume, unless states otherwise, that $\lambda_C(t)$ is lower than the default intensity of counterparty n, $\lambda_n(t)$, i.e., $\lambda_C(t) < \lambda_n(t)$, for all n.[31]

5 The Impact of clearing on credit risk

Having developed a framework for credit risk and its cost, we now proceed to analyze the impact of clearing on credit risk. We will conduct our analysis in stages, first looking at the impact of clearing on the exposure of a portfolio, then on its value, on its risk, and finally on the cost of risk of a given portfolio.

5.1 Impact on exposure

The current credit exposure of a non-cleared portfolio is given by

$$CE_c(\theta(t)) := \sum_{n=1}^{N} \max\left(\sum_{i=1}^{I} \theta_{in}(t) v_{in}^d(t) - M_{On}(\theta_n(t)), 0\right) \tag{5.1}$$

In other words, the firm has positive credit exposure to a counterparty n if $\sum_{i=1}^{I} \theta_{in} v_{in}^d - M_{On}(t)$ is positive. Otherwise, credit exposure is zero.

[31] Some of the rationale behind this assumption is given in this section as well as in Section 5.6. A more formal analysis is the subject of separate work.

Note that credit exposure in relation to derivatives is different from credit exposure in relation to a loan. Whereas in the case of a loan, credit exposure at time $t + \Delta t$ is known a time t, for derivatives this is typically not the case. We now assume that we have a portfolio of non-cleared derivatives, which is transferred to a clearing house. This means that the clearing house is substituted for all the different counterparties. Equation 5.1 changes to

$$CE_c(\theta_C(t)) = \max\left(\sum_{i=1}^{I}\sum_{n=1}^{N}\theta_{in}v_{in}^{d} - M_V(\theta(t)), 0\right) \tag{5.2}$$

where $M_V(\theta_C(t))$ represents the amount of collateral received from the clearing house (in the form of variation margin). Note that in the above equation we assume, for the moment, that v_{in}^{d} does not change when the derivative is transferred from an arbitrary counterparty to a clearing house.

Proposition 5.1. *Suppose* $M_O(\theta(t)) < M_V(\theta_C(t))$ *for a given portfolio of derivatives* $\theta(t)$. *The current credit exposure of the cleared portfolio,* $CE_c(\theta_C(t))$, *is always lower than or equal to the current credit exposure of a similar non-cleared portfolio,* $CE_c(\theta(t))$, *i.e.,*

$$CE_c(\theta_C(t)) \leq CE_c(\theta(t)) \quad \forall t \tag{5.3}$$

whereby $CE_c(\theta_C(t))$ *and* $CE_c(\theta(t))$ *are defined as in Equations 5.2 and 5.1, respectively.*

Proof. See Appendix. □

5.2 Impact on portfolio value

In the previous section, we assumed that the value of a derivative i traded with counterparty n, $v_{in}^d(t)$, does not change when the derivative is transferred to a clearing house. It implies that the default intensity of counterparty n, $\lambda_n(t)$, is equal to the default intensity of the clearing house, $\lambda_C(t)$. This, however, is generally not the case, i.e., the credit quality of a clearing house is typically higher than the credit qualities of most of the counterparties. Therefore, as previously stated, we assume that $\lambda_C(t) < \lambda_n(t)$ for all n.

As shown previously, the default intensity influences the value of a derivative. A lower default intensity implies a higher value of the derivative, as can easily be seen in Equation 2.12.

Definition 5.2. *The value of a derivative security that is transferred to a clearing house is given by*

$$v_C^d(t) = v(t)\left(e^{-\lambda_C(T-t)} + \delta\left(1 - e^{-\lambda_C(T-t)}\right)\right) \tag{5.4}$$

where $v(t)$ is the value of a similar default-free security and λ_C is the default intensity of the clearing house.[32]

[32] We point out at this stage that the transfer of an OTC-traded derivative contract to a clearing house affects the cash flow pattern of the contract (due to variation margin), and thus its value. The spread between an un-cleared forward contract and a cleared forward contract (=futures contract) is known as the forward-future spread and is well-understood in the absence of credit risk. Murawski (2002) reviews the related literature and extends existing results by incorporating default risk into the forward-future spread. In a default-free environment, this spread is zero if interest rates are deterministic or if the covariance between interest rates and underlying is zero. We assume throughout this paper that the risk-free rate of interest is deterministic, and that default risk is reflected by λ. In other words, we assume that the spread between forward and futures contracts is only due to default risk.

We can now state the following proposition:

Proposition 5.3. *The value of a derivative security i traded with counterparty n, v^d_{in}, changes to $v^d_{iC}(t)$ when the derivative is transferred to a clearing house. Assuming that $\lambda_n > \lambda_C$, we have that*

$$v^d_{iC}(t) > v^d_{in}(t) \quad \forall i \tag{5.5}$$

where $v^d_{iC}(t)$ and $v^d_{in}(t)$ are defined as in Equations 5.4 and 2.12, respectively. Similarly, the value of a portfolio of derivative securities $i \in 1, \ldots, I$, $V(\theta_n(t))$, as defined in Equation 3.2 changes to $V(\theta_C(t))$, as defined in Equation 4.1, when transferred to a clearing house. Again assuming $\lambda_n > \lambda_C$ for all n, we get

$$V(\theta_C(t)) > V(\theta_n(t)) \tag{5.6}$$

Proof. Follows directly from the definitions in Equations 2.12 and 5.4 and the assumption that $\lambda_n > \lambda_C$. □

Proposition 5.3 states that the value of a derivative increases when the derivative is transferred to a clearing house. Similarly, the value of a portfolio of derivatives increases when it is transferred to a clearing house.

This result is important. When a market participant decides to transfer derivatives – struck with arbitrary counterparties – to a clearing house, it immediately increases the value of these derivatives.

Obviously, a change in the value of a derivative resulting from a transfer of the derivative to the clearing house also affects the credit exposure of the market participant (cf. Equation 5.2).

Definition 5.4. *The increase in portfolio value due to clearing according to Proposition 5.3 is denoted by* $\Delta^C(\theta(t))$.

5.3 Impact on credit risk

In this section we look at the impact of clearing on the credit risk of a derivatives portfolio. Recall from Section 3.2 that we defined credit risk as the Expected Shortfall of the portfolio's loss function, L, at level α, or

$$\rho_\alpha^C(\theta(t)) = \mathrm{ES}_\alpha(\theta(t))$$

where $\rho_\alpha^C(\theta(t))$, $\mathrm{ES}_\alpha(\theta(t))$, and α are defined as in Section 3.2.

Making the same assumptions as in the previous section, we get the following result.

Proposition 5.5. *Let* L_n *be the loss function of a portfolio* θ_n *of derivatives with counterparty* n*, and* L_C *be the loss function of a similar portfolio* θ_C *of cleared derivatives. Let* λ_n *be the default intensity of a counterparty* n*,* λ_C *be the default intensity of a clearing house, with* $\lambda_C < \lambda_n$ *for all* n*. We then have*

$$\rho_\alpha^C(\theta_C(t)) < \rho_\alpha^C(\theta_n(t)) \quad \forall n \tag{5.7}$$

where $\rho_\alpha^C(\theta_C(t))$ *is defined as in Equation 3.9.*

Proof. See Appendix. □

This result states that the credit risk of a cleared portfolio is lower than the credit risk of a similar uncleared portfolio.

5.4 Impact on cost of credit risk

We now want to look at the economic impact of the results developed in the previous sections. In Section 3.3, we defined the cost of risk $C(\rho_\alpha^C(\theta(t)))$ as

$$C_E(\rho_\alpha^C(\theta(t))) = c(\rho_\alpha^C(\theta(t)))$$

where $\rho_\alpha^C(\theta(t))$ denotes credit risk, and c denotes some cost function. From the results in the previous section, the following is immediate:

Proposition 5.6. *Let $\rho_\alpha^C(\theta)$ be the credit risk of a portfolio of non-cleared derivatives, and let $\rho_\alpha^C(\theta_C)$ be the credit risk of a similar cleared portfolio, whereby $\rho_\alpha^C(\theta) > \rho_\alpha^C(\theta_C)$. From the definition of the cost of risk C_E we get*

$$C_E(\rho_\alpha^C(\theta_C(t))) < C_E(\rho_\alpha^C(\theta(t))) \tag{5.8}$$

Proof. Straightforward. □

The result states that the cost of credit risk of a cleared portfolio is lower than the cost of credit risk of a similar non-cleared portfolio.

5.5 Comparison

We now collect the results obtained so far to compare the cost of credit risk of a un-cleared portfolio to the cost of a similar cleared portfolio and the cost of clearing.

The cost of a cleared portfolio is given by

$$C_C(\theta_C(t)) = C_E(\theta_C(t)) + C_K(\theta_C(t)) - \Delta^C(\theta_C(t)) \tag{5.9}$$

where $C_K(\theta_C(t)) = C_M(\theta_C(t)) + C_F(\theta_C(t))$. This expression needs to be compared to the cost of a similar non-cleared portfolio,

$$C_O(\theta(t)) = C_E(\theta(t)) + C_K(\theta(t)) \tag{5.10}$$

where $C_K(\theta(t)) = C_M(\theta(t))$ and C_M is the cost of collateral of the un-cleared portfolio. Clearing is beneficial if

$$C_C(\theta_C(t)) < C_O(\theta(t)).$$

A decisive parameter in the analysis is the clearing fee, $C_F(\theta_C(t))$. One would assume that in an efficient market the agent is indifferent between clearing and no clearing, i.e., $C_C(\theta_C(t)) = C_O(\theta(t))$. Assuming that this relationship holds, we can state the clearing fee as follows:

$$C_F(\theta_C(t)) = \big[C_E(\theta(t)) - C_E(\theta_C(t))\big] + \big[C_M(\theta(t)) - C_M(\theta_C(t))\big] + \Delta^C(\theta_C(t)). \tag{5.11}$$

The above equation expresses that, in an efficient market, the clearing fee equals the benefits gained from clearing.

5.6 Interpretation

Our analysis shows that general statements whether clearing is beneficial are not possible.[33] As pointed out already, the two important parameters in the

[33] Baer, France, and Moser (1996) analyze clearing using a more restrictive model than we do. According to their analysis, a clearing house is generally Pareto improving. However, they only consider margins and their opportunity cost as well as credit losses in their analysis. They ignore the cost of credit risk and the impact of clearing on the value of

evaluation of clearing are the credit quality of the clearing house, represented by λ_C, and the cost of clearing, C_K. In the remainder of this section, we make some remarks on these parameters.

In most of our analysis we assumed $\lambda_C < \lambda_n$ for all n, i.e., we assumed that the probability of default of the clearing house is smaller than the probabilities of default of the market participants, during some time interval. This assumption is in line with practice. However, if the relationship does not hold, the results in Section 5 do not hold any longer in general.

In our model, λ_C is exogenously given. Heuristically, it reflects the probability that one or more clients of the clearing house default and the resulting losses for the clearing house are larger than its capital. Bernanke (1990) and Bates and Craine (1998) analyze the financial stability of clearing houses during the 1987 crash, when one of the most severe market movements of the recent past occurred. Bates and Craine (1998) state that the clearing house of the Chicago Mercantile Exchange had severe difficulties during the 1987 crash and survived with "some help from the Federal Reserve".[34] The analysis shows that clearing houses are not infallible. In other words, λ_C should be assumed to be strictly positive.

Unfortunately, there is no systematic analysis of the credit quality of clearing houses available, at least not to our knowledge. As stated previously, some clearing houses are rated, such as the Board of Trade Clearing Corporation

derivatives. Furthermore, they do not take into account clearing fees. It has to be pointed out that their results would not generally hold under less restrictive assumptions.

[34]Strictly speaking, the crisis in 1987 was first and foremost a liquidity crisis. However, the situation would have turned into a credit issue if one or more of the counterparties had defaulted due to liquidity problems.

('AAA'), but this is not generally the case. Furthermore, these ratings might not be reliable.

If the relationship $\lambda_C < \lambda_n$ does not hold, the impact of clearing on the value of the portfolio is ambiguous and depends on the relationship of λ_C and λ_n for the n different counterparties. The impact on credit risk and cost of credit risk is also ambiguous. In other words, it is no longer possible to state general results.[35]

One might point out diversification effects to support the assumption of $\lambda_C < \lambda_n$ for all n. Firstly, default risk should be split up into an *idiosyncratic* component and a *systematic* component. It is not clear whether the idiosyncratic part can be fully "diversified away" in a clearing house portfolio, as clearing house clients are all exposed to the same risks and business is highly interrelated.

Even if the clearing house was able to completely diversify the idiosyncratic component of default risk, it would still be exposed to the systematic part of default risk.[36] The systematic part can be aggravated by the *systemic* risk of the financial system. Systemic risk can loosely be defined as the risk that the failure of a counterparty to meet its obligations when due will cause other counterparties to fail to meet their obligations when due (loss propagation).[37] It might actually be the case that a clearing house increases systemic risk in a market. On the other hand, a clearing house, if structured correctly, might

[35] Hanley, McCann, and Moser (1995) show, in a slightly different context and a more restrictive setting, that risk-reducing benefits of a central counterparty arise if the credit quality of the central counterparty is at least equal to that of the most creditworthy participant.

[36] Very often, systematic risk is not insurable.

[37] Cf. Bank for International Settlements (1998).

also be able to reduce systemic risk.[38]

To summarize, we can state that our assumption of $\lambda_C < \lambda_n$ for all n is in line with market practice but does not have a theoretical foundation.

The next important parameter is the cost of clearing, or, more specifically, the cost of capital of margins, and the clearing fees. Let us assume that clearing fees are driven by risk only, not by operational expenses. Then, the size of margins and clearing fees are (or, should be) dependent, i.e., the higher the margins are for a given position, the lower is the residual risk for the clearing house, and the lower should be the clearing fees. To our knowledge, there is no theoretical analysis of the relationship between margin size and clearing fees.

This issue is particularly interesting when different market structures are considered, such as monopoly or perfect competition. Market structure might have an effect on margin rates, M_I, and clearing fees, C_F. One might conjecture that margin rates and clearing fees are comparatively higher in case of a monopoly than in case of perfect competition. In fact, competition between clearing houses might lead to a 'race to the bottom', thus increasing the credit risk the clearing house holds, as well as its probability of default, λ_C. Again, we do not know of any work that has been done in this direction.

[38] Cf. for example Hanley, McCann, and Moser (1995).

6 Numerical example

In this section we apply the framework presented in the previous sections to demonstrate the effects by way of a numerical example. The example comprises two scenarios. In the first scenario, a trader has entered into a derivative contract with a counterparty, and decides to clear this contract. In the second scenario, the trader has two positions in the same contract, a long position with one counterparty and an offsetting short position with another counterparty. The two counterparties have different credit qualities. Again, the trader gives up the portfolio to the clearing house. We compute the cost of credit of holding the portfolio and the changes due to clearing.

6.1 Set-up

We consider a position comprising a 10-year USD LIBOR swap contract with a fixed leg of 8.00% and a floating leg of 50 basis points. The notional value of the contract is USD 100 million. In the first scenario the trader holds a short position with counterparty A. This counterparty's credit quality is characterized by $\lambda_A = 0.0001$ (see Equation 2.12). In the second scenario, the trader holds a short position with counterparty A and an offsetting long position with counterparty B ($\lambda_B = 0.001$).

Table 1 shows the evolution of the value of the position during a 10-day period. The table contains two values, a default risk-free value (benchmark) and the defaultable value taking into account the credit quality of counterparty A. The valuation in the default-free case is based on the Heath-Jarrow-Morton approach. The calculation of the defaultable value is based on Equation 2.12, assuming a recovery rate of zero.

The table also shows the credit risk ρ^C of the position at a 95% confidence level (see Equation 3.9, i.e., $\alpha = 0.95$). The calculation of the cost of credit risk is based on Equation 3.10 assuming $C_E(\rho^C_{95\%}) = 0.1\rho^C_{95\%}$ on a full-year basis.

We assume that the credit quality of the clearing house is characterized by $\lambda_C = 0.0001$. Additionally, we assume that initial margin, M_I, for the contract is USD 1 million (see Definition 4.2), and that it is identical for long and short positions. We further assume that the capital cost for margins, C_M (see Definition 4.5) is 10% per year. The clearing fee, C_F, is 10 basis points of notional value.

In the following, we analyze the cost of carrying the portfolio given the two scenarios over a period of 10 days. We compare the cost of the un-cleared (OTC) case to the cost of the cleared case. The numerical example resembles the SwapClear service offered by the London Clearing House, which is probably among those clearing services for OTC transactions with the highest volume.[39]

6.2 Scenario 1

In this scenario, the trader holds a short position with counterparty A (see above). Table 1 shows the evolution of the value of the portfolio, its credit risk, the margin requirement (collateral), and the cost of credit risk. The cost of credit risk of the portfolio on an annualized basis is USD 100,320 or

[39] Unfortunately, the London Clearing House did not make available margin rates or fee structures for SwapClear citing confidentiality reasons. We therefore had to rely on our own model that resembles the SwapClear model in principle.

5.51% of average portfolio value.[40]

We now consider the case where the position is given up for clearing, i.e., the contract is transferred to a clearing house. The clearing house charges initial margin as well as variation margin, both of which have to be covered by collateral. Furthermore, the clearing house charges a clearing fee for every contract cleared. The evolution of the portfolio value and related costs is exhibited in Table 2.

The annualized cost of holding the portfolio is USD 206,843 or 11.47% of its average value. This includes the clearing fee, C_F, and the value increase resulting from clearing, Δ^C. In this scenario, the cleared portfolio is more expensive than the uncleared portfolio. The reasons for this are as follows: Firstly, counterparty A is of very high credit quality, which means that the position has very little credit risk at the outset, and that the change of portfolio value resulting from clearing is very small. Secondly, the portfolio contains only one position, i.e., there are no effects from multilateral netting of offsetting contracts. Finally, we assumed that the trader has to post a similar amount of initial margin in both the OTC case and the cleared case. In summary, the cost of clearing fee by far overcompensates the benefits.

6.3 Scenario 2

In the second scenario, we consider a portfolio with two positions which offset each other. One position is held with counterparty A (high credit quality)

[40] Annualized cost of credit risk was calculated by multiplying the sum of the daily cost during the 10-day holding period by 250/10.

and the other position is held with counterparty B (low credit quality). This scenario is exhibited in Table 3. It should be noted that the trader has to post collateral with both counterparties.
The annualized cost of credit risk of holding the portfolio is USD 209,147. We point out that although the portfolio consists of opposite positions of equal size in the same instrument, the total value of the portfolio is not zero. This is due to the effect the different credit qualities have on the values of the instruments. In other words, a hedge of a position has to take into account both market and credit risk.

As before, we now consider the case where the position is given up for clearing. As the portfolio consists of two offsetting positions, the transfer of the portfolio to the clearing house results in a net-zero position with the clearing house. Now price moves are offset and the value of the portfolio is zero at all times. Therefore, initial, M_I, and variation margin, M_V, are zero, too. The credit risk of the position is negligible due to the high credit rating of the clearing house, i.e., $C_E = 0$. Therefore, the only cost arising with the position is the clearing fee ($C_F = 0.200$). The increase of portfolio value due to clearing, $\Delta^C v^d$, is USD 189,292 on average during the time period of the example. We thus conclude that the cost of credit of holding the position is USD 10,708, compared to USD 209,147 in the case of no clearing, i.e., a gain of USD 198,439.

We conclude that clearing tends to be economically disadvantageous for positions with counterparties of high credit quality and one-sided exposures.

In such a case, the reduction in credit risk by clearing is insignificant, and so is the value increase. On the other hand, traders with balanced portfolios and with exposures to counterparties with lower credit qualities benefit significantly from clearing. Here, credit risk of the portfolio is reduced considerably. Additionally, the value increase resulting from the transfer of the positions to the clearing house is usually substantial. In such a case, benefits from clearing tend to be higher than the costs of clearing.

7 Conclusion

We developed a framework to determine the effects of clearing on the credit risk of a derivatives portfolio. More specifically, we determine the cost of credit risk of such a portfolio. The cost of credit risk is the cost that arise from carrying a certain position on a balance sheet. We then compare the cost of credit risk of a portfolio of un-cleared (OTC-traded) derivatives to a portfolio of similar but cleared derivatives. When comparing the cost of these two portfolios, we also take into account the change in portfolio value due to clearing as well as any clearing fees that arise.

When we compare the case of the un-cleared portfolio to a cleared portfolio, we make the assumption that the credit quality of the clearing house is higher than the credit qualities of the market participants. This assumption is in line with market practice. However, it does not have a theoretical foundation and should be handled with care. If the assumption is relaxed, our results do not hold anymore in general.

We show that a cleared portfolio has lower or equal credit exposure than a similar un-cleared portfolio. Under the assumption on credit qualities mentioned above, a cleared portfolio also has lower credit risk and lower cost of credit risk. We also show that the value of the cleared portfolio is higher than that of the un-cleared portfolio.

We observe that two parameters play an important role in our analysis, mainly the credit quality of the clearing house and the clearing fees. Whereas we assume these parameters to be exogenously given, they deserve some detailed theoretical analysis. This is going to be the subject of future work.

We also observe that, in an efficient market, the agent should be indifferent between clearing an no clearing. In other words, the cost of credit risk of the cleared portfolio should be the same as the cost of credit risk of the un-cleared portfolio. In practice, this is often not the case. More theoretical work should be done on this issue addressing the questions under which conditions the agent is indifferent; and how the market structure of clearing houses affects the cost of clearing, and thus its value.

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A Proofs

A.1 Proof of Proposition 5.1

Proof. See Equations 5.1 and 5.2 for the definitions of $CE_c(\theta(t))$ and $CE_c(\theta_C(t))$. As $M_O(\theta(t)) < M_V(\theta(t))$,

$$\sum_{n=1}^{N} \max\left(\sum_{i=1}^{I} \theta_{in} v_{in}^d - M_{On}(t), 0\right) \geq \max\left(\sum_{i=1}^{I}\sum_{n=1}^{N} \theta_{in} v_{in}^d - M_V(\theta_C(t)), 0\right) \tag{A.1}$$

This is the claim in the proposition. □

A.2 Proof of Proposition 5.5

Proof. Let λ_n and λ_C be defined as previously, and assume $\lambda_n < \lambda_C$. We have shown that $CE_c(\lambda_n) \geq CE_c(\lambda_C)$. We have also shown that $V(\theta_n(t)) < V(\theta_C(t))$. Thus, $CE_c(\lambda_n) \geq CE_c(\lambda_C)$. Assuming $\delta_C \geq \delta_n$, we get $L_n > L_C$. From the monotonicity property of Expected Shortfall follows $ES_\alpha(L_n) > ES_\alpha(L_C)$ for some α. Hence, $\rho_\alpha^C(\theta_n(t)) > \rho_\alpha^C(\theta_C(t))$. □

B Tables

Day	v	v^d	Δv^d	$\rho_{95\%}^C$	M_O	C_E
1	-0.607	-0.613	—	4,391	1.000	401.8
2	-1.283	-1.295	-0.682	3,716	1.000	401.5
3	-0.882	-0.890	0.405	4,116	1.000	401.6
4	-1.419	-1.433	-0.543	3,579	1.000	401.4
5	-1.420	-1.434	-0.001	3,578	1.000	401.4
6	-1.941	-1.961	-0.527	3,057	1.000	401.2
7	-2.509	-2.534	-0.574	2,489	1.000	401.0
8	-2.849	-2.877	-0.343	2,150	1.000	400.9
9	-2.196	-2.218	0.659	2,803	1.000	401.1
10	-2.903	-2.932	-0.714	2,096	1.000	400.8

Table 1: Scenario 1 without clearing. v: value of instrument without default risk (USD million); v^d: value of instrument traded with counterparty A (USD million); Δv^d: change of portfolio value (USD million); $\rho_{95\%}^C$: credit risk of position (USD); M_O: margin (collateral) requirement (USD million); C_E: cost of credit risk *per day* (USD).

Day	v	v_C^d	Δv_C^d	$\rho_{95\%}^C$	C_E	M_I	M_V	C_M	C_F	C_K
1	-0.607	-0.607	—	0.000	0.000	1.000	0.000	400	100,000	100,400
2	-1.283	-1.284	-0.676	0.000	0.000	1.000	-0.676	670	—	670
3	-0.882	-0.882	0.401	0.000	0.000	1.000	-0.401	240	—	240
4	-1.419	-1.421	-0.538	0.000	0.000	1.000	0.538	615	—	615
5	-1.420	-1.421	-0.001	0.000	0.000	1.000	0.001	400	—	400
6	-1.941	-1.943	-0.522	0.000	0.000	1.000	0.521	609	—	609
7	-2.509	-2.512	-0.569	0.000	0.000	1.000	0.568	627	—	627
8	-2.849	-2.851	-0.340	0.000	0.000	1.000	0.339	536	—	536
9	-2.196	-2.198	0.654	0.000	0.000	1.000	-0.653	139	—	139
10	-2.903	-2.906	-0.708	0.000	0.000	1.000	0.707	683	—	638

Table 2: Scenario 1 with clearing. v: value of instrument without default risk (USD million); v_C^d: value of cleared instrument (USD million); Δv_C^d: Change of instrument value (USD million); $\rho_{95\%}^C$: credit risk of position (USD); C_E: cost of credit risk *per day* (USD); M_I: initial margin (USD million); M_V: variation margin (USD million); C_M: capital cost of margins (USD); C_F: clearing fee (USD); C_K: total cost of clearing (USD).

Day	v	v_A^d	v_B^d	$\sum v^d$	Δv^d	$\rho_{95\%}^C$	M_O	C_E
1	-0.607	-0.613	0.549	-0.064	—	103,421	2.000	841.4
2	-1.283	-1.295	1.161	-0.135	-0.071	96,655	2.000	839.7
3	-0.882	-0.890	0.798	-0.093	0.042	100,669	2.000	840.3
4	-1.419	-1.433	1.284	-0.149	-0.057	95,286	2.000	838.1
5	-1.420	-1.434	1.285	-0.149	0.000	95,279	2.000	838.1
6	-1.941	-1.961	1.757	-0.204	-0.055	90,058	2.000	836.0
7	-2.509	-2.534	2.271	-0.264	-0.060	84,370	2.000	833.7
8	-2.849	-2.877	2.578	-0.299	-0.036	80,974	2.000	832.4
9	-2.196	-2.218	1.987	-0.231	0.069	87,511	2.000	835.0
10	-2.903	-2.932	2.627	-0.305	-0.074	80,429	2.000	832.2

Table 3: Scenario 2 without clearing. v: value of instrument without default risk (USD million); v_A^d: value of instrument traded with counterparty A (USD million); v_B^d: value of instrument traded with counterparty B (USD million); $\sum v^d$: total portfolio value (USD million); Δv^d: change of portfolio value (USD million); $\rho_{95\%}^C$: credit risk of position (USD); M_O: margin (collateral) requirement (USD million); C_E: cost of credit risk per day (USD).