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Loss Aversion in Aggregate Macroeconomic Time Series

Rina Rosenblatt - Wisch

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Rina Rosenblatt-Wisch[a]

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Abstract

Prospect theory has been the focus of increasing attention in many fields of economics. However, it has scarcely been addressed in macroeconomic growth models. In an earlier paper we introduced prospect theory into a stochastic growth model. This paper focuses on linking the Euler equation induced by such a prospect theory growth model to real macroeconomic data. We will follow the approach of Generalized Method of Moments (GMM) estimation to test the implications of a non-linear prospect utility Euler equation. Our results indicate that loss aversion can be traced in aggregate macroeconomic time series.

JEL-Classification: E21; C32.
Keywords: Ramsey growth model; loss aversion; prospect theory; GMM.

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[a]Institute for Empirical Research in Economics, University of Zurich, Blümlisalpstrasse 10, 8006 Zürich, Switzerland.
Email: rosenblatt@iew.unizh.ch

1 Introduction

Kahneman and Tversky's prospect theory has already found wide application in conjunction with financial markets and individual portfolio choices. So far, it has rarely entered macroeconomic growth models and as such found application to aggregate macroeconomic time series. Prospect theory builds on individual behavior under uncertainty and it is a fair question whether the concept of loss aversion can be empirically validated in the aggregate economy.

In the following paper, we set up a macroeconomic growth framework where the representative agent is modelled as a prospect utility maximizer. Our aim is to test non-linear moment conditions of a stochastic prospect utility Euler equation and to examine whether the implications of Kahneman and Tversky's prospect theory (1979/1992) can be observed in real time series data. This paper develops a method to bring the stochastic prospect Euler equation into a form suitable to perform GMM estimation - an estimation procedure for Euler equations introduced by Hansen and Singleton (1982). This will be done by approximating Kahneman and Tversky's power utility function by a piecewise-linear utility function and modelling the loss aversion coefficient as a switching function. The paper suggests a reconciliation between prospect theory, in particular loss aversion and reference point dependence, and aggregate macroeconomic time series data.

In the next section of the paper the model is introduced and a stochastic prospect utility Euler equation is developed suitable to perform GMM estimation to it. The model will be followed up by estimation results and testing for overidentification before summarizing and concluding.

2 The Model

2.1 A Prospect Utility Function

Inspired by Kahneman and Tversky's (1979/1992) prospect theory, we will focus on a non-time-separable utility function. For this utility function we will derive a stochastic Euler equation and bring it to a form such that GMM can be applied in order to test for loss aversion in real macroeconomic time series data.

In Kahneman and Tversky's prospect theory (1979/1992) agents value their prospects in terms of gains and losses relative to a reference point. They are loss averse, which means that they are more averse to losses than gain seeking on the other hand. Furthermore, they perform subjective, non-linear probability transformation where they overweigh small probabilities and underweigh high probabilities. Kahneman and Tversky suggest a value function which is concave in the region of gains and convex for losses. To capture the effect of loss aversion it is steeper in the region of losses.

Out of this perception Kahneman and Tversky (1992) specify the following two-part value power function:

$$v(x) = \begin{cases} x^{\hat{\alpha}} & \text{if } x \geq 0, \\ -\lambda(-x)^{\hat{\beta}} & \text{if } x < 0, \end{cases} \tag{1}$$

where x represents a gain or a loss and $\lambda > 1$ captures loss aversion indicating the fact that losses hurt more than gains. Kahneman and Tversky (1992) estimated in an experiment the following values for the parameters: $\hat{\alpha} = \hat{\beta} = 0.88$ and $\lambda = 2.25$.

Under cumulative prospect theory the value V of a lottery is evaluated as a weighted average of the following form:[1]

$$V = \sum_{i\,\epsilon\,gains} w_i^+ v(x_i) + \sum_{i\,\epsilon\,losses} w_i^- v(x_i), \tag{2}$$

where the decision weights w are not the objective probabilities of the lottery, but are calculated by using the following functional form:

$$w^+(\pi) = \frac{\pi^{\hat{\gamma}}}{(\pi^{\hat{\gamma}} + (1-\pi)^{\hat{\gamma}})^{\frac{1}{\hat{\gamma}}}}, \quad w^-(\pi) = \frac{\pi^{\hat{\delta}}}{(\pi^{\hat{\delta}} + (1-\pi)^{\hat{\delta}})^{\frac{1}{\hat{\delta}}}} \tag{3}$$

with $\hat{\gamma}$ estimated to be 0.61 and $\hat{\delta}$ to be 0.69. The decision weights are calculated as $w_i^{\pm} = w^{\pm}(\pi_i) - w^{\pm}(\pi_{i^*})$ where π_{i^*} is the probability of the outcomes that are strictly better (worse) than i, and π_i on the other hand is the probability of all outcomes at least as good (bad) as i.

We focus in the following case on loss aversion and that the agent generates utility out of differences. We therefore suggest a linear value function over losses and gains with a kink at the reference point, where losses and gains are meant to be negative or positive changes in consumption relative to a reference point. The piecewise-linear approximation is a common approach and

[1]See Polkovnichenko (2003).

is also used for example by Aït-Sahalia and Brandt (2001) to derive an asset pricing Euler equation, which is then used for GMM estimation.

Hence, we define a piecewise-linear prospect utility function:

$$u(\Delta c_t) = \begin{cases} \Delta c_t & \text{if } \Delta c_t \geq 0, \\ \lambda \Delta c_t & \text{if } \Delta c_t < 0, \end{cases} \tag{4}$$

where $\Delta c_t = c_t - c_{t-1}$. Also, $\lambda > 1$ to capture loss aversion.

Note that:

- $\frac{\partial u(\Delta c_t)}{\partial \Delta c_t} > 0$ for $\Delta c_t \neq 0$
- $\frac{\partial u(\Delta c_t)}{\partial \Delta c_t} < \frac{\partial u(-\Delta c_t)}{\partial (-\Delta c_t)}$.

The social planner[2] solves

$$\max_{\Delta c_t, k_{t+1}} E \sum_{t=0}^{\infty} \beta^t u(\Delta c_t) \tag{5}$$

subject to the constraint

$$f(k_t) + (1-\delta)k_t = c_t + k_{t+1}, \tag{6}$$

where the production function $f(k_t)$ is strictly increasing and concave and the production shocks A_t (later introduced) are assumed to enter multiplicative into the production function. β is the discount factor and $0 < \beta < 1$.

Δc_t can be expressed as

$$\Delta c_t = f(k_t) + (1-\delta)k_t - k_{t+1} - f(k_{t-1}) - (1-\delta)k_{t-1} + k_t. \tag{7}$$

We substitute the constraint into the objective function, thus the social planner solves

$$\max_{k_{t+1}} E \sum_{t=0}^{\infty} \beta^t u(f(k_t) + (1-\delta)k_t - k_{t+1} - f(k_{t-1}) - (1-\delta)k_{t-1} + k_t). \tag{8}$$

[2]Markets are complete and agents behave competitively so that the First Fundamental Theorem of Welfare Economics holds.

This can be done under the condition that there is an interior solution to the above problem. Having linear utility corner solutions could be an issue. However, the social planner approach unites maximization of households and firms. Even though utility is linear with $\lambda > 1$, the production function is concave and hence, the social planner chooses an interior solution.

The stochastic Euler equation has the following form

$$\frac{\partial u(\Delta c_t)}{\partial \Delta c_t} = E_t \left\{ \begin{array}{c} \beta \frac{\partial u(\Delta c_{t+1})}{\partial \Delta c_{t+1}} \left(\frac{\partial f(k_{t+1})}{\partial k_{t+1}} + (1-\delta) + 1 \right) \\ -\beta^2 \frac{\partial u(\Delta c_{t+2})}{\partial \Delta c_{t+2}} \left(\frac{\partial f(k_{t+1})}{\partial k_{t+1}} + (1-\delta) \right) \end{array} \right\}. \tag{9}$$

This stochastic Euler equation is now the main focus of our investigation. Hansen and Singleton (1982) introduce the concept of testing the implications of stochastic Euler equations directly with Generalized Method of Moments. One advantage of GMM is that it does not require full specification of the underlying economy. It is an econometric estimation procedure where it is possible to estimate parameters in dynamic objective functions without explicitly having to solve for the stochastic equilibrium. GMM estimation allows us to derive parameter estimation of the stochastic Euler equation and to test for overidentification.

We must perform some reformulation of the above Euler equation in order to be able to apply GMM. Our utility function has to be a continuously differentiable function which can be represented by just one function over the entire set of gains and losses. As we note above, our utility function in (4) is not differentiable at the reference point. In order to perform GMM we have to smooth it such that it is also differentiable at the kink. We achieve this goal by transforming the utility function such that the loss aversion coefficient and the utility part form an entity. This can be done by setting up the loss aversion coefficient as a switching function. Under the assumption of loss aversion, λ in equation (4) should be bigger than 1 in the region of losses and exactly 1 in the region of gains. It should switch its value as close as possible to the reference point. Such a switching function for the loss aversion coefficient λ can be represented by

$$\tilde{f}(x) = 1 + \frac{\gamma}{1 + e^{\mu x}}, \tag{10}$$

where plotting the function, $\tilde{f}(x) \in [1, \gamma + 1]$ and μ represents the speed of switching. Figure 1 and 2 show the relationship. The higher μ the faster the switching around zero. The higher γ the higher the value range of the loss aversion coefficient λ.

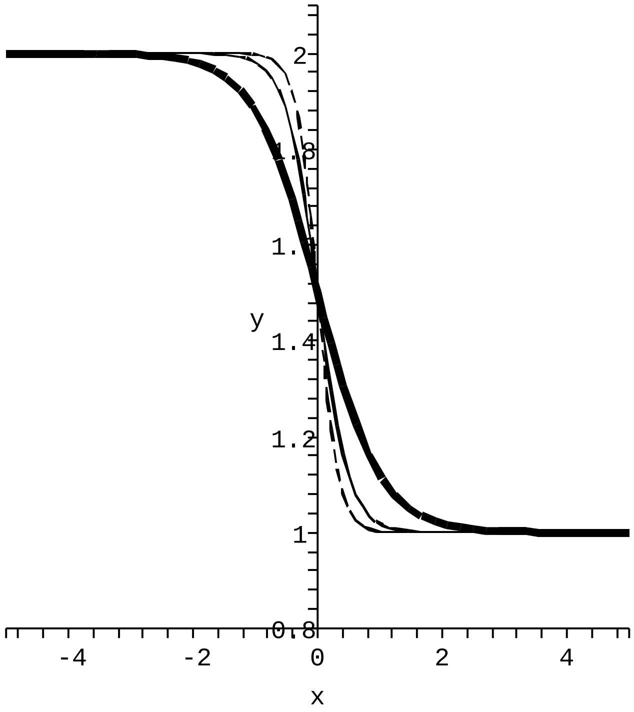

Figure 1: μ responsible for the switching speed around the reference point with $\mu = 2$ (bold line), $\mu = 4$ (solid line) and $\mu = 6$ (dashed line).

Equation (10) yields a smooth function to express the loss aversion coefficient λ in our model. Now we want to substitute λ in the piecewise-linear utility function (4) with the above switching function (10). Plugging equation (10) into equation (4) yields for all x representing Δc

$$\hat{f}(x) = x(1 + \frac{\gamma}{1 + e^{\mu x}}). \tag{11}$$

The graph of equation (11) is shown in Figure 3. This prospect utility function in Figure 3 has similar shape to our first specified piecewise-linear utility function in equation (4). However, since we replace the loss aversion coefficient by the switching function we find a continuously twice differentiable

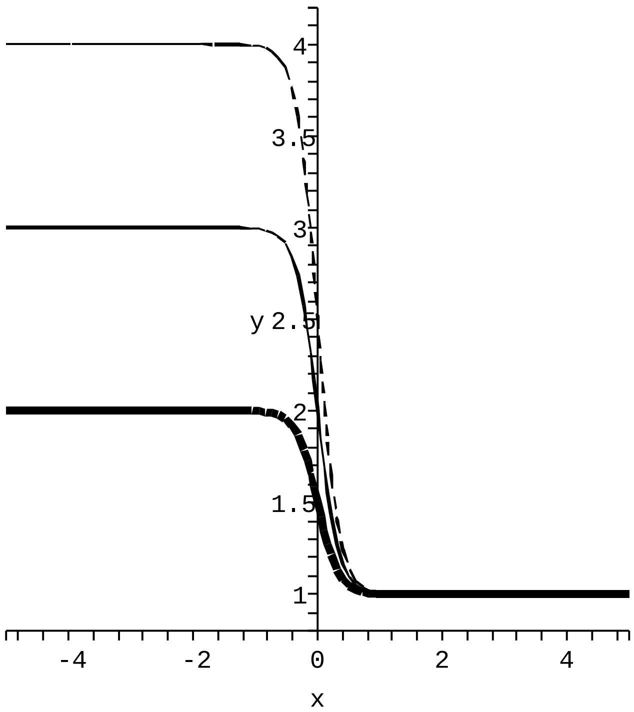

Figure 2: γ assigning the interval of the loss aversion coefficient λ with $\gamma = 1$ (bold line), $\gamma = 2$ (solid line) and $\gamma = 3$ (dashed line).

utility function. The first and second order derivatives are shown in Figure 4, respectively Figure 5.

Differentiation of equation (11) is needed for rewriting the Euler equation:

$$\hat{f}'(x) = 1 + \frac{\gamma}{1 + e^{\mu x}} - \frac{x\gamma\mu e^{\mu x}}{(1 + e^{\mu x})^2}. \tag{12}$$

Setting $x = \Delta c$ it follows from equation (4) and (11)

$$\hat{u}(\Delta c) = (\Delta c)(1 + \frac{\gamma}{1 + e^{\mu(\Delta c)}}) \tag{13}$$

and from equation (4), (11) and (12)

$$\hat{u}'(\Delta c) = 1 + \frac{\gamma}{1 + e^{\mu(\Delta c)}} - \frac{(\Delta c)\gamma\mu e^{\mu(\Delta c)}}{(1 + e^{\mu(\Delta c)})^2}. \tag{14}$$

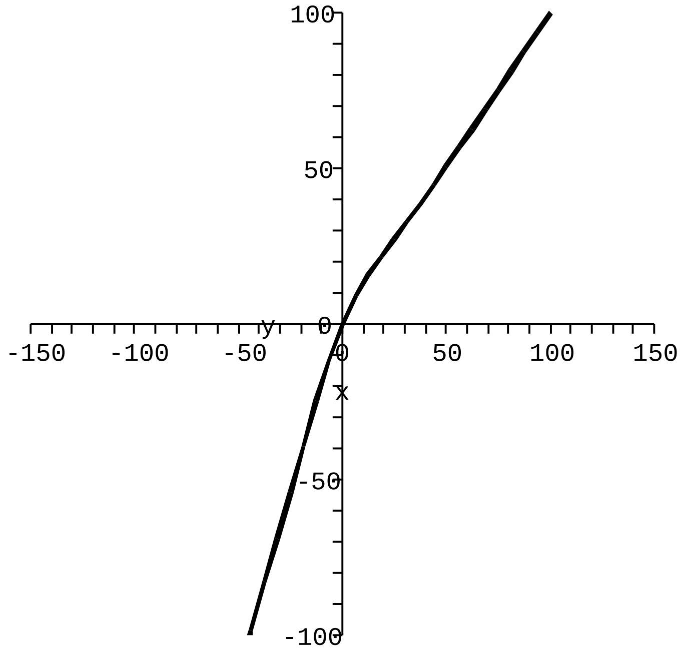

Figure 3: A continuously differentiable prospect utility function with $\gamma = 1.25$ and $\mu = 0.1$.

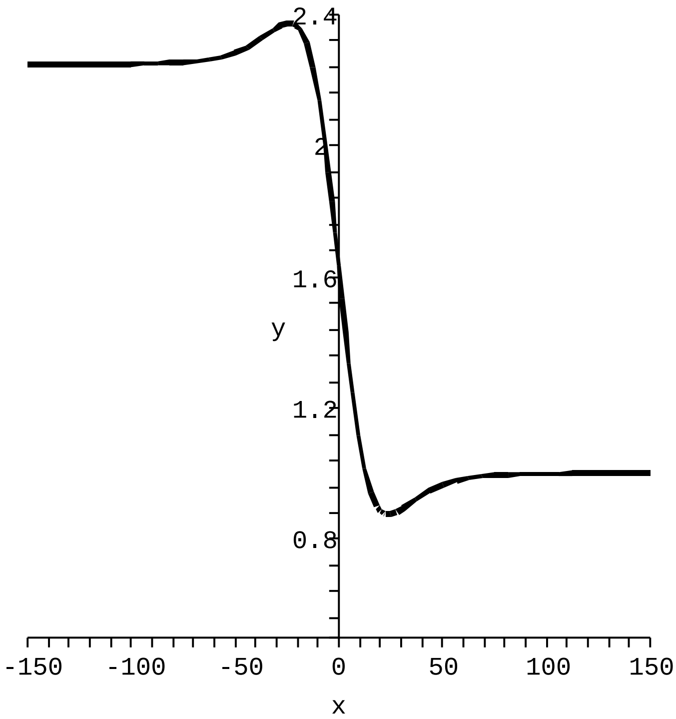

Figure 4: First order derivative of the continuously differentiable prospect utility function with $\gamma = 1.25$ and $\mu = 0.1$.

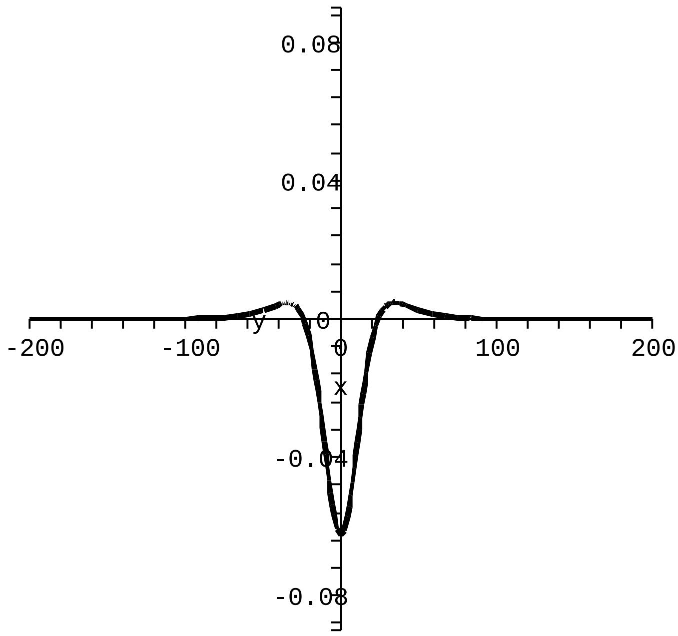

Figure 5: Second order derivative of the continuously differentiable prospect utility function with $\gamma = 1.25$ and $\mu = 0.1$.

Plugging equation (14) into the Euler equation yields

$$
\begin{aligned}
&1+\frac{\gamma}{1+e^{\mu(\Delta c_t)}}-\frac{(\Delta c_t)\gamma\mu e^{\mu(\Delta c_t)}}{(1+e^{\mu(\Delta c_t)})^2}= \\
&E_t\left\{\begin{array}{l}\beta\left(1+\frac{\gamma}{1+e^{\mu(\Delta c_{t+1})}}-\frac{(\Delta c_{t+1})\gamma\mu e^{\mu(\Delta c_{t+1})}}{(1+e^{\mu(\Delta c_{t+1})})^2}\right)\left(\frac{\partial f(k_{t+1})}{\partial k_{t+1}}+(1-\delta)+1\right) \\ -\beta^2\left(1+\frac{\gamma}{1+e^{\mu(\Delta c_{t+2})}}-\frac{(\Delta c_{t+2})\gamma\mu e^{\mu(\Delta c_{t+2})}}{(1+e^{\mu(\Delta c_{t+2})})^2}\right)\left(\frac{\partial f(k_{t+1})}{\partial k_{t+1}}+(1-\delta)\right)\end{array}\right\}.
\end{aligned}
\tag{15}
$$

This is the form we need in order to apply GMM.
In the following estimations we want to test whether a model of loss aversion fits the data. Loss aversion in our Euler equation is represented by γ, respectively, loss aversion is calculated as $\lambda = 1 + \gamma$. Setting $\gamma = 0$ in the above Euler equation (15), respectively $\lambda = 1$ yields

$$\frac{1}{\beta} = E_t\left(\frac{\partial f(k_{t+1})}{\partial k_{t+1}} + (1-\delta)\right). \tag{16}$$

This is the first order condition for the corresponding Cass-Koopmans-Ramsey model with linear utility.[3] So testing for γ is also an implicit test against/for the standard Ramsey model.

3 GMM Estimation Results and Testing for Overidentification

We use US time series from OECD data basis. Personal consumption expenditure, gross domestic product and gross capital formation as a proxy for the capital stock. All data is in real terms, on fixed prices for the year 2000, and is seasonally adjusted. To bring the data into intensive form we use quarterly data of civilian employment, sixteen years and over, provided by the US Department of Labor: Bureau of Labor Statistics. The data range is from $1955Q1$ to $2004Q4$ and is adjusted automatically during the empirical estimations.

Assuming a Cobb-Douglas production function, the uncertainty in our model stems from the production part of the economy. To derive the technological shocks we calculate the Solow residual from the data. The Solow residual will then be used as the technological shock in our Euler equation.

[3]See also Rosenblatt-Wisch (2005).

The production function is

$$F(A_tK_t, L_t) = Y_t = A_tK_t^{\alpha}L_t^{1-\alpha} \tag{17}$$

and in intensive form, dividing by L_t, we obtain

$$f(k_t) = y_t = A_tk_t^{\alpha}, \tag{18}$$

where $y_t = Y_t/L_t$ and $k_t = K_t/L_t$. Taking logs and first differences, the Solow residual can then be expressed as

$$\Delta ln(A_t) = \Delta ln(y_t) - \Delta\alpha ln(k_t). \tag{19}$$

α represents the capital share in the production function. A common value assumed is $\alpha = 0.33$.[4] This value will be used in our estimations. Also, we set $\mu = 0.1$ for computational efficiency as used in Figure 3 (et sqq.) and the depreciation rate $\delta = 1$.[5] Introducing the Cobb-Douglas type production function into the Euler equation and setting the depreciation rate $\delta = 1$ yields:

$$\begin{aligned} &1 + \frac{\gamma}{1+e^{\mu(\Delta c_t)}} - \frac{(\Delta c_t)\gamma\mu e^{\mu(\Delta c_t)}}{(1+e^{\mu(\Delta c_t)})^2} = \\ &E_t \left\{ \begin{array}{c} \beta\left(1 + \frac{\gamma}{1+e^{\mu(\Delta c_{t+1})}} - \frac{(\Delta c_{t+1})\gamma\mu e^{\mu(\Delta c_{t+1})}}{(1+e^{\mu(\Delta c_{t+1})})^2}\right)\left(\alpha A_{t+1}k_{t+1}^{(\alpha-1)} + 1\right) \\ -\beta^2\left(1 + \frac{\gamma}{1+e^{\mu(\Delta c_{t+2})}} - \frac{(\Delta c_{t+2})\gamma\mu e^{\mu(\Delta c_{t+2})}}{(1+e^{\mu(\Delta c_{t+2})})^2}\right)\left(\alpha A_{t+1}k_{t+1}^{(\alpha-1)}\right) \end{array} \right\}. \end{aligned} \tag{20}$$

Our estimations will be built on this Euler equation. It is common in macroeconomic time series that the error terms are autocorrelated over time. We therefore use as default setting Bartlett kernel, Newey and West's fixed bandwidth (4), no prewithening. This setting ensures a heteroscedastic and autocorrelation consistent weighting matrix for heteroscedastic and autocorrelated error terms of unknown form. The Bartlett kernel yields a positive semi-definite estimator of the covariance matrix. In general, GMM requires a positive-semi definite covariance matrix estimator. Negative variance estimates are not desired and also problematic in the sense of convergence to

[4]See for example Abel and Bernanke (2001) or Hall and Taylor (1997).

[5]The depreciation rate enters the calculations of the capital formation stock data (OECD basis) and is as such a part of our physical capital available in the production process.

a negative variance.[6] Further, as it is often the case in macroeconomic time series, we perform our empirical investigation in small samples. Applicable especially for small sample estimation, we use Hansen, Heaton and Yaron's (1996) simultaneous updating procedure of the weighting matrix and the coefficients - implying convergence after a few iterations such that the criterion function is minimized.

An advantage of GMM estimation is that we do not have to know, nor to specify, the full economic setting of the underlying economy. Nevertheless, a few assumptions have to be made: GMM relies on stationarity of the components. Plotting our consumption series we can already guess that

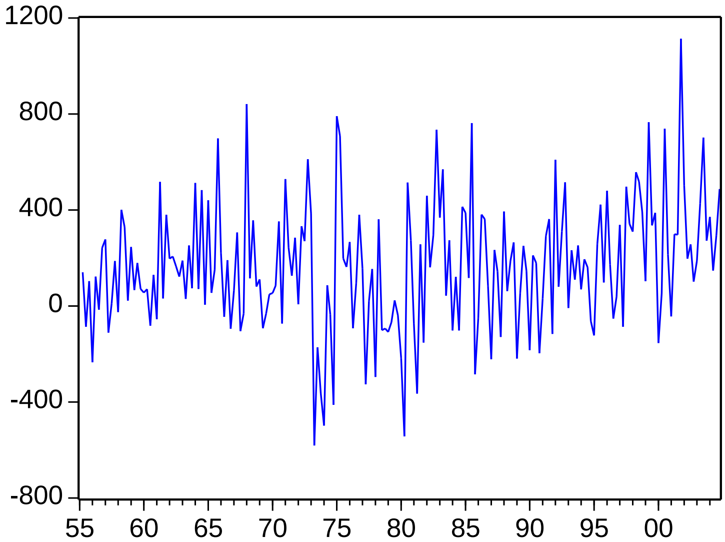

Figure 6: First difference consumption series with quarterly update.

it is first difference stationary. We provide a graph of the first difference consumption path in Figure 6.

To this series we apply Dickey-Fuller test which does not imply a unit root. In addition, performing Kwiatowski-Phillips-Schmidt-Shin test for stationarity, the null of stationarity cannot be rejected at least at the 1% level for all lagged series of consumption we use in the following estimations. Taking

[6]See Mátyás (1999), p. 69 and 77 et sqq.

the exponential of the Solow residual (Figure 7) in terms of growth rates of technological progress multiplied by the marginal productivity of the growth rate of capital, we also find a stationary process for the production part of our Euler equation.

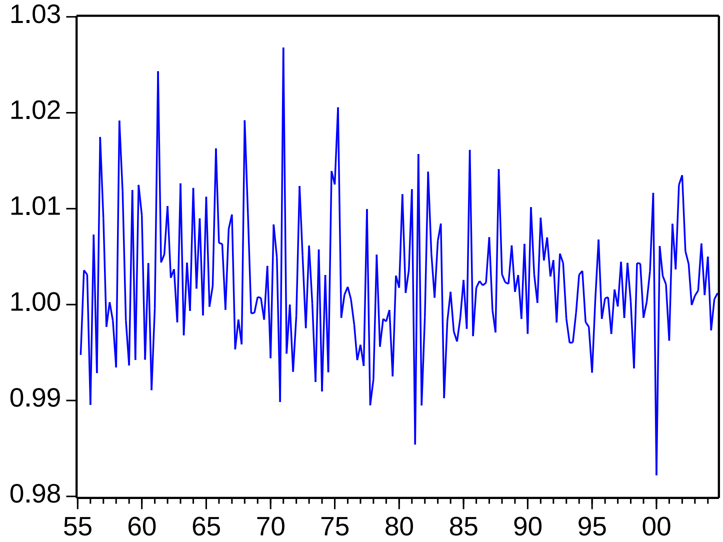

Figure 7: The exponential of the Solow residual.

As mentioned above, we set the rate of capital share of output to $\alpha = 0.33$. We will run the regression for different discount factors β.
As defined in equation (4) our prospect maximizer derives his utility out of differences in consumption. He updates the reference point dynamically. But the question is: which reference point is taken from the past? It could well be, that his reference point is updated quarterly. Although, this time horizon could be too short and he may take as the reference point a consumption level he had a year ago. So we will run our estimations for different reference updating horizons. We constantly check for continuous stationarity of the series. This is given in all the variations of the reference point and also in the production part of the Euler equation.

In the first round, we assume a just identified model with one parameter γ to be estimated and one equation. The J-statistic is then by definition 0. Otherwise, taking instrumental variables into account, in a correctly speci-

fied model the J-statistic is approximately χ^2-distributed.
In Table 1 we test for different discount factors and different adjustment of the reference point. All results are significant at the 1%, respectively 5% level. It is interesting to note that the further the adjustment of the reference point lies in the past, the higher the estimated value of γ and in line with it, the loss aversion coefficient. This effect can be explained by higher variation in the consumption difference the longer the time period. It is important to note that by referring to equation (10), the loss aversion coefficient can be approximately calculated by $(1 + \gamma)$. For the more frequent updating procedure we can see that the estimated γ is more significant, which could be an indication of greater evidence for an updating of the consumption reference point more frequently. The pattern is similar for all values of β.
Comparing the γ values of the same lag structure for the different discount factors β, i.e. reading Table 1 by its columns, the higher the discount factor, the lower the loss aversion implied by the data. This means that fixing a data point, one effect comes from the loss aversion part the other from the discount factor. This is in line with the results of Rosenblatt-Wisch and Schenk-Hoppé (2005), where a deterministic version of a prospect utility maximizer is discussed. There, one notes that the β and the loss aversion coefficient work in the same direction - the higher the discount factor/the higher the loss aversion coefficient, the more future losses hurt. So fixing a given consumption path, there are two forces at work which make future losses more crucial - one part is a higher loss aversion coefficient, the other a higher β. So it is not surprising that ceteris paribus lowering one component, the other one has to be increased and vice versa to explain a given consumption path.

Reference point adjustment	1 quarter	2 quarters	4 quarters
$\beta = 0.90$			
γ	0.8517*	1.2958*	3.2653**
Standard Dev.	0.1223	0.2623	1.2955
J-stats	0	0	0
$\beta = 0.95$			
γ	0.5283*	0.7883*	1.9063**
Standard Dev.	0.0930	0.1844	0.8124
J-stats	0	0	0
$\beta = 0.97$			
γ	0.3830*	0.5656*	1.3306**
Standard Dev.	0.0808	0.1525	0.6018
J-stats	0	0	0

*/** denote statistical significance at the 1%, respectively 5% level.

Table 1: Estimates of γ for different discount factors and different adjustment of the reference point in a just identified system.

We now look at those estimations where we use instrumental variables. Table 2 displays the calculations using as instrumental variables the first lagged value of the respective consumption difference and Table 3 the ones for the first lagged value of capital growth. The weak instrument problem is minimized with the Hansen, Heaton and Yaron type weighting matrix. We check whether the overidentifying restrictions hold by calculating the J-statistics with the respective p-values.

Both tables indicate a similar pattern as discussed above: for a given reference updating scheme, the higher the β the lower the corresponding loss aversion value. Reading the tables by their rows, again, the more time elapses between updating the reference point, the higher the loss aversion. However, we observe some deviation in the 2 quarters column of Table 2 - γ is too small for $\beta = 0.97$ in comparison to the pattern in Table 1. The corresponding J-statistic and p-value in Table 2 indicate that the overidentifying restrictions for this specification do not hold at the 1% level. The model seems to be misspecified. In this case, the estimated value of γ and its significance is spurious. As for the other estimation results in Table 2 the J-statistics and p-values reflect that the overidentifying restrictions hold at the 1% level and the model cannot be rejected as such. Nevertheless, to comment on the

stability of the model, the higher the p-values the more explanatory power our model displays. For example, a p-value of 50% indicates that one would reject the model falsely in 50% of cases even though the specifications are correct. As already mentioned, for certain specifications our p-values are such that we cannot reject the null of a χ^2-distributed J-statistic at a common significance level. However, for a half-yearly updating in Table 2 our p-values are rather small and in Table 3 we do not find any p-values above 50% at all. Overall, Table 2 displays higher p-values than Table 3: for a reference point updating of 1 quarter and all three discount factors β, the p-values are strikingly high and the estimated values of γ are significant at the 1% level and positive. A yearly updating scheme for $\beta = 0.97$ also yields a respectable p-value above 50% as well as a positive and significant γ at the 5% level. This is strong evidence that for a reference updating of a quarter, respectively a year, loss aversion can indeed be found in the data.
The loss aversion parameter closest to Kahneman and Tversky's experimentally supported value of 2.25 is generated by a yearly reference point updating with a discount factor of $\beta = 0.97$ and the corresponding $\gamma = 1.3817$.

Table 4 yields the estimation results for the same settings as in Table 1, 2 and 3, which means that we use the same parameter values and iterative scheme for the weighting matrix in the GMM calculations. But now we check whether a combination of the instrumental variables provides evidence for a correctly specified model. We use as instrumental variables the first lagged value of consumption difference and the first lagged value of capital growth. The estimation results indicate that the overidentifying restrictions hold for most updating schemes at the 1% level, but we find rather low p-values (all below 30%). Again, this representation produces the common pattern first observed in Table 1, namely that the more time has elapsed before updating the reference point, the higher the loss aversion found in the data, and the higher the discount factor, the lower the loss aversion parameter.

Apparently, comparing Table 2 and Table 3, using lagged difference of consumption as instrumental variable produces more explanatory power than including the lagged values of capital growth. We therefore run a fifth estimation (Table 5) and take the first, second and third lagged values of consumption difference as the set of instrumental variables. Again, we observe complementary of the discount factor β and the loss aversion λ and increasing loss aversion as time advances. Nevertheless, the p-values of the J-statistics are all below 50% and do not give strong indication of a correctly specified model.

Reference point adjustment	1 quarter	2 quarters	4 quarters
$\beta = 0.90$			
γ	0.8892*	1.1745*	2.9759**
Standard Dev.	0.1313	0.2376	1.1927
J-stats	0.0624	3.3717	4.4977
p-value	0.8028	0.0663	0.0339
$\beta = 0.95$			
γ	0.5560*	0.6140*	1.9152**
Standard Dev.	0.1005	0.1444	0.8217
J-stats	0.1426	5.2114	1.5187
p-value	0.7057	0.0224	0.2178
$\beta = 0.97$			
γ	0.4058*	0.3493*	1.3817**
Standard Dev.	0.0879	0.0955	0.6217
J-stats	0.2006	7.1087	0.4206
p-value	0.6542	0.0077	0.5167

*/** denote statistical significance at the 1%, respectively 5% level.

Table 2: Estimates of γ for different discount factors and different adjustment of the reference point with the first lagged value of consumption difference as instrumental variable.

Reference point adjustment	1 quarter	2 quarters	4 quarters
$\beta = 0.90$			
γ	0.7820*	1.1613*	2.9390**
Standard Dev.	0.1142	0.2343	1.1690
J-stats	9.6104	3.4938	2.3157
p-value	0.0082	0.1743	0.3142
$\beta = 0.95$			
γ	0.3981*	0.6817*	1.5881**
Standard Dev.	0.0748	0.1607	0.6743
J-stats	11.6702	2.9330	2.5086
p-value	0.0029	0.2307	0.2853
$\beta = 0.97$			
γ	0.1931*	0.4711*	1.0285**
Standard Dev.	0.0468	0.1299	0.4698
J-stats	16.2270	2.5940	2.6411
p-value	0.0003	0.2734	0.2670

*/** denote statistical significance at the 1%, respectively 5% level.

Table 3: Estimates of γ for different discount factors and different adjustment of the reference point with the first lagged value of capital growth as instrumental variable.

Reference point adjustment	1 quarter	2 quarters	4 quarters
$\beta = 0.90$			
γ	0.8058*	1.0864*	2.8210**
Standard Dev.	0.1202	0.2188	1.1252
J-stats	8.5800	6.3206	5.4840
p-value	0.0137	0.0424	0.0644
$\beta = 0.95$			
γ	0.4258*	0.5648*	1.6804**
Standard Dev.	0.2188	0.1331	0.7048
J-stats	10.3084	7.2800	3.2249
p-value	0.0058	0.0263	0.1994
$\beta = 0.97$			
γ	0.2278*	0.3217*	1.1214**
Standard Dev.	0.0545	0.0884	0.4860
J-stats	13.6692	8.8538	2.5551
p-value	0.0011	0.0120	0.2787

*/** denote statistical significance at the 1%, respectively 5% level.

Table 4: Estimates of γ for different discount factors and different adjustment of the reference point with the first lagged values of consumption difference and capital growth as instrumental variables.

Reference point adjustment	1 quarter	2 quarters	4 quarters
$\beta = 0.90$			
γ	0.8636*	1.2104*	2.9760**
Standard Dev.	0.1302	0.2492	1.1568
J-stats	4.5472	12.5426	6.7953
p-value	0.2081	0.0057	0.0787
$\beta = 0.95$			
γ	0.5217*	0.3695*	1.8517**
Standard Dev.	0.0980	0.0950	0.7626
J-stats	4.2463	21.3113	4.1921
p-value	0.2361	0.0001	0.2414
$\beta = 0.97$			
γ	0.3606*	0.0366*	1.2774**
Standard Dev.	0.0830	0.0116	0.5476
J-stats	4.2640	38.2746	3.0452
p-value	0.2343	0.0000	0.3847

*/** denote statistical significance at the 1%, respectively 5% level.

Table 5: Estimates of γ for different discount factors and different adjustment of the reference point with the first, second and third lagged values of consumption difference as instrumental variables.

GMM is an approach to estimate a model directly from its first order conditions. As such, it has not been free of critique since it performs testing of a necessary and not sufficient condition of optimality. Nevertheless, it has been widely used to render empirical implications through first order conditions. Especially in macroeconomic models there is often the problem of small sample bias. To handle this bias we used Hansen, Heaton and Yaron's suggested iteration scheme of the weighting function.

For the just identified model and also respectively for the overidentified model with the first lagged value of consumption difference as the instrumental variable (Table 2) we find strong evidence of a positive γ resulting in a loss aversion coefficient $\lambda > 1$. We come closest to the value of Kahneman and Tversky's loss aversion coefficient of 2.25 for a yearly updating scheme and a discount factor of 0.97. All estimated values of γ are significantly different from zero and positive. This is strong indication against the standard Ramsey model and in favor of an incorporation of loss aversion to fit the data better.

4 Conclusion

We introduced prospect theory into a neoclassical growth model and derived an intertemporal Euler equation which is defined not only by comparing current marginal utility of consumption today to marginal utility tomorrow, but also to marginal utility the day after tomorrow. Our aim was to link the prospect utility Euler equation to real economic data and investigate whether the loss aversion parameter can be found in macroeconomic time series. To achieve this, we linearized Kahneman and Tversky's power function and represented the loss aversion parameter by a switching function. As such, our piecewise-linear utility function is differentiable even at the reference point. Also, we needed a representation of the utility function which is the same for the domain of gains and losses in order to make the Euler equation testable under GMM.

Our GMM estimation results indicate that the higher we set the discount factor the lower the loss aversion found in the data. This is consistent with theoretical/numerical results we find in a deterministic prospect utility growth model where we observe complementary of the discount factor and the loss aversion coefficient.[7] We observe a loss aversion parameter $\lambda > 1$, which

[7] See Rosenblatt-Wisch and Schenk-Hoppé (2005).

is increasing the longer the updating horizon becomes. In particular, we find strong evidence of loss aversion for a quarterly and yearly updating scheme of the reference point. Furthermore, it comes closest to Kahneman and Tversky's experimentally validated value of 2.25 for a representative agent, who sets his reference point on a yearly basis and has a discount factor around 0.97. So even with a linearized version of our prospect Euler equation we can empirically reconcile Kahneman and Tversky's findings of loss aversion and thinking in differences and macroeconomic theory of growth.

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