Trade credit, collateral liquidation and borrowing constraints

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Trade credit, collateral liquidation and borrowing constraints∗

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Abstract

The paper investigates the determinants of trade credit and its interactions with borrowing constraints and the input combination of the firm, within an incomplete contract setting in which firms use a two-input technology and collateralised credit contracts. Assuming that the supplier is better able to extract value from existing assets and that she has an information advantage relative to other creditors, the paper derives the following predictions: (1) financially unconstrained firms (with unused bank credit lines) take trade credit for a liquidation motive; (2) the reliance on trade credit does not depend on the degree of credit rationing, if inputs are liquid enough; (3) firms buying goods make more purchases on account than firms buying services, while suppliers of services offer more trade credit than suppliers of standardised goods; (4) suppliers lend inputs to their customers but not cash; (5) larger reliance on trade credit is associated with a more intensive use of tangible inputs; (6) better creditor protection decreases both the use of trade credit and input tangibility.

Keywords: trade credit, collateral, financial constraints, asset tangibility, creditor protection.


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Introduction

Firms raise funding not only from specialised financial intermediaries but also from suppliers, generally by delaying payments for input provision. The empirical evidence on trade credit poses several questions that are hard to reconcile with existing theories. First, what justifies its widespread use by financially unconstrained firms having access to seemingly cheaper alternative sources of funding? Second, why is the reliance on trade credit not always increasing in the degree of credit rationing? Third, why do suppliers extend credit by allowing delayed payment for their products, but seldom by lending cash? Last, does input lending have an impact on the borrower’s choice of inputs? And, relatedly, are the financing and input choices affected by the degree of creditor protection? The present paper addresses all of these questions in a unified setting.

There is a general consensus that trade credit is widespread among firms facing borrowing constraints. This idea follows from the presumption that trade credit is more expensive than bank loans. According to this view, the dependence on trade credit should increase in credit rationing. Although it is difficult to investigate the relation between the use of trade credit and credit rationing, given the lack of reliable empirical measures of borrowing constraints, the empirical evidence is not generally consistent with this common belief. Petersen and Rajan (1997) document that the U.S. large firms, which are less likely to be credit-constrained, rely heavily on trade credit and do so to a larger extent than small firms: accounts payable average 11.6% and 4.4% of sales for large and small firms, respectively. Similarly, for the Italian manufacturing sector, Marotta (2001) documents that trade credit finances on average 38.1% of the input purchases of non-rationed firms, as opposed to the 37.5% of rationed ones.

A common feature in the use of trade credit, which is independent of the degree of credit rationing, is that the supplier’s lending activity is closely tied to the value of the input transaction, i.e. suppliers

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1The evidence on trade credit as a more expensive source of financing than bank loans is mostly anecdotal (Petersen and Rajan, 1997; Ng, Smith and Smith, 1999; Wilner, 2000). In support of it, scholars generally invoke the canonical “2/10 net 30” agreement (a 2% discount for payment within 10 days, with the net price charged for payment within 30 days) that implies an effective interest rate above 40% for those who do not take the discount. However, it is not clear how widespread this kind of agreement is in the buyer-seller relationship.

2Petersen and Rajan (1997) also find that firms that have been denied credit in the previous year receive more trade credit. However, the coefficient is not statistically significant.

3Marotta (2001) uses data from a survey conducted by the bank Mediocredito Centrale in 1994. Credit constrained firms are identified by two questions: “In 1994, has the firm applied for, but not obtained, more bank loans?” and “in 1994, would the firm have accepted tighter terms (higher interest rates or higher collateral requirements) to obtain more bank loan?”
lend inputs, but they seldom lend cash. Given that not all inputs can be purchased on account, it is reasonable to expect that the use of trade credit goes together with some bias in the input combination chosen by the entrepreneur. This seems confirmed by scattered evidence on financing and technological choices. Some papers document a larger use of trade credit in countries with lower degrees of creditor protection, such as developing countries (see, among others, Rajan and Zingales, 1995; La Porta et al., 1998; Fisman and Love, 2003; Frank and Maksimovic, 2004). Further evidence shows that firms in developing countries have a higher proportion of fixed assets and fewer intangibles than firms in developed countries (e.g., Demirguc-Kunt and Maksimovic, 2001). Although fragmented, these findings suggest the existence of a cross-country relationship between financing and input choices and identify the degree of creditor protection as a possible explanation for this relationship.

To account for the above stylised facts, we construct a model where opportunistic firms, facing uncertain demand, choose between sources of external funding (bank and trade credit) and inputs with different degree of observability and collateral value (tangibles and intangibles). Being opportunistic, firms may face borrowing constraints. Banks are specialised intermediaries and have a cost advantage in financing firms. Suppliers have both an information and a liquidation advantage. The first consists in observing input transactions costlessly. This allows suppliers to provide credit to relax the firm’s financial constraints, i.e. for incentive reasons. The second advantage derives from the supplier’s ability to extract a higher liquidation value from inputs in case of distress. Uncertainty and multiple inputs in a model with moral hazard are key to address the open questions listed above.

A novel feature of our analysis is that it can explain why firms with unused bank credit lines may demand trade credit from their suppliers: even such firms may benefit from the liquidation advantage of their supplier. This advantage makes trade credit cheaper than bank loans, thus offsetting the lower cost that banks face in raising funds on the deposit market.

While the liquidation advantage is by itself sufficient to explain the demand for trade credit by financially unconstrained firms, the interaction between the liquidation and the information advantage helps to understand why reliance on trade credit does not always increase in the tightness of financing constraints. Our analysis shows that financially constrained firms may take trade credit for both reasons. If it is for incentive motives, rationed firms finance a larger share of their inputs by trade credit relative to non-rationed ones, in line with the existing theoretical literature. Conversely, when the liquidation motive dominates, the share of inputs purchased on account stays constant across firms.

4For example, intangible assets cannot be financed in general by trade credit.
with different degrees of credit rationing.

Moreover, we show that the relation between trade credit use and financial constraints depends crucially on the characteristics of the inputs. Firms using inputs with a high degree of liquidity (e.g., standardised inputs) or a high collateral value (e.g., differentiated inputs) are more likely to take trade credit to exploit the liquidation advantage of the supplier. Conversely, the incentive motive is more likely to dominate among financially constrained firms purchasing illiquid inputs with a low collateral value (e.g., services). Our analysis also allows us to derive several testable predictions on how demand and supply of trade credit vary across industries: firms buying goods (both differentiated and standardised) make more purchases on account than firms buying services, while suppliers of services offer more trade credit than suppliers of standardised goods.

Regardless the motives underlying the use of trade credit, our analysis shows that suppliers always finance the inputs they sell but they never lend cash. This follows from the assumption that suppliers can only observe the input transaction in which they are involved. Indeed, if they could observe also the input purchase from other suppliers, cash lending would arise endogenously. To our knowledge, the only available evidence of cash lending concerns Japanese trading companies (Uesugi and Yamashiro, 2004). These companies typically feature a strong involvement of suppliers in the firm’s activity due to an organisational structure that guarantees a continuous information flow from clients to suppliers. This feature is consistent with our theoretical finding.

The lack of cash lending by suppliers implies that trade credit can only be used to finance specific inputs that in our setting are tangibles. It follows that whenever trade credit is used to relax financial constraints, a rationed entrepreneur can benefit from it only by distorting the input combination towards these inputs. This introduces a link between financing and input decisions. By exploring this link, our analysis generates several new results. In particular, a more intensive use of trade credit goes together with a technology biased towards tangible assets, and this bias becomes stronger as the legal protection of creditors weakens. These predictions reconcile the scattered pieces of cross-country evidence discussed above (Rajan and Zingales, 1995; La Porta et al., 1998; Demirgüç-Kunt and Maksimovic, 2001).

The rest of the paper is organised as follows. Section 1 provides a sketch of the related literature. Section 2 describes the model. Section 3 analyses the determinants of trade credit, distinguishing between liquidation and incentive motives. Section 4 presents and discusses the results. Section 5 explores the effect of incorporating the specific content of the bankruptcy and commercial laws on our
predictions. Section 6 concludes.

1 Related literature

One of the main objectives of the literature on trade credit has been to explain why agents should want to borrow from firms rather than from financial intermediaries. The traditional explanation has been that trade credit serves a non financial role. More precisely, it allows to reduce transaction costs (Ferris, 1981), implement price discrimination across customers with different creditworthiness (Brennan et al., 1988), facilitate the establishment of long term relationships with customers (Summers and Wilson, 2002), and even provide a warranty for product quality when customers cannot observe product characteristics (Long et al., 1993).

Although these non-financial theories can explain the existence of trade credit, they do not deliver any prediction on how borrowing constraints affect the demand for trade credit, since credit rationing is not explicitly modeled in any of these papers. Financial theories have attempted to fill this gap (Biais and Gollier, 1997; Burkart and Ellingsen, 2004, among others). In these theories the supplier has an information advantage over financial institutions in lending to the buyer. Burkart and Ellingsen (2004), whose analysis is closest to ours, construct a model in which banks have an intermediation advantage, while suppliers have an information advantage which mitigates their exposure to borrowers’ opportunistic behaviour. It turns out that sufficiently rich firms, facing no incentive problems, never use trade credit, while, poorer firms, which do face incentive problems, experience bank credit rationing. For these firms, suppliers’ information advantage becomes relevant, as they can relax borrowing constraints by extending trade credit to their customers. Similarly, Biais and Gollier (1997) construct a screening model in which the seller’s provision of trade credit signals the creditworthiness of the buyer and thus mitigates credit rationing.

Hence, both of these papers, and more generally existing financial theories of trade credit, fail to explain: (i) why trade credit is used also by financially unconstrained firms; and (ii) why reliance on trade credit does not necessarily increase with the severity of financial constraints, as documented by the empirical literature (Petersen and Rajan, 1997; Marotta, 2001). In order to distinguish between rationed and non-rationed firms, we model the information advantage as in Burkart and Ellingsen (2004) but interact it with a liquidation advantage, which can explain why even wealthy firms may wish to use trade credit. The liquidation advantage of suppliers, when it exceeds the bank’s intermediation advantage, warrants reliance on trade credit by rationed and unrationed firms alike. This squares with
the evidence that firms facing different degrees of credit rationing tend to rely to the same extent on trade credit.

The idea that trade credit can offer a way to exploit the supplier’s liquidation advantage has been proposed and tested in various empirical contributions (Mian and Smith, 1992; Petersen and Rajan, 1997, among others). Frank and Maksimovic (2004) have also theoretically modeled the effects of such advantage, and shown that it makes trade credit cheaper than bank financing. However, in their setting bank credit is never rationed, so that no prediction regarding the demand for trade credit by financially unconstrained firms can be derived.\(^5\)

Finally, the existing literature has disregarded the relations between financing and input decisions and has offered no explanation for why firms only lend inputs. The use of a multi-input technology allows us to fill these gaps.

## 2 The model

A risk-neutral entrepreneur has an investment project which uses a tangible and an intangible input to produce a verifiable output. The tangible input can be interpreted as raw material as well as physical capital, while intangibles as skilled labour, for example employees working in R&D units. Let \(q_k\) and \(q_L\) denote the purchased amount of tangible and intangible inputs respectively and \(I_k \leq q_k, I_L \leq q_L\), the amount of such inputs that is invested. The purchase of inputs is observable only to their respective suppliers. The amount invested is unobservable to any party and is transformed into a verifiable state contingent output \(y_\sigma\), with \(\sigma \in \{G, B\}\) and \(y_G > y_B = 0\). The good state (\(\sigma = G\)) occurs with probability \(p\). Uncertainty affects production through demand (i.e., production is demand-driven). At times of high demand, invested inputs produce output according to the increasing and strictly concave production function \(f_G(I_k, I_L)\). At times of low demand, no output is produced and the firm is worth only the scrap value of unused inputs, which can therefore be pledged as a collateral to financiers.\(^6\) Inputs are substitutes, but a positive amount of each is essential for production.

The entrepreneur is a price taker both in the inputs and in the output markets. The output price

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\(^5\)In their model, suppliers must borrow from banks in order to extend trade credit to their customers. This intermediary role of suppliers creates an adverse selection problem that induces banks to ration credit to suppliers. These, in turn, will ration creditworthy customers, who will be induced to turn to bank credit. Hence, banks will not ration credit to customer firms.

\(^6\)This implies that ex-post diversion is not feasible. However, allowing for this case does not alter our qualitative results, as long as there is a minimal share of the assets that cannot be hidden (e.g., the premises of the firm, or heavy machinery).
is normalised to 1, as well as the prices of tangible and intangible inputs.\textsuperscript{7}

Although the entrepreneur has observable internal wealth ($A$), this is not large enough to finance the first-best investment.\textsuperscript{8} Hence, the entrepreneur needs external funding from competitive banks ($L_B \geq 0$) and/or suppliers ($L_S \geq 0$) to undertake the project.

Banks and suppliers of each input play different roles. Banks lend cash. The supplier of intangibles provides the input, which is fully paid for in cash. The supplier of tangibles, instead, not only sells the input, but can also act as a financier, lending both inputs and cash.

**Moral hazard.** Unobservability of investment to all parties and of input purchases to parties other than the respective suppliers introduce a problem of moral hazard for the entrepreneur: the funds raised, either in cash or in kind, might not be invested in the venture, but diverted to private uses.\textsuperscript{9} If diversion occurs, creditors get paid only to the extent that project returns are eventually available. However, the supplier can observe whether inputs have been purchased. This advantage together with the lower liquidity of inputs relative to cash imply that the entrepreneur’s moral hazard problem is less severe if funding is raised from the supplier rather than from the bank. In particular, one unit of cash gives the entrepreneur a return $\phi < 1$ if diverted, where $\phi$ can be interpreted as the degree of vulnerability of creditor rights. One unit of the tangible input $q_k$ gives instead a return $\phi \beta_k$ if diverted, where $\beta_k < 1$ denotes the tangible input liquidity. When $\beta_k$ is close to 1, the input can be resold at a price close to its purchase price and transformed into monetary benefits. This applies to standardised products, which can be used by many different customers and thus have a high re-sale value. Conversely, perishable goods, services or inputs tailored to the specific needs of the buyer (differentiated) are less liquid and thus have a low $\beta_k$. Last, one unit of the intangible input $q_L$ gives a zero return if diverted. This implies that it is not possible to extract monetary benefits from workers by assigning them to tasks different from those they were employed for.\textsuperscript{10} In many countries, such practices are indeed prohibited by the employment protection legislation.

**Collateral value.** Inputs are also valuable if repossessed in the event of firm’s default. We

\textsuperscript{7}This normalisation is without loss of generality since we use a partial equilibrium setting.

\textsuperscript{8}We define the first-best investment as the values of $I_k$ and $I_L$ chosen by the entrepreneur when he faces no borrowing constraints.

\textsuperscript{9}The assumption of full unobservability of input purchase to parties other than the direct supplier implies that the bank cannot condition the contract on $q_k$ or on a share of that. This is a useful simplification but is not crucial to obtain our results. We only need that the supplier has some information/monitoring advantage relative to the bank. This can consist in getting more accurate information, or in getting the same information at a lower cost. Both situations are reasonable given the specific nature of the firm-supplier relationship, i.e., the supplier provides the input.

\textsuperscript{10}This assumption is without loss of generality. In Section 3, it will be shown that, even if they had positive degree of liquidity, intangibles would never be purchased for diversion purposes.
assume that only tangible inputs can be pledged to financiers, while intangibles have zero collateral value. Hence, the total value of pledgeable collateral is $C = \beta_k I_k$. However, financiers have different liquidation abilities. We define $\beta_i C$ as the liquidation value extracted by each financier in case of default, with $i = B, S$ referring to the bank or the supplier, respectively. Since the supplier has a better knowledge of the resale market, we assume that she has a better liquidation technology relative to the bank, i.e. $\beta_S > \beta_B$.

Finally, the cost of raising one unit of funds on the market is assumed to be higher for the supplier than for the bank ($r_B < r_S$). This appears realistic for several reasons: first, it is consistent with the role of banks as specialised financial intermediaries. Second, suppliers are likely to be themselves credit constrained and to face a higher cost of funds than banks. Finally, it might be the effect of the supplier having bargaining power due to her cost advantages (information and liquidation) over other financiers.\footnote{In an earlier version of the paper, we show that our analysis can be interpreted as a reduced form of a model in which $r_B = r_S$ and the supplier has bargaining power due to her information and liquidation advantages over the bank. The bargaining power allows the supplier to charge a higher interest rate than the bank.}

**Contracts.** The entrepreneur-bank contract specifies: the credit granted by the bank $L_B$; the entrepreneur’s repayment obligation $R_B^\sigma \geq 0$, which depends on the realised revenues and on the size of the loan; the share of the collateral obtained in case of default $\gamma$.

The contract between the entrepreneur and the supplier of the tangible input specifies: the credit granted by the supplier $L_S$; the input provision $q_k$; the entrepreneur’s repayment obligation $R_S^\sigma \geq 0$, which depends on the realised revenues and on the size of the loan; the share of the collateral obtained in case of default $(1 - \gamma)$. Notice that, unlike the bank, the supplier can condition the contract also on the input purchase $q_k$.

Last, given that the intangible input is fully paid for when it is purchased, the contract between entrepreneur and supplier specifies the purchased amount of the input $q_L$.

We assume that each party is protected by limited liability.

**Time line**

1. Competitive banks and suppliers make contract offers, given entrepreneur’s wealth;

2. the entrepreneur accepts or rejects;

3. conditional on acceptance, the investment and diversion decisions are taken;
4. uncertainty resolves;
5. the payoff realises and repayments are made.

3 Determinants of trade credit

Firms carry out production, which is financed with internal funds, with the cash provided by banks, or with the cash or in-kind resources lent by suppliers of the tangible input. Since banks have a comparative advantage in raising funds \((r_B < r_S)\), entrepreneurs would prefer bank financing to trade credit. However, the moral hazard problem introduces a limit on the amount of funding that can be raised from banks, i.e., the entrepreneur may face borrowing constraints on bank credit. Trade credit has two advantages relative to bank’s financing. First, the supplier is better at liquidating the inputs if repossessed from a defaulting firm. Second, lending inputs rather than cash reduces the scope for diversion due to their lower liquidity. This mitigates the entrepreneur’s moral hazard problem and relaxes the borrowing constraints with banks. Hence, trade credit arises for two motives: a liquidation motive (i.e., to exploit the supplier liquidation technology) and an incentive motive (i.e., to relax financial constraints created by moral hazard problems). In this section, we discuss the conditions under which each of the two motives becomes relevant and the way they interact.

Firms maximise profits, which can be split into two components: the return from production \((EP)\) and the return from diversion \((D)\). The expected return from production is given by:

\[
EP = p \left[ f_G (I_k, I_L) - R^B_G - R^S_G \right] + (1 - p) \left[ f_B (I_k, I_L) - R^B_B - R^S_B \right]
\]

Since output is zero in the bad state, limited liability implies that the repayments to banks and suppliers in this state are both zero \((R^B_B = R^S_B = 0)\).

The return from diversion is equal to:

\[
D = \phi \{ \beta_k (q_k - I_k) \} + [A + L_B + L_S - q_k]
\]

where the term in round brackets denotes the return from tangible input diversion, net of the amount invested in production, and the term in square brackets denotes the return from residual cash diversion

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12 A remark regarding the terminology is warranted here. We will henceforth use the term trade credit to identify the credit, either in cash or in-kind, provided by the supplier. To be rigorous, however, the term trade credit should be used only to define in-kind finance and should not include any cash lending. We find however that in equilibrium the supplier never lends cash but only inputs, which makes our terminology consistent. We will address this issue in Section 4.5.

13 Banks and suppliers can still get a repayment in the bad state by having the right to a share of the scrap value of unused inputs.
(the amount of cash not spent on the input purchase). Notice that an opportunistic entrepreneur only purchases tangibles \( q_k \geq I_k \geq 0 \) and never intangibles for diversion purposes.\(^{14}\) Moreover, the inefficient diversion technology \( \phi < 1 \) implies that partial diversion is never optimal. Thus, either all funds (and inputs) are used for investment \( (D = 0) \), or they are diverted, in which case none of the purchased inputs is invested: \( I_k = 0.\)^{15}

To prevent the entrepreneur from diverting all resources in equilibrium, we require that the return from investment exceeds the maximum return from cash and input diversion, i.e.,:

\[
EP \geq \phi (A + LB) 
\]

\[
EP \geq \phi [\beta_k q_k + A + LB -(q_k - LS)] 
\]

where (2) is the incentive compatibility condition vis-à-vis the bank, which prevents the entrepreneur from diverting internal funds as well as the credit raised from the bank, while (3) is the incentive constraint vis-à-vis the supplier, preventing the entrepreneur from diverting inputs, plus any spare cash left after the input purchase. If conditions (2) and (3) hold, there is no diversion in equilibrium, so that \( D = 0 \) and \( q_k = I_k \).

Notice that if the vulnerability of creditor rights is low enough \( (\phi \) small), even a zero-wealth entrepreneur faces no incentive problems and carries out the optimal investment using only external financing. To avoid this uninteresting case, we introduce the following assumption:

**Assumption 1** : \( \phi > \phi_0.\)^{16}

\(^{14}\)This is obvious since intangibles have zero liquidation value by assumption. However, this result holds for any positive liquidity degree of the intangible. The argument is the following. Being paid in cash, intangible inputs cannot be used to raise credit from suppliers. Moreover, their purchase is unobservable to banks and/or suppliers of tangibles, which implies that neither of these contracts can be conditioned on the amount of intangibles actually purchased. Last, intangibles are less liquid than cash, which implies that the return from diverting them is lower than the return from diverting cash. Hence, an opportunistic entrepreneur would never use cash to buy intangibles and then divert them, i.e. \( q_L = 0 \) and thus \( I_L = 0 \). A similar argument can be used to show that the entrepreneur does purchase tangibles for diversion purposes, even though they are less liquid than cash. This happens because the supplier is willing to extend credit to the firm and the relevant contract can be conditioned on the input transaction taking place. Thus, an opportunistic entrepreneur will purchase the desired \( q_k \) from the supplier and keep the residual resources in cash.

\(^{15}\)To get the corner solution, we use the assumption that diversion of any type of resources implies a social loss, i.e., \( \phi < 1 \). The argument is the following. Suppose the entrepreneur invests in the venture an amount sufficient to repay the loan in full. Diverting the marginal unit gives a return \( \phi \beta_k \). Investing it in production, the firm gets the expected marginal product, which in the first best equals \( r_B [1 - \beta_k \beta_S / r_S] \). If \( \phi < 1 \), the return from diversion is lower than the return from production. Thus, the entrepreneur always prefers to invest this unit, and more so for the inframarginal units. Suppose now that the entrepreneur invests in the venture an amount not sufficient to repay the loan in full. Because output is observable, any return from production will be claimed by creditors and the entrepreneur will get a return only from the residual resources not invested and diverted. It is then better to divert them all.

\(^{16}\)The value of \( \phi_0 \) is defined in Appendix 2.
Banks and suppliers participate to the venture if their expected returns cover at least the opportunity cost of funds, i.e.:

\[ pR^B_G + (1 - p) \gamma \beta B C \geq L_B r_B \]  \hspace{1cm}(4)

\[ pR^S_G + (1 - p) (1 - \gamma) \beta S C \geq L_S r_S \]  \hspace{1cm}(5)

where \( \gamma \) and \((1 - \gamma)\) are the shares of the collateral accruing to the bank and the supplier, respectively. Competition in the banking sector and among suppliers implies that (4) and (5) bind. Notice that, because \( \beta_S > \beta_B \), pledging the collateral to the supplier relaxes her participation constraint more than the bank’s, thus reducing the total repayments due by the entrepreneur in the good state and increasing the total surplus. However, \( r_B < r_S \) implies that the entrepreneur prefers bank credit to trade credit, i.e., \( L_S = 0 \). Giving the supplier the right to the proceeds from liquidation without taking trade credit implies, using constraint (5), that the repayment of the supplier in the good state is negative, \( R^S_G < 0 \). The supplier acts therefore as a liquidator. Being interested in her role as a financier, we do not allow for such contracts in this analysis and require repayments to be non-negative. Moreover, we also impose repayments to be non-decreasing in revenues:\(^{17}\)

\[ R^S_G \geq (1 - \gamma) \beta S C \]  \hspace{1cm}(6)

Solving the binding constraint (5) for \( R^S_G \), the non-decreasing repayments condition (6) implies a lower bound on trade credit equal to the collateral value of the inputs pledged to the supplier:

\[ L_S \geq (1 - \gamma) \frac{\beta_S \beta_k}{r_S} I_k. \]  \hspace{1cm}(7)

However, it is worthwhile for the entrepreneur to get funding from the supplier \((L_S > 0)\) only if the cost of trade credit \( r_S \), discounted for the saving in repayments obtained by pledging the collateral to the supplier, is lower than the cost of bank credit \( r_B \), i.e.:

**Assumption 2** \( r_S \leq r_B \left\{ \frac{\beta_S}{p \beta S + (1 - p) \beta B} \right\} \).

When this condition holds, the higher cost of trade credit is offset by the higher proceeds collected in case of liquidation. Under Assumption 2, we derive the following lemma:

\(^{17}\)This condition is standard in the literature (Innes, 1990). In our setup it is necessary to prevent firm-bank collusion. The argument is the following: when the bad state occurs, the firm might manipulate the books to pretend the good state to have occurred. The unused inputs could then be liquidated by the bank and the proceeds shared with the firm.
Lemma 1  At equilibrium, $\gamma = 0$, i.e., the right to repossess and liquidate the collateral goes to the supplier.

Proof. See Appendix 2. Henceforth, we report proofs of lemmas and propositions in Appendix 2, unless otherwise stated. ■

The last constraint faced by the entrepreneur is that input purchase is no larger than available funds, i.e.:
\[
I_L + I_k \leq A + L_B + L_S \tag{8}
\]

Under Lemma 1, solving the binding constraints (4) and (5) for $R^B_G$ and $R^S_G$ and substituting out in (1), gives the entrepreneur’s problem as:
\[
\max_{L_B, L_S, I_k, I_L} EP = pf_G(I_k, I_L) - L_Br_B - L_Sr_S + (1 - p)\beta_SC \tag{9}
\]

subject to the incentive constraints (2) and (3), the non-decreasing repayments condition (7) and the resource constraint (8).

The liquidation motive. When the liquidation advantage is the only determinant of trade credit use, (6) holds with equality and (7) sets the trade credit demand equal to the collateral’s discounted value to the supplier:
\[
L_{S,LM} = \frac{\beta_S\beta_k}{r_S} I_k. \tag{10}
\]

Each unit of trade credit below this amount costs less than bank credit, since the supplier exploits the higher liquidation revenues accruing in the bad state to decrease the repayment asked in the good state. In particular, using the binding constraints (4) and (5) and condition (6), we derive the price of one unit of trade credit and of bank credit as $r_S$ and $r_B/p$, respectively. Under Assumption 2, $r_S < r_B/p$. An extra unit of trade credit above the level set in (10) costs more than bank credit, since there is no more collateral to pledge. This level is thus the optimal amount of trade credit taken for liquidation motives.

The incentive motive. Besides extracting more value from existing assets, trade credit also allows the entrepreneur to relax financial constraints. Since diverting inputs is less profitable than diverting cash, the supplier is less exposed to borrowers’ opportunistic behaviour relative to banks and may be willing to provide credit even when the bank is not. When this happens, the demand for
trade credit is above the minimum (i.e., (7) is slack) and trade credit is taken for incentive motives. However, suppliers are not willing to meet any entrepreneur’s request, since supplying too many inputs on credit may induce the entrepreneur to divert them all. The maximum trade credit line extended to a rationed entrepreneur is defined by:

$$L_{S,IM_{max}} = (1 - \beta_k) I_k.$$  \hspace{1cm} (11)

obtained when both incentive constraints (2) and (3) bind. Any extra unit of trade credit above this amount can be obtained only if the entrepreneur gives up some liquid resources, i.e. reduces his bank credit exposition.

**Two alternative regimes.** From the above discussion it follows that there is a demand for trade credit that depends only on input characteristics, and a demand for trade credit that depends instead on the financial constraints faced by the firm, and therefore on the entrepreneur’s wealth. Two regimes may then arise, depending on whether the demand for liquidation motives (10) exceeds the maximum credit line extended to a constrained entrepreneur (11). Interestingly, this condition can be redefined exclusively in terms of the parameters of the model as $\frac{\beta_k \beta_S}{r_S} \leq 1 - \beta_k$. If

$$\frac{\beta_k \beta_S}{r_S} < 1 - \beta_k$$  \hspace{1cm} (12)

i.e., the fractional scrap value of the collateral (left hand side) is lower than the share of inputs that cannot be diverted (right hand side), entrepreneurs with different levels of wealth have different demands for trade credit. Wealthy entrepreneurs take trade credit only for liquidation motives, while less wealthy ones use it to relax borrowing constraints. In particular, there exists a level of wealth at which the funding raised from the bank (and the supplier) is so high that the bank has to ration the entrepreneur to prevent opportunistic behaviour. However, at this level, the supplier is still willing to provide credit because of the less severe incentive problem she faces. This credit line can then be used to relax borrowing constraints and keep investment constant. Since the trade credit demand for liquidation motives is negligible when compared to the one for incentive, we define this regime as the one with dominant incentive motive.

In the complementary parameter space, namely when:

$$\frac{\beta_k \beta_S}{r_S} \geq 1 - \beta_k$$  \hspace{1cm} (13)
we are in the regime featuring dominant liquidation motive, in which the amount of trade credit demanded for liquidation motives is very high. In common with the previous regime, sufficiently wealthy entrepreneurs still demand trade credit only for liquidation motives. Unlike the previous regime, though, because the amount of inputs financed on credit is already very high, there exists a level of wealth at which an extra unit of credit, be it in-cash or in-kind, makes it profitable for the entrepreneur to divert both cash and inputs. At this level both the bank and the supplier have to ration the entrepreneur to prevent opportunistic behaviour. This implies that trade credit is never demanded to relax financial constraints, but only to exploit the supplier’s liquidation advantage, for any level of wealth.

Having defined the determinants of trade credit use, the next section is devoted to the presentation of the results.

4 Results

This section presents our results organised in six parts. In Section 4.1, we focus on the trade credit demand of financially unconstrained firms and we isolate the liquidation motive. Section 4.2 considers all firms, identifies two regimes and focuses on how trade credit varies with borrowing constraints across the two regimes. Section 4.3 links the dominance of each regime to observable industry characteristics. In Section 4.4, we describe the contracts between the entrepreneur and the two financiers (the supplier and the bank). In Section 4.5, we discuss the issue of cash lending by suppliers. Finally, Section 4.6 investigates the relation between financing, technology and borrowing constraints.

4.1 The liquidation motive of financially unconstrained firms

In this section, we consider firms with no incentive problems, i.e., facing no borrowing constraints on bank credit. We show that, in spite of the bank’s intermediation advantage, the supplier’s better liquidation technology provides a reason to take trade credit.

**Proposition 1** Firms that are not credit rationed by banks take trade credit to exploit their supplier’s liquidation advantage. The amount of trade credit used equals the collateral value of tangible inputs pledged to the supplier (equation 10).

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18These firms are financially unconstrained. They need bank financing since their initial wealth is not high enough to carry out the project without raising any external funding. However, their wealth is sufficiently high to remove any incentive problem with the bank, so that they have their loan demand fully satisfied without being rationed.
Proposition 1 fills a gap in the literature. As explained in Section 1, earlier theories rationalise the existence of trade credit, but not its use by unconstrained firms, a fact that is documented by empirical studies (Petersen and Rajan (1997) for the U.S. economy, Miwa and Ramseyer (2005) for Japan and Marotta (2001) for the Italian manufacturing sector).

Proposition 1 also states that the use of trade credit is tied to the input collateral value when this is pledged to the supplier. This is because the supplier’s liquidation advantage makes trade credit cheaper than bank loans only up to an amount equal to the collateral value.\(^{19}\) Therefore our liquidation story requires that: i) the input has a positive collateral value; ii) it is worth sufficiently more in the hands of the supplier than in those of the bank in case of default, which by Lemma 1 implies a supplier’s contractual seniority; iii) the bankruptcy law does not alter the contractually agreed claims held by creditors. We will further analyse these points in Section 4.3, which is devoted to the role of input characteristics, and in Section 5, in which we discuss how the design of bankruptcy codes may alter our findings.

Our result builds on the trade-off between suppliers’ liquidation advantage and their higher cost of funds compared to banks.\(^{20}\) While there is evidence that the provision of trade credit is increasing in the liquidation value of the inputs sold by the supplier (Mian and Smith, 1992; Petersen and Rajan, 1997), there is no direct empirical support that suppliers’ liquidation advantage offsets banks’ intermediation advantage.\(^{21}\) Testing this trade-off would require information on the interest rates charged by suppliers on trade credit. Unfortunately, such firm-level data are not available. This explains why the existing empirical literature on the cost of trade credit is ambiguous. Scholars argue

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\(^{19}\)There are other papers arguing that trade credit may be cheap. However, in these papers cheap trade credit has implications which greatly differ from ours. Burkart and Ellingsen (2004), for example, argue that if suppliers are unconstrained in the bank credit market, the trade credit interest rate might be equal to the bank rate. If there is a wedge between the banks’ deposit rates and lending rates, the equilibrium trade credit interest rate may end up strictly below the bank rate. However, if this occurs, trade credit crowds out bank credit, i.e. the firm should be entirely financed by trade credit, which does not seem realistic. In our model this scenario never occurs and the two sources of funding always coexist. In Frank and Maksimovic (2004), trade credit is cheap because of the liquidation advantage, as in our model. However, they do not get any prediction for the trade credit demand by unconstrained firms.

\(^{20}\)The magnitude of the bank’s intermediation advantage (\(r_S/r_B\)) relative to the size of the supplier’s liquidation advantage (\(\beta_S/\beta_B\)) is crucial. If banks and suppliers are equally good in liquidating the assets (\(\beta_S = \beta_B\)), then any positive, although small, intermediation advantage of the bank implies that the liquidation motive never arises. Conversely, if the supplier is better able to liquidate than the bank, this advantage can be large enough to counterbalance the bank intermediation advantage. Assumption 2 explicitly states the necessary conditions for this case to occur.

\(^{21}\)Petersen and Rajan (1997) document that the supplier’s liquidation advantage is one of the determinant of trade credit use. They use the fraction of the firm’s inventory that are not finished goods as a proxy for the supplier’s liquidation advantage. The intuition is that once the customer has transformed the inputs into finished goods, the liquidation advantage is partially lost.
that the most common buyer-seller agreements imply high implicit interest rates (e.g., the canonical 2/10 net 30 would imply an interest rate above 40%). But recent papers suggest that trade credit is not more expensive than bank credit. For example, Miwa and Ramseyer (2005) claim that there is no evidence that sellers use the “extravagant cash discounts” mentioned above. Moreover, they find that firms borrow heavily from their suppliers at implicit rates that track the explicit rates banks would charge them. Similarly, Marotta (2001) documents that trade credit provided by Italian manufacturing firms is, if anything, only slightly more expensive than bank credit. In summary, while evidence and intuition support the conjecture of a supplier liquidation advantage, data limitations do not allow to show compellingly whether this suffices to offset the lower funding costs of banks.

4.2 Trade credit and borrowing constraints

In the previous section we focused on financially unconstrained firms, finding that their trade credit demand is entirely explained by the liquidation motive. In this section, we include into the analysis constrained firms and show that trade credit may emerge for liquidation motives or for incentive reasons. In one regime, where the incentive motive dominates (\((12)\) holds), rich entrepreneurs take trade credit for liquidation motives (LM), while less wealthy entrepreneurs take it for incentive motives (IM). The share of inputs purchased on credit is non-increasing in wealth and higher for entrepreneurs that are credit rationed. In the second regime, where the liquidation motive dominates (\((13)\) holds), all entrepreneurs - independently of their wealth - take trade credit for liquidation reasons and the share of inputs purchased on credit is the same across rationed and non-rationed firms.

Our theoretical results reconcile an apparent contrast between existing theoretical literature and empirical evidence. On the one hand, by arguing that trade credit is taken to mitigate bank credit rationing, the theoretical literature (Biais and Gollier, 1997; Burkart and Ellingsen, 2004) has highlighted a positive relation between reliance on trade credit and borrowing constraints. On the other hand, some empirical literature has shown that reliance on trade credit is virtually unaffected by the degree of credit rationing (Petersen and Rajan, 1997; Marotta, 2001). Our theoretical model accounts for both these cases.

**Proposition 2 - Dominant incentive regime** - When the incentive motive dominates, there exist three critical levels of wealth, \(A_1 < A_2 < A_3\) such that:

(i) for \(A \geq A_3\) entrepreneurs finance the first-best investment \((I_{FB}, I_{FB}^L)\) and take trade credit for
liquidation motives and bank credit as a residual. The share of inputs purchased on credit is equal to the scrap value of tangible inputs \( (\beta_k \beta_s r_s) \);

(ii) for \( A_2 \leq A < A_3 \), entrepreneurs are credit rationed by banks, invest \( I_k(A) \in [I_k^*, I_k^{FB}) \) and \( I_L(A) \in [I_L^*, I_L^{FB}) \) and take trade credit for liquidation motives, with a share \( \frac{\beta_k \beta_s r_s}{r_s} \);

(iii) for \( A_1 \leq A < A_2 \), entrepreneurs invest \( I_k^* < I_k^{FB} \), and \( I_L^* < I_L^{FB} \), and take trade credit for incentive motives. The share of inputs purchased on credit is decreasing in wealth and within the interval \( \left( \frac{\beta_k \beta_s r_s}{r_s}, 1 - \beta_k \right) \);

(iv) for \( A < A_1 \), entrepreneurs invest \( I_k(A) < I_k^* \) and \( I_L(A) < I_L^* \); they are constrained on both credit lines and take trade credit for incentive motives. The share of inputs purchased on credit is constant and equal to the proportion of it that cannot be diverted \( 1 - \beta_k \).

Figure 1 illustrates Proposition 2. The population of entrepreneurs is distributed in four wealth areas with different degrees of credit rationing. For each area, the figure shows the reason behind the demand for trade credit (liquidation versus incentive motives), as well as the share of inputs purchased on account. Sufficiently rich entrepreneurs (\( A \geq A_3 \)) finance the first-best investment by taking a constant amount of trade credit, equal to the discounted value of collateralised assets, and a variable amount of bank credit.\(^{22}\) As wealth approaches \( A_3 \), the amount of trade credit stays constant, while the amount of bank credit increases to compensate for the lack of internal wealth. For \( A < A_3 \), the amount of cash raised is so high that banks have to ration the entrepreneur to prevent opportunistic behaviour. Suppliers are still willing to sell inputs on credit because they face a less severe incentive problem. However, for \( A_2 \leq A < A_3 \), firms do not increase trade credit demand yet, since the cost of an extra unit is still higher than the cost of bank credit. Thus, they are forced to reduce the investment level below the first-best, as well as the absolute level of trade credit and bank finance, but they keep the share of inputs purchased on account constant. Only for wealth below \( A_2 \), when the shadow cost of bank credit exceeds the marginal cost of trade credit, they start demanding trade credit also for incentive motives, i.e., to relax financial constraints and keep the investment constant. Thus, the amount of bank credit stays constant, but both the absolute level of trade credit and the share of tangible inputs purchased on account increase up to their maximum. This is reached at \( A = A_1 \) when also the incentive constraint vis-à-vis the supplier binds. For \( A < A_1 \), the entrepreneur

\(^{22}\)Recall that trade credit is cheaper than bank credit for amounts no higher than the liquidation value of the collateral.
Entrepreneur’s wealth

% inputs purchased on credit

constrained on TC and BC

constrained on BC

unconstrained

TC for IM

TC for IM

TC for LM

TC for LM

Figure 1: The regime where the incentive motive dominates is constrained on both credit lines and is forced to further reduce the investment level. Both trade and bank credit decrease, but the share of inputs purchased on credit stays constant and equal to its maximum \(1 - \beta_k\). In summary, across the wealth areas described in Figure 1, the share of input purchased on account is non decreasing in the degree of credit rationing.

**Proposition 3 - Dominant liquidation regime** - When the liquidation motive dominates, there exists a critical level of wealth, \(\hat{A}_1\), such that:

(i) for \(A \geq \hat{A}_1\) entrepreneurs are not rationed on either credit line. They finance the first-best investment \((\hat{I}_{k}^{FB}, \hat{I}_{L}^{FB})\) taking trade credit for liquidation motives and bank credit as a residual;

(ii) for \(A < \hat{A}_1\), entrepreneurs are rationed both on bank credit and trade credit. They invest \(\hat{I}_{k}(A) < \hat{I}_{k}^{FB}\) and \(\hat{I}_{L}(A) < \hat{I}_{L}^{FB}\) taking trade credit for liquidation motives;

in either case, the share of inputs purchased on credit equals the scrap value of tangible inputs \((\frac{\beta_k \beta_S}{r_S})\).

Figure 2 illustrates Proposition 3 and has the same interpretation as Figure 1. In this case, there are only two wealth areas. For \(A \geq \hat{A}_1\), firms are wealthy enough to finance the first-best investment.
without exhausting their credit lines. They use a constant amount of trade credit, equal to the scrap value of collateral assets \((\beta_S \beta_k / r_S) I_k\), and, as wealth decreases, an increasing amount of bank credit. The funding raised from banks stops when \(A = \hat{A}_1\). At this level of wealth, because the amount of inputs financed on credit is already very high, the total funding obtained is so large that an extra amount of it, be it in cash or in kind, would induce the entrepreneur to divert all resources. Thus, for \(A < \hat{A}_1\), entrepreneurs are forced to reduce both sources of external financing as well as the investment level. In contrast with the previous regime, they keep financing a constant share of input by trade credit equal to \(\beta_S \beta_k / r_S\) for any level of wealth. They have no incentive to alter it, since this would increase the total cost of financing: each unit of trade credit above the scrap value of collateral assets is more expensive than bank loans; similarly, each unit below this amount can be substituted only with more costly bank credit. Thus, in contrast with earlier financial theories, trade credit use is independent of the firm’s financial constraints: both rationed and non-rationed firms purchase the same share of inputs on account, in line with existing evidence. In this second regime, trade credit is never demanded to mitigate borrowing constraints, but only for liquidation motives.

The dominance of one regime over the other depends on input characteristics. We explore thoroughly this link in the next section.
4.3 The role of input characteristics

We now extend the analysis presented in Section 4.2 by discussing the role of input characteristics for the dominance of each one of the two regimes. This extension has clear economic interpretation and provides several testable predictions. According to our analysis (see inequalities (12) and (13)), the dominance of one regime depends on the liquidity of the tangible input \( \beta_k \) and its collateral value when pledged to the supplier \( \beta_S \). In particular, the incentive motive is more likely to arise among firms purchasing illiquid inputs or with low collateral value, which corresponds to the case described in Figure 1. Conversely, the liquidation motive dominates among firms using relatively liquid inputs or with high collateral value. This case corresponds to Figure 2.

Since the two characteristics of the input reflect to some extent industry characteristics, we can use them to classify goods in different categories/industries. One possible classification would be to distinguish services (low liquidity and low collateral value), standardised goods (high liquidity and low collateral value), and differentiated products (low liquidity and high collateral value). Using this classification, our theory provides three testable predictions on how the use of trade credit varies across industries.

**Prediction 1.** Firms buying services make more purchases on account the tighter the credit constraints, while firms buying goods (both standardised and differentiated) finance the same share of their purchases on account independently of their degree of credit rationing. This prediction can be derived by comparing the pattern of trade credit use across wealth areas between Figures 1 and 2.

**Prediction 2.** Firms buying goods (both standardised and differentiated) make more purchases on account than firms buying services. This prediction can be derived by focusing on the right hand sides of Figures 1 and 2, which isolate the use of trade credit by wealthy firms. Since these firms are unconstrained in the use of trade credit, we interpret them as the demand side of the trade credit market. This finding is consistent with the empirical evidence provided by Burkart et al. (2004, p. 20) for firms taking trade credit.

**Prediction 3.** Suppliers of services offer more trade credit than suppliers of standardised goods. This prediction is derived by comparing the left hand sides of Figures 1 and 2, which isolate the maximum share of inputs purchased on account by poor firms. Being constrained on trade credit, these firms are up against the supply side of the trade credit market. Since this prediction compares inputs with the same collateral value but different liquidity degrees, it is useful to represent the maximum
share of inputs purchased on account, namely $L_S/I_k = \max\{1 - \beta_k, \frac{\beta_S r_S}{r_S + \beta_S}\}$, as a function of the input liquidity, $\beta_k$. This relation is represented in Figure 3. The pattern is clearly non-monotonic, with a V-shape, in contrast to Burkart and Ellingsen (2004), who find a pattern always decreasing in the liquidity parameter. In particular, there exists a threshold degree of liquidity, $\hat{\beta} = \frac{r_S}{r_S + \beta_S}$, such that if $\beta_k < \hat{\beta}$, the maximum share of tangible inputs financed by trade credit decreases in $\beta_k$. This situation corresponds to the dashed line in Figure 3. Conversely, if $\beta_k \geq \hat{\beta}$, this share increases with liquidity, which corresponds to the bold line in the same figure. The two patterns capture the two motives underlying the reliance on trade credit by less wealthy firms. When the incentive motive dominates, this relation is negative. The reason is that the supplier’s information advantage becomes more important the more the inputs differ from cash, i.e., the lower their liquidity degree in case of diversion. Conversely, when the liquidation motive is the driver, trade credit use is increasing in input liquidity, since the supplier’s advantage becomes more important the higher the collateral value of the input in case of default. Producers of services are identified by the upper part of the dashed line ($\beta_k \simeq 0$) while producers of standardised goods by the upper part of the bold line ($\beta_k \simeq 1$). Thus, producers of services are willing to finance a larger share of inputs. This prediction is consistent with the empirical evidence documented by Burkart et al. (2004, pp. 15, 17) for firms supplying trade credit.
4.4 Properties of the financial contracts

The reasons underlying the demand for trade credit - liquidation versus incentive motive - also affect the properties of the financial contract between the entrepreneur and the financiers, as described in the following proposition:

**Proposition 4** The supplier gets a flat repayment contract when trade credit is demanded for the liquidation motive and an increasing repayment contract when the incentive motive becomes dominant. The bank always gets a contract with repayments increasing in cash flows.

The proof is straightforward and can be derived from the following discussion. By Lemma 1, the supplier always gets full priority in case of repossession of the collateral. Two cases may then arise. First, trade credit is demanded for liquidation motives ($A \geq A_2$ in Figure 1; any level of wealth in Figure 2): the supplier gets the same return across states, equal to the scrap value of unused inputs. This contract can be seen as secured debt. Second, trade credit is demanded for incentive motives ($A < A_2$ in Figure 1): the value of the unused scrap inputs is not sufficient to repay the supplier for the loan granted. An extra unit of trade credit can be provided only if a higher repayment is promised in the good state. Therefore the supplier gets an increasing repayment contract, with an extra return for any unit of trade credit taken above the collateral value. This contract can be seen as ordinary debt. Last, the bank always gets a contract with repayments increasing in cash flows. The reason is that the bank gets a positive return only in the good state, given that, by Lemma 1, the collateral is always repossessed by the supplier. If compared to supplier’s debt, bank debt can be interpreted as subordinated debt. However, it can also have an equity interpretation assuming that the bank gets an equity stake of the surplus after the supplier has been paid. This second interpretation leaves all of our results unchanged.

4.5 Do suppliers ever lend cash?

Propositions 2 and 3 show that the share of tangible inputs financed by suppliers is always less than one. This means that, despite the information advantage, suppliers extend credit for their products, but they do not lend cash to finance the purchase of other inputs. However, there is some empirical evidence that cash lending does arise in Japan (Uesugi and Yamashiro, 2004). This raises two questions: first, is the information advantage concerning the input sold by the supplier always insufficient to induce
her to lend cash? And second, what is special about the Japanese trade credit market? This section addresses both questions.

In our analysis, the lack of cash lending crucially depends on the assumption that the information advantage concerns only the purchase of the inputs provided by the supplier, in our case tangibles. If we assume that this advantage extends also to the other input (for example both creditors can partially but asymmetrically observe the intangible input purchase), cash lending arises. Denoting by $\delta_B$ and $\delta_S$ the degree of observability of intangibles by bank and supplier respectively, with $\delta_S > \delta_B$, the incentive constraints (2) and (3) are replaced by:

$$\begin{align*}
    EP & \geq \phi (A + L_B - \delta_B I_L) \\
    EP & \geq \phi (\beta_k I_k + A + L_B + (L_S - I_k - \delta_S I_L))
\end{align*}$$

Using the resource constraint (8) and assuming that both incentive constraints bind, we can find the maximum credit line offered by suppliers and banks as follows:\(^{23}\)

$$\begin{align*}
    L_S &= (1 - \beta_k) I_k + (\delta_S - \delta_B) I_L \\
    L_B &= I_k \beta_k + (1 - \delta_S + \delta_B) I_L - A
\end{align*}$$

Thus, we find that the supplier not only provides the inputs and delays repayment for a share equal to $1 - \beta_k$ of their value, but also provides an amount of cash to finance the intangibles equal to $\delta_S - \delta_B$ of their value. This result implies that, in order to have cash lending, the supplier must have an information advantage over the bank also on the intangible input purchase. The bigger this advantage $\delta_S - \delta_B$, the larger the amount of cash lending.

Interestingly, Uesugi and Yamashiro (2004) document that in Japan cash lending is provided by trading companies. These are large integrated firms, dealing with a variety of commodities and carrying out various business transactions, including sometimes all the stages of the production and commercialisation of a good. Thus, Japanese trading companies can supply raw materials to manufacturing firms, but also work as sales agents for the same firms. Commodity transactions are supported by a variety of financial services, from trade credit to long-term and short-term loans, loan guarantees and investment in equities.\(^{24}\) The supplier therefore provides many types of services to

\(^{23}\)Notice that cash lending can arise only when there is an incentive motive behind the demand for trade credit. There is no scope for borrowing cash, if trade credit is used for liquidation motives.

\(^{24}\)Examples include Mitsubishi Companies, Mitsui and Toyota Tsusho Corporation. Similar types of business are rarely seen in the rest of the world, except for Korea and China. See Uesugi and Yamashiro (2004) for a detailed description of their organisational structure.
the same buyer. This organisational structure guarantees a continuous flow of information that allows the supplier to better monitor the behaviour of its customer. In line with our intuition, the Japanese evidence suggests that cash lending arises when the supplier information advantage extends to various aspects of the firm’s activity, and is not confined exclusively to the firm-supplier relationship.

4.6 Input tangibility, financial decisions and creditor protection

One of our main results derived in the previous sections is that suppliers lend inputs but not cash. This result implies that trade credit only finances the purchase of tangible inputs. It follows that when a constrained entrepreneur wants to relax borrowing constraints by using trade credit, he also distorts the input mix towards tangibles. This bias introduces a link between financing and input choices across different levels of wealth and thus across different degrees of borrowing constraints. This section explores this link and investigates the impact of changes in creditor protection. We show that a larger use of trade credit goes together with an input bias towards tangible assets. This bias becomes stronger when creditor vulnerability increases. The intuition is the following: since bank financing is more sensitive to moral hazard, weaker creditor protection increases the relative cost of bank financing and induces rationed entrepreneurs to rely relatively more on trade credit and to shift towards technologies that make more intensive use of tangible inputs. If we assume that countries differ in the degree of creditor protection, our analysis suggests that firms in countries with a weaker protection rely more on trade credit and have a technological bias towards tangibles. This prediction is consistent with two distinct pieces of evidence documenting, on the one hand, a larger use of trade credit in countries with lower degree of creditor protection, among which developing countries (see among others, Rajan and Zingales, 1995; La Porta et al., 1998; Fisman and Love, 2003; Frank and Maksimovic, 2004); on the other hand, firms in developing countries having a higher proportion of fixed assets to total assets and fewer intangible assets than firms in developed countries (e.g., Demirguc-Kunt and Maksimovic, 2001). Our paper provides therefore a theory which is capable of reconciling both of these distinct findings.

We develop this intuition in the next two propositions, which relate asset tangibility, $I_k/I_L$, and trade credit intensity, $L_S/(A + L_B + L_S)$, with the firm wealth, $A$, and with the degree of creditor vulnerability, $\phi$. We restrict our analysis to homogeneous functions, which have the property that the optimal input combination only depends on the input price ratio, in our case $P_k/P_L$ (tangible over
Proposition 5  Both asset tangibility and trade credit intensity are non-increasing in wealth.

Proposition 6  Greater vulnerability of creditor rights increases both asset tangibility and trade credit intensity for any $A < A_1$ and $A_2 \leq A < A_3$; it increases trade credit intensity while it leaves constant asset tangibility for any $A_1 \leq A < A_2$; it has no effect on either of them for any $A \geq A_3$.

The content of the above propositions is represented in Figures 4 and 5, which display trade credit intensity (Figure 4) and input tangibility (Figure 5) for different wealth levels. Firms with $A \geq A_3$ are unconstrained on both credit lines. Thus, both the price ratios between trade and bank credit and between inputs are invariant in wealth. It follows that both the trade credit intensity and the input tangibility stay constant for levels of wealth above $A_3$. When wealth falls below $A_3$, the moral hazard problem towards the bank binds. Reductions in wealth in the interval $A_2 < A < A_3$ increase the shadow cost of bank credit, and therefore decrease the price ratio between the two sources of funding. Firms give up more bank credit than trade credit, thereby increasing trade credit intensity (bold line in the interval $A_2 < A < A_3$ of Figure 5). Such increment affects also the two input prices, but with different magnitudes. It fully translates into a higher price of intangibles, given that they are totally financed by bank credit, and only partially into a higher price of tangibles, given that only the share $(1 - \beta k \beta S / \tau S)$ is financed with bank credit. The input price ratio thus falls for decreasing levels of wealth, thereby inducing entrepreneurs to increase input tangibility (bold line in the interval $A_2 < A < A_3$ of Figure 5). When wealth falls below $A_2$, the shadow cost of bank credit equals the cost of trade credit. In the interval $A_1 < A < A_2$, firms are indifferent between either source of financing. However, while they are constrained by banks, they are still unconstrained by suppliers and can therefore take trade credit at a constant price to compensate the decrease in wealth. Thus, trade credit intensity increases (bold line in the interval $A_1 < A < A_2$ of Figure 4). This extra credit is used to finance the purchase of tangibles, thereby freeing resources to finance intangibles and leaving the input combination unchanged (bold line in the interval $A_1 < A < A_2$ of Figure 5). Finally, when wealth falls below $A_1$, entrepreneurs become financially constrained on both credit lines. The prices of both sources of funding increase, but more so for bank credit than for trade credit, due to their differential exposure to moral hazard. Given that the tangible input is partly financed by trade credit,

25 The two propositions refer to the case in which $(1 - \beta k) > \frac{\beta k \beta S / \tau S}{\tau}$ (dominating incentive motive). However, qualitatively similar results hold also for the complementary case (dominating liquidation motive).
while the intangible is fully financed by bank credit, the input price ratio decreases, thereby increasing input tangibility (bold line in the area $A < A_1$ of Figure 5).

Consider now how trade credit intensity and input tangibility respond to an increase in creditor rights vulnerability (dotted lines in Figures 4 and 5). Notice first that any increase in $\phi$ moves to the right all the threshold levels of $A$, given that all the incentive constraints become binding at higher levels of wealth. Firms with $A \geq \tilde{A}_3$ are unconstrained on both credit lines and neither trade credit intensity nor asset tangibility vary. When wealth decreases ($\tilde{A}_2 \leq A < \tilde{A}_3$), the incentive constraint towards the bank becomes stringent and the shadow cost of bank credit rises. Thus, both the price ratio between bank and trade credit and between inputs increase, inducing entrepreneurs to rely more
on trade credit and to shift towards a technology that relies more on tangible inputs (the dotted lines shift upwards in both graphs). When $\bar{A}_1 \leq A < \bar{A}_2$, the two sources of finance have the same price, but firms are unconstrained by suppliers. They can therefore use trade credit to keep investment and input combination constant (the dotted line does not shift upwards in Figure 5), while increasing trade credit intensity (the dotted line shifts upwards in Figure 4). When $A < \bar{A}_1$, the change in $\phi$ makes the entrepreneur’s moral hazard problem more severe *vis-à-vis* both bank and supplier. The prices of the two sources of finance and of the two input increase, but the increment is larger for bank credit (intangibles) than for trade credit (tangibles), since only the fraction $\beta_k$ of tangibles can be diverted. Thus, both trade credit intensity and asset tangibility increase, as shown by the upward shift of the dotted lines in both figures.

5 The role of the priority rule

The liquidation story discussed in Sections 4.1 and 4.2 presupposes that, in case of bankruptcy, priority should be assigned on efficiency basis, i.e. to the supplier (Lemma 1). However, the legal system may prevent the supplier from seizing particular goods, thereby neutralising the liquidation motive behind the demand for trade credit and thus “contractual seniority.” One way to obtain this outcome is to design debtor- rather than creditor-oriented bankruptcy codes, thereby subordinating all creditor rights, including suppliers’ rights, to the firm’s survival. A second more specific way is to establish, within given bankruptcy codes, priority rules that privilege certain creditors over suppliers. Although a complete and detailed analysis of this topic is beyond the scope of this paper, in this section we discuss these two aspects of bankruptcy codes, to understand how they may alter our results.

Regarding the first aspect, the French bankruptcy law, for instance, has the stated objective of helping distressed firms (Biais and Mariotti, 2003) and favouring their reorganisation, with an *automatic stay* of secured creditors that prevents them from removing their collateral during the reorganisation period. The German bankruptcy law has similar provisions, with a greater role assigned to creditors in the reorganisation decision. These two systems can be seen as debtor-oriented, as opposed to the Anglo-Saxon codes that are traditionally creditor-oriented. In the U.S. bankruptcy law, managers have the right to choose between filing for bankruptcy under Chapter 7 (liquidation procedure) or under Chapter 11 (reorganisation procedure). Liquidation can therefore occur without prior reorganisation. Moreover, according to Franks *et al.* (1996), the majority of U.S. bankruptcies
are actually processed through Chapter 7. In the U.K., there are two bankruptcy regimes: the receivership and the administration order. Under the first, a creditor holding a general secured interest in the firm’s assets, known as floating charge, may appoint a receiver if the firms defaults with the right to sell any assets of the firm to repay the debt owed, except those which are subject to another creditor’s lien. To prevent the liquidation of the firm’s assets, an administration order can be issued by appointing a bankruptcy official with the task of proposing a reorganisation plan to the creditors committee. However, unlike in the U.S., an administration order cannot block a receivership procedure that has already started, unless the floating charge holder consents.

Regarding the more specific issue of the priority rule, it is generally true that trade credit is a junior credit, unless it is secured, in which case the supplier can reclaim in bankruptcy any good that has not been transformed into output. This poses a limitation on the type of goods which can be secured, generally not intermediate goods or services, but rather durable goods. One might therefore expect the demand for trade credit to be driven, among other things, by the possibility for the seller to create a lien on the good sold, and therefore by input characteristics. Notice that this prediction is fully in line with our analysis, where we find that the liquidation motive is stronger whenever the scrap value of the inputs is larger (dominant liquidation motive). In countries like the U.K. and the U.S., trade creditors do have also specific liquidation rights. In the U.K., suppliers can incorporate into the sale contract a Retention of Title clause, that allows them to reclaim all the goods supplied on credit in case of bankruptcy, as long as they are distinguishable from other suppliers’ goods. With such Title they become first in the order of seniority along with the holders of a fixed charge (Franks and Sussman, 2005). In the U.S., even when the firm is not under a bankruptcy procedure, the Uniform Commercial Code gives the seller the right to reclaim the goods sold to an insolvent buyer, within ten days after the receipt.

If the effects of such legal provisions are incorporated into our model, the liquidation motive will play a role depending on both input and bankruptcy codes characteristics. In particular, it should be more important when the good is durable, when it is not transformed into finished goods, and in countries with bankruptcy codes protecting contractual rights in general and supplier’s debt claims in particular. Notice that this discussion acknowledges the role that legal institutions have on the

26 They report that in the Central District of the California Bankruptcy Court there were 57,752 Chapter 7 cases pending as compared with only 6,739 Chapter 11 cases.
27 For example those subject to a fixed charge, i.e. with a security on a specific asset such as heavy machinery.
28 This deadline does not apply if misrepresentation of solvency has been made to the seller in writing. See Garvin (1996) for more details.
demand for trade credit, in line with previous studies. However, our paper identifies a new channel, as well as new testable predictions. In our model, the channel through which legal institutions affect the use of trade credit is the degree of legal protection granted to the supplier and the economic force is the liquidation motive. It follows that a stronger legal protection granted to the supplier induces a larger demand for trade credit, \textit{ceteris paribus}. Conversely, in the related literature, legal institutions affect the reliance on trade credit through the legal protection granted to the banking sector, rather than to the supplier, and the economic driver is the incentive motive. Thus, better legal protection reduces credit rationing and thus induces lower (rather than higher) demand for trade credit.

6 Conclusion

The paper uses an incomplete contract approach with two-input technology and collateralised credit contracts to investigate the determinants of trade credit and its interactions with borrowing constraints, input combination and creditor protection.

We rationalise the use of trade credit by financially unconstrained firms based on the seller’s advantage in liquidating the inputs in the event of default. This complements previous financial theories of trade credit, which explain its use only by rationed firms relying on an incentive motive.

By interacting liquidation and incentive motives, we also show that, as financial constraints become tighter, the share of inputs purchased on credit might stay constant or increase. The dominance of one case over the other depends on input characteristics, like liquidity or collateral value. We then derive new testable predictions on how trade credit varies among firms using differentiated or standardised inputs, rather than services.

In addition, we find that suppliers only lend the inputs they sell and we identify the conditions for them to lend cash. Finally, we show that input and financing decisions are strictly related and both react to changes in creditor protection. More intensive use of trade credit goes together with an input combination biased towards tangible assets. Weaker creditor protection increases both the reliance on trade credit and the input tangibility.

Our analysis could be extended in several directions. An obvious one would be to test empirically the predictions derived in Section 4.6 on the relation among input choices, financing decisions and legal institutions. From a theoretical point of view, it would be interesting to explore the effects of the supplier conditioning the input provision to the purchase of complementary inputs. In our model this assumption would imply that the intangible input is partially observable. This generates cash lending
by the supplier with potential implications on both input choices and financing decisions which are still unexplored.
Appendix 1

Table of notation

\[ y_\sigma = f_\sigma(\cdot, \cdot) : \text{state-contingent output with } \sigma \in \{G, B\} \]
\[ p : \text{probability of state } \sigma = G \]
\[ q_k : \text{purchase of tangible input} \]
\[ q_L : \text{purchase of intangible input} \]
\[ I_k : \text{investment in tangible input} \]
\[ I_L : \text{investment in intangible input} \]
\[ \beta_k : \text{degree of liquidity of the tangible input} \]
\[ C = \beta_k I_k : \text{collateral value of the tangible input} \]
\[ \phi : \text{degree of creditor rights vulnerability} \]
\[ A : \text{entrepreneur’s wealth} \]
\[ L_B : \text{bank credit} \]
\[ L_S : \text{supplier credit} \]
\[ \beta_S : \text{value of one unit of collateral asset to the supplier} \]
\[ \beta_B : \text{value of one unit of collateral asset to the bank} \]
\[ r_B : \text{bank’s cost of raising one unit of funds on the market} \]
\[ r_S : \text{supplier’s cost of raising one unit of funds on the market} \]
\[ R_B^\sigma : \text{state-contingent repayment due to the bank} \]
\[ R_S^\sigma : \text{state-contingent repayment due to the supplier} \]
\[ \gamma, (1 - \gamma) : \text{share of collateral obtained in case of default by bank and supplier, respectively} \]

Appendix 2

To prove our results, let us define the following constrained maximisation problem \( \mathcal{P}_G \):

\[
\max_{L_B, L_S, I_k, I_L, R_B^G, R_S^G, \gamma} EP = p \left[ f_G(I_k, I_L) - R_B^G - R_S^G \right] 
\tag{14}
\]

\[
st EP \geq \max \left\{ \phi(A + L_B), \phi(\beta_k I_k + A + L_B - (I_k - L_S)) \right\} 
\tag{15}
\]

\[
L_S \geq (1 - \beta_k) I_k 
\tag{16}
\]

\[
pR_B^G + (1 - p) \gamma \beta_B C = L_B r_B 
\tag{17}
\]

\[
pR_S^G + (1 - p) (1 - \gamma) \beta_S C = L_S r_S 
\tag{18}
\]

\[
A + L_B + L_S = I_L + I_k 
\tag{19}
\]

\[
R_S^G \geq (1 - \gamma) \beta_S C 
\tag{20}
\]

Notice that the return from diversion in the incentive constraint (15) is expressed as the maximum between cash-only diversion and input-and-cash diversion. Which one of the two is higher depends on how many inputs the entrepreneur buys on credit in equilibrium, i.e. how much trade credit he uses. If \( L_S \leq (1 - \beta_k) I_k \), the return from cash diversion is no less than the return from input-and-cash diversion. The relevant temptation facing the entrepreneur in this case is to divert all cash.
\((A + L_B \geq \beta_k I_k + A + L_B - (I_k - L_S))\). If \(L_S > (1 - \beta_k) I_k\), instead, the return from cash diversion is strictly less than the return from input-and-cash diversion. The entrepreneur may then be tempted to borrow cash from the bank, buy inputs on credit from the supplier and divert both cash and inputs. Whether \(L_S \geq (1 - \beta_k) I_k\) depends on the amount of trade credit that is taken for liquidation motives.

**Definition 1** The threshold level of \(\phi\) below which a zero-wealth firm can carry out the level of investment which is optimal by using both bank credit and trade credit is given by \(\phi = \frac{pf_o(I_k, I_L) - (I_L + I_k)r_B + (r_B - r_S)L_S + (1 - p)(\gamma S_B + (1 - \gamma) S_k) \beta_k I_k}{I_L + I_k - L_S}\), where \(I_k, I_L\) solve the first order conditions of programme \(P_G\), with the incentive constraint (15) slack and \(L_S = \max\left\{ (1 - \beta_k) I_k, \frac{\beta k \beta_s}{r_S} I_k \right\}\).

**Proof of Lemma 1.** Under assumption 2, the entrepreneur takes trade credit to exploit the supplier’s liquidation advantage. Solving (17) and (18) for \(R^B_k\) and \(R^S\), (20) sets the minimum demand for trade credit as \(L_S \geq (1 - \gamma) \frac{\beta k \beta_s}{r_S} I_k\). Assuming this to be binding and using it in \(P_G\), we get:

\[
\max_{I_k, I_L: L_B > \gamma} EP = pf_G(I_k, I_L) - L_B r_B - p (1 - \gamma) \beta_s C + (1 - p) \gamma \beta B C
\]

\[
st EP \geq \max \left\{ \phi (A + L_B), \phi (A + L_B + \frac{(1 - \gamma) \beta_s C}{r_S} - (1 - \beta_k) I_k) \right\}
\]

\[
A + L_B + \frac{(1 - \gamma) \beta_s C}{r_S} = I_L + I_k
\]

Solving (23) for \(L_B\), we define programme \(P_F\):

\[
\max_{I_k, I_L, \gamma} EP_F = p [f_G(I_k, I_L) - (1 - \gamma) \beta_s C] - \left( I_L + I_k - A - \frac{1 - \gamma}{r_S} \beta_s C \right) r_B + (1 - p) \gamma \beta B C
\]

\[
st EP_F \geq \max \left\{ \phi (I_L + I_k - \frac{1 - \gamma}{r_S} \beta_s C), \phi (I_L + \beta_k I_k) \right\}
\]

Defining \(\lambda_1\) as the multiplier of constraint (25), the Lagrangean is:

\[
\Lambda_F = EP_F + \lambda_1 \left[ EP_F - \max \left\{ \phi (I_L + I_k - \frac{1 - \gamma}{r_S} \beta_s C), \phi (\beta_k I_k + I_L) \right\} \right]
\]

Differentiating \(\Lambda_F\) wrt \(\gamma\):

\[
\frac{\partial \Lambda_F}{\partial \gamma} = \left( p (\beta S - \beta B) - \frac{1 - \gamma}{r_S} (r_B \beta S - r_S \beta B) \right) (1 + \lambda_1) - \lambda_1 \max \left\{ \frac{\beta \beta_s}{r_S} \phi, 0 \right\} \leq 0
\]

Under Assumption 2, \(\frac{\partial \Lambda_F}{\partial \gamma} \leq 0\), which implies that \(\gamma = 0\) and proves the lemma.\(^{29}\)

**Proof of Proposition 1.** The proof is straightforward, using Lemma 1. Suppose the entrepreneur is sufficiently rich to finance the first-best investment with his own wealth and with the funding provided by banks and suppliers, i.e. wealth exceeds the threshold level at which the incentive constraint (22) either \(\text{vis-à-vis}\) the bank (\(A_3\) in Fig. 1), or \(\text{vis-à-vis}\) the supplier (\(A_1\) in Fig. 2) is binding. Because above this threshold trade credit is taken only to exploit the supplier’s liquidation advantage, (20) is binding. Using \(\gamma = 0\) and the binding supplier’s participation constraint (18), the demand for trade credit is \(L_S = \frac{\beta k \beta_s}{r_S} I_k\), as defined in (10), which proves the proposition. \(\blacksquare\)

\(^{29}\)When \(\frac{\partial \Lambda_F}{\partial \gamma} = 0\), \(\gamma \in [0,1]\). We take it to be zero.
Proof of Proposition 2. (Dominant incentive regime) When $\frac{\beta_S \beta_k}{r_S} < (1 - \beta_k)$, $L_S \in \left[\frac{\beta_k \beta_S}{r_S} I_k, (1 - \beta_k) I_k\right]$ and the relevant incentive constraint in (15) is the one vis-à-vis the bank.

The proposition is proved in steps: we first prove that a) $I_i(A_{A_2 < A_1}) < I_i^* < I_i(A_{A_2 \leq A < A_3}) < I_i^{FB}$, $i = k, L$; then that b) the critical levels $A_1, A_2$ and $A_3$, exist and are unique. To establish part a), we first focus on $A \geq A_2$, where the entrepreneur demands trade credit only for liquidation motives, and then on $A < A_2$, where the entrepreneur demands trade credit also for incentive motives.

$A \geq A_2$.

The entrepreneur takes trade credit only to exploit the supplier’s liquidation advantage and $L_S = \frac{\beta_k \beta_S}{r_s} I_k < (1 - \beta_k) I_k$. The problem is to maximise programme $P_F$, where the relevant incentive constraint in (25) is the one vis-à-vis the bank. Using Lemma 1, the FOC’s are:

\[
\frac{\partial \Delta_F}{\partial I_k} = p \frac{\partial f_G}{\partial I_k} - r_B - \beta_k \beta_S \left(p - \frac{r_B}{r_S}\right) = \frac{\lambda_1}{1 + \lambda_1} \phi \left(1 - \frac{\beta_S \beta_k}{r_S}\right) \tag{28}
\]
\[
\frac{\partial \Delta_F}{\partial I_L} = p \frac{\partial f_G}{\partial I_L} - r_B = \frac{\lambda_1}{1 + \lambda_1} \phi \tag{29}
\]
\[
\frac{\partial \Delta_F}{\partial I_L} = EP_F \geq \phi \left[I_L + \left(1 - \frac{\beta_S \beta_k}{r_S}\right) I_k\right] \tag{30}
\]

Conditions (28) and (29) can also be written as:

\[
\frac{r_S}{r_S - \beta_S \beta_k} \left(\frac{\partial f_G}{\partial I_k} - \beta_k \beta_S\right) = \frac{\partial f_G}{\partial I_L} \tag{31}
\]

Within $A \geq A_2$, we further distinguish two wealth areas: $A \geq A_3$ and $A_2 \leq A < A_3$.

$A \geq A_3$ : In this case the incentive constraint (25) is slack and the firm invests $I_k^{FB}, I_L^{FB}$ solving (28) and (29) with $\lambda_1 = 0$. The optimal financial contract has the following properties:

\[
R_G^S = \beta_S \beta_k I_k^{FB}
\]
\[
L_S = \frac{\beta_k \beta_S}{r_S} I_k^{FB}
\]
\[
L_B = I_L^{FB} + \left(1 - \frac{\beta_S \beta_k}{r_S}\right) I_k^{FB} - A
\]
\[
R_G^B = \frac{r_B}{p} \left(I_L^{FB} + \left(1 - \frac{\beta_S \beta_k}{r_S}\right) I_k^{FB} - A\right)
\]

$A_2 \leq A < A_3$ : The incentive constraint (25) is binding and the firm invests $I_i(A) \in [I_i^*, I_i^{FB})$, where $I_i(A), i = k, L$, solve (28) and (29) with $\lambda_1 > 0$.

To prove that $I_i^* < I_i^{FB}$, consider the FOC’s (28) and (29). Relative to the first-best ($\lambda_1 = 0$), there is now an increase in the cost of both factors. In order to derive the implications of such rise on the levels of $I_i$, we totally differentiate (28) and (29), and get:

\[
\begin{bmatrix}
\frac{\partial^2 f(\cdot)}{\partial I_k^2} & \frac{\partial^2 f(\cdot)}{\partial I_k \partial I_L} \\
\frac{\partial^2 f(\cdot)}{\partial I_L \partial I_k} & \frac{\partial^2 f(\cdot)}{\partial I_L^2}
\end{bmatrix}
\begin{bmatrix}
dI_k \\
dI_L
\end{bmatrix}
= \begin{bmatrix}
dP_k \\
dP_L
\end{bmatrix} \tag{M1}
\]

where $dP_k > 0, dP_L > 0$ are the changes in the cost of factors induced by a change in one of their determinants. Inverting (M1), we can solve for the vector of unknowns:

\[
\begin{bmatrix}
dI_k \\
dI_L
\end{bmatrix}
= \frac{1}{h}
\begin{bmatrix}
\frac{\partial^2 f(\cdot)}{\partial I_L^2} & -\frac{\partial^2 f(\cdot)}{\partial I_L \partial I_k} \\
-\frac{\partial^2 f(\cdot)}{\partial I_L \partial I_k} & \frac{\partial^2 f(\cdot)}{\partial I_k^2}
\end{bmatrix}
\begin{bmatrix}
dP_k \\
dP_L
\end{bmatrix} \tag{M2}
\]

32
where $H$ is the determinant of the Hessian, which is positive assuming the Hessian to be negative semidefinite. Thus, if factors are substitutes, i.e. $\frac{\partial^2 f(\cdot)}{\partial I_k \partial I_L} > 0$, then

$$
dI_k = \frac{1}{\mathcal{P}} \left( \frac{\partial^2 f(\cdot)}{\partial I_k^2} dP_k - \frac{\partial^2 f(\cdot)}{\partial I_L \partial I_k} dP_L \right) < 0
$$

$$
dI_L = \frac{1}{\mathcal{P}} \left( \frac{\partial^2 f(\cdot)}{\partial I_k \partial I_L} dP_k + \frac{\partial^2 f(\cdot)}{\partial I_L^2} dP_L \right) < 0
$$

which implies that both factors are under-invested.

The optimal financial contract has the following properties:

$$
R^G_{C} = \beta_S \beta_k I_k (A),
$$

$$
L_S = \frac{1}{\tau_S} \beta_S \beta_k I_k (A),
$$

$$
L_B = I_L (A) + \frac{1}{\tau_S} (r_S - \beta_S \beta_k) I_k (A) - A,
$$

$$
R^B_{C} = \frac{r_p}{\mathcal{P}} \left( I_L (A) + \frac{1}{\tau_S} (r_S - \beta_S \beta_k) I_k (A) - A \right)
$$

where $I_k (A), I_L (A)$ satisfy (28) and (29) with $\lambda_1 > 0$.

$A < A_2$.

The entrepreneur is still constrained on bank credit, but, unlike the case in which $A_2 \leq A < A_3$, the shadow price of bank credit is so high that he finds it worthwhile to take trade credit not only for liquidation, but also for incentive motives. Thus

$$
L_S > \frac{\beta_k \beta_S}{\tau_S} I_k. \tag{31}
$$

However, to persuade the supplier to increase financing, the entrepreneur has to offer her a contract with repayments increasing in cash flows. Thus, the non-decreasing repayments condition (20) is slack. The optimal contract solves programme $P_G$ subject to the binding incentive constraint (15) vis-à-vis the bank and to constraint (16) as $L_S \leq (1 - \beta_k) I_k$. Solving the resource constraint (19) for $L_S$, programme $P_G$ can be written as:

$$
\max_{I_k, I_L, L_B} EP_I = pf_G (I_k, I_L) - L_B r_B - (I_k + I_L - A - L_B) r_S + (1 - p) \beta_S \beta_k I_k \tag{32}
$$

s.t. $EP_I = \phi (A + L_B) \tag{33}$

$$
L_B \geq I_L + \beta_k I_k - A \tag{34}
$$

which, using binding (33), becomes:

$$
\max_{I_k, I_L} EP_{GI} = pf_G (I_k, I_L) - (I_L - A) r_B - [(1 - \beta_k) r_S + \beta_k (r_B - (1 - p) \beta_k)] I_k \tag{35}
$$

s.t. $EP_{GI} \geq \phi \{\beta_k I_k + I_L\}$

where (35) is the global incentive constraint. Setting up the Lagrangean $\Lambda_G = EP_{GI} + \lambda_2 [EP_{GI} - \phi (\beta_k I_k + I_L)]$, the FOC’s are:

$$
\frac{\partial \Lambda_G}{\partial I_k} : pf_G + \beta_S \beta_k (1 - p) - r_S (1 - \beta_k) - \beta_k \left( r_B + \phi \frac{\lambda_2}{1 + \lambda_2} \right) = 0 \tag{36}
$$

$$
\frac{\partial \Lambda_G}{\partial I_L} : pf_G - r_B - \frac{\lambda_2}{1 + \lambda_2} \phi = 0 \tag{37}
$$

$$
\frac{\partial \Lambda_G}{\partial \lambda_2} : EP_{GI} - \phi (\beta_k I_k + I_L) \geq 0 \tag{38}
$$

---

30 Notice that this result holds also for the case in which factors are complements, provided the Hessian has a dominant diagonal.

31 This is feasible since the amount of trade credit taken for liquidation does not exhaust the maximum credit line offered by the supplier to a rationed entrepreneur (11) (recall that we are in the case in which $1 - \beta_k > \frac{\beta_k \beta_S}{\tau_S}$).
where \( \lambda_2 \) is the multiplier of the global incentive constraint. Conditions (36) and (37) can also be written as:

\[
p p^\partial f_G / \partial t_k + \beta_S \beta_k (1 - p) - p \beta_k p^\partial f_G / \partial t_L = r_S (1 - \beta_k).
\]

(39)

Within \( A < A_2 \), we can further distinguish between two wealth areas: \( A_1 \leq A < A_2 \) and \( A < A_1 \).

\( A_1 \leq A < A_2 \): The incentive constraint (35) is slack \((\lambda_2 = 0)\). This implies that the entrepreneur can keep investing \( I^*_L, I^*_L \) even for decreasing levels of wealth until (35) becomes binding. The properties of the optimal contract are defined by:

\[
L_S \in \left( \frac{\beta_S \beta_k}{r_S} I^*_k, (1 - \beta_k) I^*_k \right)
\]

\[
L_B \in \left( I^*_L + \beta_k I^*_k - A, I^*_L + I^*_k \left( 1 - \frac{\beta_S \beta_k}{r_S} \right) - A \right)
\]

\[
R^S_G \in \left( \left( \frac{\beta_S \beta_k}{r_S} - (1 - p) \beta_S \beta_k \right) I^*_k \left( 1 - \frac{\beta_S \beta_k}{r_S} \right), \left( (1 - \beta_k) r_S - (1 - p) \beta_S \beta_k \right) I^*_k \left( 1 - \frac{\beta_S \beta_k}{r_S} \right) - A \right)
\]

\[
R^B_G \in [\frac{r_B}{p} (I^*_L + \beta_k I^*_k - A), \frac{r_B}{p} I^*_L + I^*_k \left( 1 - \frac{\beta_S \beta_k}{r_S} \right) - A]
\]

where \( I^*_k, I^*_L \) solve (36) and (37) with \( \lambda_2 = 0 \).

\( A < A_1 \): The incentive constraint vis-à-vis the supplier becomes binding \((\lambda_2 > 0)\) and (36) and (37) imply that \( I_k (A) < I^*_k \), and \( I_L (A) < I^*_L \).

The contract has the following properties:

\[
L_S = (1 - \beta_k) I_k (A)
\]

\[
L_B = I_L + \beta_k I_k (A) - A
\]

\[
R^S_G = \frac{1}{p} \left( (1 - \beta_k) r_S - (1 - p) \beta_S \beta_k \right) I_k (A)
\]

\[
R^B_G = \frac{1}{p} (I_L (A) + \beta_k I_k (A) - A) r_B
\]

where \( I_k (A), I_L (A) \) solve (36) and (37) with (35) binding. ■

Part (b) is proved using the following lemma:

**Lemma 2** For any \( r_B + \phi > r_S \) and \( 1 - \beta_k > \frac{\beta_S \beta_k}{r_S} \), there exists a triple of threshold values \( A_i (\phi, \beta_k, \beta_S) \), \( i = 1, 2, 3 \), such that:

1. \( p f_G (I^*_k, I^*_L) - \bar{L} B r_B - p \beta_S \beta_k I^*_k - \phi (A + \bar{L}) = 0 \) for \( A = A_3 (\phi, \beta_k, \beta_S) \);
2. \( p f_G (I^*_k, I^*_L) - \bar{L} B r_B - p \beta_S \beta_k I^*_k - \phi (A + \bar{L}) = 0 \) for \( A = A_2 (\phi, \beta_k, \beta_S) \);
3. \( p f_G (I^*_k, I^*_L) - \bar{L} B r_B - \bar{L} S r_S + (1 - p) \beta_S \beta_k I^*_k - \phi (A + \bar{L}) = 0 \) and \( \bar{L} S = (1 - \beta_k) I^*_k \) for \( A = A_1 (\phi, \beta_k, \beta_S) \);
4. \( A_3 > A_2 > A_1 > 0 \).

**Proof.**

\[
32 \text{ Notice that while the level of the two inputs is constant in the above interval, the repayments due to bank and supplier in the two states vary with wealth.}
\]

\[
33 \text{The proof of this result is analogous to the one obtained for the case in which } A_2 \leq A < A_3 \text{ and thus omitted.}
\]
1. : The threshold $A_3 (\phi, \beta_k, \beta_S)$ is the minimum wealth that allows the entrepreneur to invest $I^*_k, I^*_L$ fully exploiting the bank credit line and taking trade credit only for liquidation motives.\(^{34}\) Thus $A_3$ must satisfy:

$$A_3 = \frac{1}{r_B} \left\{ (\phi + r_B) \left( I^*_L + I^*_L - \frac{\beta_S \beta_k}{r_S} I^*_k \right) - p \left[ f_G \left( I^*_k, I^*_L \right) - \beta_S \beta_k I^*_k \right] \right\} \quad (40)$$

To prove that this threshold exists and is unique we need to show that: (1a) $0 + \bar{L}_B + L_{S, LM} < I^*_k + I^*_L$, which follows from Assumption 1 ($\phi > \bar{\phi}$); (1b) $\bar{L}_B$ is continuously increasing in $A$. To establish part (1b), it is useful to define the following functions, obtained by taking the derivatives of constraint (25) wrt $I_k$ and $I_L$ respectively:

$$h_{k1} = p \frac{\partial f_G}{\partial I_k} - (r_B + \phi) \left( 1 - \frac{\beta_S \beta_k}{r_S} \right) - p \beta_S \beta_k \quad (41)$$

$$h_{L1} = p \frac{\partial f_G}{\partial I_L} - r_B - \phi \quad (42)$$

Constraint (25) is only binding if $h_{k1}, h_{L1} < 0$, otherwise $I_k$ and $I_L$ could be further increased without violating the constraint.

Using (8) and (10), we deduce that $I_k = \frac{A + \bar{L}_B - I^*_k}{(r_S - \beta_S \beta_k) r_S}$. The maximum bank credit line $\bar{L}_B$, given by the binding constraint (22),\(^{35}\) can therefore be written as a function of $I_L$, $\bar{L}_B$ and $A$:

$$p f_G \left( \left( \frac{A + \bar{L}_B - I^*_k}{(r_S - \beta_S \beta_k) r_S}, I_L \right) - \bar{L}_B r_B - p \beta_S \beta_k r_S \left( \frac{A + \bar{L}_B - I^*_k}{(r_S - \beta_S \beta_k)} \right) - \phi \left( A + \bar{L}_B \right) = 0.\right.$$  

Totally differentiating:

$$\left\{ \frac{p r_S}{(r_S - \beta_S \beta_k)} \frac{\partial f_G (\cdot)}{\partial I_k} - r_B - \frac{\partial f_G (\cdot)}{\partial I_k} - \frac{p \beta_S \beta_k r_S}{(r_S - \beta_S \beta_k)} - \phi \right\} dI_L + \left\{ \frac{p r_S}{(r_S - \beta_S \beta_k)} \frac{\partial f_G (\cdot)}{\partial I_k} - r_B - \frac{\partial f_G (\cdot)}{\partial I_k} - \frac{p \beta_S \beta_k r_S}{(r_S - \beta_S \beta_k)} - \phi \right\} dA \right\} = 0$$

and noting that the multiplier of $dI_L$ is null by (31), we can solve for $\frac{dL_B}{dA} = \frac{p}{(r_S - \beta_S \beta_k) \frac{\partial f_G (\cdot)}{\partial I_k} - r_B - \frac{\partial f_G (\cdot)}{\partial I_k} - \frac{p \beta_S \beta_k r_S}{(r_S - \beta_S \beta_k)} - \phi}$.

The denominator is negative whenever constraint (25) binds, i.e. when $h_{k1} < 0$ (otherwise it would be possible to raise the credit limit $\bar{L}_B$, and thus raise either $I_k$ or $I_L$, without violating it). The sign of the numerator can be inferred by rearranging condition (28) as follows:

$$\frac{p r_S}{(r_S - \beta_S \beta_k)} \frac{\partial f_G (\cdot)}{\partial I_k} - \frac{\partial f_G (\cdot)}{\partial I_k} - \frac{p \beta_S \beta_k r_S}{(r_S - \beta_S \beta_k)} - \phi = r_B - \frac{1}{1 + h_{k1}} \bar{\phi}.\right.$$  

Because the right hand side is positive ($\frac{1}{1 + h_{k1}} < 1$), the numerator of $\frac{dL_B}{dA}$ is also positive and the whole expression is positive.

2. : The threshold $A_2 (\phi, \beta_k, \beta_S)$ is the minimum wealth that allows the entrepreneur to invest $I^*_k < I^*_k + I^*_L < I^*_L$ fully exploiting the bank credit line and taking trade credit still for liquidation motives. The level $A_2$ must satisfy:

$$A_2 = \frac{1}{r_B} \left\{ (\phi + r_B) \left( I^*_L + I^*_k - \frac{\beta_S \beta_k}{r_S} I^*_k \right) - p \left[ f_G \left( I^*_k, I^*_L \right) - \beta_S \beta_k I^*_k \right] \right\} \quad (43)$$

The proof of existence and uniqueness of $A_2$ is analogous to the proof of point 1 and is omitted.

\(^{34}\)This amounts to say that $A_3 + \bar{L}_B + L_{S, LM} = A_3 + \bar{L}_B + \frac{\beta_S \beta_k}{r_S} I^*_k = I^*_k + I^*_L$.

\(^{35}\)Recall that we are in the case in which the relevant incentive constraint is the one vis-à-vis the bank.

\(^{36}\)It can be deduced by rearranging (41).
The incentive constraint (33) can therefore be written as a function of $I$. Totally differentiating, we get:

$$
dI = \beta_k r_B r_S - (1 - \beta_k) r_S - (1 - p) \beta_S \beta_k I^*_k. \tag{45}
$$

To prove that $A_1$ exists and is unique we need to show that: (3a) at zero wealth the amount of funding raised by the bank and the supplier is strictly less than the second-best investment, i.e. $0 + \bar{L}_B + \bar{L}_S = I^*_k + I^*_L$; (3b) $\bar{L}_B$ and $\bar{L}_S$ are continuously increasing in $A$. Part (3a) follows from Assumption 1 ($\phi > \bar{\phi}$). To establish part (3b) it is helpful to define the following functions, obtained taking the derivative of (35) wrt $I_k$ and $I_L$:

$$
h_{k2} = p \frac{\partial f_G}{\partial I_k} - \beta_k r_B - (1 - \beta_k) r_S + (1 - p) \beta_S \beta_k - \phi \beta_k \tag{46}
$$

$$
h_{L2} = p \frac{\partial f_G}{\partial I_L} - (r_B + \phi). \tag{47}
$$

Constraint (35) is only binding if $h_{k2}, h_{L2} < 0$, otherwise $I_k$ and $I_L$ could be further increased without violating it.

We first prove that $\frac{dI_k}{dA} > 0$. Using (8) and (11), it follows that $I_k = \frac{A + L_B - L_L}{\beta_k}$. Substituting out in the incentive constraint (33), this can be written as a function of $I_L$, $\bar{L}_B$ and $A$:

$$
pf_G \left( \frac{A + L_B - L_L}{\beta_k}, I_L \right) - \bar{L}_B r_B - ((1 - \beta_k) r_S - (1 - p) \beta_S \beta_k) \left( \frac{A + L_B - L_L}{\beta_k} \right) = \phi (A + \bar{L}_B) \tag{48}
$$

Totally differentiating, we obtain the following:

$$
\frac{1}{\beta_k} \left( p \frac{\partial f_G}{\partial I_k} - \beta_k (r_B + \phi) - (1 - \beta_k) r_S + (1 - p) \beta_S \beta_k \right) \left( dI_B + dA \right) + r_B dA + \\
\frac{1}{\beta_k} \left( p \frac{\partial f_G}{\partial I_L} + \frac{\partial f_G}{\partial I_k} + \beta_k ((1 - \beta_k) r_S - (1 - p) \beta_S \beta_k) \right) dI_L = 0.
$$

Using (39), the multiplier of $dI_L$ is zero, while the multiplier of $dI_B$ is $h_{k2}$. Solving for $\frac{dI_k}{dA}$ and rearranging, we get

$$
\frac{dI_k}{dA} = - \left( \frac{p \frac{\partial f_G}{\partial I_k} - r_S (1 - \beta_k) + (1 - p) \beta_S \beta_k - \beta_k \phi)}{h_{k2}} \right), \tag{49}
$$

whose sign depends on the sign of the numerator, given that the denominator is negative. Using the FOC on $I_k$, (36), and $r_B > \phi$, we deduce that the sign of the numerator is always positive, whence $\frac{dI_k}{dA} > 0$.

To complete the proof we need to show that $\frac{dI_L}{dA} > 0$. To prove this, we use the same procedure used to show that $\frac{dI_k}{dA} > 0$. Using (8) and (11), it follows that $I_k = \frac{L_S}{1 - \beta_k}$ and $L_B = \frac{\beta_k}{1 - \beta_k} L_S - A + I_L$. The incentive constraint (33) can therefore be written as a function of $I_L$, $\bar{L}_S$ and $A$:

$$
pf_G \left( \frac{L_S}{1 - \beta_k}, I_L \right) - \left( \frac{\beta_k}{1 - \beta_k} (r_B + \phi) + r_S - (1 - p) \beta_S \beta_k \right) \bar{L}_S + A r_B - (r_B + \phi) I_L = 0. \tag{50}
$$

Totally differentiating, we get:

$$
\frac{1}{1 - \beta_k} \left( p \frac{\partial f_G}{\partial I_k} - \beta_k [r_B + \phi - (1 - p) \beta_S] - r_S (1 - \beta_k) \right) d\bar{L}_S + \left( p \frac{\partial f_G}{\partial I_L} - (r_B + \phi) \right) dI_L + r_B dA = 0
$$

which, using $h_{k2}$ and $h_{L2}$, we write as:

$$
\frac{1}{1 - \beta_k} h_{k2} d\bar{L}_S + h_{L2} dI_L + r_B dA = 0 \tag{51}
$$
Totally differentiating (39), we get: 
\[ \frac{dI_L}{dA} \left( p \left( f_{kk} - \beta_k f_{Lk} \right) dL_S + p \left( f_{kL} - \beta_k f_{LL} \right) dI_L \right) = 0. \]

Solving for 
\[ dI_L = -\frac{1}{1-\beta_k} \left( f_{kk} - \beta_k f_{Lk} \right) \left( f_{kL} - \beta_k f_{LL} \right)^{-1} dL_S, \]

and substituting out in (49), we can solve for 
\[ \frac{dL_S}{dA} = -r_B (1 - \beta_k) \left( h_{k2} - \frac{f_{kk} - \beta_k f_{Lk}}{f_{kL} - \beta_k f_{LL}} h_{L2} \right)^{-1} > 0. \]

4. \( A_3 > A_2 > A_1 > 0. \)

To prove that \( A_3 > A_2, \) we have to confront the levels of wealth obtained from the binding incentive constraint (25) when \( I_i = I_i^{FB} \) and \( I_i = I_i^*, \) \( i = k, L, \) respectively. This amounts to calculate the effect of a change in \( I_k \) or \( I_L \) on \( A \) leaving the incentive constraint unaltered. Totally differentiating the incentive constraint, we get:
\[
p \left( \frac{\partial f_G}{\partial I_k} dI_k + \frac{\partial f_G}{\partial I_L} dI_L \right) - (r_B + \phi) \left( 1 - \frac{\beta_k \beta_S}{r_S} \right) \left( dI_k + dI_L \right) - p \beta_S \beta_k dI_k + r_B dA = 0 \]

whence 
\[
\frac{dA}{dI_k} = -\frac{1}{r_B} \left\{ p \frac{\partial f_G(\cdot)}{\partial I_k} \right\} - \left( r_B + \phi \right) \left( 1 - \frac{\beta_k \beta_S}{r_S} \right) + p \beta_S \beta_k \}
\]

and 
\[
\frac{dA}{dI_L} = -\frac{1}{r_B} \left( p \frac{\partial f_G(\cdot)}{\partial I_L} - r_B - \phi \right). \]

Whenever the incentive constraint binds, the terms in brackets, \( h_{k1}, h_{L1} \) are negative, which implies that 
\[
\frac{dA}{dI_k}, \frac{dA}{dI_L} > 0. \]

Thus, as \( I_k, I_L \) decrease, \( A \) decreases, which proves that \( A_3 > A_2. \)

To prove that \( A_2 > A_1, \) we compare (25) and (35). Since within this wealth area the level of investment is unchanged and equal to \( I_k^*, I_L^* \), we only need to compare parameters. This leads to 
\[
A_2 - A_1 = \frac{1}{r_B} \left( 1 - \beta_k - \frac{\beta_S \beta_k}{r_S} \right) (r_B + \phi - r_S) I_k^* > 0. \]

Hence, \( A_2 > A_1. \)

Finally, \( A_1 > 0 \) follows from Assumption 1 \( (\phi > \bar{\phi}). \)

**Proof of Proposition 3.** (Dominant liquidation regime) When \( \frac{\beta_S \beta_k}{r_S} \geq (1 - \beta_k), \)
\( L_S = \frac{\beta_k \beta_S}{r_S} I_k \) and the relevant incentive constraint in (15) is the one \( \text{vis-à-vis} \) the supplier.

The line of the proof is similar to that followed in the proof of Proposition 2. Given that (20) is binding, the maximisation problem for any level of wealth is the one given by programme \( P_F. \) Setting up the Lagrangean (26) with \( \gamma = 0 \) gives the following FOC’s:

\[
\frac{\partial L}{\partial I_k} = p \frac{\partial f_G}{\partial I_k} - r_B - \beta_k \beta_S \left( p - \frac{r_B}{r_S} \right) = \frac{\lambda_1}{1 + \lambda_1} \beta \]

\[
\frac{\partial L}{\partial I_L} = p \frac{\partial f_G}{\partial I_L} - r_B = \frac{\lambda_1}{1 + \lambda_1} \beta \]

\[
E \frac{\partial F}{\partial I_k} \geq \beta (I_L + \beta_k I_k) \]

where (50) and (51) can also be written as:
\[
\frac{1}{\beta_k} \left\{ p \frac{\partial f_G}{\partial I_k} - r_B - \beta_k \left( p \beta_S - \frac{r_B}{r_S} \beta_S \right) \right\} = p \frac{\partial f_G}{\partial I_L} - r_B \]

We can distinguish between two cases, according to whether \( A \geq \hat{A}_1 \) or \( A < \hat{A}_1. \)

\( \hat{A}_1 \): The incentive constraint (25) is slack and \( \hat{I}_k^{FB}, \hat{I}_L^{FB} \) solve (50) and (51) with \( \lambda_1 = 0. \) The optimal financial contract has the following properties:

\[
R_G^S = \beta_S \beta_k \hat{I}_k^{FB}, \]

\[
L_S = \frac{1}{r_S} \beta_S \beta_k \hat{I}_k^{FB}, \]

\[
L_B = \hat{I}_L^{FB} + \frac{1}{r_S} (r_S - \beta_S \beta_k) \hat{I}_k^{FB} - A, \]

\[
R_G^B = \frac{r_B}{p} \left[ \hat{I}_L^{FB} + \frac{1}{r_S} (r_S - \beta_S \beta_k) \hat{I}_k^{FB} - A \right] \]

37
Thus, the supplier gets flat repayments across states for the funding provided, getting the collateral in case of default, while the bank gets an increasing repayment contract.

\( A < \hat{A}_1 \): The incentive constraint (25) becomes binding and \( \hat{I}_k, \hat{I}_L \) solve (50) and (51) with \( \lambda_1 > 0 \). Under the assumption that factors are substitutes, (50) and (51) imply that \( \hat{I}_k < \hat{I}_k^{FB} \), and \( \hat{I}_L < \hat{I}_L^{FB} \).

The contract has the following properties:

\[
\begin{align*}
R_G^S &= \beta_S \beta_k \hat{I}_k, \\
\hat{L}_S &= \frac{1}{r_s} \beta_S \beta_k \hat{I}_k, \\
\hat{L}_B &= \hat{I}_L + \frac{1}{r_s} (r_S - \beta_S \beta_k) \hat{I}_k - A, \\
R_G^B &= \frac{r_B}{p} \left( \hat{I}_L + \frac{1}{r_s} (r_S - \beta_S \beta_k) \hat{I}_k - A \right)
\end{align*}
\]

Part (b) is proved using the following lemma:

**Lemma 3** For any \( 1 - \beta_k \leq \frac{\beta_S \beta_k}{r_s} \) there exists a unique threshold value \( \hat{A}_1 (\phi, \beta_k, \beta_S) \) such that

\[
p f_{G} \left( \hat{I}_k^{FB}, \hat{I}_L^{FB} \right) - \hat{L}_B r_B - \hat{L}_S r_S + (1 - p) \beta_S \beta_k \hat{I}_k^{FB} - \phi \left\{ A + \hat{L}_B + \hat{L}_S - (1 - \beta_k) \hat{I}_k^{FB} \right\} = 0 \text{ and } \hat{L}_S = \frac{\beta_S \beta_k}{r_s} \hat{I}_k^{FB}.
\]

**Proof.** The threshold \( \hat{A}_1 (\phi, \beta_k, \beta_S) \) is the minimum wealth that allows the entrepreneur to invest \( \hat{I}_k^{FB}, \hat{I}_L^{FB} \) fully exploiting both credit lines.\(^3^8\) This level must satisfy:

\[
\hat{A}_1 = \frac{1}{r_B} \left\{ (\phi + r_B) \hat{I}_L^{FB} + \left( 1 - \frac{\beta_S \beta_k}{r_s} \right) r_B + \phi \beta_k + p \beta_S \beta_k \right\} \hat{I}_k^{FB} - p \left[ f_{G} \left( \hat{I}_k^{FB}, \hat{I}_L^{FB} \right) \right] - \phi = 0
\]

To prove that this threshold exists and is unique we need to show that: \((i)\) \( 0 + \hat{L}_B + \hat{L}_S < \hat{I}_k^{FB} + \hat{I}_L^{FB} \), which follows from assumption 1 (\( \phi > 0 \)); \((ii)\) \( \hat{L}_B \) and \( \hat{L}_S \) are continuously increasing in \( A \).

To establish part \((ii)\), it is useful to define the following functions, obtained by taking the derivatives of (25) wrt \( \hat{I}_k \) and \( \hat{I}_L \) respectively:

\[
\begin{align*}
h_{k3} &= \frac{\partial f_{G}(\cdot)}{\partial \hat{I}_k} - \frac{1}{r_s} (r_S - \beta_S \beta_k) r_B + pr_S \beta_S \beta_k + \phi r_S \beta_k \\
h_{L3} &= \frac{\partial f_{G}(\cdot)}{\partial \hat{I}_L} - r_B - \phi \end{align*}
\]

Constraint (25) is only binding if \( h_{k3}, h_{L3} < 0 \), otherwise \( \hat{I}_k \) and \( \hat{I}_L \) could be further increased without violating the constraint.

We first prove that \( \frac{\partial \hat{L}_B}{\partial A} > 0 \). Using (8) and (10), we deduce that \( I_k = \frac{A + \hat{L}_B - \hat{I}_B}{(r_S - \beta_S \beta_k)} \). The binding incentive constraint (25) can be written as a function of \( \hat{I}_L, \hat{L}_B \) and \( A \):

\[
p f_{G} \left( \frac{A + \hat{L}_B - \hat{I}_B}{r_S - \beta_S \beta_k}, \hat{I}_L \right) - \hat{I}_L (r_B + \phi) + Ar_B = \left\{ \left( 1 - \frac{\beta_S \beta_k}{r_s} \right) r_B + p \beta_S \beta_k + \phi \beta_k \right\} \frac{A + \hat{L}_B - \hat{I}_B}{r_S - \beta_S \beta_k} r_S.\]

Totally differentiating

\[
\begin{align*}
\left\{ p \left( 1 - \frac{\beta_S \beta_k}{r_s} \right) \frac{\partial f_{G}(\cdot)}{\partial \hat{I}_L} - p \beta_S \beta_k - \phi \left( 1 - \beta_k - \frac{\beta_S \beta_k}{r_s} \right) + p \beta_S \beta_k \right\} d\hat{I}_L + \left\{ p \frac{\partial f_{G}(\cdot)}{\partial \hat{I}_L} - \left[ \left( 1 - \frac{\beta_S \beta_k}{r_s} \right) r_B + p \beta_S \beta_k + \phi \beta_k \right] \right\} d\hat{L}_B + \left\{ p \left( 1 - \frac{\beta_S \beta_k}{r_s} \right) r_B + p \beta_S \beta_k + \phi \beta_k \right\} dA = 0
\end{align*}
\]

\(^3^7\)The proof of this result is analogous to the one obtained for the case in which \( A_2 \leq A < A_3 \) for Proposition 2 and thus omitted.

\(^3^8\)This amounts to say that \( \hat{A}_1 + \hat{L}_B + \hat{L}_S = \hat{A}_1 + \hat{L}_B + \frac{\beta_S \beta_k}{r_s} \hat{I}_k^{FB} = \hat{I}_k^{FB} + \hat{I}_L^{FB} \).

38
Adding and subtracting \(1 - \frac{\beta_S \beta_k}{r_S}\) \(r_B\) and using (54) and (55), the multiplier of \(dI_L\) can also be written as

\[
(1 - \frac{\beta_S \beta_k}{r_S}) h_{L3} - h_{k3}
\]  

(57)

After some manipulations,\(^\text{39}\) this reduces to

\[
(1 - \beta_k - \frac{\beta_S \beta_k}{r_S}) h_{L3}
\]  

(58)

Given that \(1 - \beta_k < \frac{\beta_S \beta_k}{r_S}\) and knowing that \(h_{L3} < 0\) whenever the incentive constraint binds, we deduce that (57), and thus the multiplier of \(dI_L\) in (56), is positive. Hence, (56) writes as

\[
(1 - \beta_k - \frac{\beta_S \beta_k}{r_S}) h_{L3} dI_L + h_{k3} d\bar{L}_B + \left\{ p \frac{\partial f_L}{\partial I_L} - [p \beta_S \beta_k + \phi \beta_k]\right\} dA = 0
\]  

(59)

Totally differentiating (53) and recalling that \(I_k = \frac{A + L_B - L_k}{(r_S - \beta_S \beta_k)}\), we get:

\[
\frac{r_S}{(r_S - \beta_S \beta_k)} (f_{kk} - \beta_k f_{lk}) \left( dA + d\bar{L}_B - dI_L \right) + (f_{kl} - \beta_k f_{ll}) dI_L = 0.
\]

Solving for \(dI_L\), we get

\[
\frac{dI_L}{dA} = -\frac{r_S}{(r_S - \beta_S \beta_k)} \left( f_{kk} - \beta_k f_{lk} \right) dL_B =
\]

\[
\left\{ \frac{1 - \beta_k - \frac{\beta_S \beta_k}{r_S}}{r_S - \beta_S \beta_k} \right\} h_{L3} dL_B - \left\{ p \frac{\partial f_L}{\partial I_L} - [p \beta_S \beta_k + \phi \beta_k]\right\} dA
\]

whence

\[
\frac{dL_B}{dA} = \frac{\left(1 - \beta_k - \frac{\beta_S \beta_k}{r_S}\right) r_S (f_{kk} - \beta_k f_{lk}) h_{L3} - \left\{ p \frac{\partial f_L}{\partial I_L} - [p \beta_S \beta_k + \phi \beta_k]\right\} dA}{h_{k3} dI_L - \left(1 - \beta_k - \frac{\beta_S \beta_k}{r_S}\right) r_S (f_{kk} - \beta_k f_{lk}) h_{L3}}
\]

Using \(h_{k3}, h_{L3}, f_{ii} < 0\) and \(f_{ij} > 0\) and using the FOC on \(I_k\) (50), we deduce that the numerator of \(\frac{\partial L_B}{\partial A}\) is negative. The sign of \(\frac{\partial L_B}{\partial A}\) depends on the sign of the denominator. Using the equality of (57) with (58), we can write the denominator as

\[
den\left(\frac{dL_B}{dA}\right) = h_{k3} dI_L - \frac{r_S}{(r_S - \beta_S \beta_k)} (f_{kk} - \beta_k f_{lk}) \left(1 - \frac{\beta_S \beta_k}{r_S}\right) h_{L3} - h_{k3}.
\]

This reduces to

\[
den\left(\frac{dL_B}{dA}\right) = h_{k3} (f_{kk} - \beta_k f_{lk}) - \frac{r_S}{(r_S - \beta_S \beta_k)} (f_{kk} - \beta_k f_{lk}) \left(1 - \frac{\beta_S \beta_k}{r_S}\right) h_{L3} < 0
\]

\(^{39}\)By adding and subtracting \(\beta_k \left(p \frac{\partial f_L}{\partial I_L} - r_B\right)\) to (57) and using (53).
which is unambiguously negative. This completes the proof that \( \frac{\partial L_B}{\partial A} > 0 \).

The last step is to show that \( \frac{\partial L_S}{\partial A} > 0 \). Using (8) and (10), we deduce that \( L_B = I_L - \left(1 - \frac{r_S}{\beta_S \beta_k} \right) L_S - A \) and \( I_L \). The incentive constraint \( \text{vis-à-vis} \) the supplier (22) can therefore be written as a function of \( I_L \), \( L_S \) and \( A \):

\[
pf_G \left( \frac{r_S}{\beta_S \beta_k} L_S, I_L \right) - I_L, (r_B + \phi) - \frac{L_S}{\beta_S \beta_k} \left( (r_S - \beta_S \beta_k) r_B + (p \beta_S + \phi) \beta_k r_S \right) + A r_B = 0
\]

Totally differentiating:

\[
\left\{ p \frac{\partial f_G(c_c)}{\partial I_L} - (r_B + \phi) \right\} dI_L + r_B dA + \frac{r_S}{\beta_S \beta_k} \left\{ \frac{p \partial f_G(c_c)}{\partial I_L} + \frac{1}{r_S} \left[ (r_S - \beta_S \beta_k) r_B + \beta_k r_S (p \beta_S + \phi) \right] \right\} dL_S = 0
\]

which, using \( h_k, h_l \), we can write as:

\[
h_k l_d I_L + r_B dA + \frac{r_S}{\beta_S \beta_k} h_k dL_S = 0
\]

Totally differentiating (53), we get:

\[
dI_L = \frac{r_S}{\beta_S \beta_k} \left( f_{kk} (f_k - \beta_k f_{kL}) (f_{kL} - \beta_k f_{LL}) \right) dL_S.\]

Substituting this out in (60) and solving for \( \frac{\partial L_S}{\partial A} = r_B \left( h_{k_d} l_d I_L + \frac{r_S}{\beta_S \beta_k} f_{kk} (f_k - \beta_k f_{kL}) (f_{kL} - \beta_k f_{LL}) \right) \) depends on wealth only through the share of trade credit increases until it reaches \( \beta_k \).

\[
\text{Proof of Proposition 5.} \quad \text{Under the assumption that the production function is homogeneous, the input combination} (I_k/I_L) \text{ only depends on the input price ratio} (P_k/P_L). \quad \text{Using the proof of Proposition 2, we can write} P_k/P_L \text{ as a function of the parameters of the model. Let us consider the four wealth areas separately.}
\]

When \( A \geq A_3 \), \( \frac{P_k}{P_L} = \frac{r_B - \beta_k \beta_S \left( \frac{r_S}{r_S} - p \right)}{s_B} \) and \( \frac{L_S}{L_S + L_B + A} = \beta_S \beta_k \left( \frac{r_S}{r_S} + P_k \right) + r_S \right) \). Notice that trade credit intensity is increasing in input tangibility. Given \( \frac{\partial (P_k/P_L)}{\partial A} = 0 \) and given that trade credit intensity \( (L_S/\left( A + L_B + L_S \right)) \) depends on wealth only through \( I_k/I_L \), both input tangibility and trade credit intensity are independent of \( A \).

When \( A_2 \leq A < A_3 \): \( \left( \frac{P_k}{P_L} \right)_{A_2 \leq A < A_3} = \frac{r_B + \frac{\phi_1}{1 + \lambda_1} (1 - \frac{\beta_0 \beta_k}{r_S}) - \beta_S \beta_k (r_B \frac{r_S}{r_S} - p)}{r_B + \frac{\phi_1}{1 + \lambda_1} (1 - \frac{\beta_0 \beta_k}{r_S})} \)...

0. Given that \( L_S/\left( A + L_B + L_S \right) \) depends on wealth only through \( I_k/I_L \), both input tangibility and trade credit intensity are decreasing in \( A \).

When \( A_1 \leq A < A_2 \): \( \left( \frac{P_k}{P_L} \right)_{A_1 \leq A < A_2} = \left( \frac{\beta_k r_B + (1 - \beta_k)r_S - \beta_0 \beta_k (1 - p)}{r_S} \right) \)...

where \((\beta_k \beta_S/r_S) \leq \mu \leq (1 - \beta_k)\) and varies with \( A \). Since \( \frac{\partial (P_k/P_L)_{A_1 \leq A < A_2}}{\partial A} = 0 \) and \( \frac{\partial \mu}{\partial A} \leq 0 \),

40This follows from Proposition 2 and from Figure 2. The intuition is the following: when \( A = A_2 \), the firm uses a share of trade credit equal to \( \beta_S \beta_k/r_S \) and the shadow cost of bank credit equals the cost of trade credit. Since the firm is constrained on bank credit but still unconstrained on trade credit, any reduction in wealth is compensated by a rise in trade credit, keeping investment at \( I_k, I_L \). Thus the share of trade trade credit increases until it reaches \( (1 - \beta_k) \) when \( A = A_1 \).
while input tangibility is independent of \( A \), trade credit intensity is decreasing in \( A \).

When \( A < A_1 \): \( \frac{P_b}{P_L} \), \( A_1 < A < A_2 \), also \( \frac{L_S}{L_S + L_B + A} = \frac{(1 - \beta_k)}{(L_k + B)^{-1}} \). Notice that trade credit intensity is increasing in asset tangibility. It follows that the whole expression is negative. This implies that asset tangibility increases. Since \( \phi \) affects trade credit intensity only through the input combination, also trade credit intensity is independent of \( \phi \).

Proof of Proposition 6. Notice that, from the proof of Proposition 5, trade credit intensity is an increasing function of input tangibility. Let us consider separately the four relevant wealth areas.

When \( A \geq A_3 \), \( \frac{\partial(P_b/P_L)}{\partial \phi} = 0 \). It follows that \( I_k/I_L \) is independent of \( \phi \). However since \( \phi \) affects trade credit intensity only through the input combination, also trade credit intensity is independent of \( \phi \).

When \( A_2 \leq A < A_3 \), the sign of the derivative \( \frac{\partial(P_b/P_L)}{\partial \phi} \) depends on the sign of \( \left( \lambda_1 + \phi \frac{\partial \lambda_1/\partial \phi}{(1 + \lambda_1)^2} \right) \left( \beta_S \beta_k (1 - p) - (1 - \beta_k) r_S \right) \). Since \( (\partial \lambda_1/\partial \phi) > 0 \) and \( (\beta_k - 1) < 0 \), the whole expression is negative. This implies that asset tangibility increases in \( \phi \). Since a change in \( \phi \) affects trade credit intensity through the input combination, also \( L_S/(A + L_B + L_S) \) is increasing in \( \phi \).

When \( A_1 \leq A < A_2 \), \( \frac{\partial(P_b/P_L)}{\partial \phi} = 0 \), which implies that \( I_k/I_L \) is independent of \( \phi \). However, since \( \frac{\partial \mu}{\partial \phi} > 0 \), \( L_S/(A + L_B + L_S) \) is increasing in \( \phi \). When \( \phi \) increases, the shadow cost of bank credit equals the cost of trade credit at \( \tilde{A}_2 > A_2 \). For decreasing \( A \), the firm substitutes bank credit with trade credit, thereby increasing the absolute level of trade credit from \( \beta_S \beta_k I_k^* / r_S \) at \( A = \tilde{A}_1 > A_1 \).

When \( A < A_1 \), the sign of the derivative \( \frac{\partial(P_b/P_L)}{\partial \phi} \) depends on the sign of \( \left( \lambda_1 + \phi \frac{\partial \lambda_1/\partial \phi}{(1 + \lambda_1)^2} \right) \left( \beta_S \beta_k (1 - p) - (1 - \beta_k) r_S \right) \). Since \( (\partial \lambda_1/\partial \phi) > 0 \) and the term in square brackets is negative, \(^{41}\) the whole expression is negative. This implies that asset tangibility increases. Since \( \phi \) affects trade credit intensity only through the input combination, also \( L_S/(A + L_B + L_S) \) is increasing in \( \phi \).

\(^{41}\) Given that we are in the case in which \( (1 - \beta_k) > \beta_k \beta_S/r_S \).
References


