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The Conditional Value of R&D Investments

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Abstract

Companies' investments in research and development (R&D) are usually associated with better growth opportunities incorporated in the firms' market valuation. This study focuses on the question how does the firms' market value attributable to R&D investments depend on the firms' ability to employ intangible capital profitably. The results suggest that the R&D activities of firms with positive profits receive a higher market valuation than the R&D projects of firms with negative profits. Hence, investors appear to disregard the optimism of managers boosting R&D investments in the face of negative profits and prefer to focus on the risks associated with such investments. In general, this effect remains stable over time, but the investors' sensitivity to shifts in R&D investments and earnings changes over the business cycle.

Keywords: R&D, Intangible Assets, Corporate Finance
JEL Classification: M41, G12

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1 Introduction

The market value of firm's shares reflects the value of all its net assets. In some industries, the main part of the firm's value may reflect primarily its intangible assets representing a non-physical claim on future cash flows, e.g. patents, copyrights, trademarks. The most commonly used indicator of cumulated intangibles is research and development (R&D) expenditures. R&D investments contribute directly to the development of new products and aid indirectly the successful adoption of technologies developed outside the firm. In particular, R&D investments aim to improve the profitability of the firm. If R&D expenditures are able to create capital that enables firms to develop innovations furthering growth they should be reflected in firms' market valuation.

The impact of R&D investments on the future economic performance of the firm is highly unpredictable since it is affected by market and technology uncertainty but also by firm's ability to exploit emerging opportunities created by the uncertain environment.[1] The firm's ability to manage the uncertainties in its environment successfully is particularly important for the valuation of its R&D projects because R&D projects become profitable only when the goods in which the R&D is embodied are sold and productivity gains are realized. In knowledge-driven industries, firms' reported earnings might be not directly reflected in the market value of the firm. Particularly in the context of R&D investments, earnings are not only a capital constraint but also a signal for firms' ability to employ R&D capital profitably. If this signal is informative, investors would discriminate between firms reporting different earnings when valuing R&D activities. Simply boosting R&D expenditures would not be enough to generate higher growth expectations and the market value of the firm would not increase proportionally to firms' R&D expenditures.

The results in this study suggest that the market value of R&D investments depends significantly on companies' earnings. Firms reporting positive earnings receive a higher market rent for their R&D activities than firms with lower earnings. Further, comparing the R&D elasticity of firms with positive and negative earnings, the results indicate that investors do not fully share the strong optimism of managers deciding to invest in R&D projects under the pressure of current negative earnings and cutting costs. Instead, investors appear to be more concerned with the risks associated with the R&D investments.

Previous studies on the relationship between R&D investments and market value has identified two main reasons why R&D investments might influence the market value of the company. The first link between expected returns and R&D arises from the notion that R&D expenditures create intangible assets. The idea is based on the theoretical concept that in equilibrium the market value of the firm is equal to the book value of the assets composing the firm. Deviation from this relationship arises either because the market is not in an equilibrium or there is an unmeasured source of

[1]There are certainly significant interactions between these effects, though the complexity of the relationships suggest to focus initially on the firms' characteristics reflected in their accounting reports.

rents driving a wedge between the market and book value of the assets (see Hall[5]). The question, whether this wedge is associated to R&D expenditures has been the subject of several studies. In general, discrepancies in the estimated relationship between firms' market value and their R&D investments are mainly caused by differences in the variables included in the estimation equation.[2] The sources of rents considered in addition to R&D expenditures are for example patenting activities (see Griliches[3], Megna and Klock[18], Pakes[19]), advertising expenditures, sales growth (see Hall[5], Hirschey and Weygandt[10]), market concentration (see Hirschey and Weygandt[10], Jaffe[11]), or monopoly power (see Johnson and Pazderka[13]). The particular importance of earnings when estimating the value of R&D expenditures is addressed by Sougiannis[20]. He raises the question whether past R&D expenditures are reflected in the market valuation directly or indirectly through their impact on earnings. His results show that the indirect impact is much stronger than the direct one, i.e. the rents associated with R&D expenditures are better explained by the earnings they generate rather than by the R&D investments themselves.

The second potential link is closely related to the risk characteristics of R&D investments. While the costs affect firms' profits immediately the benefits are often ambiguous and likely to materialize in subsequent periods. As a result, investors may become overoptimistic about the innovative potential of R&D intensive firms systematically overlooking the possibility that many R&D projects are not profitable. On the other hand, if investors are myopic and value firms by the face value of their financial statements, the value of R&D capital will be on average underpriced by the market. In both cases, the market value of R&D investments would differ from its 'fair' value representing the discounted future cash flows the firm can achieve as a result of these investments.

Several studies analyze how do investors value risky R&D investments. Chan, Lakonishok, and Sougiannis[2] for example analyze the performance of portfolios based on different firm characteristics and conclude that simply doing R&D by itself does not give rise to differential stock price performance, on average. Specifically, the market appear to be sluggish revising its expectations about the prospects of R&D activities by firms with poor past returns. This result indicates that investors do not share the optimism of managers spending heavily on R&D despite poor market returns and pressure on cost cuts. The market valuation of R&D expenditures is further analyzed by Lev and Sougiannis [15][14]. They show that the estimated R&D capital-to-market variable subsumes the role of the book-to-market ratio, though, the subsequent returns related to the R&D capital are due to a risk factor associated with R&D, rather than a result of mispricing.

Mainly encouraged by the results of Sougiannis[20] showing that investors use earnings to elicit information on the value of R&D expenditures, this study goes further analyzing the question how investors assess the profitability of firms' R&D activities in the context of their reported earnings. Differences across companies with respect to their earnings are reflected in a continuous non-linear function. Using this function as a condition when estimating the value of R&D investments reflected

[2] Hall[6] and Mairesse and Sassenou[16] provide summaries of the results.

in the market capitalization of the company allows drawing conclusions on investors' sensitivity to changes in firms' R&D investments in the context of continuous earnings changes. Characteristics as "high" and "low" earnings firms are then not exogenously specified but determined by the data.[3]

The paper is organized as follows. The formal statement of the problem is provided in section 2. Section 3 describes the sample selection procedure; it also discusses some empirical properties of the data. The results of the estimated equations are discussed in section 4. Section 5 provides an interpretation of the results. The main conclusions are summarized in section 6.

2 The Problem

The typical model of market value used in previous studies hypothesizes that the market value of the firm is a function of its assets (see Hall[6], Hall and Kim[7], Hall and Hayashi[8], Johnson and Pazderka[13]). There are two types of assets: tangible assets TA (e.g. physical capital) and intangible assets IA (e.g. patents, copyrights, knowledge capital). Thus, the market value of the firm V_t at time t can be expressed as:

$$V_t = f(TA_t, IA_{t-\theta}, IA_{t-\theta+1}, ..., IA_t) \qquad (1)$$

where f is an unknown function describing how the assets combine to create value.[4] θ is a gestation lag reflecting the idea that the production of knowledge capital is different than the production of capital goods and it is likely to involve projects with durations of several years $\theta = 1, ..., T$, where T reflects the age of the firm.

Adapting a multiplicative separable specification for the function f, the market value function (1) can be written as:

$$V_t = (TA_t)^{\beta_1} \sum_{\theta=1}^{T} (IA_{t-\theta})^{\beta_{2,\theta}} \qquad (2)$$

TA_t are the real assets of the company such as fixed assets and inventories. They are measured by the book values of these items and represent the net capital stock of the company. The value of the intangible assets IA are not reported and must be estimated. One possibility to estimate the value of intangible capital is to refer to firm's R&D expenditures and use them as an indicator of innovation and growth power.[5] Using current and past R&D expenditures as a proxy for intangible

[3]The idea of using a non-linear relationship when estimating the importance of variables is not new. In a different context, McConnell and Servaes[17] for example use a quadratic regression and show that the relation between corporate value and leverage is nonlinear, i.e. it is negative for "high" growth firms and positive for "low" growth firms.

[4]This function is linear (in the logs) if assets provide constant returns to scale.

[5]Several studies demonstrate that R&D expenditures creates intangible capital (see, for example, Hall[5], Hirschey and Weigandt[10]).

capital (2) and taking the natural logarithms of both sides, we obtain:

$$\ln V_t = \hat{\beta}_{1,t} \ln TA_t + \sum_{\theta=1}^{T} \hat{\beta}_{2,t} \ln RD_{t-\theta} \qquad (3)$$

The ratio $\frac{V_t}{TA_t}$, respectively the difference $\ln V_t - \ln TA_t$, reflects the quality of firm's current and anticipated projects as perceived by investors. The company should acquire more assets if the market valuation of those assets is greater then the replacement costs usually reflected in the book value of the assets. In other words, new investments are considered by investors as profitable if they are used so as to create at least as much value as the cost of reproducing the new assets. Therefore, one can learn whether R&D expenditures give rise to intangible capital by simply studying the relationship between firms' R&D investments and their market value.

In the simplest case, this relationship is linear, so that every unit money spent on R&D is transformed in market value by a multiple. Clearly, this multiple can vary across industries and over time as previous studies have already reported. Hall[6] and Mairesse and Sassenou[16] provide summaries of the estimated coefficients in dependence on the variables additionally included in the estimation equation. Though, to our knowledge, none of these studies explain how this multiple depends on firm's characteristics. This is important since all investments in R&D are not necessary good, the question is if there are firm specific factors that systematically explain why the market gives more credits to some firms and less to others although all of them invest in R&D. Since R&D expenditures are dedicated to improve the current and future earnings of the firm, the simplest way to learn something about the value of R&D activities is to look at firms' profits. Given that the market differentiates between companies investing in R&D, two firms with different R&D earnings contribution, should also differ in their market valuations.

Various methods aid testing this intuition. The simplest one is to split the sample in firms with high and firms with low profits. The main problem with this approach is that splitting firms requires setting up a certain criteria in advance. Therefore the criteria cannot be endogenously determined. For example, setting the cut off by zero and dividing firms in two groups, one with positive and one with negative earnings, will be inconsistent with the data if investors apply another criteria to order firms. Applying the wrong cut off criteria would lead to rejecting the hypothesis that investors differentiate between firms doing R&D and to the erroneous conclusion that R&D activities by all firms within an industry have the same value. An additional disadvantage of the approach is that dividing firms in groups necessary reduces the sample of observations within each group. This is disadvantageous for interpreting the results in terms of significance.

To overcome the problems associated with applying predefined criteria, we suggest a model based on one equation including all firms in the sample for a given period. Differences in firms' profitability can be described using a function with similar properties as the indicator function but without requiring a decision for the cut off point in advance. One candidate with this property is a non-linear function that can 'switch' between zero and one but also allow for the existence of an interval where the function can take values between zero and one. Additionally, the function must include a shift parameter, which determines the cut off or 'switching' level.

One example for a non-linear function with the desired properties is:

$$\psi(X) = (1 + e^{-a-bX})^{-1} \tag{4}$$

The function is s-shaped and takes values between zero and one. Its tightness depends on the parameter b. Larger values of b reduce the interval, where the function takes values between zero and one. Then, $\psi(X)$ behaves like an indicator function. The smaller the parameter b, the flatter is the function. In the extreme case, it is linear. a is a shift parameter. It determines the level of X where the function 'switches'. A general form of the function and a discussion on its properties is included in the appendix.

For the purpose of this study testing whether investors evaluate R&D investments of firms in dependence of their profitability, the function $\psi(X)$ is particularly helpful in different aspects. First, it allows to determine the switching point endogenously from the data. This is important for our analysis since we do not know for sure what is 'high' and what is 'low' profitability from investors point of view and over time. Second, since the function is continuous, it allows estimating how the market value of the company changes to small shifts in the R&D expenditures and earnings simultaneously. Third, applying this functional form to estimate the impact of earnings on the market value of R&D, the study is able to draw conclusions on how does R&D elasticity change over time for firms with different earnings levels. Finally, the specification allows testing for non-linear dependence using a linear model.

The model defined in equation (3) is extended as follows:

$$\ln V_t = \hat{\beta}_{0,t} + \hat{\beta}_{1,t} \ln TA_t + \hat{\beta}_{2,t} \ln RD_t + \hat{\beta}_{3,t}\psi(X_t) \ln RD_t + \varepsilon_t \tag{5}$$

where X_t is a random variable reflecting the earnings of the company in time t.[6] The difference to the linear model in equation (3) is in the third term. It introduces an *indirect* relationship between R&D investments and market value as a non-linear function of firm's profitability measured by X_t. This relationship is different from the indirect relationship studied by Sougiannis[20] in two aspects. First, it is simultaneously determined by one equation instead of a system of equations stating first the link between R&D investments and earnings and then between earnings and market value. Second, it allows drawing conclusions on the value of R&D investments conditioned on earnings. In contrast, applying the system of equations as used by Sougiannis[20] one can compare the informativeness of earnings and R&D for investors valuing the R&D activities of the firm. However, conclusions on the particular impact of earnings on the value of firms' R&D investments are not offhand possible.

[6] Alternatively, one can take other profitability measures, e.g. operating profits. Though, the basic results (not reported here) do not differ substantially.

Since product development usually takes several years an additional issue when explaining the link between the market value of the firm and its R&D activities is the importance of previous R&D investments. The problem is the exact deprecation rate, respectively the percentage of past R&D investments, which are still associated with earnings growth in the future as reflected in the market price of the company. An alternative to using lagged R&D investments as indicator for expected earnings growth is to focus on realized earnings reflecting previous R&D investments returning products that increase earnings as previously expected by investors. A necessary condition for this argument to hold is that firms' profits are strongly research-driven and firms invest in R&D on a regular basis. If firms invest in R&D occasionally, the earnings in the next period would not reflect their profitability correctly since product development usually takes several years. However, if firms run various R&D projects requiring continuous investing in R&D, the observed earnings reflect these projects that were successful.

To capture the effect of previous R&D investments in the model, equation (5) is modified slightly to:

$$\ln V_t = \hat{\beta}_{0,t} + \hat{\beta}_{1,t} \ln TA_t + \hat{\beta}_{2,t} \ln RD_t + \hat{\beta}_{3,t}\psi(X_t) \ln RD_t + \hat{\beta}_{4,t}\psi(X_t) + \varepsilon_t \tag{6}$$

Since this transformation is additive without including the variable *R&D*, it does not have any impact on the *elasticity* of market value to current R&D expenditures, which is equal to:

$$\frac{\partial \ln V_t}{\partial \ln RD_t} = \hat{\beta}_{2,t} + \hat{\beta}_{3,t}\psi(X_t) \tag{7}$$

The regression parameters $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{4,t}$ in equation (6) are expected to be positive. The intercept $\hat{\beta}_{0,t}$ captures the valuation effect of variables not included in the equation, which may be positive, negative, or zero. The parameter measuring the indirect effect of R&D on market value can be also positive, negative, or zero.

3 Sample Selection Procedure and Data Description

The database for this study includes accounting and pricing data of companies belonging to the pharmaceutical industry as specified and reported by Datastream. This industry is particularly interesting because R&D investments are the lifeblood for the companies, i.e. one can expect that investments in R&D are one of the main sources of their future profits.

The data covers the period from 1990 to 2004. Firms are included in the sample for year t if data are available on market value, total assets and R&D expenditures for a financial year ending in year t. There are no restriction on the market capitalization of the companies in order to utilize the maximum possible sample in the following tests. Unobserved heterogeneity and selection bias is undoubtedly an issue, though most of the previous empirical studies do not deal with it. Moreover, the literature, which has adjusted for selectivity, conclude that Ordinary Least Square (OLS) results are probably not too seriously biased (see Bosworth and Rogers[1]).

The sample consists of 148 US pharmaceutical companies with market capitalization ranging from USD 0.05 Mill. to USD 201755 Mill. (as of the end 2004). The broad range of market capitalization reflects the companies' diversity particularly with respect to their R&D investment activities. During the period from 1997 to 2004, some firms invest more than USD 2000 Mill. and others spending less than USD 0.011 Mill. (see Table 1).

Table 1: *Summary statistics of R&D expenditures (1997-2004)*

Thousand USD	1997	1998	1999	2000	2001	2002	2003	2004
Mean	149'025	130'024	120'969	123'235	135'698	151'165	160'843	185'509
Median	4'835	4'400	4'291	4'940	5'299	6'283	7'091	7'158
Maximum	1'905'000	2'140'000	2'279'000	2'776'000	4'435'000	4'847'000	5'176'000	7'070'000
Minimum	7	44	51	17	11	11	11	11
Std.Dev.	426'943	425'819	434'781	468'783	559'762	622'775	661'640	812'383
Skewness	2.90	3.43	3.84	4.20	5.19	5.20	5.30	6.04
Kurtosis	9.94	13.48	16.53	19.97	32.48	32.23	33.38	43.96
Observations	78	101	121	134	146	148	148	148

Since 1998, the mean and the median of R&D expenditures increases continuously. Though, the standard deviation, skewness and kurtosis increases as well. With the time, the wedge among the level of R&D activities deepen so that more companies become outliers. Figures 1 illustrates this result. Figure 2 shows the R&D distribution without far outliers. As one can easily see, the level of R&D expenditures for half of the companies is much lower (about USD 30 Mill.) than the level of R&D investments of the far outliers (more than USD 1000 Mill.).

Figure 1: *R&D expenditures*

The boxplot summarizes the distribution of R&D expenditures across companies for each year in the period 1995-2004. The data with the symbols 'o' and '∗' represent outliers.

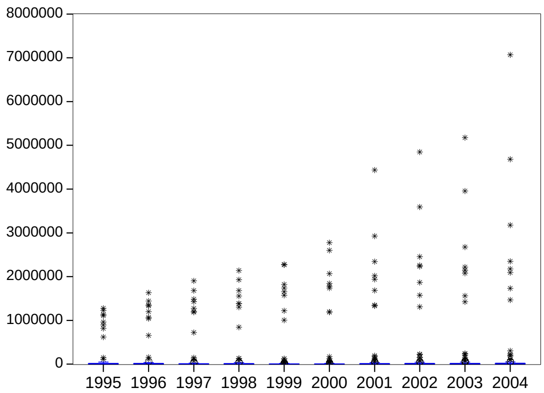
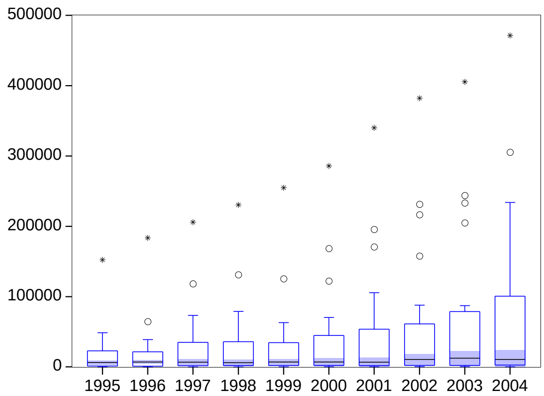


Figure 2: *R&D expenditures without large outliers*

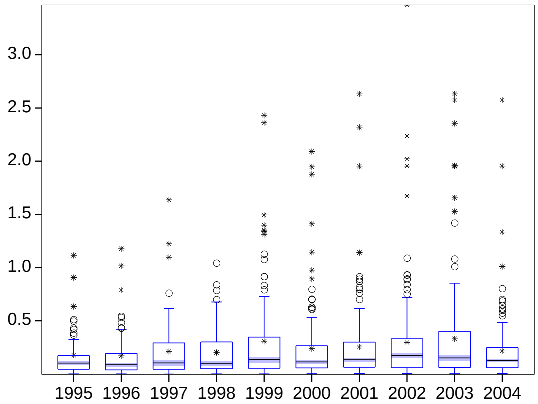
The box portion of the boxplot represents the first and third quartiles (middle 50 percent of the data). The median is depicted using a line through the center of the box, while the mean is drawn using the symbol '∗'.



One possible explanation for the large differences in the level of R&D expenditures across firms is that bigger companies usually have larger capacity to extend their R&D investments than smaller firms. To eliminate this effect in analyzing companies' heterogeneity, the R&D expenditures are normalized with the book value of total assets as reported by the companies at the end of each year. Figure 3 shows the distribution of the ratio R&D expenditures to total assets in a boxplot. The median is almost constant over time. There are still some outliers, however, they are smaller than the outliers in the R&D distribution (see Figure 1).

Figure 3: *R&D expenditures to total assets (1995 - 2004)*

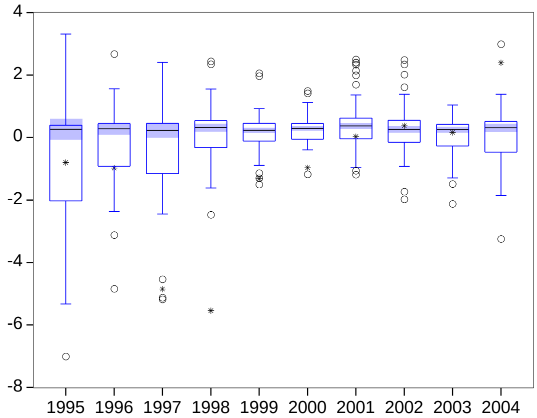
The boxplot summarizes the distribution of the ratio R&D expenditures to total assets across companies for each year in the period 19905-2004. The box portion of the boxplot represents the first and third quartiles (middle 50 percent of the data). The median is depicted using a line through the center of the box, while the mean is drawn using the symbol '$*$' within the box. The data with the symbols '$\circ$' and '$*$' are outliers.



The more interesting question for this study is the importance of firm earnings for the market value of R&D investments. To approach the question descriptively, Figure 4 plots the distribution of firms R&D expenditures relative to their reported earnings after taxes but before R&D expenditures.

Figure 4: *R&D expenditures to earnings (1995 - 2004)*

The boxplot summarizes the distribution of the ratio R&D expenditures to earnings (after taxes and before R&D expenditures) across companies for each year in the period 1995-2004. The box portion of the boxplot represents the first and third quartiles (middle 50 percent of the data). The median is depicted using a line through the center of the box, while the mean is drawn using the symbol '$*$'. The data with the symbols '$\circ$' are outliers. Far outliers have been neglected.



Until 1998 and after 2003, the middle fifty percent of the companies continued to invest strongly in R&D although their adjusted earnings have been negative in the current period. For the period between 1998 and 2003, the R&D expenditures of firms with positive and negative earnings do not differ substantially. Intuitively, firms deciding to invest more intensively in R&D in the face of negative earnings must be very confident in the prospects of their investments. The question is whether and to which extend do investors share the optimism of managers and reward their R&D investments.

4 Results

All tests are performed using (linear) Ordinary Least Squares (OLS) regression with multiple variables. The variables are defined as follows:

- V_t is a vector including the market capitalization of all firms at the end of the first quarter of year $t+1$.[7]
- TA_t a vector including the total assets as reported by the firms in the sample on the end of year t
- RD_t is a vector including the R&D expenditures of firms reported on the end of year t
- X_t is a vector including the after tax earnings before R&D expenditures (adjusted earnings) as reported by the firms on the end of year t

To minimize the problem of heteroscedasticity, all variables are included with their logarithmic values. Additionally, the standard errors of the estimated coefficients are reported after adjusting for heteroscedasticity according to the White test. Autocorrelation is not an issue, since the model is specified for a cross-sectional sample with no lags over the time.

First, we estimate the market value of intangible capital as specified by equation (3). Then, we test the causal dependence between R&D expenditures and adjusted earnings and confirm the adequacy of the specification in equation (6). The market value of R&D investments conditioned on the reported earnings is estimated with two different methods. The first splits the sample of firms in two groups in dependence of their earnings and estimate the market value of R&D expenditures of the firms within each group. The second approach estimates the market value of firms' R&D investments in dependence on the level of their adjusted earnings directly by using the non-linear specification from equation (6).

[7] The underlying assumption is that investors receive the accounting reports for the current year within the first quarter of the next one. Accounting information is reflected in market prices as soon as it is available.

4.1 The value of R&D investments as intangible capital

Adopting the notion that firm's value is determined by the capitalized value of its asset, the paper proceeds estimating the market value of firms' intangible assets approximated by their investments in R&D. Table 2 reports the estimated coefficients.

Table 2: *The market value of intangible assets*

$$\ln V_t = \hat{\beta}_{0,t} + \hat{\beta}_{1,t} \ln RD_t + \varepsilon$$

P-values are reported under every coefficient in parentheses. The (centered) R^2 statistic explains the variation in $\ln V_t$ after fitting the constant. SSR is the sum of squared residuals. Standard errors are White-heteroscedasticity consistent.

$\log(V_t)$	$\hat{\beta}_{0,t}$	$\hat{\beta}_{1,t}$	R^2	SSR	N
2004	2.4314	1.0371	0.6248	463	148
	(0.000)	(0.000)			
2003	2.7376	1.0242	0.6443	394	142
	(0.000)	(0.000)			
2002	1.4594	1.0777	0.6108	486	140
	(0.030)	(0.000)			
2001	1.9090	1.1000	0.7530	244	136
	(0.109)	(0.000)			
2000	2.5110	1.0561	0.7028	274	130
	(0.000)	(0.000)			
1999	-4.5628	0.8907	0.6812	185	106
	(0.000)	(0.000)			
1998	3.7359	0.9526	0.6890	191	85
	(0.000)	(0.000)			
1997	5.5772	0.7878	0.6457	144	70
	(0.000)	(0.000)			
1996	4.9461	0.8516	0.7378	98	64
	(0.000)	(0.000)			

Overall, the market value elasticity with respect to R&D investments is significantly different from zero for every year. It is continuously increasing over time taking values from 0.8 to 1.1. The highest value is reached for the reporting year 2001, just before the overall industry price index drops down (see Figure 23). The estimated coefficients may be overstated due to the omission of the tangible assets as an explanatory variable because of its high correlation with the R&D investments. This issue is taken into account in the further analysis by including the adjusted earnings after taxes, which can serve as a proxy for firms' size.

To get an intuition if there is more information on the elasticity parameter $\hat{\beta}_{1,t}$, the market value of R&D investments is conditioned on the reported earnings. The simplest way to get an idea on the relevance of earnings for the market value of R&D expenditures is to plot the variables. The sample of companies is divided in two groups: one containing firms reporting positive earnings ($X_t > 0$) and one containing firms reporting negative earnings in the current period ($X_t < 0$). The R&D expenditures and market values of the firms in both groups are plotted together for each of the reporting years. If investors value R&D projects in dependance on the current earnings, then the relationship between R&D investments and the market value would be different in both groups.

Figures 5 to 10 visualizes the plausibility of this intuition for the sample of data from 1999 to 2004.

The plots show that the market value of R&D is not well determined by a simple linear function. In particular, the market value of firms with negative earnings do not always increase with the level of R&D spending. One part of the firms with negative earnings receives similar valuation for their R&D activities as firms reporting positive earnings, though another part of the firms with negative earnings do not. Thus, estimating the market value of R&D investments using a simple linear regression over the whole sample of firms without considering the importance of earnings characteristics would not be very precise if one aims to draw a conclusion on the market value of R&D investments. The accuracy of the results can be improved by conditioning the market value of R&D investments on the firms' earnings.

The simplest way to do this is to split the sample of firms in groups with different level of earnings.[8] The approach has the disadvantage that it reduces the number of observations within each group. Additionally, it does not provide any results on the sensitivity of the estimated coefficients with respect to different earnings levels. To overcome this problems, this study includes a third dimension in the analysis between market value and R&D investments describing the impact of earnings. The results are discussed in the following.

[8]Another possibility to take account for different earnings levels is suggested by Johnson and Pazderka[13]. They simply exclude companies reporting negative earnings and compare the results.

Figure 5: *R&D market value (2004)*

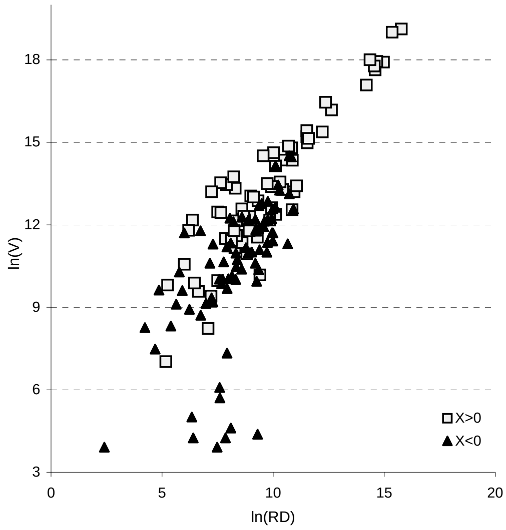


Figure 6: *R&D market value (2003)*

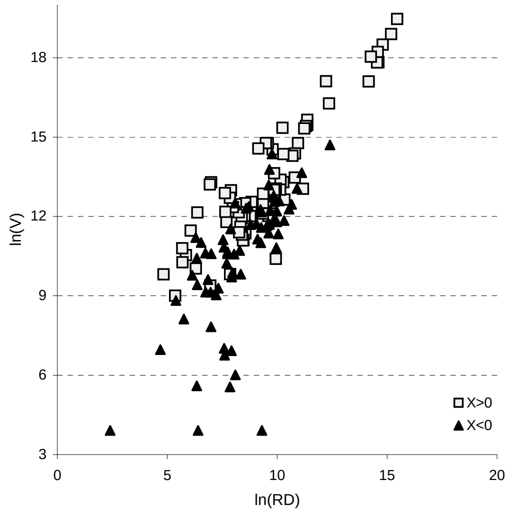


Figure 7: *R&D market value (2002)*

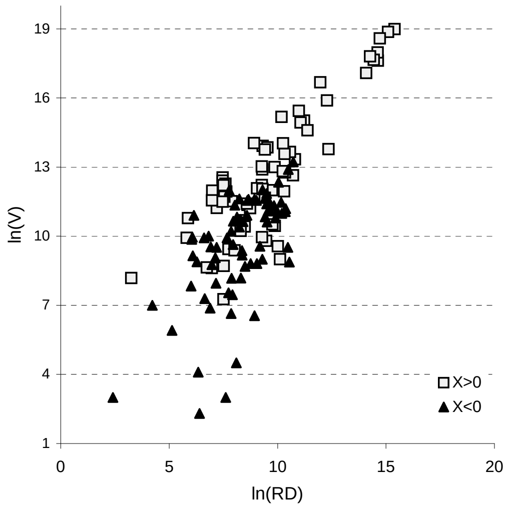


Figure 8: *R&D market value (2001)*

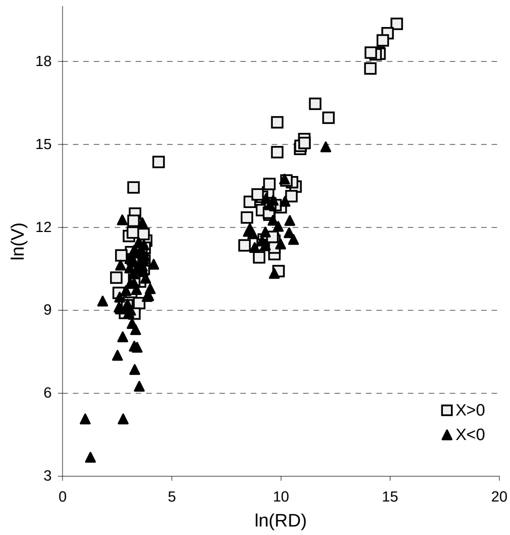


Figure 9: *R&D market value (2000)*

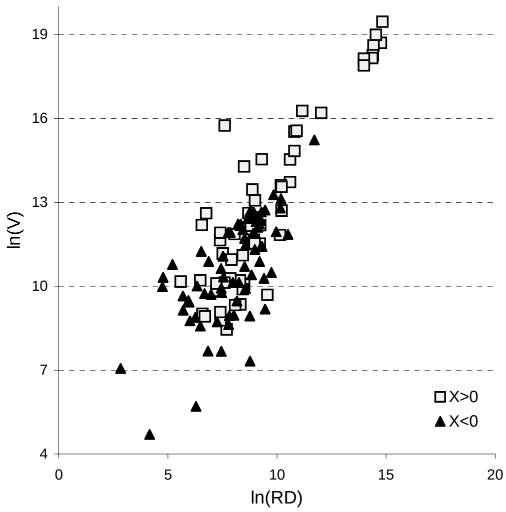
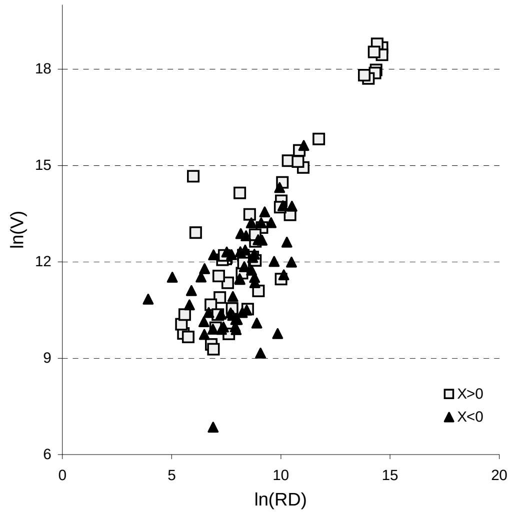


Figure 10: *R&D market value (1999)*



4.2 The causal dependence between R&D investments and adjusted earnings

Before estimating how the market value of firms' R&D expenditures varies in dependence of the reported adjusted earnings, we test whether the estimating equation (6) is specified correctly. In particular, the following tests aim to verify the current level of adjusted earnings as variable reflecting the value of past R&D expenditures for the company. The tests are specified such as to estimate whether changes in past R&D investments cause changes in adjusted earnings as assumed in equation (6) or changes in past adjusted earnings cause changes in current R&D expenditures. The results are reported in Table 3.

Table 3: *The causal dependance between R&D investments and adjusted earnings*

(1) $\Delta RD_t = \hat{\alpha}_{0,t} + \hat{\alpha}_{1,t}\Delta RD_{t-1} + \hat{\alpha}_{2,t}\Delta RD_{t-2} + \hat{\beta}_{1,t}\Delta X_{t-1} + \hat{\beta}_{2,t}\Delta X_{t-2} + \varepsilon$
(2) $\Delta X_t = \hat{\alpha}_{0,t} + \hat{\alpha}_{1,t}\Delta X_{t-1} + \hat{\alpha}_{2,t}\Delta X_{t-2} + \hat{\beta}_{1,t}\Delta RD_{t-1} + \hat{\beta}_{2,t}\Delta RD_{t-2} + \varepsilon$

Equation (1) tests if changes in past earnings cause changes in R&D expenditures. Equation (2) tests if changes in past R&D expenditures cause changes in adjusted earnings. The tests are performed using differences in the variables in order to insure that the variables included in the regressions are stationary.
P-values are reported under every coefficient in parentheses. The (centered) R^2 statistic explains the variation in ΔRD_t and ΔX_t after fitting the constant. Standard errors are White-heteroscedasticity consistent.

(1)	$\hat{\alpha}_{0,t}$	$\hat{\alpha}_{1,t}$	$\hat{\alpha}_{2,t}$	$\hat{\beta}_{1,t}$	$\hat{\beta}_{2,t}$	R^2	N
2004	-3032	0.7645	1.1068	-0.3431	0.2811	0.7845	132
	(0.426)	(0.207)	(0.001)	(0.216)	(0.057)		
2003	862	0.1714	-0.1643	0.0531	0.1364	0.8151	132
	(0.478)	(0.135)	(0.016)	(0.000)	(0.036)		
2002	4342	-0.1479	0.9667	0.1035	-0.0045	0.6673	119
	(0.084)	(0.561)	(0.034)	(0.547)	(0.814)		
2001	-3792	0.5308	2.2907	0.0354	-0.2313	0.7234	98
	(0.549)	(0.411)	(0.186)	(0.102)	(0.252)		
(2)							
2004	-8195	-1.4883	-2.1984	2.7742	5.7041	0.7672	132
	(0.574)	(0.002)	(0.002)	(0.422)	(0.001)		
2003	4662	0.4848	0.0138	1.4537	-0.6641	0.8153	132
	(0.442)	(0.000)	(0.960)	(0.007)	(0.093)		
2002	-16034	0.2196	-0.8882	2.6087	2.2173	0.8557	119
	(0.259)	(0.720)	(0.000)	(0.000)	(0.395)		
2001	1528	0.0273	0.3350	3.9078	-1.2559	0.8021	98
	(0.855)	(0.241)	(0.201)	(0.000)	(0.572)		

Comparing the significance of the coefficients $\hat{\beta}_{1,t}$ and $\hat{\beta}_{2,t}$ in the first equation, we can conclude that changes in past earnings do not cause significant changes in the current R&D expenditures except in year 2003.[9] For this year, changes in the past R&D expenditures cause also changes in current earnings so that the causal dependance between the variable is eliminated. The significance of the coefficients $\hat{\beta}_{1,t}$ and $\hat{\beta}_{2,t}$ in the second equation suggests that lagged changes in R&D expenditures cause changes in current adjusted earnings. This causal dependance is in line with the

[9] This result is valid if one requires that the coefficients are significant different from zero at the 5% level.

assumption that the value of past R&D expenditures can be captured by their impact on adjusted earnings as formulated in equation (6).

4.3 The value of R&D investments in firms with positive and negative earnings

The results from the first subsection suggest that firms investing in R&D receive higher market valuation on average. Though, not every firm can be successful in its R&D activities and do best all the time. Therefore, we expect to see that smart investors differentiate between firms investing in R&D by considering their current profitability as reflected in the published earnings after taxes. The intuition behind this idea is visualized in Figures 5 to 10. This subsection reports the results of an empirical test estimating the significance of the intuition.

The test is performed by simply splitting the sample of firms in two groups: one including companies reporting positive earnings (Group A) and another one including firms reporting negative earnings in the current period (Group B). An indicator function $\mathbb{I}$ determines to which group a firm belongs. The market value of R&D expenditures is then estimated separately for each group. Table 4 summarizes the results.

Table 4: *The market value of R&D of firms with different adjusted earnings*

(Group A): $\ln V_t = \hat{\beta}_{0,t} + \hat{\beta}_{1,t} \ln RD_t \mathbb{I}_{X_t>0} + \varepsilon_t$
(Group B): $\ln V_t = \hat{\beta}_{0,t} + \hat{\beta}_{1,t} \ln RD_t \mathbb{I}_{X_t<0} + \varepsilon_t$
where $\mathbb{I}_{X_t>0} = \begin{cases} 1 \text{ for } X_t > 0 \\ 0 \text{ for } X_t < 0 \end{cases}$ and $\mathbb{I}_{X_t<0} = \begin{cases} 1 \text{ for } X_t < 0 \\ 0 \text{ for } X_t > 0 \end{cases}$

P-values are reported under every coefficient in parentheses. The (centered) R^2 statistic explains the variation in $\log V_t$ after fitting the constant. Standard errors are White-heteroscedasticity consistent. The total assets are excluded from the estimation equation since they are highly correlated with the level of R&D investments.

Group A	$\hat{\beta}_{0,t}$	$\hat{\beta}_{1,t}$	R^2	N	Group B	$\hat{\beta}_{0,t}$	$\hat{\beta}_{1,t}$	R^2	N
2004	4.3692	0.9140	0.8206	71		2.5962	0.9345	0.3888	77
	(0.000)	(0.000)				(0.015)	(0.000)		
2003	4.8828	0.8824	0.8173	73		2.2711	0.9800	0.4693	69
	(0.000)	(0.000)				(0.012)	(0.000)		
2002	3.4256	0.9591	0.7210	68		2.1729	0.8958	0.3762	72
	(0.000)	(0.000)				(0.058)	(0.000)		
2001	8.4592	0.5568	0.6685	67		8.0289	0.4500	0.4558	69
	(0.000)	(0.000)				(0.000)	(0.000)		
2000	2.3358	1.1136	0.7845	57		4.3845	0.7837	0.4584	72
	(0.001)	(0.000)				(0.000)	(0.000)		
1999	4.4852	0.9414	0.8284	50		7.0608	0.5420	0.2625	55
	(0.000)	(0.000)				(0.000)	(0.001)		
1998	4.3358	0.9509	0.8128	51		5.4772	0.6351	0.3324	34
	(0.000)	(0.000)				(0.000)	(0.000)		

Until 2002, firms reporting positive earnings receive a higher market valuation for their R&D activities than firms reporting negative earnings. After 2001, the difference in the coefficients $\hat{\beta}_{1,t}$ is not significant at the 5% level.

4.4 The value of R&D investments conditioned on the level of firms' earnings

To take a closer look on the significance of the relationship between the market value of R&D investments and firms earnings a non-linear functional form describing the sensitivity of market value to R&D investments in dependence on small changes in firms' profits is introduced. The advantage of this approach compared to the previous one is that the estimation does not separate firms imposing assumptions on the criteria that might be relevant for investors while evaluating firms' R&D activities in the context of their profits. Instead, these criteria are the output of an estimation searching for the best fit with the data. The results are reported in Table 5.

All coefficients besides of the intercept are significant different form zero at the 5% level. The R^2 statistic is in each year better than the statistic in the simple case estimating the market value of R&D investments without conditioning on firms' earnings (see Table 2). Moreover, the sum of squared residuals is in each year lower. If one neglects companies with negative earnings the test does not always fit better the data. Though, for this sample, this would mean to exclude approximately one half of the companies each year (see the last column in Table 4).

Table 5: *The market value of R&D conditioned on firms' adjusted earnings*

$$\ln V_t = \hat{\beta}_{0,t} + \hat{\beta}_{1,t} \ln RD_t + \hat{\beta}_{2,t} \frac{1}{1+e^{-a-bX_t}} + \hat{\beta}_{3,t} \frac{1}{1+e^{-a-bX_t}} \ln RD_t + \varepsilon_t$$

P-values are reported under every coefficient in parentheses. The (centered) R^2 statistic explains the variation in $\log V_t$ after fitting the constant. Standard errors are White-heteroscedasticity consistent. SSR is the sum of squared residuals. The coefficients a and b solve an optimization problem minimizing the p-value of the t-statistic. Clearly, these values are not unique. It is possible that there are other values for a and b, for which the relationship between the variables is significant as well.

$\log V_t$	$\hat{\beta}_{0,t}$	$\hat{\beta}_{1,t}$	$\hat{\beta}_{2,t}$	$\hat{\beta}_{3,t}$	a	b	R^2	SSR	N
2004	2.8155	0.9092	5.9923	-0.3206	2	-0.1^4	0.6875	339	148
	(0.005)	(0.000)	(0.001)	(0.041)					
2003	-2.7507	1.3262	9.7669	-0.5940	-0.5	-0.1^4	0.7588	266	142
	(0.215)	(0.000)	(0.001)	(0.025)					
2002	-11.845	1.5590	24.9899	-1.2452	-0.5	-0.1^5	0.8206	369	140
	(0.003)	(0.000)	(0.000)	(0.006)					
2001	-1.1326	1.1244	12.5293	-0.6468	0.5	-0.1^5	0.7969	200	136
	(0.522)	(0.000)	(0.001)	(0.022)					
2000	1.3066	0.9570	18.6557	-1.0667	1.5	-0.1^5	0.7634	216	128
	(0.270)	(0.000)	(0.000)	(0.003)					
1999	3.9806	0.7774	22.3340	-1.3429	2	-0.1^5	0.7385	151	106
	(0.000)	(0.000)	(0.006)	(0.012)					
1998	4.9831	0.6583	38.0382	-2.3825	3	-0.1^5	0.7747	138	85
	(0.000)	(0.000)	(0.007)	(0.014)					
1997	6.9526	0.4355	30.9268	-1.8288	2.5	-0.1^5	0.8075	77	69
	(0.000)	(0.008)	(0.012)	(0.031)					

The best way to interpret these results is to show the relationship between the market value of the firms, its R&D investments and adjusted earning in a three-dimensional plot (see Figure 11 to 22). The level of R&D expenditures is plotted on the x-axis, adjusted earnings are on the y-axis, and the vertical axis measures the market value of the firms. For each year there are two plots, one (on the left side) representing the estimated relationship for 95% of the observations and one (on the right side) with a smaller scale representing 75% (3. Quartile) of the firms.

In the following discussion we first consider the importance of adjusted earnings (y-axis) for the market value of R&D expenditures. Then, we focus on firms with positive earnings and analyze how the market value of the firm changes when the firm boost its R&D investments. Finally, the results are interpreted for the smaller sample of firms including 75% of the observations.

For 95% of the firms (plots on the left side of the pages), the level of adjusted earnings has a significant impact on the market value of their R&D expenditures. The effect is observed particularly for companies reporting positive earnings. In general, the market value of these companies increases with the level of their R&D investments. Though, the effect changes over time. With the strong increase in R&D expenditures in 1998, even firms reporting negative earnings receive a higher market valuation for their R&D expenditures (see Figure 21). This effect mitigates over time and earnings become more and more important for investors valuing the R&D activities of the firms. For the period from 2000 to 2002, the market value of R&D investments increases smoothly with the reported earnings. Though, the marginal market return on R&D investments by firms in particular with the highest positive earnings is decreasing. Since 2003, the marginal return of R&D investing by all profitable firms becomes constant.

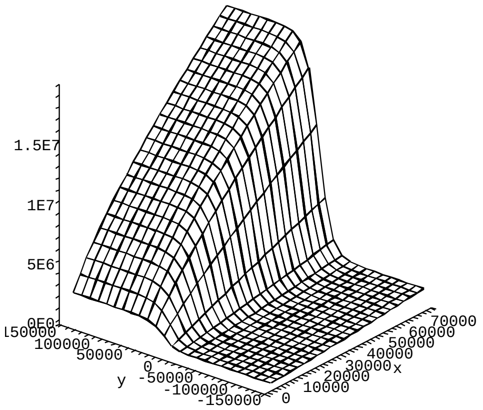


Figure 11: *Conditional R&D value, 2004*

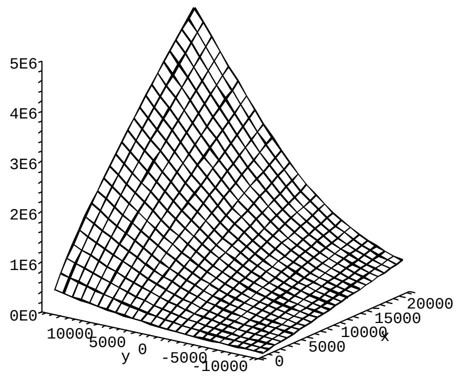


Figure 12: *Conditional R&D value, 2004, 3.Quartile*

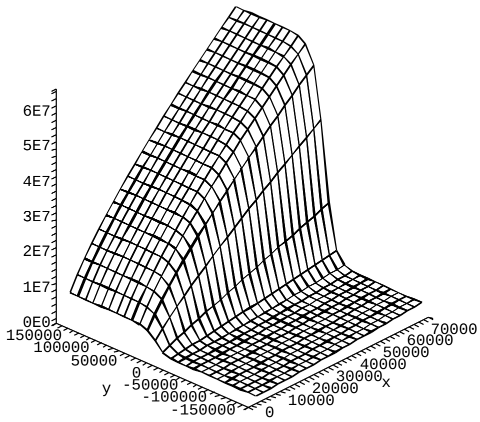


Figure 13: *Conditional R&D value, 2003*

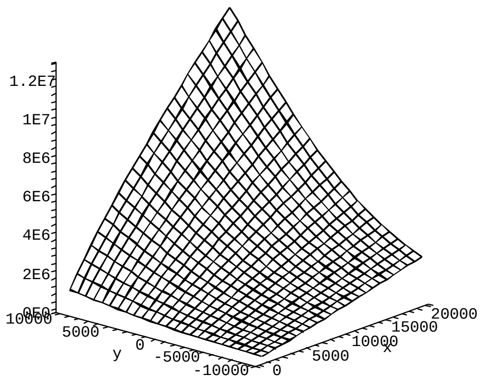


Figure 14: *Conditional R&D value, 2003, 3.Quartile*

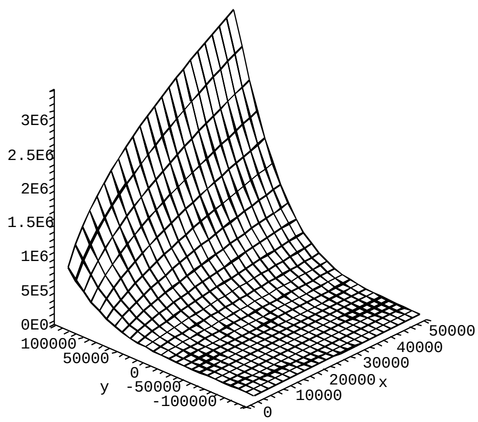


Figure 15: *Conditional R&D value, 2002*

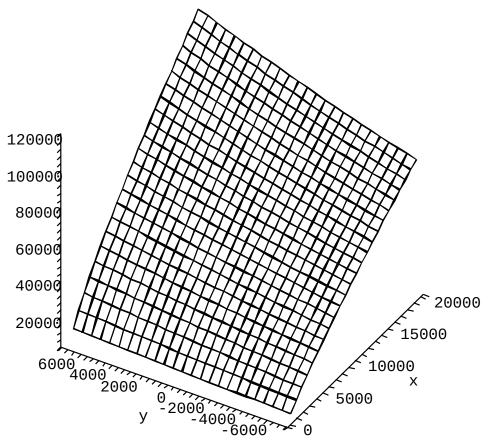


Figure 16: *Conditional R&D value, 2002, 3.Quartile*

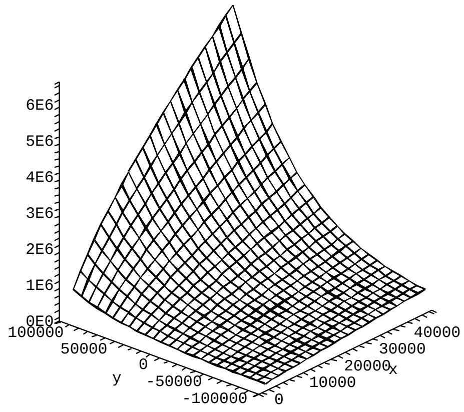


Figure 17: *Conditional R&D value, 2001*

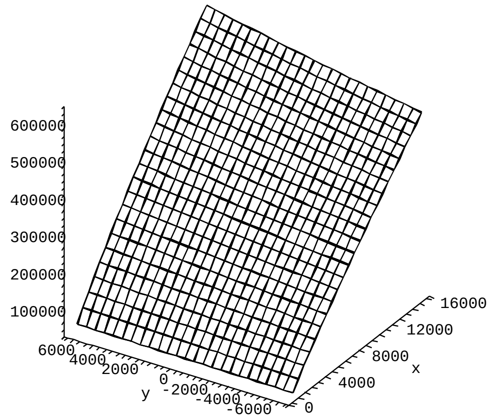


Figure 18: *Conditional R&D value, 2001, 3.Quartile*

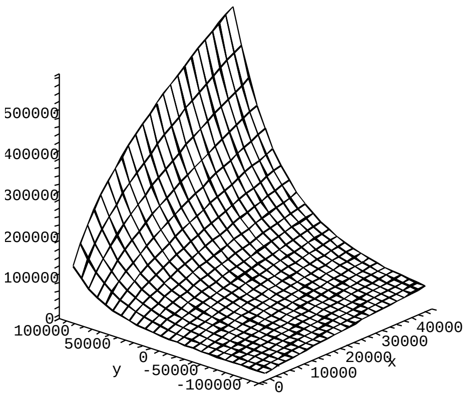


Figure 19: *Conditional R&D value, 2000*

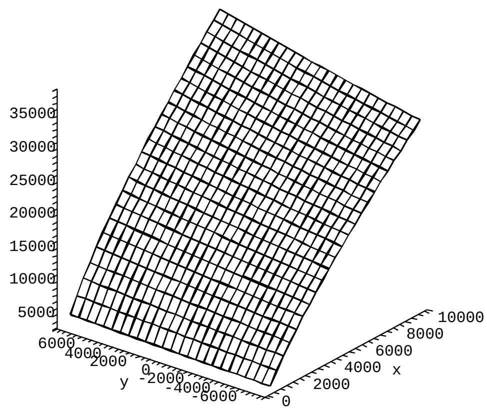


Figure 20: *Conditional R&D value, 2000, 3.Quartile*

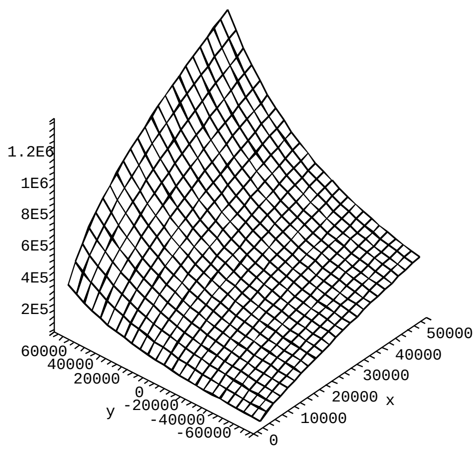


Figure 21: *Conditional R&D value, 1999*

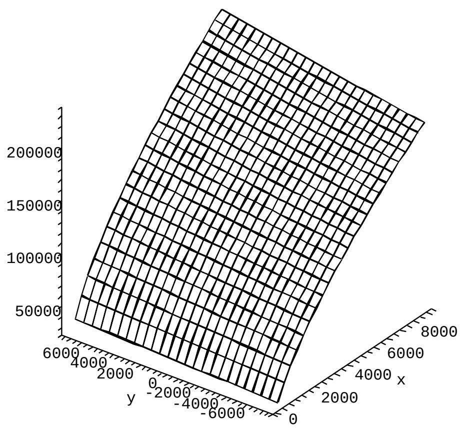
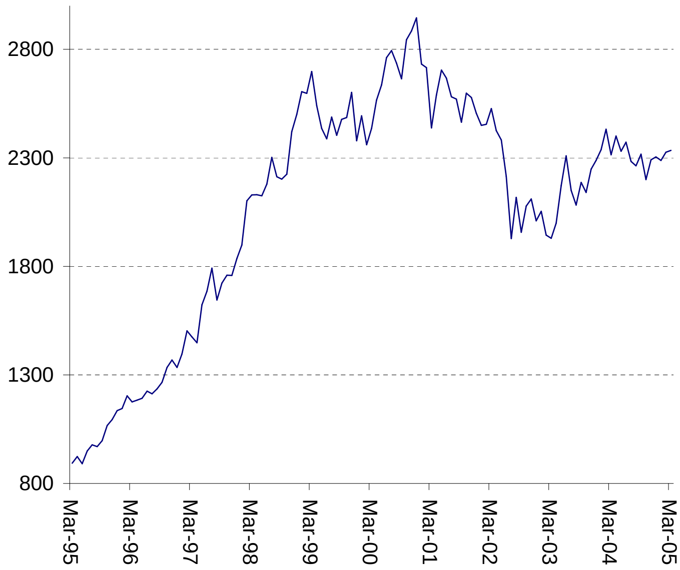


Figure 22: *Conditional R&D value, 1999, 3.Quartile*

To get an intuition on why the relative importance of earnings changes over time when investors estimate the market value of firms' R&D one can look for example at the industry price index.

Figure 23: *Datastream World Pharmaceutical Index (1995 - 2005)*



There are three important market phases for the current discussion. The first one is the year 2000. In this year the market value of the pharmaceutical companies worldwide increased strongly. The second phase is the period from the first quarter 2001 to the first quarter 2003. During these two years, pharmaceutical companies lost a significant part of their market valuation. The first signs of a slight recovery can be seen in the last phase, which lasts from the first quarter 2003 to the first quarter 2005.

The strong increase of market valuation during the first phase reduces the risk premia required by investors so that the net present value of riskier projects particularly those run by firms with negative projects increases. As a result, the sensitivity of the market value of R&D investments to the current reported earnings decreases (see Figure 21). With the sharp decrease in the market valuation of the companies in the first quarter 2001, the risk premia required by investors for holding shares of companies with negative earnings increases, so that the net present value of their R&D projects decreases (see Figure 19). During the second phase, the R&D projects of firms with negative are discounted stronger than the R&D investments of firms with positive earnings according to the higher risk premia required by investors (see Figures 17 and 15). During the last phase, the level of reported earnings do not have any impact on the market value of R&D expenditures as long as the earnings are positive and the market value of the firm increases proportionally with firms' R&D expenditures (see Figures 13 and 11).

The importance of negative earnings for the market value of R&D investment is better observable for firms with R&D investments and earnings within the third quartile (75% of the observations). Investors analyzing firms with low levels of R&D investments do not condition the value of these investments on the firms' earnings. The stronger firms decide to invest in R&D, the more important are the earnings for the market value of their R&D investments. With the recovery in the overall market valuation in the last phase, the R&D projects of firms with positive earnings gain more in value than the projects of firms with negative earnings (see Figures 14 and 12). That is, firms with positive earnings profit stronger from the reduction in the overall risk premia than firms with negative earnings.

Overall, in periods of sharp increase of the overall market valuation, the R&D projects of firms with negative earnings profits stronger from the decrease in the risk premia required by investors. When the risk premia increases, the net present value of riskier R&D projects run by firms with negative earnings decreases. A slight recovery in the overall market valuation, i.e. a lower risk premia, increases the net present value of R&D projects in particular for firms with positive earnings. As long as the firm report earnings over the threshold its market value increases proportionally to its R&D investments.

5 Discussion

Why should the value of R&D investments as reflected in the market value of the firm increase with its earnings? One possible explanation is related to the financial restrictions of the firm. The profits earned by the company are necessary to finance R&D investments. For most well-established corporations, R&D spending is not strongly dependent upon internal cash flow, but pharmaceutical companies are probably an exception (see Himmelberg and Petersen[9]). Thus, the higher the profits, the less restricted is the company with respect to covering further investments, which might be required in the following periods. The value of this flexibility is embodied in the value of current R&D investments conditioned on firm's earnings as reflected in the market value of the company.

Another explanation for the positive relationship between firm's profits and the value of its R&D investments is related to the skewness of firms' profits - only a minority of new products lead to exceptional earnings, most of the products return less than the capitalized cost of their R&D investments (see Grabowski and Vernon[4]). The products contribution to earnings depends not only on the size and duration of the investments but also on firm's abilities to manage them efficiently. In the pharmaceutical industry, investments in R&D are the lifeblood of the companies. Additionally, the product development usually requires continuous investments over several years. From this perspective, firm's current earnings can be seen as indicators for the profitability of past R&D investments, i.e. firm's abilities to manage R&D projects efficiently. Conditioning current R&D investments on this information, firms with higher earnings indicating a better implementation of past R&D projects are expected to receive a higher valuation for their current R&D activities.

Overall, positive earnings may relax the capital constraint of the company but also signal firm's abilities to manage R&D projects efficiently.

The effect of negative earnings on the market value of firm's R&D investments is more puzzling. Intuitively, negative earnings are not necessary bad since they force managers to be very careful when selecting further R&D projects although past R&D expenditures still do not return earnings. On the other hand, early stage firms with few products in development may continue to invest although the results are less than promising just because managers are reluctant to return funds to shareholders. Additionally, if managers care about losses as suggested by Kahneman and Tversky, they would probably feel comfortable gambling-to-get-back-to-even. This increases also the probability for a default. The larger the R&D spending when earnings are negative the higher is the default risk, the more likely is it that investors focus stronger on the probability that the firm can not sustain the planed R&D growth and have to bankrupt. Overall, managerial decision to invest in R&D despite negative earnings may reflect managerial optimism in the prospect of the current projects. Though, from investors' perspective managerial incentives to take more risks and continue poor projects "throwing good money after bad" appears to be stronger, so that firms reporting negative earnings receive a lower market value for their R&D investments than firms reporting positive earnings.

6 Conclusion

In modern economics many firms invest strongly in intangible assets in particular through R&D. This paper addresses the question whether firms' reported earnings are relevant for the market value of their R&D investments. The empirical evidence reported in this study confirms that there is a certain direct link between R&D investments and the market value of the firm as reported in previous studies. In particular, the study shows that the effect depends on the current profits of the companies. Firms reporting positive profits receive a higher market valuation for their R&D activities than firms reporting negative profits. This effect is significant and persistent over time. Though, in different market phases the investors' sensitivity to shifts in R&D investments and profits changes over the business cycle.

The results are highly significant for the sample of firms in the US pharmaceutical industry. Further tests with firms in other knowledge-driven industries (e.g. biotechnology, semiconductors) can provide insights to the question whether the observed effects are common for firms in R&D-intensive sectors.

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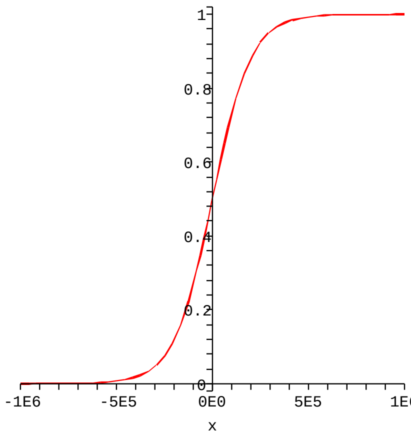
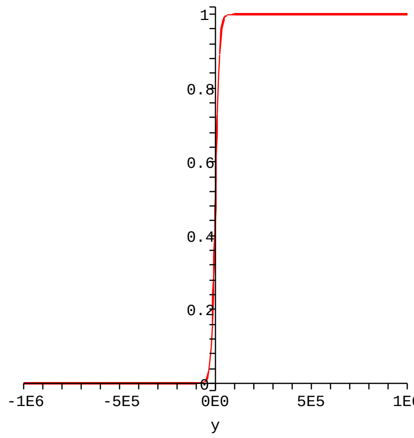
A Appendix

A.1 Properties of the non-linear function

The non-linear function defined in equation (4) is visualized in the following figures.



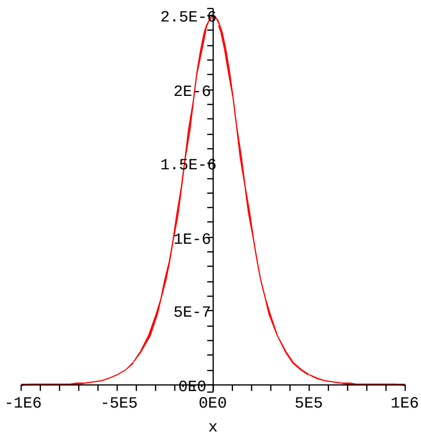


The function is plotted for values $a = 0$, $b = 0.00001$ (on the left side) and $a = 0$, $b = 0.0001$ (on the right site) and $X \in [-1'000'000; 1'000'000]$ as a reasonable range for the adjusted earnings. Larger values of b reduce the interval, where the function takes value between zero and one. The function becomes similar to the indicator function.

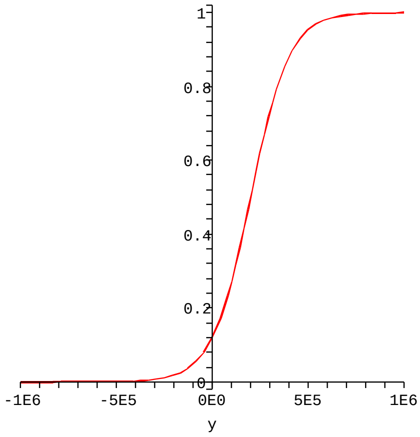
The first derivative of the function is:

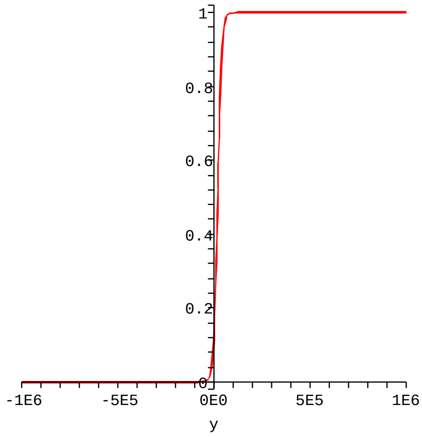
$$g'(x) = \frac{b\exp(-bx)}{(1+\exp(-bx))^2} \tag{8}$$

It is increasing for $x < 0$ and decreasing for $x > 0$ as represented in the figure below (with $a = 0$ and $b = 0.0001$). This is equivalent to the assumption that the marginal returns of R&D investments are increasing for firms with negative profits and decreasing for firms with positive profits. This is plausible, since the pressure of firms with negative earnings to cut costs is stronger than of firms with positive profits, so that these firms have to be more careful in selecting their R&D project, which would increase the probability of success.



The parameter a is a shift parameter. It determines the position of the curve along the x-axis. For example. for $a = 2$ and $b = 0.00001$ respectively $b = 0.0001$, the curves look as follows.





The first derivative is then:

$$g'(x) = \frac{b\exp(a - bx)}{(1 + \exp(a - bx))^2} \tag{9}$$

or graphically: