Computational Aspects of Prospect Theory with Asset Pricing Applications

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Computational Aspects of Prospect Theory with Asset Pricing Applications*

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Abstract

We develop an algorithm to compute asset allocations for Kahneman and Tversky’s (1979) prospect theory. An application to benchmark data as in Fama and French (1992) shows that the equity premium puzzle is resolved for parameter values similar to those found in the laboratory experiments of Kahneman and Tversky (1979). While previous studies like Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), and Grüne and Semmler (2005) only used myopic loss aversion to explain the equity premium puzzle our paper extends this explanation of the equity premium puzzle by incorporating changing risk aversion. Our extension allows reducing the degree of loss aversion from 2.353 to 2.25, which is the value found by Kahneman and Tversky (1979) while increasing the risk aversion from 1 to 0.894, which is a slightly higher value than the 0.88 found by Kahneman and Tversky (1979). The equivalence of these parameter settings is robust to incorporating the size and the value portfolios of Fama and French (1992). However, the optimal prospect theory portfolios found on this larger set of assets differ drastically from the optimal mean-variance portfolio.

Keywords: prospect theory, asset pricing, equity premium puzzle, global optimization, non–smooth problems, numerical algorithms

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1 Introduction

A large equity premium is one of the more robust findings in financial economics (Mehra and Prescott (1985)). On US-data, for example, over the period of 1802 to 1998 Siegel (1998) reports an excess return of U.S. Stocks over U.S. Bonds of about 7% to 8% p.a.. Similar results have been found for other periods and across other countries. This empirical finding is puzzling because it is hard to reconcile with plausible parameter values for agents’ risk aversion in the standard consumption based asset pricing model originating from Lucas (1978). That is to say, the volatility of consumption, which, as Hansen and Jagannathan (1991) have shown, is an upper bound for the equity premium, is not found to be sufficiently high to allow for the large equity premium.

The huge number of solutions that have been suggested to resolve the equity premium puzzle is another puzzling aspect of the equity premium. It is impossible to review this literature in a few words without being accused for serious omissions. Therefore we only highlight a few suggested solutions that relate most closely to our paper. For a recent comprehensive treatment of all suggested solutions see the recent Handbook of Finance on this topic edited by Mehra (2006). One line of research points out that the Hansen and Jagannathan bound is increased by choosing more appropriate proxies for consumption that include fluctuations in financial wealth (Lettau and Ludvigson (2001)), fluctuations in housing wealth (Piazzesi, Schneider, and Tuzel (2005)) or other such proxies. Another line of research points out that introducing more links in the utility function between consumption of different periods can also resolve the equity premium puzzle (Constantinides (1990), Constantinides, Donaldson, and R. Mehra (2005)). Yet a different strategy is to generate extra fluctuations by including behavioural aspects like myopic loss aversion (Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001), and Grüne and Semmler (2005)).

Our paper follows the behavioural approach to the equity premium puzzle. This explanation of the equity premium puzzle is based on two main ideas, myopia and loss aversion. First, investors evaluate risky assets by the gains and losses on a short horizon but not by the final wealth the investor achieves. To illustrate this point consider a fair lottery in which you can double your yearly income with a 50% chance while you can loose a percentage, say x% of your income in the other 50% cases. What is the highest loss x that you would be willing to incur to accept playing this lottery? Shefrin (1999) reports that the average answer typically is x = 23%. However, if your yearly income is something like 50% of your wealth then an expected utility maximizer with a constant relative risk aversion (CRRA) of at least -75.58 would generate the average answer x=23%. Note that 50% is a quite small fraction of a person’s income as of his total wealth. The high number of risk aversion however increases the smaller the fraction of the yearly income as of total wealth. If on the other hand the respondent to this question ignores his background wealth and only evaluates the lottery from the gains and losses a CRRA of -2.22 would already be sufficient to generate

\[ 0.5 \left( \frac{0.5+2}{W} \right) + 0.5 \left( \frac{0.5+1}{W} \right) \]

is solved for \( x = 0.23 \) and \( \alpha = 75.58 \).

\[ 0.5 \left( \frac{2w^{\alpha}}{\alpha} \right) + 0.5 \left( \frac{(1-x)w^{\alpha}}{\alpha} \right) = \left( \frac{w^{\alpha}}{\alpha} \right) \]

is solved for \( x = 0.23 \) and \( \alpha = -2.22 \).
x=23%. Hence thinking in terms of gains and losses already goes a good way in reducing risk aversions to match realistic observations. That is to say if one substitutes consumption by changes in wealth from investing then the Hansen and Jagannathan bound is considerably increased. Moreover, due to mean-reversion in equity returns, computing gains and losses on a short horizon in spite of the equity premium observed in long time series an investment in equities evaluated on a short horizon does not look very attractive. Finally, the behavioural explanation replaces the standard CRRA utility with a value function that has loss aversion and changing risk aversion. Loss aversion describes the finding that a loss of, say 100$ needs to be compensated by a gain of at least 200$ for the investor to be satisfied again with the investment. Changing risk aversion models the finding that a lottery in which you can only gain is typically not preferred to the certain payoff equal to the expected value of the lottery while when facing a sure loss people prefer to gamble, i.e. they prefer a lottery with expected value equal to the sure loss. These fundamental findings have been observed robustly in laboratory experiments but they also determine many actual decisions made by investors. The value function suggested by Kahneman and Tversky (1979) is the following piecewise power function:

\[ v(x) = \begin{cases} 
  x^\alpha & \text{for } x \geq 0 \\
  -\beta(-x)^\alpha & \text{for } x < 0 
\end{cases} \]

where \( \alpha \approx 0.88 \) and \( \beta \approx 2.25 \), \( \text{(1)} \)

which is concave for gains \( (x > 0) \) and convex for losses \( (x < 0) \). Moreover, note that the value function is kinked at \( x = 0 \).

While the studies of Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), and Grüne and Semmler (2005) only used myopic loss aversion to explain the equity premium puzzle, i.e. they define the value function on changes in wealth and they set \( \alpha = 1 \), our paper extends this explanation of the equity premium puzzle by incorporating the other important aspect of prospect theory: Changing risk aversion, i.e. we allow for \( \alpha < 1 \). A priori it is unclear in which direction the inclusion of changing risk aversion will drive the result because the value function becomes more concave in some region and more convex in the other region. Therefore, we analyse standard annual U.S. asset market data from 1927 to 2002 that has been grouped into benchmark portfolios according to the methodology of Fama and French (1992), Fama and French (1993). Our extension to changing risk aversion allows reducing the degree of loss aversion from 2.353 to 2.25 while increasing the risk aversion from 1 to 0.894. Hence the parameter values we find are more similar to those found by Kahneman and Tversky (1979), which were 2.25 for the loss aversion and 0.88 for the risk aversion. The degree to which these parameter values coincide with those found in the laboratory is amazing because the former are determined on aggregate financial data while the latter have been determined by investigating individual decision problems. However, as we explain below, due to a robustness problem when maximizing prospect theory these specific values should also not be overemphasized.

\[ ^3\text{We are most grateful to Thierry Post for having made available this excellent data set.} \]
One reason for the omission of changing risk aversion in the previous studies may be that prospect theory including this feature becomes much more difficult to apply. Since the value function is then convex for losses the objective function is no longer quasi–concave and the first order condition may only describe local optima. Indeed on our data we found local optima for prospect theory with changing risk aversion. The computational part of this paper describes a fast, efficient and robust method to nevertheless compute optimal prospect theory portfolios. Finally, we analyse the equivalence of the parameter settings mentioned above on the complete data set now including the Fama and French (1992) size and value sorted portfolios. It is found that the optimal prospect theory portfolios are still quite similar and that they do differ drastically from the optimal mean-variance portfolio.

The rest of the paper is organized as follows. Section 2 introduces the model setup and the algorithm for determining prospect theory optimal allocations. Section 3 presents an asset pricing application dealing with the equity premium puzzle. Section 4 concludes.

2 Computational Aspects of Prospect Theory

In this section we analyse the prospect theory of Kahneman and Tversky (1979) from a computational point of view. One computational difficulty is loss aversion which leads to a non differentiability at the reference point. In the case of a piecewise linear value function the differentiability problem can however easily dealt with because for $\alpha = 1$ maximization of the prospect utility amounts to solving a linear program. For the more general case we will evoke some smoothing techniques to get around the non–differentiability. The next problem that arises for $\alpha < 1$ is that the objective function is not quasi–concave since the value function is convex for losses and concave for gains. As an effect local optima can arise. We solve this problem by choosing randomly selected starting points for our algorithm.

2.1 The General Computational Problem

The Kahneman–Tversky value function (1) can also be formulated in the following compact form

$$v(x) = (x^+)^{\alpha} - \beta (x^-)^{\alpha}$$

with $x^+ = \max\{0, x\}$ and $x^- = \max\{0, -x\}$ denoting the positive and negative parts of the real number $x$, respectively. Figure 1 displays $v(x)$ with the parameter selection $\alpha = 0.88$ and $\beta = 2.25$. For formulating the asset allocation problem we consider $n$ assets with asset weights $\lambda_j$, $j = 1, \ldots, n$, for which we have the constraint $\sum_{j=1}^{n} \lambda_j = 1$ and will focus on the case $\lambda_j \geq 0 \ \forall j$, that means, we assume that short sales are not allowed. Regarding data for the asset returns, we presuppose that scenarios $r^s_j$ are given for the net return of asset $j$ in scenario $s$, $j = 1, \ldots, n$, $s = 1, \ldots, S$. The portfolio return in scenario $s$ will be $r^s := \sum_{j=1}^{n} r^s_j \lambda_j = (r^s)^T \lambda$ with $r^s$ standing for the vector of portfolio returns in scenario $s$ and
\( \lambda \) denoting the vector of the asset weights. For the sake of simplicity of presentation, we assume also that the return scenarios \( r^s \) are equally probable. Then, the prospect theory asset allocation problem consists of maximizing the objective function

\[
V(x^1, \ldots, x^S) := \frac{1}{S} \sum_{s=1}^{S} v(x^s) = \frac{1}{S} \sum_{s=1}^{S} \left[ ((x^s)^+)^\alpha - \beta ((x^s)^-)^\alpha \right]
\]

subject to the constraints

\[
\begin{cases}
 x^s - \sum_{j=1}^{n} r^s_j \lambda_j = 0, & s = 1, \ldots, S \\
 \sum_{j=1}^{n} \lambda_j = 1 \\
 \lambda_j \geq 0, & j = 1, \ldots, n.
\end{cases}
\]

### 2.2 Piecewise linear value function

In this subsection we consider the case when in the Kahneman-Tversky value-function \( \alpha = 1 \) holds. This is the value function discussed in Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), and Gröne and Semmler (2005). In this case we have

\[
v(x) = \begin{cases} 
 x & \text{for } x \geq 0 \\
 \beta x & \text{for } x < 0
\end{cases}
\]

or in a compact form

\[
v(x) = x^+ - \beta x^-.
\]

This is a piecewise linear function with a kink at \( x = 0 \). If \( \beta \geq 1 \) holds, which we assume in the sequel then \( v(x) \) is a concave function. Figure 2 shows this function for \( \beta = 2.25 \).

The sum of concave functions being concave, it follows that \( V(x^1, \ldots, x^S) \) is a multivariate

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4Prospect theory assumes that investors distort probabilities by means of subjective weighting functions. When working with sample data, it is however natural to consider each sample as one equally likely state. Under the assumption of equally probable states of nature, the distortions will not affect optimal portfolio choices, thus the subjective weighting function is dropped in our presentation.

5In what follows, we also assume that investors evaluate gains and losses with respect to the zero return.
Figure 2: Piecewise linear value function with $\beta = 2.25$.

A concave function. It is a nonlinear function which is non-differentiable at points where $x^s = 0$ holds for some $s$. Figure 3 shows the graph and contour lines of $V(x^1, x^2) = v(x^1) + v(x^2)$ with points of non-differentiability along two lines. For a concave function the upper level sets are concave. In the figure this means that the contour lines, viewed from the southwest corner, are convex curves. Thus our asset allocation problem belongs to the class of convex optimisation problems for which, theoretically, several powerful algorithms exist. Nevertheless, the objective function is non-smooth, which makes the problem difficult to solve numerically for problems with a large number of assets. It is a well-known fact in nonlinear programming that maximizing a piecewise linear concave function subject to linear constraints can be reformulated as a linear programming problem. Thus, fortunately, the asset allocation problem can be equivalently formulated in our case as a linear optimisation problem which enables solving large-scale asset allocation problems. For our problem the transformation is particularly simple. We introduce auxiliary variables $y^s$ for representing $(x^s)^+$ and $z^s$ for representing $(x^s)^-$, respectively. Utilizing the fact that $x^s = (x^s)^+ - (x^s)^-$ holds generally, we obtain the following equivalent formulation as a linear programming problem in the $n + 2S$ variables $(\lambda, y^1, \ldots, y^S, z^1, \ldots, z^S)$: Maximize the linear function $\frac{1}{S} \sum_{s=1}^S (y^s - \beta z^s)$, subject to the following constraints: we replace the first constraint in the general formulation by $y^s - z^s - \sum_{j=1}^n r^s_j \lambda_j = 0$, $s = 1, \ldots, S$, keep the constraints solely involving $\lambda$ and require $y^s \geq 0$ and $z^s \geq 0$ for $s = 1, \ldots, S$. Note that, besides having obtained a computationally attractive alternative formulation, the transformation has also

Figure 3: Graph and contour lines of the sum of two piecewise linear value functions.
eliminated the inconvenient feature of non-differentiability.

2.3 Computational difficulties in the general case

In the case $\alpha \neq 1$, the transformation outlined in the previous section does not work. We will consider the general case with the Kahneman-Tversky parameter settings $\alpha = 0.88$ and $\beta = 2.25$. In the general case, we face two kinds of difficulties from the computational point of view.

On the one hand, the objective function is not differentiable at points where $x^s = 0$ holds for some $s$. Figure 4 displays $V(x_1, x_2) = v(x_1) + v(x_2)$ where we have, similarly to the piecewise linear case, points of non-differentiability along two lines. Consequently, the problem belongs to the class of non-smooth optimisation problems. The size of problems that can be efficiently solved by solvers (implemented algorithms) for this problem class is much smaller than for the smooth case. In addition to this, available algorithms for non-smooth problems essentially rely on the assumption that the objective function is concave. This is not the case for our asset allocation problem. Thus we arrive at the second kind of numerical difficulty that we face regarding the asset allocation problem. A glance at the graph of $V(x_1, x_2)$ in Figure 4 confirms that the surface is now curved. Viewing the contour lines in the left-hand-side picture from the south-west corner viewpoint it is clear that some of the contour lines are no more convex curves. This implies that there are some upper level sets that are non-convex sets. Consequently, $V(x_1, x_2)$ cannot be a concave function. Moreover, $V(x_1, x_2)$ is not even quasi-concave, quasi-concavity being defined by the property that all upper level sets are convex sets. For quasi-concavity and for further types of generalized concavity discussed below see, for instance, Avriel, Diewert, Schaible, and Zang (1988).

To confirm the missing quasi-concavity, as suggested by Figure 4, we proceed by considering the value function along a line, that means, we take $f(\kappa) := V(x_1 + \kappa u_1, x_2 + \kappa u_2)$. Supposing that $V(x_1, x_2)$ is quasi-concave, $f$ would be a univariate quasi-concave function, for any choice of $x_1, x_2, u_1, u_2$. Let us choose $x_1 = -1, x_2 = -1, u_1 = 1.1$, and $u_2 = -1$. Figure 5 displays $f(\kappa)$ for $-2 \leq \kappa \leq 2$. It is immediately clear that the function in the Figure is not quasi-concave (we omit the obvious mathematical proof). Thus, we conclude that $V(x_1, x_2)$ is not a quasi-concave function. On the other hand, pseudo-concavity implies

Figure 4: Graph and contour lines of the sum of two Kahneman–Tversky value functions

Figure 5: Graph of quasi-concavity function
Figure 5: The value function $V$ along a line: $f(\kappa) = V(x^1 + \kappa u^1, x^2 + \kappa u^2)$

quasi–concavity. Consequently, $V(x^1, x^2)$ is not a pseudo–concave function. This implies that our asset allocation problem may have several local maxima which are not global solutions of the problem.

Let us remark that the value function $v(x)$, being a strictly increasing function, is obviously pseudo–concave. The point is that, unlike for concave functions, summing up pseudo–concave functions does not preserve the pseudo–concave property. As our example shows, this may even happen in the case when we construct multivariate functions by addition, based on the same univariate pseudo–concave function as in our case $V(x^1, x^2) = v(x^1) + v(x^2)$.

For smaller values of $\alpha$, the missing quasi–concavity appears more markedly as it can be seen in Figure 6 which displays $V$ with $\alpha = 0.7$.

Figure 6: Graph and contour lines of $V(x^1, x^2)$ with $\beta = 2.25$ and $\alpha = 0.7$

2.4 Overcoming the numerical difficulties

For dealing with non–smoothness we have applied smoothing to the value function in the vicinity of 0, by employing cubic splines. More closely we proceed as follows.

Let $\delta > 0$ and $p(x) = ax^3 + bx^2 + cx + d$ be a cubic polynomial. The four coefficients of the polynomial are computed from the four equations $p(-\delta) = v(-\delta)$, $p'(-\delta) = v'(-\delta)$, $p(\delta) = v(\delta)$, $p'(\delta) = v'(\delta)$, thus ensuring that both the function values and the first derivatives of $p$ and $v$ are equal, at both endpoints of the interval $[-\delta, \delta]$. The value function is subsequently
replaced by the smoothed value function

\[ v_\delta(x) := \begin{cases} p(x) & \text{if } x \in [-\delta, \delta] \\ v(x) & \text{otherwise} \end{cases} \]  

leading to the following approximation of the objective function of the asset allocation problem

\[ V_\delta(x^1, \ldots, x^s) := \frac{1}{S} \sum_{s=1}^{S} v_\delta(x^s). \]  

Denoting the maximal approximation error over the interval \([-\delta, \delta]\) by \(\varepsilon\), we have

\[ V_\delta(x^1, \ldots, x^s) \leq \frac{1}{S} \sum_{s=1}^{S} (v(x^s) + \varepsilon) = \frac{1}{S} \sum_{s=1}^{S} v(x^s) + \varepsilon = V(x^1, \ldots, x^s) + \varepsilon \]

and similarly we get \(V_\delta(x^1, \ldots, x^s) \geq V(x^1, \ldots, x^s) - \varepsilon\). Thus, replacing our objective function with \(V_\delta(x^1, \ldots, x^s)\) results in an \(\varepsilon\)-optimal solution, with respect to the true optimal objective function value. Choosing \(\delta > 0\) small enough, \(\varepsilon > 0\) can be made arbitrarily small in theory. In practice, \(\delta = 0.00001\) turned out to be small enough.

For solving the smoothed problems we have employed the general-purpose solver Minos 5.4 (Murtagh and Saunders (1978), Murtagh and Saunders (1995)), designed for smooth nonlinear programming problems.

The second difficulty, the possible presence of several local optima, has been dealt with as follows. First \(N\) starting points have been randomly generated on the unit simplex \(\{\lambda \mid \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \forall j\}\), according to the uniform distribution over the unit simplex. For this we employed an algorithm of Rubinstein (1982). The following procedure has been repeated \(N\) times:

- \(n\) pseudo–random numbers \(z_1, \ldots, z_n\) are generated according to the uniform distribution on \([0,1]\).
- These are transformed as \(y_j = -\log(z_j) \ \forall j\), corresponding to the exponential distribution with parameter 1.
- Finally, the normalization \(\lambda_j = \frac{y_j}{\hat{y}}\) with \(\hat{y} := \sum_{j=1}^{n} y_j\) results in a random vector \(\lambda\), corresponding to the uniform distribution over the unit simplex.

Subsequently, the solver Minos has been started up from the \(N\) starting points in turn, resulting in \(N\) (locally optimal) asset allocations with corresponding optimal objective values \(\hat{V}_1, \ldots, \hat{V}_N\). The allocation with the highest \(\hat{V}_j\) value has been chosen as the solution of the problem. The optimisation algorithm outlined above belongs to the class of multistart
random search methods, see Törn and Zilinskas (1989).

Obviously, the quality of the solution largely depends on the proper choice of $N$. In practice, we took an initial $N$ and started up the solver. This has been repeated with increased $N$ till the solution did not change.

2.5 Outlook on planned developments of the method

The next step in the algorithm development, still based on the general-purpose solver Minos, will be the inclusion of adaptive elements into the procedure for selecting starting points. This procedure we plan to design in the spirit of the adaptive grid method of Grüne and Semmler (2003). On the long range we plan to develop a special-purpose algorithm for finding the global optimum, based on the structure of the problem.

3 Asset Pricing Applications

Replacing the piecewise linear value function with the piecewise power function is important because it does incorporate risk taking for losses and risk aversion for gains, as it is robustly observed in laboratory findings. However, due to the loss in quasi-concavity it is not clear a priori which asset pricing implications are introduced this way. In this section we check whether the approximation of the piecewise power value function by a piecewise linear function can be justified. On standard data for a broad stock and a broad bond index, as it can be found on the homepage of Kenneth French, we find that introducing changing risk aversion does bring the parameter values of the representative asset pricing model closer to those found in the laboratory. Hence by doing so we gain on both sides the representative agent has a richer behaviour and its parameter values are closer to those observed in the laboratory. Thereafter, we compare both the piecewise linear and the piecewise power function on a richer set of assets including the standard Fama and French size and value portfolios. It is found that the resulting asset allocations are still similar but they are much different to the optimal mean-variance portfolio. The data we use has the following summary statistics (See Appendix): The annual real equity premium is about 6.4% but the equity index is also much more volatile than the bond index. Both indices also differ in kurtosis and skewness. The size and the value portfolios show the well known size and value effect, i.e. the high excess return of small cap to large cap and of value to growth stocks.

To analyse the Equity Premium Puzzle we adopt the following methodology. Working with the representative consumer model we cannot use the standard Euler equation approach because due to the lack of quasi-concavity it may not describe the sufficient condition for the optimal asset allocation. Instead we use our algorithm to solve for globally optimal prospect theory portfolios as described above. The task is to find the parameters $\alpha$ and $\beta$ of the value function such that the representative consumer holds the stock and the bond index in proportion to their market capitalization. For the relative size of the bond and the stock market we follow Bandourinan and Winkelmann (2003) who estimate the bond equity proportion to be approximately 50:50. First we investigate the piecewise linear value function on the bond
and the stock market index described above. After some iterations we find a loss aversion of about 2.353. Figure 7 shows that the optimal asset allocation is not perfectly robust around this value but the bond stock split of the market portfolio is also only an approximation. Then we investigate the piecewise power value function for a loss aversion as found in the laboratory (2.25) and search for a risk aversion in order to also get the 50:50 split. The value found is about 0.894. Again around this value the optimal asset allocation shows a jump. The size of the jumps for the piecewise power value function are however not larger than those for the piecewise linear function.

Finally, we fix these parameter values and compare the portfolio choice of the two prospect theory functions with that of a mean-variance investor maximizing the ratio of mean to variance. We find for the piecewise linear value function\(^6\): ME1=38.19%, BM8=14.37%, BM9=47.44% and for the piecewise power: ME1=36.07%, BM8=12.63%, BM9=51.29% while we find the optimal mean-variance portfolio (maximal ratio of mean to standard deviation) to be: ME1= 4.61%, BM5=4.71%, BM8=19.23% and Bond=71.44%. Hence prospect theory optimization problems lead to quite similar results as compared to the mean-variance portfolio.

4 Conclusion

We developed an algorithm to compute asset allocations for Kahneman and Tversky’s (1979) prospect theory. An application to benchmark data as in Fama and French (1992) shows that the equity premium puzzle is resolved for parameter values similar to those found in

\(^6\)ME denotes the size portfolios. It reads as market to equity portfolio, BM denotes the value portfolios. It reads as book to market portfolio.
the laboratory experiments of Kahneman and Tversky (1979). While previous studies like Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), and Grüne and Semmler (2005) only used myopic loss aversion to explain the equity premium puzzle our paper extends this explanation of the equity premium puzzle by incorporating changing risk aversion.

The introduction of changing risk aversion bears some considerable cost in terms of computational efforts but it comes at no cost for the economic result. To the contrary: the values found are even more similar that those found in the laboratory, however both value functions considered lead to jumps in the optimal asset allocation - also in the area of the values found on the data. The challenge for further research is to find value functions that account for loss aversion and changing risk aversion but that show more robustness when optimized over realistic data. The piecewise exponential value function by De Giorgi, Hens, and Levy (2004) may be a candidate also for this matter.

Appendix

Table 1 shows descriptive statistics (average, standard deviation, skewness, excess kurtosis and max and min) for the annual real returns of the value–weighted CRSP all–share market portfolio, the intermediate government bond index of Ibbotson and the size and value decile portfolios from Kenneth French’ data library. The sample period is from January 1927 to December 2002 (76 yearly observations).
Table 1: Descriptive statistics

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<td>Equity</td>
<td>8.59</td>
<td>21.05</td>
<td>-0.19</td>
<td>-0.36</td>
<td>-40.13</td>
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References


L. Grüne and W. Semmler. Solving asset pricing models with stochastic dynamic programming, 2003. mimeo Bielefeld University, Germany.

L. Grüne and W. Semmler. Asset prices and loss aversion, 2005. mimeo Bielefeld University, Germany.


