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Structural Credit Modeling and Boundary Sensitivity. Maybe One Half is not Enough!

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Structural Credit Modeling and Boundary Sensitivity
Maybe One Half is not Enough !

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Abstract

We analyze the role played by the boundary value for the *sensitivity* of the *creditworthiness predictions* in methodologies based on Merton [1974]. We run Monte-Carlo simulations with two various samples of firms - American, European - in order to build confidence intervals for the estimator of the fractional long-term debt component parameter of the default barrier. The robustness of these confidence intervals is tested by a resampling method for an efficient determination of the a-quantile. In addition to this main task, kernel empirical distributions for the *built*-default probabilities are performed by using the Silvermann's method. This paper can be seen as a model risk study that emphasizes the possible errors induced by the use of a constant fractional long-term debt component part in the actual definition of the barrier offered by the KMV™ Corporation.

Keywords : *structural credit modeling, Merton's Model (1974) ,boundary calibration, Monte-Carlo simulations, Resampling methods, Kernel densities.*

JEL Classification : G13, G22.

Introduction

In this article we focus on the calibration of the boundary[1] parameter in the context of the structural credit models. The structual credit models family underlies all models derived from Merton's firm-value model (1974). More particularly, that family underlies the most important industry-models such those proposed by the KMV Corporation and CreditMetrics.

In those models, default of an obligor occurs if a latent variable, often considered as the value of obligor's assets, falls below a particular threshold, often interpreted as the value of obligor's liabilities. Credit risk can be thus regarded as a put option on the value of the firm's asset in the perspective of the structural approach and hence, the corporate bond should be priced as a default free bond minus the put option value with strike price K written on the assets of the firm. The calibration of this strike price is the aim of the paper. But, at this level we have to mention the specifity of the current work. As already mentioned, the structural credit models include the most important industry-models used with a growing interest by the risk managers of the major banks. In the context of the seminal paper of Merton (1974), practical issues closely linked to the implementation of the modeling have not been considered and could not be considered[2]. In addition to this first empirical issue, we have to point out that Merton's firm-value model (1974) has been mostly concerned with debt pricing, not with *creditworthiness forecast.* Many other previous papers already noticed the gap between, on a one hand, the growing and dense developments of the theoritical credit modelings and, on the other hand, the attempts of empirical tests. Here, we would like to observe that the wide variety of the previous empirical studies have been conducted in order to appreciate the accuracy of the structural models from the pricing perspective only. We mean that, in our present knowledge, few papers attempted to appreciate the forecast sensitivity of the major industry-models from the perspective of the *creditworthiness* paths predictions of the firms.

In line with the latter observation, this paper investigates more deeply the quantitative framework developed by the KMV Corporation[3]. Indeed, the actual definitions of the boudary values assumed by the major commercial applications of credit risk management tools - based on a structural framework - appear like a black box for the user. The lack of transparency regarding the calibration of the boundary value is the main focus of this paper.

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[1] contrarily to Leland (2002), *'boundary'* refers to the liabilities and not to the assets value.

[2] The lack of consistent data was and is yet a significant issue for the calibration of credit risk models.

[3] We do not present in this paper the complete framework developed by the KMV Corporation. For a complete presentation, we refer to Crouhy, Galai and Mark (2001) pp.368-383 or Leland (2002), working paper, p.28 .

This paper can be seen as a model risk study that emphasizes the possible errors induced by the use of a constant fractional long-term debt component part in the actual definition of the barrier offered by the KMV™ Corporation.. The use of a constant parameter to define the boundary value may not be sufficient to implement an efficient *early warning* monitor.

The paper has been built into four sections. The first derives the seminal paper of Merton (1974) and shows how the original pricing modeling has been adapted to creditworthiness monitoring purposes. The second presents our theoretical methodology in order to build robust α-quantiles for the estimator of the fractional long-term debt component. Section three describes our samples and explains how these data have been used to perform the empirical study. Section four synthesizes the numerical results and points out the major conclusions of the article. Section five concludes the paper.

1. The structural Credit Modeling and its adaptation to creditworthiness monitoring systems.

In this first part, we attempt an updated presentation of the structural credit class and focus on the derivation of the original framework of Merton (1974). In a second step, we insist on the assumptions made to adapt such credit models to creditworthiness monitoring purposes. We think that a shift has been operated when the seminal work of Merton (1974) has been adapted for practical purposes.

1.1 The structural credit risk modelling framework

The fundamental idea on which the firm-value models are based is to consider that the default process occurs *"not with a bang but with a whimper"* to quote the famous sentence of T.S Eliot [4]. All the models underlied by the structural credit modelings class assume a given latent variable, the assets value of the obligor, supposed to be completely observable or not by the investors, and consider that default occurs when this latent variable reaches a critical threshold given by the obligor's liabilities value. Because these models consider the characteristics of the firm, they are called structural models. One of the first models for *pricing* credit-sensitive bonds or similar instruments[5] is developed by Merton (1974). We now present this original framework.

Following the original notations, F denotes simply the value of the debt issue and satisfies the *parabolic partial differential* equation :

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + rVF_V - rF - F_t = 0 \qquad [1]$$

[4] the reference to this sentence has been introduced for the first time by Duffie&Lando (1998).

[5] for a survey of Merton's model applied to mortgages, see Kau and Keenan (1995).

The two boundary conditions and the initial condition necessary to solve that *Forward-Planck* equation are :

. non negativity condition $$F(0,t) = f(0,t) = 0 \qquad [2]$$

. regularity condition $$\frac{F(V,t)}{V} \leq 1 \qquad [3]$$

. initial condition $$F(V,0) = \min[V, B] \qquad [4]$$

Given [2]-[4] one could solve [1] directly for F by standard methods of *Fourier* transforms or separation of variables. Merton (1974) proposed to avoid those calculations by observing that the current problem can be replaced by another one already solved in the litterature.

Indeed, we note that - $f(V,t) \stackrel{\Delta}{=} V - F(V,t)$ and one can substitute for F in [1] and [2]-[4] to deduce the partial differential equation for f. We may write simply :

$$\frac{1}{2} s^2 V^2 f_{VV} + rVf_V - rf - f_t = 0 \qquad [5]$$

Conditions [2] & [3] are stables and the initial condition for that new PDE may be rewriten as :

$$f(V,0) = \max[0, V - B] \qquad [6]$$

By applying the comparative statics results of Black & Scholes (1973) due to the isomorphic price relationship between levered equity of the firm and the non-dividend- paying common stock call option , Merton (1974) obtained :

$$f(V,t) = V.\Theta.[x_1] - B.e^{-rt}.\Theta.[x_2] \qquad [7]$$

where :

$$\Theta[x] \stackrel{\Delta}{=} \frac{1}{\sqrt{2p}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dz$$

and the standard limits are given by :

$$x_1 \overset{\Delta}{=} \frac{Log\left[\frac{V}{B}\right] + \left(r + \frac{1}{2}s^{2}\right)t}{s\sqrt{t}}$$

$$x_2 \overset{\Delta}{=} x_1 - s\sqrt{t}$$

We know that $f(V,t) \overset{\Delta}{=} V - F(V,t)$ and hence, the value of the debt issue can be written as :

$$F(V,t) = Be^{-rt}\left(\Theta\left[h_2\left(d, s^{2}t\right)\right] + \frac{1}{d}\Theta\left[h_1\left(d, s^{2}t\right)\right]\right) \qquad [8]$$

with :

$$d \overset{\Delta}{=} \frac{Be^{-rt}}{V} \qquad [9]$$

and

$$h1\left(d, s^{2}t\right) \overset{\Delta}{=} \frac{-\left[\frac{1}{2}s^{2}t - \log(d)\right]}{s\sqrt{t}} \qquad [10]$$

$$h2\left(d, s^{2}t\right) \overset{\Delta}{=} \frac{-\left[\frac{1}{2}s^{2}t + \log(d)\right]}{s\sqrt{t}} \qquad [11]$$

Of course, the closed form solution [8] is obtained under very stringent conditions[6]. Particularly, it is well known that the default cannot occur before maturity of the debt ; the debt of the firm can only consists of a zero-coupon single class, different maturity dates or seniority levels are not considered. In addition, it is implicitly assumed that bankruptcy can occur only if the firm value is below the face value of the debt.

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[6] for a complete review of these stringent conditions we refer to Bielecki & Rutkowski (2002) and Ammann (2001). In the review of the literature we made below, we considered the extensions of Merton model in a strict sense and hence do not consider here neither the particular case of endogeneous default models nor the case with counterparty default risk.

Merton's credit risk model for zero-coupon bonds has been extended in several ways. Black&Cox (1976) derive the american option pricing framework of Merton (1974) and hence consider that default may occur before the debt maturity. Geske(1977) and Geske and Johnson (1984) derive closed-form solutions for risky coupon bonds. Geske (1977) also provides a formula for subordinate debt within this compound option pricing framework. Ho and Singer (1982) analyse the effect on different indenture provisions such as time to maturity, priority rules and payment schedules on the credit risk of bonds in the modeling of Merton (1974). Ho and Singer (1984) analyse the effect of sinking fund provisions on the price of risky debt. Cox, Ingersoll and Ross (1980) apply the Merton approach to the valuation of credit-risky- variable rate debt to identify variable coupon payout structures that eliminate or reduce interest rate risk. Claessens and Pennacchi (1996) derive implicit default probabilities from the price of Brady bonds using the Merton model. Chance (1990) examines the duration of defaultable zero bonds within the framework of Merton (1974). Shimko, Tejima and van Deventer (1993) derive closed form solutions for risky bonds when interest rates evolve according to Vasicek (1977). Zhou (1997) introduced the possibility of jump occurences in the assets dynamics and provided a framework in which expected and unexpected defaults can coexist.

The number of empirical studies which have been conducted testing Merton model (1974) is rather thin as already mentioned previously. Jones, Mason and Rosenfeld (1984) examine firms with simple capital structures to test the explanatory power of Merton's model with respect to bond prices. Weinstein(1983) show that the model only explain a small part of the systematic risk in corporate bond returns. Applied to mortgage loans, a good explanatory power is reported by Titman and Sorous (1989). Delianedis and Geske (2001) use Merton's model to investigate the possible components of credit spreads and find as a major result that default probability only explain a small part of the observed spreads. Eom, Huang and Helwege (2001) lead a comparative analysis between the five major structural models and find that the Black, Scholes and Merton (1974) perform very poorly in the explanation of the observed spreads particularly for very short term maturities. Ericsson and Reneby (2002) consider that the lack of accuracy of Merton's model pointed out by the previous empirical papers comes from the misspecification of the calibration of the standard deviation of the assets value dynamics. They develop a maximum likelihood test in order to calibrate this parameter and find that the results are strongly supportive of the maximum likelihood approach. In fact, the inefficiency of the traditional estimator may explain the failure of past attempts to implement structural bond pricing models. Vasallou and Xing (2002) use the modeling of Merton (1974) in order to infer the possible impact of credit risk on equity returns.Gemmill (2002) report results strongly supportive of Merton (1974) but only for zero-coupon bonds and very simple capital structure based firms.

Except for the empirical study lead by Vasallou and Xing (2002),the major idea shared by all those empirical tests is to appreciate the accuracy of Merton model (1974) and its extensions with respect to their pricing explanatory power. In other words, all those empirical studies try to understand how accurate is the structural modeling in order to map theoritical spreads into observed spreads.

But, in 1995 when KMV™ Corporation introduced the famous CreditMonitor™ presented for industry purposes as based on the original framework of the modeling of Merton (1974), we think that a shift of meaning has been introduced in the use of the structural credit models. In fact, no particular studies (except those lead internally by the KMV Corporation) have been conducted to appreciate how this type of modeling must be adapted to creditworthiness monitoring purposes. However in a recent paper Leland (2002) attempts to isolate key differences which distinguish the models and second to elucidate the differences in predicted default probabilities that arise from alternative approaches when inputs are similar. One of the important results obtained by Leland (2002) suggests that the choice of the fixed fraction ß of the principal value does not seem critical for the predicted probabilities when ß has been chosen in order to match the endogenous model base case. We consider this paper as very close to our present aims but think that the choice of the fixed fraction of the principal value merits a renewal of attention.

In the second sub-section of this part, we explain how structural credit pricing models can be mapped into creditworthiness monitoring instruments.

1.2 From the modeling of Merton (1974) to the Expected Default Frequencies™

On the basis of the original *assumption A.8*, the dynamics for the value of the firm, *V*, through time, - and under the historical probability **P**- can be described by a diffusion-type stochastic process with stochastic differential equation described by :

$$dV = (aV - C)dt + sVdz \qquad [12]$$

After having mapped **P** to **Q**[7] - the classic equivalent probability measure under which the dynamics of the state variable is a martingale - and supposed that there are no coupon payments - i.e, $C = 0$ - [2] can be simply written as :

$$dV = rVdt + sVdz^{Q} \qquad [13]$$

where *dz* is a standard *Wiener* incremental process.

[7] due to this risk neutral specification, we recognize that a part of the bias analysed in the numerical results comes directly from this specification. We should have worked with respect to instantaneous historical drifts and not with a risk adjusted drift since our current task is related to creditworthiness forecasts and not pricing purposes. However, such a computational task became untractable in regard of the numerical procedures length.

By a very simple application of the Ito's Lemma, [2] can be expressed in a similar way by :

$$LnV = LnV_0 + (r - \frac{s^2}{2})t + s\sqrt{t}e \qquad [14]$$

and $$e \rightarrow N(0,1) \ i.i.d \qquad [14 \#]$$

In the structural modeling of Merton (1974) , the default is pronouced when the assets value of the firm falls below a given level of boundary, noted *B*. If we denote by p_j^i the probability for the firm *i* to default given a level B_j of the boundary value, we can formally write the following steps :

$$p_j^i = \mathrm{P}\left[LnV \leq LnB_j \middle| V_0\right] \qquad [15]$$

$$\Leftrightarrow \qquad p_j^i = \mathrm{P}\left[LnV_0 + (r - \frac{s^2}{2})t + s\sqrt{t}e \leq LnB_j\right] \qquad [16]$$

$$\Leftrightarrow \qquad p_j^i = \mathrm{P}\left[-\frac{Ln\frac{V_0}{B_j} + (r - \frac{s^2}{2})t}{s\sqrt{t}} \geq e\right] \qquad [17]$$

$$\Leftrightarrow \qquad p_j^i = \mathrm{N}\left[-\frac{Ln\frac{V_0}{B_j} + (r - \frac{s^2}{2})t}{s\sqrt{t}}\right] \qquad [18]$$

where **N***(.)* simply expresses the standard normal cumulative function [8].

[8] the derivation of [18] from [17] is allowed using assumption [14 #] as a sufficient condition.

We agree and fully recognise that the approach developed by the monitoring product of the KMV Corporation is slightly different of that exposed above[9]. In fact, in order to avoid the issues related to non gaussians environments [10], KMV proposed to map a built quantity, denoted DD, the so-called distance-to-default, along a large historical distribution of default probabilities. A univoque relationship [11] is hence defined between the scaled measure DD (see the footnote 12) and *EDF*™ ; that is, the expected default probabilities which have been obtained in the presentation above by the quantity defined in [18].
However, we decided to pursue our calibration approach of the boundary with the quantity defined by [18] and not with the specific approach of the KMV Corporation. Indeed, it is sufficiently obvious that default data for large sample of firms are not easily available and hence we are unable to test the real mapping offered by KMV but only a possible estimator produced by the quantity expressed in [18].
The former remark leads us to work directly with the framework derived above and to define a methodology exposed in the next section. Our final aim will be to appreciate quantitatively, through the definition of confidence intervals, the possible errors due to the use of a constant parameter defining the boundary value, denoted B_j in the quantity defined by [18].

2. Methodological Considerations.

We develop a three-steps methodology which can be summarised as follows. Monte-Carlo simulations are first run in order to build the potential distributions of default probabilities for each firm, during the first step. We compute empirical distribution functions on the basis of their kernel densities during the second step, using the Silverman's method. We finally perform resampling techniques in order to reduce the standard errors of the a-quantiles estimators, during the last step.

[9] a) as wentioned in the *footnote 5* of Vasalou and Xing (2002) : There are [at least] two main differences between our approach and the one used by KMV. They use a more complicated method to assess the asset volatility than we do, which incorporates Bayesian adjustments for the country, industry and size of the firm. They also allow for convertibles and preferred stocks in the capital structure of the firm, whereas we only allow for equity , and short and long-term debt.

b) as mentioned in the *footnote 6* of the same authors : Our procedure also differs from the one used in KMV with respect to the way we calculate the distance to default. Whereas we use the formula that follows from the Black-Scholes model, KMV uses the one below:
DD = (Market value of Assets – Default Point)/(Market value of Assets * Asset Volatility).

[10] as a particular subset of this very dense family, one can consider that the specific environment of the assets value distributions is clearly not gaussian.
[11] the graphical representation of this relationship could be found in Crouhy, Galai and Mark (2001) pp. 375 *figure 9.5*

2.1 The Monte-Carlo simulations

During this first step, we would like to generate a vector $\underline{\mathbf{x}} = (x_1, ..., x_d) \in \mathbf{R}^d$ where the *i-th* component of the vector $\underline{\mathbf{x}}$, denoted x_i, is the realisation of the random variable X_i, supposed to be a measurable function defined on the probability space $(W, \mathbf{F}, P)$. More precisely, the vector $\underline{\mathbf{x}}$ is a random vector in the sense that each component and for instance x_i is the outcome of a random generation.

Introduce the processes $\{B_t^1\}_{0 \le t \le T}$ and $\{B_t^2\}_{0 \le t \le T}$ [12] such that $\{B_t^1\}_{0 \le t \le T}$ is supposed to model the short term component of the total liabilities and $\{B_t^2\}_{0 \le t \le T}$ is supposed to model the long term component of the total liabilities. We suppose that the processes $\{B_t^1\}_{0 \le t \le T}$ and $\{B_t^2\}_{0 \le t \le T}$ are both F_t-measurables with $F_s = s(B_s), s < t$ and from accounting rules[13], we are allowed to write down that $B_s = B_s^1 \cup B_s^2$ since $P[[B_s^1 \cap B_s^2] \neq \emptyset] = 0$. Due to the commutativity of measures on the set operations, we can think of B_s as the total liabilities expressed as the sum of the short term component and of the long term component in a natural way.

We can now express precisely how x_i can be defined with respect to the processes $\{B_t^1\}_{0 \le t \le T}$, $\{B_t^2\}_{0 \le t \le T}$ and a random parameter a_i supposed to be extracted from a uniformly distributed density, defined by :

$$f(x) = \frac{1}{a - b} \mathbf{1}_{[\mathbf{a}, \mathbf{b}]}(x)$$

and simply denoted $\mathbf{U}_{[\mathbf{a}, \mathbf{b}]}$. We mention below the interval of interest **[a,b]** and point out that we work in fact with the centered uniform law.

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[12] $\{B_t^1\}_{0 \le t \le T}$ and $\{B_t^2\}_{0 \le t \le T}$ are discrete processes with lenght intervals of time equal to one year in our database. Of course, shorter intervals could be considered to include quartely data, for instance.

[13] we mean that the International Accounting Standard Council edicts precise ways to categorise the various components of the debt in the balance sheet of the firm. Suppose that $P[[B_s^1 \cap B_s^2] = e] > 0$ is equivalent to introduce a modeling in which accounting misappropriations could be possible.

With respect to the processes $\{B_t^1\}_{0 \le t \le T}$ and $\{B_t^2\}_{0 \le t \le T}$, we build the quantity [19] and consider [19] as a first step in the Monte-Carlo simulations :

$$\tilde{B}_i = \overline{B}_t^1 + \tilde{a}_i \, \overline{B}_t^2 \qquad [19]$$

We mentioned that $\{B_t^1\}_{0 \le t \le T}$ and $\{B_t^2\}_{0 \le t \le T}$ are F_t -measurables. At time t, $\overline{B}_t^1$ and $\overline{B}_t^2$ are then supposed to be observable and therefore are not random quantities. We already mentioned that $\tilde{a}_i$ must be considered as the random component included in the quantity [19]. More precisely $\tilde{a}_i$ is the central parameter in our current modeling. Lookig at [19], $\tilde{a}_i$ expresses the fractional part of the long-term component of the debt. As a particular realisation of $\tilde{a}_i$, we notice that $\tilde{a}_i$ equals 1/2 - the expectation of a uniformly distributed density, on the specific centered interval [0,1] , is the actual definition of the boundary given by the KMV Corporation. We can find an economic justification of this value in the following extract of a KMV publishing:

> " We have found that the *default point*, the asset value at which the firm will default, generally lies somewhere between total liabilities and current, or short-term, liabilities."
>
> Modeling default Risk, KMV Publishing - 2002

A complete justification of the particular value taken by $\tilde{a}_i$ can be found in Crouhy, Galai and Mark (2001), pp 371-372 :

> " Using samples of hundred companies, KMV observed that firms default when the value of the value of assets value reaches a level that is somewhere between the total liabilities and the value of short-term debt. Therefore, the tail of the distribution of assets value below total debt value may not be an accurate measure of the actual probability of default. Loss of accuracy may also result from factors such as the non-normality of the asset-return distribution, and the simplifying assumptions about the capital structure of the firm.
>
> This may be aggravated if a company is able to draw on (otherwise unobservable) lines of credit. If the company is in distress, using these lines may (unexpectedly) increase its liabilities while providing the necessary cash to honor promised payments. For all these

reasons, KMV implements an intermediate phase before computing the probabilities of default. "

If we agree with the explanations provided to explain the loss of accuracy due to the non log-normality of assets returns and the simplifying capital structure, we do not understand in what sense the second part of explanations could be supportive of the use of a constant fractional parameter for the modeling of the long-term debt component. On the contrary, we argue that such explanations, pointing out the role played by distress factors, are rather supportive of a more flexible modeling for the fractional part of the long-term debt. Indeed, one can argue that a fixed parameter is not the most convenient modelling in order to track the default dynamics with a flexible and sensitive approach. Credit risk managers must have the possibility to appreciate the possible consequences induced by alternative values for the fixed part of the default barrier in regard of events described by Crouhy, Galai and Mark (2001) in the life-cycle of the firm.

In light of the definitions of [19], the Monte-Carlo simulations for each $\tilde{B}_i$ have been performed 2000 times for each single firm. Hence we build a random vector $\vec{\tilde{B}} = (b_1, ..., b_n) \in \mathbf{R}^n$ where n simply denotes the number of simulations. Since we perform such simulations for each single company j ($j=1,...,m$), we obtain the random matrix, denoted $\mathrm{B}_{[n\times m]}$ and defined by :

$$\mathrm{B}_{[n\times m]} = \left(\tilde{B}_{ij}\right) \qquad [20]$$

where $\left\{\tilde{B}_{i\bullet}\right\}$ is given by [19].

The second step during the simulations has been to define default probabilities vectors on the basis of the random default barriers vectors previously computed. Hence, we have to map the random matrix $\mathrm{B}_{[n\times m]}$, defined by [20] into the random matrix $\mathrm{P}_{[n\times m]}$ defined by :

$$\mathrm{P}_{[n\times m]} = \left(\tilde{p}_{ij}\right)$$

and $\left\{\tilde{p}_{\bullet j}\right\}$ is given by a function, simply denoted f,defined as follow :

$$f \; : \; \mathbf{R}^n \rightarrow \mathbf{R}^n$$
$$\vec{\tilde{B}} \mapsto \vec{\tilde{p}}$$

$\vec{\tilde{B}}^j = \left(\tilde{B}_1,..., \tilde{B}_n\right)'_j \in \mathbf{R}^n$ and $\vec{\tilde{p}}^j = \left(\tilde{p}_1,..., \tilde{p}_n\right)'_j \in \mathbf{R}^n$. $\vec{\tilde{B}}^j$ stands for the vector of random default barrier for the n firms of the sample and $\vec{\tilde{p}}^j$ stands for the vector of default probabilities computed from the specification of f .As a more formal expression for the function f, defined above, we define the following quantity :

$$f\left(\tilde{B}_j\right) = \mathbf{P}\left(\inf_{s<t} V(t) - \tilde{B}_j \le 0 \middle| \mathrm{X}[\bullet]\right) \qquad [21]$$

where :

$\Xi[\bullet]$ simply denotes a set of caracteristics of the firm.

$V(t)$ stands for the assets value of the firm at time t

We expresse that last quantity to underline the specificity of the current simulations. Here, we assume that the misspecification of the default probabilities comes exclusively from the misspecification of the boundary value.

We think necessary to sum up this first step since the following steps are based on this first step and of course, the results depend largely of this simulation step. First, we identify two components for the default barrier. The first component models the short-term debt and the second models the long-term debt. Second, we define the default barrier using equation [19] and introduce a random component due to the presence of $\tilde{a}_i$. Third, we run 2000 times Monte-Carlo simulations and thus obtained a first vector of size $n = 2000$ including various default barrier values with respect to the values of $\tilde{a}_i$. Fourth, by using the relation [18], we obtain a second vector of size $n=2000$ including the default probabilities associated with the random default barriers. We then use this vector of default probabilities in the rest of the paper. Nevertheless, we mention that we can consider the forthcoming tests both as tests of $\tilde{a}_i$ and of default probabilities since, in fact, we derived these default probabilities from a former step defining the default barrier value.

In order to define the rating buckets later used in the paper, we developed an algorithm[14] to classify the default probability distributions relative to the discrete measures of default probabilities provided by Standard and Poor's.

[14] This algorithm consists into a serie of successive tests for default probabilities considered as references in the distribution. These default probabilities are the median, the default probability associated with the third quartile and the maximum of the distribution. We first test the median and define a first category , then we compare the value of the maximum with this category in order to appreciate the error made by using this first bucket. If this second test is accepted , we consider

2.2 Empirical distributions of the default probabilities with the Silverman's method.

At this step, we obtained the random vector $\underset{\sim}{\underline{B}}^{j} = \left(\tilde{B}_1,\ldots,\tilde{B}_n\right)_j' \in \mathbf{R}^n$

and with the function f, we also derived the random vector $\underset{\sim}{\underline{p}}^{j} = \left(\tilde{p}_1,\ldots,\tilde{p}_n\right)_j' \in \mathbf{R}^n$. We now want to build the empirical distribution of the default probabilities given by each vector $\underset{\sim}{\underline{p}}^{j}$. Fisrt, we think that such distributions could be meaningful in our comprehension of the possible model risks. Second, it could be useful to observe the differences in the distributions for each class of rating since our aim is to propose a more flexible modeling of the fractional part of the long-term debt. Hence, one can suppose that the analysis of the distributions based on the vectors $\underset{\sim}{\underline{p}}^{j}$ could lead to encapsulate specificities of each rating class.

This density estimation is traditionnaly performed using the kernel method, that is, the estimator :

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{(x - x_i)}{h}\right) \qquad [22]$$

where n=number of observations, x_i = observation i, h= window width (also called the smoothing parameter), and $\mathbf{K}$ = the kernel estimator. We choose as a kernel $\mathbf{K}(x) = \left(\frac{1}{\sqrt{2p}}\right)e^{-\frac{x^2}{2}}$ and[15], following the standard approach, we use for the window width the quantity $h = s(4/3)^{1/5}n^{1/5}$,

where s represents the standard deviation for the whole series.

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the whole distribution in the initial rating bucket. Otherwise, we use the third-quartile value to define an additional test in order to affine the rating bucket and repeat the first test. The final bucket is defined as the rating bucket which best fits these successive constraints.

[15] the normal kernel is used here since we are interested in testing the Gaussian environment of the default probabilities.

At this stage, we have to mention an important point. $\underline{\tilde{p}}^{j}$ is not a vector of observed real data since we have built those data with the Monte-Carlo simulations developed during the first sub-section. However, we think that we are allowed to employ the methodology of the kernel densities in order to track the modalities of these empirical distributions. As explained below in the fourth section, we think that the various modalities of the distributions can be understood as an additional argument to develop the more flexible approach that we tried to develop here.

2.3 Bootstrap method to reduce the variance size of the a -quantile estimators for the confidence intervals[16].

During this last step, we might be interested in inferences about the quantity x_q, the q-quantile of the empirical distribution of vectors $\underline{\tilde{p}}^{j}$ and our statistics might be a sample quantile such as $\hat{q} = \underline{\tilde{p}}^{j}_{[qn]}$ or $\hat{q} = \underline{\tilde{p}}^{j}_{([qn]+1)}$ where $\underline{\tilde{p}}^{j}_{(1)} < \ldots < \underline{\tilde{p}}^{j}_{(n)}$ denote order statistics of the sample. Finding the standard error of such sample-quantile can be done using a typical bootstrap technique[17]. We can resume our bootsrap methodology as follow :

- We first draw k bootsrap resamples from $\tilde{p}_1,\ldots,\tilde{p}_n$ by samples with replacement and we denote these resamples $p^{\circ i}_1,\ldots, p^{\circ i}_n$ for $i = 1,\ldots, k$.

- We evaluate the estimator for each resample to get the bootstrap replicates $\hat{q}^{\circ}_i = \hat{q}\left(p^{\circ i}_1,\ldots, p^{\circ i}_n\right)$ for $i = 1,\ldots, k$.

- The variance of the estimator $\hat{q}$ is estimated by the empirical variance of the bootstrap replicates :

$$\widehat{\mathrm{var}}^{\circ}_k\left(\hat{q}\right) = \frac{1}{k-1}\sum_{i=1}^{k}\left(\hat{q}^{\circ}_i - \bar{q}^{\circ}\right)^2$$

[16] Lecture Notes realised by Rüdiger Frey and Alexander McNeil, not published, in line with this technic are gratefully acknowledged.

[17] For a detailed discuss of the resampling technics we refer to Davison & Hinkley (1997), the seminal paper of Efron (1979) and Efron & Tibshirani (1993)

where :

$$\bar{q}^{\circ} = \frac{1}{k} \sum_{i=1}^{k} \hat{q}_i^{\circ}$$

By taking the square root of this estimated variance , we thus obtain the estimated statistic for $\hat{q}$. This particular use of the bootstrap method is the so-called variance-reducing technique and can be considered as a particular case of Monte-Carlo simulations applied to the estimation of an unbiased estimator of a certain parameter, here the a-quantile of an empirical distribution.

Thus to summarize, after having computed Monte-Carlo simulations according to the methodology described in the first sub-section, we can obtain random vectors of default probabilities. The default probabilities have been obtained by calculating the standard quantity derived from the framework of Merton (1974). Our aim is to define confidence intervals to appreciate the robustness of a constant parameter for the fractional part of the long-term debt component. To appreciate such possible errors, we first compute the kernel densities of the built-default probabilities distributions to analyse possible multi-modalities in the distributions. If such modalities apppear, we interpret such ones as an empirical proof of the need to perform a more flexible modeling of the fractional parameter. Indeed, by using a single constant parameter without considering obligor's rating class , the implicit assumption is made that all the modalities are the same unconditionnally to the rating class considered in the analysis of the creditworthiness. We finally apply a particular bootstrap technique in order to consolidate the robustness of the confidence intervals .

3. The Data

We have two data sets of firm's characteristics both for the American and the European Markets.The total number of records is approximatively 1300 firms including european and american companies. These data cover five years (1997-2001) of accounting reports and have been updated for the last time the 9th September 2002 by Comstock, a department of Standard and Poor's, providing financial informations.

We kept in our samples the observations that satisfied the following criteria :

- an asset volatility included in the range 0.15 % and 0.55%, computed as the standard deviation of asset returns. These figures have been considerably enlarged in regard of those reported by the KMV Corporation and by Leland (2002). But, since we did not find particular justifications for the range 0.21% - 0.49%, we thought that we were allowed to work on the basis of the figures computed from our database. In addition, we note that the upper bound can be supported by the results obtained by Ericsson and Reneby (2002) with respect to their maximum likelihood computations[18]. From the *figure 1*, computing the default probabilities derived from Merton (1974), we see also that the range 0.21% - 0.49% is a convenient but not necessary representative range of the entire support of gaussian default probabilities.

- Long-term debt values different from zero for all the years, it is to say from 1997 to 2001. Since our calibration task is related to the fractional part of this component in the actual definition of the credit monitoring system of the KMV Corporation, it seems reasonnable to appreciate only the situation of firms having such a component in their accounting books.

- Short-term debt values different from zero for all the years. We think that such sort leads to work with more homogeneous sub-samples. From the perspective of the computational extractions of the data, this additional condition does not seem to have a major effect upon the number of firms.[19]

[18] see the *figure 2* of this paper.
[19] less than 10 firms have been ruled out of the previous sample. Hence, the difference in percentage is about 2%.

3.1 American Data

This panel includes 560 american firms from 1997 to 2001. Some basic statistics can be found in the *table 1*, in which we report the mean, the standard error, the minimum, the maximum, the skewness and the kurtosis which lead to appreciate the statistical properties of the distributions of assets values, long-term debt and short-term debt.

3.2 European Data

This panel includes 577 european firms from 1997 to 2001. Some basic statistics can be found in the *table 2*, in which we report the mean, the standard error, the minimum, the maximum, the skewness and the kurtosis which lead to appreciate the statistical properties of the distributions of assets values, long-term debt and short-term debt.

3.3 Sub-samples definitions

We define brackets based on the distributions of default probabilities calculed with [18] for each single firm included in the american and european samples. The specific bracket (a class of rating) is considered with respect to the discrete measures provided by rating agencies like Standard and Poor's and Moody's and is determined as an output of the algorithm described in the *footnote 13* .
During this step, we split the two original samples into seven sub-samples reflecting the seven rating classes of long-term notations of Standard and Poor's, going from *AAA* to *CCC*. The buckets go from *AAA* to *CCC* since the historical default probability over a one-year horizon associated to the last class is approximatively *24.5%* and can be considered as an extreme case of default probability prediction. A bar chart presenting the number of firms by rating category can be found in the *figure 2*.
Our aim during this step is to isolate specific properties of each class of rating given a finite number of firms independently of their geographic location.

4. The Numerical Results

We point out in this last part the main results derived from the computations. As a general and major remark, we observe that the median of the default probabilities distributions, reflecting the case where $a = 1/2$, is not efficient, in general, in order to predict the possible worst *creditworthiness* cases. Second, we mention that this inefficiency sharply increases when the obligor's quality decreases. Third, we argue that the use of the resampling technique should be performed in order to compute the confidence intervals for well-below rated firms. Fourth, we think that the multi-modalities exhibited by empirical default probabilities distributions, support the necessity of a more flexible modeling of the boundary.

4.1 The default probability associated with $a = 1/2$ is not an efficient proxy of the possible worst *creditworthiness* cases.

In this section, we refer to *tables 3, 4, 6* and *7*. *Table 3* (respectively *table 4*) summarizes the numerical results for a given sub-sample of the european (respectively american) firms sample. *Table 6* (respectively *table 7*) summarizes the numerical results obtained for firms rated BBB (respectively rated AA) after having splitted the original samples (european and american) into seven sub-samples corresponding to rating classes. As mentioned previously, our aim is to isolate the specific properties of each class of rating defined with respect to the algorithm described in the *footnote 13*.
Each table summarizes the statistical description of the default probabilities distributions computed with respect to the simulations of the random default barriers vectors (see *section 2.1*). The notations used in the tables are usual. For instance, "Min" stands for the minimum of this distribution, "3rd Qu." stands for the third quartile of the distribution. "LCL mean" and "UCL mean" stand for the lower confidence limit for the mean and the upper confidence limit for the mean. The computation of these two last statistics can be seen as one of the major tasks of the paper. "Median" stands for the median of the default probability distribution. Due to the determination of this distribution, the median can be directly associated with the case $a = 1/2$. Hence, the median reported in the tables corresponds to the default probability computed by KMV™ in its monitoring instrument[20]. The comparison of the median with the mean, reported in the tables with the confidence limits, leads to appreciate the bias induced by the use of the constant value for the fractional long-term component of the default barrier.
As a particular example, take the case of E2. E2 is the second firm included in the european sample. In regard of the distribution summarized by basic statistics,we can consider that E2 is a firm rated A. In the computations, the median should be considered as the default probability obtained with a=1/2. For this class of rating, we observe that the mean can differ significantly of the previous statistic. In addition, the upper and lower bounds for the confidence intervals (LCL and UCL) support the argument that a misspecification, possibly incentive, can be induced by the constant parameter a=1/2.

[20] if they followed equation [18], but ,as already discussed, we know that it is not the case.

We have chosen to report results for firms rated *AA (table7)* and firms rated *BBB (table 6)* in order to compare the respective cases of firms said to be well-above rated and well-below rated.

As a common result shared by these two classes, the mean and the median generally differ. However, we note that the gap can significantly differ for firms rated *BBB* (we note the case of *A4* in comparison with *E224* in the *table6*) and seems to be more homogeneous for well-above rated firms.

A second key remark comes from the examination of the confidence limits. These last ones can support very different amplitudes when we analyse the case of well-below rated firms but they admit amplitudes of same order when we analyse the case of well-above firms

Based on these conclusions, we think that the use of the constant fractional value defining the role played by the long-term component, in the default barrier of the monitoring instrument developed by KMV, is not efficient in a general case. More precisely, this particular modeling can significantly underevaluate the dynamics of well-below rated firms. But, for the case of well- rated firms, we agree with the conclusions lead internally by KMV concerning the small impact on default probabilities forecasts due to variations of the fractionnal value of the long-term debt component.

4.2 The errors induced by a constant fractional long term componentcan possibly increase when the obligor's quality decreases.

Figure 5 shows that the higher the initial default probability (obtained with a=1/2, case of the median) is and worth seems to be the error induced by the use of the constant fractional coefficient. The 95th-percentiles are less and less close to the initial probability.This remark is in line with the previous observation related to the homogeneity of the standard deviation of the mean for the well above rated firms[21] (*AAA*-*AA*-*A*) and the heterogenity of the standard deviation of the mean for the well below rated firms (*BBB*-*BB*-*B*-*C*). This result supports the idea that a more flexible approach of the boundary modeling would be necessary in order to increase the accuracy of early warning systems based on the structural credit modeling of Merton (1974), especially for well-below rated firms.

[21] these discrete measures have been mapped from the figures extracted from a publication of Standard and Poor's. The methodology used in the paper in order to build these artificial buckets is explained in the part II.

4.3 A resampling method should be performed to compute the confidence intervals for low rated firms.

We observe that the estimation of robust confidence intervals could be incentively disturbed for low rated firms such *BBB-BB* and *B*. Such a remark is in line with the previous observation related to the relationship between the accuracy of the constant fractional component and the obligor's quality. Indeed, we already mentioned that the relationship presented in *figure 5* exhibits more noisy estimations for lower rating classes. Hence, the need of bootstrap computations for those classes seem to be a very logical task.. The histogram in *Figure 3* shows that the distribution of replicated median is highly skewed. A normal quantile-quantile plot can be used to further assess deviation from the normal distribution. *Figure 4* suggests that both tails of the distribution of replicated variances deviate from the normal distribution. Thus there is evidence that bootstrapping is a better approach than normal-based methods. We extract as illustrations of this method two firms respectively rated *BBB* (*firm A1*) and *B* (*firm E10*) and report the results in *table 5*. Both can be considered as poorly rated firms but the second one belongs to a rating class considered as particularly critical by investors and analysts. The confidence limits computed on the basis of the resampling method lead to better appreciate the bias included in the magnitude of the confidence intervals for the median (corresponding to the case evaluated by the monitoring instrument of KMV). The confidence limits based on the resampling method can significantly differ of the empirical confidence limits. This statement is particularly true for the *B* rated firm.

4.4 The Kernel Empirical Densities exhibit multi-modalities.

We performed the kernel empirical densities on the basis of the methodology described in part two, section three and reported these results in *figure 6*. As a general and common property shared by all the rating classes, we observe that these empirical densities do not support the stringent assumption of the gaussian distribution. We point out also that the lower the rating and the more important is the number of modalities. We think that such observation is an additional illustration of the more noisy default probabilities distributions describing the worse rating classes. To our present knowledge, no previous work reports in the credit risk litterature such a stylised fact concerning this possible property of the default probability distributions. However, it is a well known topic in the option pricing research since Ritchey (1990).

Conclusion

The aim of this paper has been to consider the calibration of the boundary parameter in the context of a particular structural credit modeling.

We think necessary to mention that the current work is based on an univariate context. In an appropriate probability space, we studied the default event dynamics for a single firm, from the perspective of the modeling of Merton (1974). Hence, specific issues associated with credit portfolio management are not studied here[22]. As a particular case of those issues, we do not explore, for instance, the analysis of multivariate *creditworthiness* dynamics. In order to focus on the particular calibration of the boundary value, we thought important to separate the univariate case from the multivariate environment to avoid delicate additional technical issues such like the appropriate modeling of the dependence between firms, for instance.

We think that the major contribution of the paper is related to the test of the consistency of the practical adaptation of the boundary value defined by Merton(1974). Mainly, -in regard to the confidence intervals obtained - we found that a definition of the boundary value based on a constant fractional part of the long-term component of the debt is clearly not sufficient to qualify the consistency of such modeling under the highlights of the early warning function attributed to those models. We could simply not give a precise definition of that boundary value in respect to a specific class of rating , for instance - such an attempt would be untractable - but we could formalise an overview of the sensitivity of that parameter. In the perspective of the growing quantitative credit tools developments, we think that such an overview is an additional empirical reference in the choice of a consistent credit monitoring modeling.

[22] However, we do not exclude the possibility of a further paper focusing on the possible impact of a misspecification of the boundary upon the default probabilities correlation values when one considers creditworthiness forecast based on a structural modeling in a mutltivariate environment.

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Figure 1 Default Probabilities computed in respect with the Assets Volatility and the Boundary Value following Merton [1974]

We compute the default probabilities - derived from [18] - and plot in 3-dimensional graphic these probabilities with respect to the assets volatility and the boundary value.

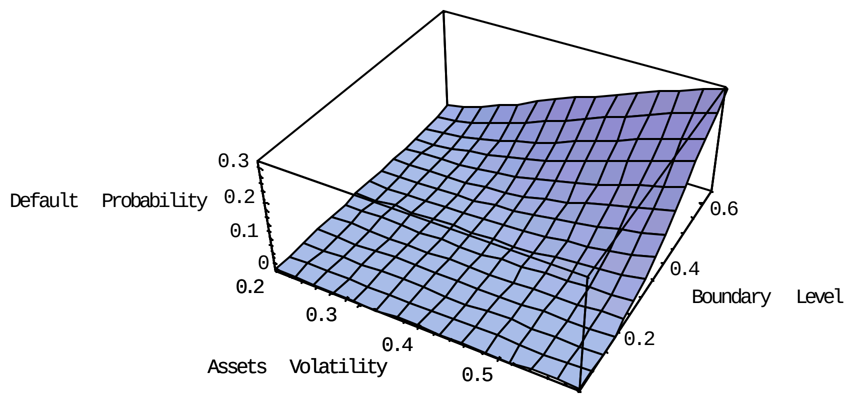

Figure 2 Whole Sample Repartition in respect to Rating Classes

After having defined our sample conformingly to the methodology exposed in the part III and computed the associated default probabilities, we associate the basic rating buckets to this sample. We use the figures offered by a publication of Standard and Poor's (september 2002) in order to define the buckets. We note that two sub-populations (AA and BBB) are over-represented in comparison to the classical figures of the rating universes

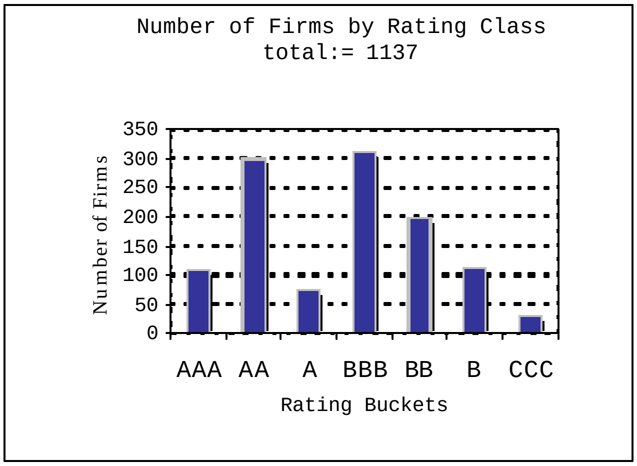

Table 1 Basic Statistics for American Firms

Some basic statistics describing the american firms panel

Millions $ — **Panel A : American Firms. Basic Statistics** — Number of observations : 560

		2001	2000	1999	1998	1997
Assets Value						
	Mean	8644.2946	7466.9533	6388.5265	5473.1082	4628.1535
	Standard Dev	44157.2269	36317.4203	29709.5213	25496.1958	21942.6372
	Min	2.453	2.473	2.019	2.579	0.55
	Max	646944	551607	388570	343620	310897
	Skewness	10.4344	10.5426	9.3076	9.7221	10.6002
	Kurtosis	123.1972	129.4225	96.3889	106.5897	130.0486
Long-Term Component						
	Mean	1654.7798	1473.0034	1169.0078	1010.1578	798.0538
	Standard Dev	5915.7316	5737.5163	3811.356	3319.8036	2706.0346
	Min	0.004	0.012	0.01	0.007	0.011
	Max	81053	80335	45017	43849	31048
	Skewness	8.0954	9.279	6.6598	7.2229	7.1711
	Kurtosis	83.0567	106.6584	54.4384	67.8745	63.7883
Short-Term Component						
	Mean	993.5406	929.2669	796.4398	639.4652	589.9402
	Standard Dev	6608.8018	6367.0593	5776.1342	4870.038	5656.0249
	Min	0.008	0.01	0.002	0.006	0.006
	Max	102484	105084	98997	91894	124236
	Skewness	11.0456	12.0551	12.9263	14.3366	19.314
	Kurtosis	139.1613	165.877	189.3134	239.866	412.9436

~ Data provided by Comstock ~ Last Update : 9th September 2002

Table 2 Basic Statistics for European Firms

Some basic statistics describing the european firms panel

Millions €		**Panel B : European Firms. Basic Statistics**				Number of observations : 577
		2001	2000	1999	1998	1997
Assets Value						
	Mean	8521.7493	8095.8777	7303.2947	8787.1793	8080.5986
	Standard Dev	56496.1016	53861.0328	42178.9081	50839.6685	40840.8087
	Min	1.995	3.776	4.02	3.302	1.453
	Max	1022513	987433	722746	901119	689568
	Skewness	14.1967	14.7672	12.6839	13.5406	11.5727
	Kurtosis	227.8357	242.747	185.4491	211.4726	164.0661
Long-Term Component						
	Mean	1467.2994	1284.3841	1287.5188	1628.0257	1292.9985
	Standard Dev	9399.8177	9976.7081	10340.5262	16584.6341	8515.4035
	Min	0.004	0.013	0.005	0.007	0.001
	Max	175400	221120	223691	382977	175958.438
	Skewness	14.0442	19.3351	18.4134	21.4626	16.2203
	Kurtosis	231.3067	415.2401	380.1628	489.2722	314.3956
Short-Term Component						
	Mean	1067.4975	877.2973	612.3	599.8271	555.412
	Standard Dev	10013.6254	7788.6438	4454.201	3020.54	2594.4484
	Min	0.015	0.014	0.012	0.011	0.009
	Max	206138	171175	96734	59899	45796
	Skewness	17.1442	19.1114	18.4114	14.4057	11.6341
	Kurtosis	326.0328	404.3345	383.6438	263.8343	175.1105

~ Data provided by Comstock ~ Last Update : 9th September 2002

Table 3 : Computations of the Confidence Intervals for the mean of the european default probabilities distributions

*** Summary Statistics for data in: def.prob.euro.data.1 ***

numeric matrix: 12 rows, 256 columns. Only the first twelve columns are reported here, denoted E1,..., E12

E2 is the second firm included in the european sample. In regard of the distribution summarized by basic statistics,we can consider that E2 is a firm rated A. In the computations, the median should be considered as the default probability obtained with a=1/2. For this class of rating, we observe that the mean can differ significantly of the previous statistic. In addition, the upper and lower bounds for the confidence intervals (LCL and UCL) support the argument that a misspecification, possibly incentive, can be induced by the constant parameter a=1/2.

			European Sample		Number of observations : 577	
	E1	E2	E3	E4	E5	E6
Min:	0.03030000	0.00980000	2.279000e-001	4.5098000	0.16480000	8.600000e-002
1st Qu.:	0.04590000	0.02390000	4.008500e-001	7.7695250	0.21747500	8.860000e-002
Mean:	0.07356760	0.07832285	7.585669e-001	12.7853210	0.29191215	9.139710e-002
Median:	0.06840000	0.05730000	6.678000e-001	11.8590000	0.28270000	9.130000e-002
3rd Qu.:	0.09810000	0.12105000	1.057125e+000	17.4628250	0.36200000	9.400000e-002
Max:	0.13760000	0.23770000	1.656500e+000	24.2493000	0.46030000	9.700000e-002
Std Dev.:	0.03093466	0.06288091	4.036953e-001	5.7017491	0.08496789	3.170339e-003
SE Mean:	0.00069172	0.00140606	9.026902e-003	0.1274950	0.00189994	7.089093e-005
LCL Mean:	0.07221103	0.07556535	7.408638e-001	12.5352840	0.28818608	9.125807e-002
UCL Mean:	0.07492417	0.08108035	7.762701e-001	13.0353579	0.29563822	9.153613e-002
Skewness:	0.41853405	0.85130995	5.210146e-001	0.3240518	0.29913126	3.513298e-002
Kurtosis:	-1.02599155	-0.39815755	-8.913440e-001	-1.1105111	-1.08061411	-1.206701e+000
	E7	E8	E9	E10	E11	E12
Min:	0.000000e+000	0.000000e+000	0.000000e+000	0.1997000	0.000200	0.11740000
1st Qu.:	0.000000e+000	0.000000e+000	3.500000e-003	1.3494250	0.000200	0.33457500
Mean:	1.025610e-002	2.612300e-003	2.696923e-001	6.5380014	0.000200	1.00878885
Median:	9.000000e-004	0.000000e+000	6.340000e-002	4.4723500	0.000200	0.78175000
3rd Qu.:	1.100000e-002	2.000000e-004	4.239500e-001	10.6744250	0.000200	1.58682500
Max:	7.910000e-002	4.650000e-002	1.494400e+000	21.0605000	0.000200	2.75620000
Std Dev.:	1.819945e-002	7.552294e-003	3.847595e-001	6.0151059	0.000000	0.76659323
SE Mean:	4.069522e-004	1.688744e-004	8.603484e-003	0.1345019	0.000000	0.01714155
LCL Mean:	9.458005e-003	2.281112e-003	2.528196e-001	6.2742228	0.000200	0.97517168
UCL Mean:	1.105419e-002	2.943488e-003	2.865651e-001	6.8017799	0.000200	1.04240602
Skewness:	2.087398e+000	3.476612e+000	1.533621e+000	0.8194055	1.000751	0.67671890
Kurtosis:	3.480077e+000	1.201859e+001	1.305557e+000	-0.5443556	-2.002003	-0.77565824

Table 4 : Computations of the Confidence Intervals for the mean of the default probabilities distributions for American firms

*** Summary Statistics for data in: defprob.america.1 ***

numeric matrix: 12 rows, 256 columns. Only a sub-sample of seven firms is reported here, denoted A8,..., A14

A8 is the eighth firm included in the american sample. In regard of the distribution summarized by basic statistics,we can consider that A8 is a firm rated BBB. In the computations, the median should be considered as the default probability obtained with a=1/2. For this class of rating, we observe that the mean can sharply differ of the previous statistic. In addition, the upper and lower bounds for the confidence intervals (LCL and UCL) support the argument that a misspecification, possibly incentive, can be induced by the constant parameter a=1/2.

	A8	A9	A10	A11	A12	A13	A14
Min:	0.00000000	0.0000000	1.303900000	0.0000000000	0.000200000	0.000000000	0.000200000
1st Qu.:	0.00060000	0.0000000	1.316600000	0.0000000000	0.003400000	0.000100000	0.005175000
Mean:	1.81677330	4.9203992	1.330039700	0.0097123000	0.108289100	0.065619600	0.202871250
Median:	0.08290000	0.1100000	1.330350000	0.0000000000	0.028850000	0.005850000	0.059000000
3rd Qu.:	1.88102500	6.4888250	1.343500000	0.0045000000	0.163175000	0.075800000	0.303725000
Max:	14.16070000	31.6032000	1.357300000	0.1212000000	0.553400000	0.491000000	1.044200000
Std Dev.:	3.30959578	8.3299137	0.015551091	0.0224101123	0.148937021	0.114325749	0.278016418
SE Mean:	0.07400481	0.1862625	0.000347733	0.0005011053	0.003330333	0.002556401	0.006216636
LCL Mean:	1.62596736	4.4401601	1.329143143	0.0084203046	0.099702532	0.059028453	0.186842953
UCL Mean:	2.00757924	5.4006382	1.330936257	0.0110042954	0.116875668	0.072210747	0.218899547
Skewness:	2.07516849	1.7268907	0.017217369	2.7972229993	1.460930623	2.011456448	1.441860306
Kurtosis:	3.36087919	1.7666674	-1.201742425	7.4620377377	0.976987760	3.116765659	0.915581802

Table 5 Confidence Intervals using the Resampling Technic

After having obtained confidence intervals with respect to the simulations performed and reported in tables 6 and 7, a resampling technique has been used in order to appreciate our own estimation errors. Two examples are shown in this table. As a result, we observe that the use of such a method can be crucial for well below firms such as a B-rated firm. This figure is an additional illustration of the comment associated with figure 4 concerning the heterogenity of well below firms and the more homogeneous behavior of the set defined by well above firms.

Number of Replications: 1000	Company A1 equivalent rating class : BBB			
Summary Statistics:				
Observed	median	Bias	Mean	SE
	0.597	0.003687	0.6007	0.02529
Empirical Percentiles:	2.5%	5%	95%	97.5%
median	0.5475	0.5592	0.6379	0.6432
BCa Confidence Limits:	2.5%	5%	95%	97.5%
median	0.5432	0.5522	0.6351	0.6389

Number of Replications: 1000	Company E10 equivalent rating class : B			
Summary Statistics:				
Observed	median	Bias	Mean	SE
	4.472	-0.001923	4.472	0.2051
Empirical Percentiles:	2.5%	5%	95%	97.5%
median	4.067	4.105	4.761	4.814
BCa Confidence Limits:	2.5%	5%	95%	97.5%
median	4.042	4.077	4.75	4.792

Figure 3 Empirical Distribution of the replicated bootstraps

Empirical distribution of the replicated bootstraps is highly skewed and support the idea of multi-modalities in the distribution of default probabilities. This stylised fact is underlined in part IV section 4 and is illustrated by the kernel densities of five classes of rating through figure 7.

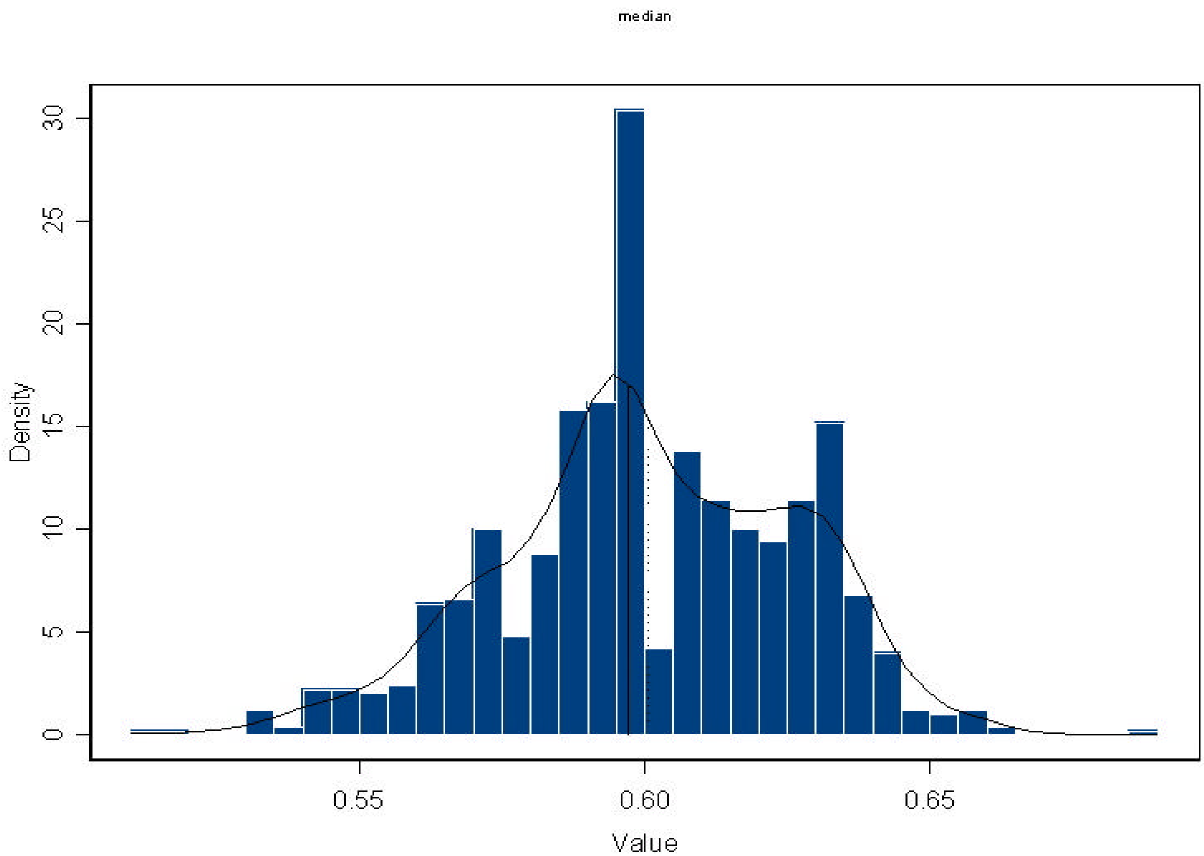

Figure 4 The Q-Q Plot of the Resampled Quantiles against the Gaussian Quantiles

The Q-Q plot of the resampled quantiles against the gaussian quantiles suggests as a classical result in statistics that both tails of the distribution of the replicated median deviated largely from the gaussian case. This stylised fact can be found in the tables 6 and 7 summarizing results of the simulations for BBB firms and AA firms respectively. There is evidence that bootstrapping is a better approach than normal-based methods especially for well below firms as mentioned in the comments asssociated with table 6.

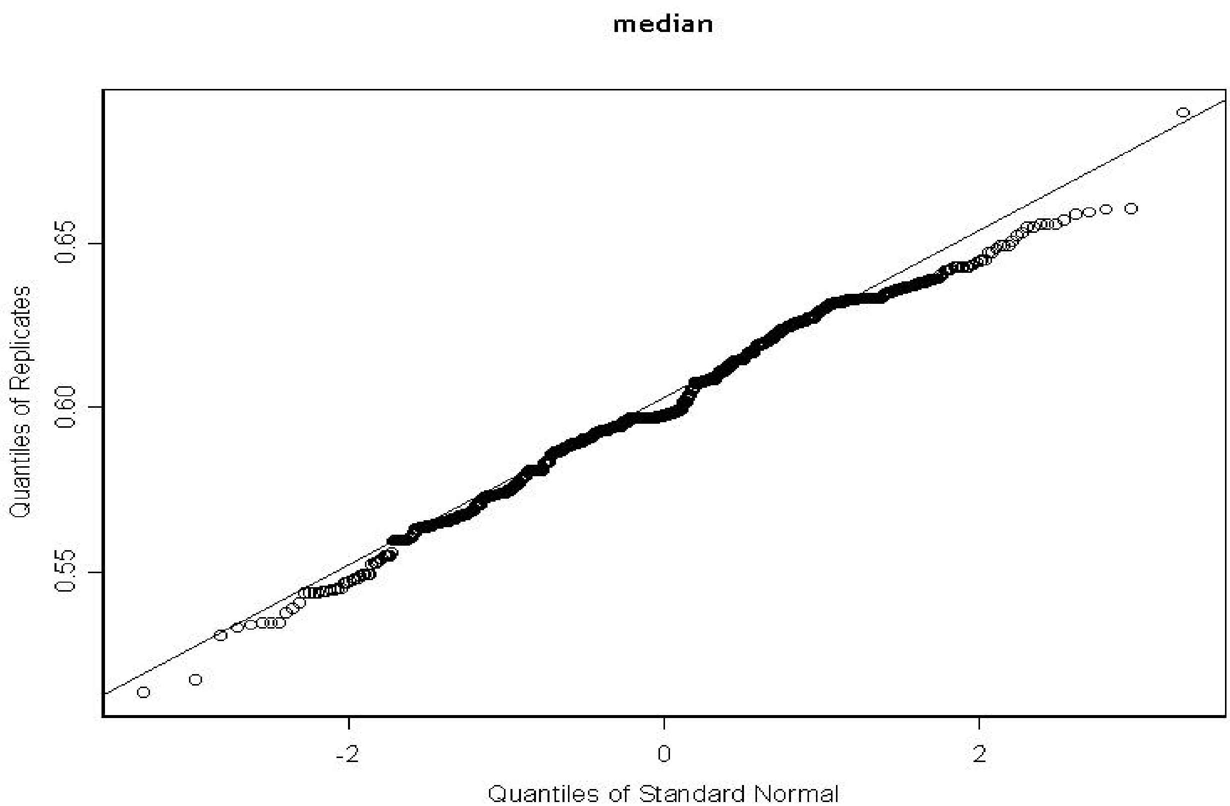

Table 6 : Computations of the Confidence Intervals for the mean of the BBB default probabilities distributions

*** Summary Statistics for data in: BBB ***

numeric matrix: 12 rows, 311 columns. Only a sub-sample of eight american firms and eight european firms has been reported here.

Table 6 presents the statistic results for the sample defined by BBB firms.
As an illustrative case, we describe the firm A16. A16 is the 16th firm of the american panel. The minimum reported in the table (0.00) means that this is the minimum value for the default probability (expressed in percentage) associated with a close to zero (a very small component of long-term debt is considered). The maximum reported in the table (6.13) means that this is the maximum value for the default probability (expressed in percentage) associated with a close to one (a very large component of long-term debt is considered). LCL mean (0.74) and UCL mean (0.92) are the standard lower and upper confidence limits for the mean computed by SPLus. The last two statistics are the skewness (1.95) and the kurtosis (2.83) strongly supportive of the non gaussian property of the default probability distribution.
We focus on three major results pointed out by this table. First, the mean and the median generally differ sharply for this class of rating. Second, the standard deviation of the mean for this class is clearly not homogeneous.Third,we observe that UCL and LCL are suport the idea that a possibly incentive misspecification is induced by using a constant fractional parameter for the long-term debt component.

	A1	A4	A12	A14	A16	A20	A21	A26
Min:	0.05950000	0.00000000	0.000200000	0.000200000	0.00000000	0.00000000	0.019600000	0.00290000
1st Qu.:	0.20352500	0.00000000	0.003400000	0.005175000	0.00010000	0.00000000	0.046925000	0.03940000
Mean:	0.85412130	0.34533725	0.108289100	0.202871250	0.83568820	0.28205755	0.128637650	0.48100410
Median:	0.59700000	0.00105000	0.028850000	0.059000000	0.03585000	0.00000000	0.097150000	0.24120000
3rd Qu.:	1.37145000	0.19037500	0.163175000	0.303725000	0.86272500	0.03550000	0.196150000	0.81720000
Max:	2.71870000	3.47820000	0.553400000	1.044200000	6.13220000	4.44550000	0.372000000	1.94820000
Std Dev.:	0.74642043	0.73744467	0.148937021	0.278016418	1.49408125	0.75724725	0.097849343	0.54172723
SE Mean:	0.01669047	0.01648976	0.003330333	0.006216636	0.03340867	0.01693256	0.002187978	0.01211339
LCL Mean:	0.81108842	0.30282184	0.099702532	0.186842953	0.74955092	0.23840047	0.122996406	0.44977226
UCL Mean:	0.89715418	0.38785266	0.116875668	0.218899547	0.92182548	0.32571463	0.134278894	0.51223594
Skewness:	0.84197248	2.46285566	1.460930623	1.441860306	1.96600765	3.29426738	0.861327595	1.09711173
Kurtosis:	-0.45119984	5.31431923	0.976987760	0.915581802	2.83096707	10.76575619	-0.375470717	0.02861278

	E224	E227	E230	E239	E242	E243	E246	E251
Min:	0.2585000000	0.00000000	0.0015000	0.58870000	0.002100000	0.00000000	0.005300000	0.010200000
1st Qu.:	0.2629000000	0.00457500	0.0182250	0.71742500	0.021725000	0.00020000	0.021700000	0.027500000
Mean:	0.2671978500	0.59566605	0.3564543	0.87302370	0.254373850	0.38540630	0.131942850	0.091997300
Median:	0.2671000000	0.10190000	0.1181000	0.86170000	0.105700000	0.01925000	0.072850000	0.064700000
3rd Qu.:	0.2716250000	0.84490000	0.5202250	1.02312500	0.413900000	0.43667500	0.209175000	0.143600000
Max:	0.2760000000	3.47430000	1.8434000	1.20600000	1.170500000	2.95160000	0.525400000	0.283300000
Std Dev.:	0.0050618803	0.89996498	0.4791224	0.17770168	0.307000012	0.68761111	0.137163728	0.075842252
SE Mean:	0.0001131871	0.02012383	0.0107135	0.00397353	0.006864729	0.01537545	0.003067074	0.001695884
LCL Mean:	0.2669060208	0.54378096	0.3288318	0.86277878	0.236674581	0.34576391	0.124035040	0.087624817
UCL Mean:	0.2674896792	0.64755114	0.3840768	0.88326862	0.272073119	0.42504869	0.139850660	0.096369783
Skewness:	0.0089508044	1.62027532	1.5184863	0.15938366	1.306370148	2.00977474	1.152143211	0.860327951
Kurtosis:	-1.1964716648	1.51781621	1.2385064	-1.17785025	0.647876301	3.11148982	0.300795446	-0.426630493

Table 7 : Computations of the Confidence Intervals for the mean of the AA default probabilities distributions

*** Summary Statistics for data in: AA ***

numeric matrix: 12 rows, 299 columns. Only a sub-sample of seven american firms and seven european firms has been extracted here.

Table 7 presents the statistic results for the sample defined by AA firms.
As an illustrative case, we describe the firm E126. E126 is the 126 th firm of the european panel. The minimum reported in the table (0.0023) means that this is the minimum value for the default probability (expressed in percentage) associated with a close to zero (a very samll component of long-term debt is considered). The maximum reported in the table (0.054) means that this is the maximum value for the default probability (expressed in percentage) associated with a close to one (a very large component of long-term debt is considered). LCL mean (0.0171) and UCL mean (0.0188) are the standard lower and upper confidence limits for the mean computed by SPLus. The last two statistics are th skewness (0.902) and the kurtosis (2.582) strongly supportive of the non gaussian property of the default probability distribution.
We focus on two major results pointed out by this table. First, the mean and the median can be considered as relatively close one of each other for this class of rating. Second, the standard deviation of the mean for this class is a little bit more homogeneous than the BBB class.

	A32	A33	A35	A36	A46	A52	A53
Min:	0.00000000000	0.0000000000	0.0000000000	0.000000e+000	0.0000000000	0.0000000000	0.00000000000
1st Qu.:	0.00000000000	0.0000000000	0.0000000000	0.000000e+000	0.0000000000	0.0000000000	0.00000000000
Mean:	0.00110850000	0.0104814000	0.0096015500	1.045450e-003	0.0108590500	0.0063693500	0.00040540000
Median:	0.00000000000	0.0006000000	0.0004000000	0.000000e+000	0.0009000000	0.0001000000	0.00000000000
3rd Qu.:	0.00052500000	0.0096250000	0.0088250000	2.000000e-004	0.0134250000	0.0047250000	0.00020000000
Max:	0.01440000000	0.0946000000	0.0868000000	1.720000e-002	0.0801000000	0.0642000000	0.00510000000
Std Dev.:	0.00262271178	0.0197827595	0.0185758624	2.866525e-003	0.0184793420	0.0132723665	0.00095110627
SE Mean:	0.00005864562	0.0004423559	0.0004153689	6.409746e-005	0.0004132106	0.0002967791	0.00002126738
LCL Mean:	0.00095729453	0.0093408776	0.0085306081	8.801881e-004	0.0097936727	0.0056041670	0.00035056650
UCL Mean:	0.00125970547	0.0116219224	0.0106724919	1.210712e-003	0.0119244273	0.0071345330	0.00046023350
Skewness:	2.96676225278	2.2790084664	2.3216203301	3.464033e+000	1.9177902832	2.5391203973	2.92626773845
Kurtosis:	8.63243902422	4.5299633037	4.7359945400	1.196940e+001	2.7593510873	5.9546339658	8.38839705755
	E121	E126	E130	E132	E133	E137	E138
Min:	0.000000000	0.0023000000	0.00000000000	0.000000000	0.00020000000	0.0071000000	0.000000000
1st Qu.:	0.000000000	0.0058000000	0.00000000000	0.000100000	0.00050000000	0.0118000000	0.000000000
Mean:	0.025690250	0.0180161500	0.00026245000	0.029054150	0.00142455000	0.0198149500	0.033273550
Median:	0.000200000	0.0128500000	0.00000000000	0.003100000	0.00100000000	0.0182000000	0.000100000
3rd Qu.:	0.016525000	0.0280000000	0.00010000000	0.032625000	0.00220000000	0.0273000000	0.018425000
Max:	0.268300000	0.0549000000	0.00330000000	0.215600000	0.00430000000	0.0394000000	0.362600000
Std Dev.:	0.055402247	0.0146422994	0.00064186525	0.050153894	0.00110640154	0.0093294379	0.073990801
SE Mean:	0.001238832	0.0003274118	0.00001435254	0.001121475	0.00002473989	0.0002086126	0.001654485
LCL Mean:	0.022496181	0.0171719872	0.00022544497	0.026162661	0.00136076336	0.0192770861	0.029007807
UCL Mean:	0.028884319	0.0188603128	0.00029945503	0.031945639	0.00148833664	0.0203528139	0.037539293
Skewness:	2.574190855	0.9029071217	2.84274159893	2.034682403	0.87326642016	0.4553662250	2.613679520
Kurtosis:	6. 6330376347	2.5820549239	4.29134689700	2.531969409	5.39266851083	5.6294639658	8.7343970575

Figure 5 The median default probability is plotted against the 95th percentile

The whole sample of firms is considered here. We plot for each american and european firm the 95th percentile of the default probability distribution against the default probability obtained for a=1/2 . The higher the default probability obtained for a=1/2 (actually used in the empirical default barrier of the monitoring product developed by the KMV Corporation) and worth seems to be the possible error induced by a constant fractional parameter. This observation is in line with one of the empirical results obtained during the simulation step. The standard deviation of the mean of the default probability distribution for the well above classes of rating appears as relatively homogeneous. As the same time, the standard deviation of the mean of the default probability distribution for the well below classes of rating is rather supportive of a strong heterogenity.

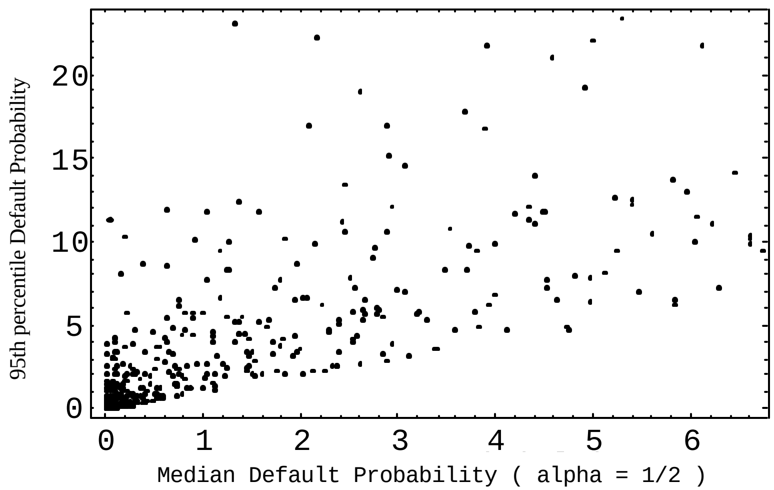

Figure 6 Kernel Densities based on the Silvermann's methodology for five Classes of Rating

Kernel densities of the built default probability distributions have been computed according to the methodology developed in part II section 3. As a general and common property shared by all the rating classes, we observe that these empirical densities do not support the stringent assumption of the gaussian distribution. We point out also that the lower the rating and more important is the number of modalities. We think that such observation is an additional illustration of the more noisy distributions describing the poorly rating classes.