Endogenous Acquisition of Information and the Equity Home Bias

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Abstract

This paper investigates the extent to which differences in information costs can explain the equity home bias puzzle. In a model where the cost of acquiring information regarding the Foreign asset is higher than for the Home asset, we show that—if cost functions are convex—the expected size of the equity home bias in terms of differences in invested amounts is positive and increasing in expected excess returns and risk, but decreasing in risk aversion. However, a calibration to US data suggests that the information cost explanation accounts only for a small fraction of the observed equity home bias.

Keywords: information acquisition, information costs, equity home bias
JEL Classification Codes: G11, G12, G15

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1 Introduction

One nearly undisputed stylized fact about international equity markets is that there is a home bias: Investors allocate more of their wealth toward domestic equity than what is implied from traditional finance theory. For example, Dahlquist et al. (2003) find that an average US investor allocates 91% of his equity portfolio to US equity, although US equity represents only 49% of the world market portfolio.

The suggested explanations for the equity home bias include regulations, taxes, asymmetric access to market prices, asymmetric information, transaction costs, information costs and investor irrationality. We provide a detailed analysis of the argument based on information costs, as alluded to by Lewis (1999), Ahearne, Griever and Warnock (2004), and Portes and Rey (2005), among others.

The effect of information costs on investors’ acquisition of international information and international portfolio allocations is not fully understood. For example, how are differences in the acquisition of domestic and foreign information related to the nature of these information costs? How are these differences related to expected excess returns, risk level and risk aversion? What is the link between investors’ acquisition of information and their international portfolio allocations? What are the magnitudes of the effects and how do they compare with empirical findings? This paper seeks to answer these research questions.

In many empirical studies, information costs are simply treated as additional costs levied on returns and thus, the roles of information costs and the roles of transaction costs are not clearly differentiated. However, as Lewis (1999) notes:

"... information costs are difficult to examine without knowledge of the
information acquisition process... ” (Lewis, 1999, p. 586).

We develop a two-country model, in which the cost of acquiring precise information regarding the payoff from the foreign asset is higher than the cost of acquiring precise information regarding the payoff from the domestic asset at all levels of precision. First, we treat equity prices as exogenous and study an individual investor’s demand for information and his asset allocations. Thereafter, we examine a noisy rational expectations equilibrium and show that the investors’ portfolio choice and information acquisition problem is similar to the one we obtain when we treat prices as exogenous.

We show that if the cost functions are concave in the precision of information, then investors spend infinite amounts on acquiring information. This can also occur with linear cost functions. With linear cost functions and parameter values that rule out infinite solutions, there can either be one solution to the investors’ information acquisition problem—not to demand any information—or there can be infinitely many solutions. However, with convex cost functions, there is a unique solution. The implications of concave or linear cost functions seem to be at odds with reality and such cost functions also make the investors’ problem ill-posed (because investors can achieve an infinite value on their objective function) or intractable (because there can be infinitely many solutions). Peress (2004) notes that one view that gives intuitive support to the assumption of convex cost functions is to consider each additional piece of information as being (positively) correlated with previously received information.

In order to evaluate the equity home bias quantitatively, we examine the case in which the Home asset and the Foreign asset are symmetric. We demonstrate that—
assuming convex cost functions—we can expect there to be an equity home bias, whose absolute size is increasing in expected excess returns and risk, but decreasing in the level of risk aversion. However—despite the intuitive appeal of the information cost argument—a calibration to US data suggests that only a small fraction of the equity home bias can be explained by differences in information costs.

By contrast, the empirical literature seems to provide evidence of the importance of information costs in explaining investors’ equity allocations. For example, Kang and Stulz (1997) find that foreign equity portfolios are tilted towards large firms, Portes and Rey (1999) provide evidence that the geography of information is an important determinant of cross-border transactions, Coval and Moskowitz (1999) show that investors tend to hold stocks of local companies, and Ahearne, Griever and Warnock (2004) demonstrate that the portion of a country’s market that has a public US listing is a major determinant of a country’s weight in US investors’ portfolios. However, none of these authors build up a theoretical model of costly information acquisition and take it to the data. Potentially, there could be other explanations to their findings, such as external habit formation (Shore and White, 2002), investors’ limited information processing abilities (van Nieuwerburgh and Veldkamp, 2006), ambiguity aversion (Epstein and Miao, 2003), etc.

Our model is a modification and an extension of Gehrig’s (1993) two-by-two ("two countries and two risky assets") model. The main difference is that while information quality is exogenous to the model of Gehrig (1993), we endogenize the information acquisition process. In a related model, Verrecchia (1982) analyzes the case in which there is one risky asset and the cost function is convex.¹ Van

¹Note, however, that while Verrecchia (1982) uses an expected utility framework, we employ
Nieuwerburgh and Veldkamp (2006) present a model, in which investors choose the quality of information subject to a capacity constraint and a no negative learning constraint, both unrelated to the budget constraint. In their model, investors choose a corner solution—to reduce variance on one risk factor as much as possible—and therefore, they amplify informational asymmetries. Thus, van Nieuwerburgh and Veldkamp (2006) argue that assumptions of information asymmetries are defensible. We build our model on the notion of costly information acquisition (Verrecchia, 1982), so that the investors’ information choice enters directly into their budget constraints, and we investigate the consequences of differences in information costs for the equity home bias.

The rest of the paper is organized as follows. In section 2, we describe the economy, and in section 3, we present our theoretical results. Section 4 contains a calibration to US data. Finally, section 5 concludes the paper.

2 The Economy

There are two countries, Home and Foreign. We let the index 1 denote the Home country, and we let the index 2 denote the Foreign country. Each country has an equal-sized continuum of investors. We let \( h_1 = [0, \frac{1}{2}] \) denote the continuum of investors residing in the Home country, and we let \( h_2 = (\frac{1}{2}, 1] \) denote the continuum of investors residing in the Foreign country. We assume that each individual investor \( j \in h_1 \cup h_2 \) is negligible. There are two risky assets (one in each country) and one risk free asset.  

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a recursive, non-expected utility framework, which induces a preference for early resolution of uncertainty.
There are two time periods. In the beginning of the first period, investors decide on the purchase of two informative signals regarding the final payoff of each asset. In the end of the first period, the signals are realized, and in the beginning of the second period, investors choose how much to invest in each asset. Finally, in the end of the second period, the asset payoffs are realized. The first period is thought of as being very short, whereas the second period is of arbitrary, but finite, length.

The risk free asset yields a fixed, constant rate of return of \( r > 0 \). In the beginning of the first period, each investor decides to purchase signals \( \tilde{s}_1^j \) and \( \tilde{s}_2^j \), where

\[
\tilde{s}_i^j = \tilde{v}_i + \tilde{\varepsilon}_i^j \quad \text{for} \quad i = 1, 2,
\]

(1)

and the error terms, \( \tilde{\varepsilon}_1^j \) and \( \tilde{\varepsilon}_2^j \), are independent draws from the distribution

\[
(\tilde{\varepsilon}_1^j, \tilde{\varepsilon}_2^j)^T \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/y_1^j & 0 \\ 0 & 1/y_2^j \end{pmatrix} \right) .
\]

(2)

Here, \( \tilde{\varepsilon}_1^j \) and \( \tilde{\varepsilon}_2^j \) are independent of \( \tilde{v}_1 \) and \( \tilde{v}_2 \).

The precisions of the signals, \( y_1^j \) and \( y_2^j \), come at a cost. For an investor residing in the Home country, the monetary cost of the precision \( y_1^j \) is \( C_{11}(y_1^j) = a_1(y_1^j)^c \) and the cost of the precision \( y_2^j \) is \( C_{12}(y_2^j) = b_1(y_2^j)^c \) where \( 0 < a_1 < b_1 \) and \( c > 0 \). That is, for such an investor, the cost of precision in the signal regarding the Home asset is lower than the cost of precision in the signal regarding the Foreign asset at all levels of precision. Conversely, for an investor residing in the Foreign country, the cost of precision in the signal regarding the Foreign asset is lower than the cost of precision in the signal regarding the Home asset. That is, for such an investor, the cost functions are \( C_{21}(y_1^j) = a_2(y_1^j)^c \) and \( C_{22}(y_2^j) = b_2(y_2^j)^c \), respectively, where \( 0 < b_2 < a_2 \). Further, since \( a_1, b_1, a_2, b_2 \) and \( c \) are strictly positive, costs are strictly
increasing in the precisions of the signals.

Each investor has initial wealth $W^j_0 > 0$. Directly after having bought and observed the signals $\tilde{s}^1_j$ and $\tilde{s}^2_j$, the investor chooses to purchase a number of shares $D^j_1$ in the Home asset, and a number of shares $D^j_2$ in the Foreign asset. He invests the remaining amount, $W^j_0 - C_{i1}(y^j_1) - C_{i2}(y^j_2) - D^j_1 P_1 - D^j_2 P_2$ in the risk free asset. Further, he has an elementary utility function over final wealth, which is of the CARA type, and he maximizes the expected certainty equivalent wealth. That is, he solves

$$\max_{y^j \in \mathbb{R}_2^+} E_1 \left[ -\ln \left( -\max_{D^j \in \mathbb{R}^2} E_2 \left[ -e^{-A\tilde{W}^j} \right] \right) \right],$$

where $\tilde{W}^j = \left( W^j_0 - C_{i1}(y^j_1) - C_{i2}(y^j_2) \right) (1+r) + D^j_1 (\tilde{v}_1 - (1+r)P_1) + D^j_2 (\tilde{v}_2 - (1+r)P_2)$, $D^j = (D^j_1, D^j_2)$ and $y^j = (y^j_1, y^j_2)$. This type of recursive, non-expected utility formulation has been developed and investigated by Kreps and Porteus (1978, 1979ab), Selden (1978) and Epstein and Zin (1989, 1991), among others. Since the so-called aggregator function ($-\ln(-x)$) is convex, our setting induces a preference for early resolution of uncertainty. The analysis in Spence and Zeckhauser (1972) suggests that investors who have standard expected utility and care about intertemporal consumption have a desire for early resolution of uncertainty. Because it would unnecessarily complicate the model, we do not model investors’ intertemporal consumption choice. By employing the preferences represented by (3) we gain tractability while, at the same time, maintaining a preference for early resolution of uncertainty.
3 Theoretical Results

In this section, we present our theoretical results regarding the portfolio choice and information acquisition problem of an individual investor when the asset prices are exogenously given as $P = (P_1, P_2)^T$. Since the problems facing Home and Foreign investors are conceptually identical, we specialize our analysis to the Home investor’s information acquisition and portfolio selection. We assume that the Home investor has a prior over equity payoffs, which is multivariate normal with mean $q = (q_1, q_2)^T$ and a variance-covariance matrix given by

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}. \quad (4)$$

In Appendix B, we show that the equilibrium implications for portfolio choice and information acquisition are similar to the ones we obtain here.

3.1 Portfolio Choice

We solve the optimization problem (3) recursively. That is, we first consider the second-period problem of selecting an optimal portfolio given the realizations of the signals. We concentrate on an individual investor residing in the Home country, and for ease of notation, we thus drop the superscript $j$.

In order to infer the asset payoffs, the investor engages in Bayesian updating. Given his prior and his signals $s = (s_1, s_2)^T$, his posterior distribution over the vector of asset payoffs $\tilde{v} = (\tilde{v}_1, \tilde{v}_2)^T$ is multivariate normal with mean

$$\tilde{\mu} = \begin{pmatrix} \tilde{\mu}_1 \\ \tilde{\mu}_2 \end{pmatrix} = q + \Sigma \Sigma_w^{-1} (s - q) \quad (5)$$
and variance-covariance matrix

\[ \hat{\Sigma} = \Sigma - \Sigma \Sigma_y^{-1} \Sigma, \]  

(6)

where

\[ \Sigma_y = \begin{pmatrix} \sigma_1^2 + \frac{1}{y_1} & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 + \frac{1}{y_2} \end{pmatrix}. \]

The second-period problem — to maximize expected utility of final wealth under CARA utility and normally distributed payoffs — has a well-known solution (Grossman, 1976):

\[ D^* = \begin{pmatrix} D^*_1 \\ D^*_2 \end{pmatrix} = \frac{1}{A} \hat{\Sigma}^{-1} (\hat{\mu} - (1 + r)P). \]  

(7)

### 3.2 Demand for Information

In the first period, the domestic investor decides on the precisions of the signals regarding the payoffs from the Home asset and the Foreign asset, respectively. His objective is to maximize the expected certainty equivalent of wealth. That is, in the beginning of the first period, he solves

\[ \max_{y \in \mathbb{R}_+^2} E_1 [-\ln (-U(y, s))] \]  

(8)

where \( y = (y_1, y_2) \) and \( U(y, s) \) is the value function from the second-period optimization. That is,

\[ U(y, s) = E_2 [-\exp \{-A (W_0 - C_{11}(y_1) - C_{12}(y_2)) (1 + r) + AD^* \tilde{v} - (1 + r)P\}]. \]  

(9)

The following lemma describes how we can reformulate the first-period problem.
Lemma 1 We can write the first-period problem in (8) as

$$\max_{y \in \mathbb{R}^2_+} \{ A(W_0 - C_{11}(y_1) - C_{12}(y_2))(1 + r) + \frac{1}{2} \left[ k_0 + \left( (q_1 - (1 + r)P_1)^2 + \sigma_1^2 \right) y_1 + \left( (q_2 - (1 + r)P_2)^2 + \sigma_2^2 \right) y_2 \right] \} \quad (10)$$

where

$$k_0 \equiv \frac{\sigma_2^2 (q_1 - (1 + r)P_1)^2 - 2 \sigma_1 \sigma_2 (q_1 - (1 + r)P_1)(q_2 - (1 + r)P_2) + \sigma_1^2 (q_2 - (1 + r)P_2)^2}{\sigma_1^2 \sigma_2^2 - \sigma_{12}^2}$$

Proof. See Appendix A. \]

Hence, we can separate the objective function $V(y_1, y_2)$ into a constant term and two functions $V_1(y_1)$ and $V_2(y_2)$, which only depend on $y_1$ and $y_2$, respectively. These functions are given by

$$V_1(y_1) = \frac{1}{2} \left( (q_1 - (1 + r)P_1)^2 + \sigma_1^2 \right) y_1 - A(1 + r)a_1 y_1^c \quad (11)$$

and

$$V_2(y_2) = \frac{1}{2} \left( (q_2 - (1 + r)P_2)^2 + \sigma_2^2 \right) y_2 - A(1 + r)b_1 y_2^c \quad (12)$$

Taking derivatives, we obtain

$$V'_1(y_1) = \frac{1}{2} \left( (q_1 - (1 + r)P_1)^2 + \sigma_1^2 \right) - A(1 + r)a_1 c y_1^{c-1}$$

and

$$V'_2(y_2) = \frac{1}{2} \left( (q_2 - (1 + r)P_2)^2 + \sigma_2^2 \right) - A(1 + r)b_1 c y_2^{c-1}.$$  

Therefore, for $0 < c < 1$, $V_i(y_i)$ is an decreasing-increasing function on the domain $y_i \geq 0$, and for $c > 1$, $V_i(y_i)$ is an increasing-decreasing function on the domain $y_i \geq 0$. Given that $c \neq 1$, the extremum points are

$$\hat{y}_1 = \left( \frac{(q_1 - (1 + r)P_1)^2 + \sigma_1^2}{2A(1 + r)a_1 c} \right)^{\frac{1}{c}} \quad (15)$$
and

\[ \hat{y}_2 = \left( \frac{(q_2 - (1 + r)P_2)^2 + \sigma_2^2}{2A(1 + r)b_1c} \right)^{-1/2}, \]

respectively. Hence, if \( 0 < c < 1 \), the objective function \( V(y_1, y_2) \) attains a global minimum on the domain \( \{(y_1, y_2) | y_1 \geq 0, y_2 \geq 0\} \) at \((\hat{y}_1, \hat{y}_2)\), whereas if \( c > 1 \), \((\hat{y}_1, \hat{y}_2)\) is instead a global maximum on that domain.

Since, for \( 0 < c < 1 \), the agent can achieve an infinite value on the objective function by letting \( y_1 \to \infty \) or \( y_2 \to \infty \), and we do not observe people investing infinite amounts in precise information, we rule out concave cost functions. For \( c = 1 \) and \( \frac{1}{2} \left( (q_1 - (1 + r)P_1)^2 + \sigma_1^2 \right) > A(1 + r)a_1 \) or \( \frac{1}{2} \left( (q_2 - (1 + r)P_2)^2 + \sigma_2^2 \right) > A(1 + r)b_1 \), the agent can likewise achieve an infinite value on the objective function by letting \( y_1 \to \infty \) or \( y_2 \to \infty \), so we rule out these parameter values as well.

Given these parameter restrictions, the problem in (3) is well-defined and in the proposition below, we determine the optimal solutions.

**Proposition 2** Treating prices as exogenous, an optimal solution to the information acquisition problem is

- **Case 1**, \( c > 1 \): \((\hat{y}_1, \hat{y}_2)\);
- **Case 2**, \( c = 1 \), \( \frac{1}{2} \left( (q_1 - (1 + r)P_1)^2 + \sigma_1^2 \right) < A(1 + r)a_1 \), and \( \frac{1}{2} \left( (q_2 - (1 + r)P_2)^2 + \sigma_2^2 \right) < A(1 + r)b_1 \): \((0, 0)\);
- **Case 3**, \( c = 1 \), \( \frac{1}{2} \left( (q_1 - (1 + r)P_1)^2 + \sigma_1^2 \right) = A(1 + r)a_1 \), and \( \frac{1}{2} \left( (q_2 - (1 + r)P_2)^2 + \sigma_2^2 \right) < A(1 + r)b_1 \): \((t, 0), t \in \mathbb{R}_+\);
- **Case 4**, \( c = 1 \), \( \frac{1}{2} \left( (q_1 - (1 + r)P_1)^2 + \sigma_1^2 \right) < A(1 + r)a_1 \), and \( \frac{1}{2} \left( (q_2 - (1 + r)P_2)^2 + \sigma_2^2 \right) = A(1 + r)b_1 \): \((0, u), u \in \mathbb{R}_+\);
- **Case 5**, \( c = 1 \), \( \frac{1}{2} \left( (q_1 - (1 + r)P_1)^2 + \sigma_1^2 \right) = A(1 + r)a_1 \), and \( \frac{1}{2} \left( (q_2 - (1 + r)P_2)^2 + \sigma_2^2 \right) = A(1 + r)b_1 \): \((t, u) \in \mathbb{R}_+^2\).
Proof. See the above discussion. ■

To gain further insight into the information acquisition process, it is helpful to consider the symmetric case, in which expected payoffs, prices and standard deviations are equal \((q_1 = q_2 = q, P_1 = P_2 = P, \text{and } \sigma_1 = \sigma_2 = \sigma)\). In this case, the investor will acquire information regarding both assets, but he will acquire more information regarding the Home asset than the Foreign asset \((y_1 > y_2)\), if his cost functions are convex \((c > 1)\), since

\[
\frac{\hat{y}_1}{y_2} = \left( \frac{b_1}{a_1} \right)^{\frac{1}{c-1}}
\]  \hfill (17)

and \(0 < a_1 < b_1\). Notably, the precision acquired regarding the Home asset relative to the precision acquired regarding the Foreign asset depends only on the features of the cost functions. It does not depend on wealth, risk aversion or risk, for example.

We can also study the difference in absolute terms. For convex \((c > 1)\) cost functions, the absolute difference is

\[
y_1 - y_2 = \left( \frac{(q - (1+r)P)^2 + \sigma^2}{2A(1+r)c} \right)^{\frac{1}{c-1}} \left( \left( \frac{1}{a_1} \right)^{\frac{1}{c-1}} - \left( \frac{1}{b_1} \right)^{\frac{1}{c-1}} \right). \hfill (18)
\]

Since \(0 < a_1 < b_1\), this difference is positive. This absolute difference is increasing in expected excess returns \((q - (1+r)P)\) and risk \((\sigma)\), whereas it is decreasing in risk aversion \((A)\). This occurs because with higher expected excess returns and higher risk, the benefits of acquiring more information regarding both assets will be higher.

Further, as the investor’s risk aversion increases, he becomes more cautious about investing in information acquisition.
In the case of symmetric assets, the investor’s information acquisition is linked to his expected asset demand through the following relations:

\[
E_1[D^*_1] = \frac{1}{A}(q - (1 + r)P) \left( \frac{1}{\sigma^2(1 + \rho)} + y_1 \right) \tag{19}
\]

\[
E_1[D^*_2] = \frac{1}{A}(q - (1 + r)P) \left( \frac{1}{\sigma^2(1 + \rho)} + y_2 \right), \tag{20}
\]

where \(\rho\) is the correlation between the asset payoffs. Thus, we have that

\[
E_1[D^*_1] - E_1[D^*_2] = \frac{1}{A}(q - (1 + r)P) (y_1 - y_2). \tag{21}
\]

Without information acquisition, the expected demands will be the same. Therefore, if this difference is greater than zero, we can say that we expect an equity home bias. With the assumption that expected equity returns exceed the return on the risk free asset, we expect an equity home bias if the precision of the acquired information regarding the Home asset’s payoff is greater than the precision of the acquired information regarding the Foreign asset’s payoff \((y_1 > y_2)\). If \(y_1 = y_2\) (as is the case when the investor chooses the corner solution \((0, 0)\)), we do not expect there to be an equity home bias.

If cost functions are convex \((c > 1)\) and expected excess returns \((q - (1 + r)P)\) are positive, then the size of the home bias (as measured by \(E_1[D^*_1] - E_1[D^*_2]\)) is positive and increasing in expected excess returns \((q - (1 + r)P)\) and risk \((\sigma)\), whereas it is decreasing in risk aversion \((A)\). As seen in equation (21), the difference in expected demands is the product of three factors, one of which is the difference in information acquisition \((y_1 - y_2)\). For an explanation of the effects of increasing expected excess returns, risk and risk aversion on this factor, we refer to the discussion below.

\footnote{The result follows from taking expectations on both sides of the demand equation (equation (7)), and inserting \(\sigma_{12} = \rho \sigma^2\) (where \(\rho\) is the coefficient of correlation between the asset payoffs).}
equation (18). The two other factors serve to amplify the effects of increasing excess returns and risk aversion; even without information acquisition, the investor will increase his holdings of both assets as expected excess returns increase, whereas he will decrease his holdings as his risk aversion increases.

4 Calibration

In order to gauge the effects of differences in information costs, we calibrate our model to the data. We use annual data from Robert Shiller’s website\(^3\) on real cum-dividend returns and real one-year interest rates in the period from 1871 to 2004. According to his data, the average annual real cum-dividend return on the S&P Composite Index is 8.26% and the volatility of these returns is 17.7%. The average real one-year US interest rate over the same period is 1.03%.

We assume that there are two risky assets, one in the Home country and one in the Foreign country. In order to be able to focus on the effects of differences in information costs, we assume that, except for information costs, they have identical characteristics. They each have a price of \( P_1 = P_2 = P = \$100 \) and an expected payoff of \( q_1 = q_2 = q = \$108 \), i.e., the expected rate of return on both assets is 8%, roughly matching average real returns on the S&P Composite Index. We set the volatility of the payoffs to \( \sigma_1 = \sigma_2 = \sigma = \$18 \), i.e., 18% of the price of the assets – matching the volatility of real returns on the S&P Composite Index. We assume a correlation of \( \rho = 0.5 \) between the domestic and the foreign asset. In order to match the average one-year real US interest rate, we set the risk free interest rate to 1%.

\(^3\)See http://www.econ.yale.edu/~shiller/data.htm. We thank Robert Shiller for making his data available through the web.
In line with Peress (2004), we set the convexity parameter $c$ to 3. Note that if we write $a_1 = K \cdot \sigma^2c$, then $K$ denotes the cost of acquiring a signal regarding the Home asset’s payoff with the same volatility as the assets. Initially, we set the parameter $K$ to $1,000$.  

We adjust the coefficient of absolute risk aversion ($A$) so that the expected total amount invested in the risky assets matches the 1994 Panel Study of Income Dynamics (PSID). According to this study, an average household in the United States with positive financial wealth and positive stockholdings had $95,881 invested in stocks, and on average, these households allocated 55% of their financial wealth to stocks (Vissing-Jørgensen, 2002). To match these numbers, we set the coefficient of absolute risk aversion to $A = 8.6 \cdot 10^{-5}$. At this level of risk aversion, the investors in our model will invest approximately $97,000 in the risky assets, assuming that it is twice as expensive to acquire information about the Foreign asset as compared to acquiring information about the Home asset.

In order to measure the equity home bias, we use the ratio of expected amounts invested in the assets:

$$
\varphi \equiv \frac{E_1[P_1D_1^*]}{E_1[P_2D_2^*]} = \frac{E_1[D_1^*]}{E_1[D_2^*]} = \frac{1 + \left(\frac{(q-(1+r)P)^2 + \sigma^2}{2A(1+r)\sigma^2c}\right)^{\frac{1}{2}} \sigma^2(1 + \rho)}{1 + \left(\frac{(q-(1+r)P)^2 + \sigma^2}{2A(1+r)b_1c}\right)^{\frac{1}{2}} \sigma^2(1 + \rho)}.
$$

Further, we define the multiple $\theta$ as $\theta \equiv b_1/a_1$. Thus, $(\theta - 1)$ measures how many times more expensive it is to acquire information about the payoff from the Foreign

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4Note, however, that Peress’ (2004) model differs from the one we employ here. He chooses $c$ to match the average share elasticity in the US. Since we use CARA preferences, the share elasticity does not depend on $c$ and therefore, a similar procedure would not be possible in our framework.

Nevertheless, we believe that $c$ should be somewhere in the neighbourhood of what Peress (2004) obtains.

5The parameter $K$ should reflect opportunity costs.
asset as compared to the cost of acquiring information about the Home asset.

Figure 1 shows the ratio of expected investments ($\varphi$) as a function of the multiple $\theta$. It suggests that only a small fraction of the equity home bias can be explained by differences in information costs: as $\theta$ varies from 1 to 10, the ratio $\varphi$ varies from 1 to only 1.89. In fact, even as the differences in information costs become infinitely large, the ratio of expected investments remains small (it approaches an asymptotic value of 3.23). As Table 1 shows, although the ratio of expected investments increases with decreasing information costs ($a_1$), the values are nowhere near the ones reported by Dahlquist et al. (2003).\footnote{Dahlquist et al. (2003) find that an average US investor allocates 91% of his equity portfolio to US equity, although US equity represents only 49% of the world market portfolio.} Even with $a_1$ approaching zero, it would require a value on $\theta$ of 81 in order to generate a ratio $\varphi$ of 9, i.e., even as $a_1$ becomes negligible, the cost of acquiring information about the Foreign asset’s payoff would have to be 80 times higher than the cost of acquiring information about the Home asset’s payoff, in order for the values to be in line with the empirical findings in Dahlquist et al. (2003).

Note, however, that for values on the convexity parameter $c$ that are close to one (but still greater than one), we can in fact achieve a realistic value on the ratio $\varphi$ for relatively small differences in information costs. As an example, consider the case in which $c = 1.2$. With $c = 1.2$, and all the other parameter values as before ($P_1 = P_2 = P = $100, $q_1 = q_2 = q = $108, $\sigma_1 = \sigma_2 = \sigma = $18, $\rho = 0.5$ and $K = $1000), absolute risk aversion ($A$) would have to be equal to $2.92 \cdot 10^{-4}$ in order for the expected amount invested in the risky assets to be in line with PSID (assuming that the cost of acquiring information regarding the Foreign asset
is twice as high as the cost of acquiring information regarding the Home asset, the investor will invest approximately $97,000 in the risky assets). With these parameter values, it would suffice with $\theta = 1.76$ to produce a $\varphi$ of 9. That is, the cost of acquiring information regarding the Foreign asset would only need to be 76% higher than the cost of acquiring information regarding the Home asset in order for the ratio of expected investments to be in line with Dahlquist et al. (2003). However, the total amount spent on acquiring information now seems implausibly large — approximately $20,000, as compared to the $5,600 spent in the case when $c = 3$ (and $\theta = 2$). Assuming higher values on the convexity parameter ($c$) and adjusting absolute risk aversion ($A$) so that the expected investments match PSID, the total amount spent on acquiring information is smaller and the values on the ratio of expected investments are lower.

5 Conclusions

Within a stylized two-country model of information acquisition, we evaluate the information cost explanation to the equity home bias puzzle, alluded to by Lewis (1999), Ahearne, Griever and Warnock (2004), and Portes and Rey (2005), among others. We show that the expected size of the equity home bias—in terms of differences in invested amounts in the Home and Foreign asset—is positive and increasing in expected excess returns and risk, but decreasing in risk aversion. Despite its intuitive appeal, a calibration to US data suggests that the information cost explanation accounts only for a small fraction of the observed equity home bias.
Figure 1. The ratio of the expected investment in the Home asset to the expected investment in the Foreign asset ($\varphi$) as a function of the ratio of foreign to domestic information costs ($\theta$). Other parameter values are: $A = 8.6 \cdot 10^{-5}$, $c = 3$, $P_1 = P_2 = P = $100, $q_1 = q_2 = q = $108, $\sigma_1 = \sigma_2 = \sigma = $18, $\rho = 0.5$ and $a_1 = $1000 \cdot 18^6$.

Table 1. The table shows the ratio of the expected investment in the Home asset to the expected investment in the Foreign asset ($\varphi$) as the ratio of foreign to domestic information costs ($\theta$) varies from 1 to 10, and the cost ($K$) of a signal with the same precision as the asset payoffs varies from $1000$ to 10 cents. The values on the other parameters are $A = 8.6 \cdot 10^{-5}$, $c = 3$, $P_1 = P_2 = P = $100, $q_1 = q_2 = q = $108, $\sigma_1 = \sigma_2 = \sigma = $18, $\rho = 0.5$ and $a_1 = K \cdot 18^6$.

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Appendix A

Proof of Lemma 1

Since, according to the investor’s beliefs in the beginning of the second period, the distribution of \( \tilde{\nu} - (1 + r)P \) is multivariate normal,

\[
\tilde{\nu} - (1 + r)P \sim MVN(\hat{\mu} - (1 + r)P, \hat{\Sigma}),
\]

we can formulate the value function in (9) as

\[
U(y, s) = -\exp \left\{ -A (W_0 - C_{11}(y_1) - C_{12}(y_2)) (1 + r) + \frac{1}{2} (\hat{\mu} - (1 + r)P)' \hat{\Sigma}^{-1} (\hat{\mu} - (1 + r)P) \right\}. \tag{23}
\]

Therefore, the first-period problem is

\[
\max_{y \in \mathbb{R}^2_+} E_1 \left[ A (W_0 - C_{11}(y_1) - C_{12}(y_2)) (1 + r) + \frac{1}{2} (\hat{\mu} - (1 + r)P)' \hat{\Sigma}^{-1} (\hat{\mu} - (1 + r)P) \right]. \tag{24}
\]

Taking the expectation of the quadratic form \((\hat{\mu} - (1 + r)P)' \hat{\Sigma}^{-1} (\hat{\mu} - (1 + r)P)\) yields

\[
E_1 \left[ (\hat{\mu} - (1 + r)P)' \hat{\Sigma}^{-1} (\hat{\mu} - (1 + r)P) \right] = Tr \left( \hat{\Sigma}^{-1} Var_1 [\hat{\mu}] \right) +
\]

\[
(q - (1 + r)P)' \hat{\Sigma}^{-1} (q - (1 + r)P), \tag{25}
\]

where \(Tr\) denotes the trace and \(Var_1 [\hat{\mu}]\) is the variance-covariance matrix of \(\hat{\mu}\) at the beginning of the first period. By equation (5),

\[
Var_1 [\hat{\mu}] = \Sigma \Sigma_y^{-1} Var_1 [s] \Sigma_y^{-1} \Sigma = \Sigma \Sigma_y^{-1} \Sigma,
\]

where the second equality follows from the fact that \(Var_1 [s] = \Sigma_y\). Hence, we can formulate the first-period problem in (24) as

\[
\max_{y \in \mathbb{R}^2_+} \left\{ A (W_0 - C_{11}(y_1) - C_{12}(y_2)) (1 + r) + \right\}
\]

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Appendix B. A Noisy Rational Expectations Equilibrium

In this note, we derive a (partial) noisy rational expectations equilibrium. In order to achieve that, we first solve the portfolio choice problem of an individual investor, basing our solution on a conjecture regarding the equilibrium price functions. Later, upon aggregation, we confirm this conjecture. Finally, we consider the investors’ information acquisition and discuss its consequences for the so-called equity home bias phenomenon.

In the equilibrium analysis, we assume that before the beginning of the first period, investors receive prior information regarding the payoffs in the form of signals \((\tilde{q}_1^j, \tilde{q}_2^j)\) given by

\[ \tilde{q}_i^j = \tilde{v}_i + \tilde{\eta}_i^j \text{ for } i = 1, 2, \]

where the error terms, \(\tilde{\eta}_1^j\) and \(\tilde{\eta}_2^j\), are independent of \(\tilde{v}_1, \tilde{v}_2, \tilde{\varepsilon}_1^j\) and \(\tilde{\varepsilon}_2^j\). They are independent draws from the distribution

\[
(\tilde{\eta}_1^j, \tilde{\eta}_2^j)^T \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right).
\]

To simplify the equilibrium analysis, we assume that the investors’ prior over the payoffs is such that its density function is given by a constant over \(\mathbb{R}^2\).\(^7\) This prior distribution captures the notion that investors’ initial beliefs regarding the payoffs

\[\frac{1}{2} \left( \frac{\sigma_2^2 (q_1 - (1 + r)P_1)^2}{\sigma_1^2 \sigma_2^2 - \sigma_1^2 \sigma_2^2} - 2 \sigma_{12} (q_1 - (1 + r)P_1) (q_2 - (1 + r)P_2) + \sigma_1^2 (q_2 - (1 + r)P_2)^2 \right) + \left( (q_1 - (1 + r)P_1)^2 + \sigma_1^2 \right) y_1 + \left( (q_2 - (1 + r)P_2)^2 + \sigma_2^2 \right) y_2. \]

\[19\]
are very diffuse, and therefore, they will base their posteriors solely on received information.

The per-capita supply of the Home country’s risky asset is $x_1 + \tilde{x}_1$, and the per-capita supply of the Foreign country’s risky asset is $x_2 + \tilde{x}_2$, where $x_1$ and $x_2$ are non-negative constants, and $\tilde{x}_1$ and $\tilde{x}_2$ are normally distributed random variables independent of $\tilde{v}_1$, $\tilde{v}_2$, $\tilde{\xi}_1^j$, $\tilde{\xi}_2^j$, $\tilde{\eta}_1^j$ and $\tilde{\eta}_2^j$, with zero mean and a variance-covariance matrix given by

$$
\Sigma_x = \begin{pmatrix}
\sigma_{x_1}^2 & 0 \\
0 & \sigma_{x_2}^2
\end{pmatrix}.
$$

The noisy parts of the supply — $\tilde{x}_1$ and $\tilde{x}_2$, respectively — ensure that the equilibrium asset prices are not fully revealing with regard to payoffs.

Upon aggregation, we impose a strong version of the law of large numbers, so that if $\tilde{Z}^j$ are independent random variables across $j \in h_1 \cup h_2$, with the same mean $E\left[\tilde{Z}^j\right] = Z$ and with uniformly bounded variances $\text{Var}\left[\tilde{Z}^j\right]$, then

$$
\int \tilde{Z}^j dj = E\left[\tilde{Z}^j\right] = Z.
$$

B.1 Portfolio Choice

The portfolio choice problem is similar to the one in the case of exogenous prices, except that the investor now also uses the (imperfect) information regarding the future payoffs contained in the equilibrium asset prices. We conjecture, and later we will verify that there is a unique linear asset market equilibrium in which asset prices are given by

$$
P_1 = \frac{1}{1 + r} (d_0 + \tilde{v}_1 - d_1 \tilde{x}_1),
$$

$$
P_2 = \frac{1}{1 + r} (\delta_0 + \tilde{v}_2 - \delta_1 \tilde{x}_2),
$$

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where \( d_0, \delta_0, d_1 \) and \( \delta_1 \) are constants (\( d_1 \neq 0, \delta_1 \neq 0 \)), \( \tilde{x}_1 \) is the stochastic part of the per-capita supply of the Home asset, and \( \tilde{x}_2 \) is the stochastic part of the per-capita supply of the Foreign asset.

Given his diffuse prior, his prior information, his private signal, and the equilibrium asset prices, the individual agent’s posterior distribution over the vector of asset payoffs is multivariate normal with mean

\[
\hat{\mu}^j = \begin{pmatrix}
\hat{\mu}_1^j \\
\hat{\mu}_2^j
\end{pmatrix} = \begin{pmatrix}
\frac{1}{\sigma_1^2} q_1^j + y_1^j s_1^j + \frac{1}{\sigma_{x1}^2} ((1+r)P_1 - d_0) \\
\frac{1}{\sigma_1^2} q_1^j + y_1^j s_1^j + \frac{1}{\sigma_{x1}^2} ((1+r)P_2 - \delta_0) \\
\frac{1}{\sigma_1^2} q_1^j + y_1^j s_1^j + \frac{1}{\sigma_{x1}^2} ((1+r)P_1 - d_0) \\
\frac{1}{\sigma_1^2} q_1^j + y_1^j s_1^j + \frac{1}{\sigma_{x1}^2} ((1+r)P_2 - \delta_0)
\end{pmatrix} \tag{33}
\]

and variance-covariance matrix

\[
\Sigma^j = \begin{pmatrix}
\left(\hat{\sigma}_1^j\right)^2 & 0 \\
0 & \left(\hat{\sigma}_2^j\right)^2
\end{pmatrix} = \begin{pmatrix}
\frac{1}{\sigma_1^2} q_1^j + y_1^j s_1^j & 0 \\
0 & \frac{1}{\sigma_2^2} q_2^j + y_2^j s_2^j
\end{pmatrix} \tag{34}
\]

Thus, the investor’s demand for the risky assets is

\[
D_1^j = \frac{\hat{\mu}_1^j - (1+r)P_1}{A \left(\hat{\sigma}_1^j\right)^2} = \frac{1}{A} \left( \frac{1}{\sigma_1^2} q_1^j + y_1^j s_1^j + \frac{1}{\sigma_{x1}^2} ((1+r)P_1 - d_0) + \right.
\]

\[
- \left( \frac{1}{\sigma_1^2} q_1^j + y_1^j s_1^j \right) (d_0 + \tilde{v}_1 - d_1 \tilde{x}_1) \right) \tag{35}
\]

\[
D_2^j = \frac{\hat{\mu}_2^j - (1+r)P_2}{A \left(\hat{\sigma}_2^j\right)^2} = \frac{1}{A} \left( \frac{1}{\sigma_2^2} q_2^j + y_2^j s_2^j + \frac{1}{\sigma_{x2}^2} ((1+r)P_2 - \delta_0) + \right.
\]

\[
- \left( \frac{1}{\sigma_2^2} q_2^j + y_2^j s_2^j \right) (\delta_0 + \tilde{v}_2 - \delta_1 \tilde{x}_1) \right) \tag{36}
\]

\begin{tabular}{l}
(Grossman, 1976).
\end{tabular}

**B.2 Equilibrium Asset Prices**

Given the investor’s demand for the risky assets expressed by equations (35) and (36), we can solve for the constants \( (d_0, d_1, \delta_0 \text{ and } \delta_1) \) in equations (31) and
(32) through the market-clearing condition, by using the method of undetermined coefficients. Hereby, we confirm the linear equilibrium price functions conjectured in equations (31) and (32), and show that they constitute a unique linear asset market equilibrium. First, however, we need to provide a precise definition of our equilibrium:

**Definition 3** A (partial) noisy rational expectations equilibrium is a set of information choices \((y_j^1, y_j^2)\), a set of asset prices \((P_1, P_2)\), and a set of portfolio allocations \((D_{1j}^*, D_{2j}^*)\), such that

1. \((P_1, P_2)\) is measurable with respect to \((\tilde{x}_1, \tilde{x}_2, \tilde{v}_1, \tilde{v}_2)\);
2. \((y_j^1, y_j^2)\) maximizes the period-1 expected certainty equivalent wealth subject to the nonnegativity constraints;
3. Given \((P_1, P_2)\) and investors’ posterior beliefs, \((D_{1j}^*, D_{2j}^*)\) solves the portfolio allocation problem for investor \(j \in h_1 \cup h_2\);
4. Asset markets clear: \(\int_j D_{1j}^* dj = \pi_i + \tilde{x}_i\) for \(i = 1, 2\) almost surely;
5. Investors’ beliefs are updated through Bayes’ law;
6. Period-1 beliefs regarding \((D_{1j}^*, D_{2j}^*)\) are consistent with the distribution of optimal portfolio allocations \((D_{1j}^*, D_{2j}^*)\), conditional on the available information.

The market clearing condition (iv) implies that

\[
\int_j D_{1j}^* dj = \frac{1}{A} \left( \frac{1}{\sigma_1^2} \int_j q_j^1 dj + \int_j y_j^1 s_j^1 dj + \frac{1}{d_1^2 \sigma_{x1}^2} ((1 + r)P_1 - d_0) \right. \\
\left. - \left( \frac{1}{\sigma_1} + \int_j y_j^1 dj + \frac{1}{d_1^2 \sigma_{x1}^2} \right) (d_0 + \tilde{v}_1 - d_1 \tilde{x}_1) \right) = \pi_1 + \tilde{x}_1
\]  

(37)
\[
\int_{\mathcal{D}_j} D^*_j \, dj = \frac{1}{A} \left( \frac{1}{\sigma^2_j} \int_{\mathcal{D}_j} q^*_j \, dj + \int_{\mathcal{D}_j} y^*_j \, dq^*_j + \frac{1}{\delta^2_1 \sigma^2_{x_j}} ((1 + r) P_2 - \delta_0) \right)
- \left( \frac{1}{\sigma^2_j} + \int_{\mathcal{D}_j} y^*_j \, dj + \frac{1}{\delta^2_1 \sigma^2_{x_j}} \right) (\delta_0 + \tilde{v}_2 - \delta_1 \tilde{x}_2) = \varphi_2 + \tilde{x}_2 \tag{38}
\]

Concentrating on the first asset, we have
\[
\int_{\mathcal{D}_j} D^*_j \, dj = \frac{1}{A} \left( \frac{\tilde{v}_1}{\sigma^2_1} + \overline{y}_1 \tilde{v}_1 + \frac{1}{d^2_1 \sigma^2_{x_1}} ((1 + r) P_1 - d_0) \right)
- \left( \frac{1}{\sigma^2_1} + \overline{y}_1 + \frac{1}{d^2_1 \sigma^2_{x_1}} \right) (d_0 + \tilde{v}_1 - d_1 \tilde{x}_1) = \varphi_1 + \tilde{x}_1, \tag{39}
\]

where \( \overline{y}_1 = \int_{\mathcal{D}_j} y^*_j \, dj \) is the "average" precision of the first signal. Thus, we obtain
\[
\frac{(1 + r) P_1}{d^2_1 \sigma^2_{x_1}} = \left( A \varphi_1 + \frac{d_0}{d^2_1 \sigma^2_{x_1}} + \frac{1}{\sigma^2_1} + \overline{y}_1 + \frac{1}{d^2_1 \sigma^2_{x_1}} d_0 \right) + \frac{1}{d^2_1 \sigma^2_{x_1}} \tilde{v}_1 +
\left( A - d_1 \left( \frac{1}{\sigma^2_1} + \overline{y}_1 + \frac{1}{d^2_1 \sigma^2_{x_1}} \right) \right) \tilde{x}_1 \tag{40}
\]

The method of undetermined coefficients yields the following system of equations for \( d_0 \) and \( d_1 \):

\[
A \varphi_1 + \frac{d_0}{d^2_1 \sigma^2_{x_1}} + \frac{1}{\sigma^2_1} + \overline{y}_1 + \frac{1}{d^2_1 \sigma^2_{x_1}} d_0 = \frac{d_0}{d^2_1 \sigma^2_{x_1}} \tag{41}
\]

\[
A - d_1 \left( \frac{1}{\sigma^2_1} + \overline{y}_1 + \frac{1}{d^2_1 \sigma^2_{x_1}} \right) = - \frac{d_1}{d^2_1 \sigma^2_{x_1}} \tag{42}
\]

Solving for \( d_1 \) in equation (42), we have
\[
d_1 = \frac{A}{\sigma^2_1 + \overline{y}_1}, \tag{43}
\]

and solving for \( d_0 \) in equation (41), we obtain
\[
d_0 = - \frac{A \varphi_1}{\sigma^2_1 + \overline{y}_1 + \frac{1}{d^2_1 \sigma^2_{x_1}}}, \tag{44}
\]
which can be solved for explicitly, given the solution of \(d_1\) in equation (43):

\[
d_0 = -\frac{A\bar{\sigma}_1}{\left(\frac{1}{\bar{\sigma}_1^2} + \bar{y}_1 + \frac{(\bar{\sigma}_1^2 + \bar{y}_1)}{A^2\bar{\sigma}_x^2}\right)^2}. \tag{45}
\]

Completely analogously, we can solve for \(\delta_0\) and \(\delta_1\), and the solutions are given by

\[
\delta_0 = -\frac{A\bar{\sigma}_2}{\left(\frac{1}{\bar{\sigma}_2^2} + \bar{y}_2 + \frac{(\bar{\sigma}_2^2 + \bar{y}_2)}{A^2\bar{\sigma}_x^2}\right)^2}, \tag{46}
\]

and

\[
\delta_1 = \frac{A}{\frac{1}{\bar{\sigma}_2^2} + \bar{y}_2}, \tag{47}
\]

where \(\bar{y}_2 = \int_j y_j^2 dj\) is the "average" precision of the second signal.

### B.3 Demand for Information

In the beginning of the first period, investors decide upon the precisions \((y_j^1\) and \(y_j^2\)) of their signals. As we will see, these choices together with the non-stochastic parts of the asset supply determine whether the equity home bias is expected to prevail in equilibrium. To simplify the analysis, we focus on convex \((c > 1)\) cost functions.

For a domestic investor, the first-period problem is

\[
\max_{y_j^1 \in \mathbb{R}^+} \left\{ E_1 \left[ A \left( W_0^j - C_{11}(y_j^1) - C_{12}(y_j^2) \right) (1 + r) \right. \right.
\]

\[
\left. \left. + \frac{1}{2} \left( \hat{\mu}_1^j - (1 + r)P_1 \right)^2 \right] \right\}. \tag{48}
\]

The lemma below shows how we can reformulate this problem.

**Lemma 4** We can write the first-period problem in (48) as

\[
\max_{y_j^1 \in \mathbb{R}^+} \left\{ A \left( W_0^j - a_1(y_j^1)^c - b_1(y_j^2)^c \right) (1 + r) + \right. \]

\[
\left. \frac{1}{2} \left( \hat{\mu}_1^j - (1 + r)P_1 \right)^2 \right\}. \tag{48}
\]
Therefore, the period-1 expectation is

\[ E \left[ \hat{\mu}_1^i - (1 + r)P_1 \right] = q_i^1 - d_0 - q_i^1 = -d_0. \]  

(51)

Further, the period-1 variance can be determined through the relation

\[ Var_1 \left[ \hat{\mu}_1^i - (1 + r)P_1 \right] = Var_1 \left[ E \left[ \hat{\mu}_1^i - (1 + r)P_1 \mid \tilde{v}_1 \right] \right] + E_1 \left[ Var_1 \left[ \hat{\mu}_1^i - (1 + r)P_1 \mid \tilde{v}_1 \right] \right]. \]

(52)

The period-1 conditional expectation is

\[ E_1 \left[ \hat{\mu}_1^i - (1 + r)P_1 \mid \tilde{v}_1 \right] = \frac{1}{\sigma_1^i} q_i^1 + \frac{y_i^1}{\sigma_1^i} \tilde{v}_1 + \frac{1}{\frac{1}{\sigma_1^i} + y_i^1 + \frac{1}{\sigma_1^i}} \tilde{v}_1 - \tilde{v}_1. \]

(53)

Thus, we have

\[ Var_1 \left[ E_1 \left[ \hat{\mu}_1^i - (1 + r)P_1 \mid \tilde{v}_1 \right] \right] = \left( \frac{y_i^1}{\frac{1}{\sigma_1^i} + y_i^1 + \frac{1}{\sigma_1^i}} + 1 \right)^2 \sigma_1^2 = \frac{\frac{1}{\sigma_1^i}}{\left( \frac{1}{\sigma_1^i} + y_i^1 + \frac{1}{\sigma_1^i} \right)^2}. \]

(54)

The period-1 conditional variance is

\[ Var_1 \left[ \hat{\mu}_1^i - (1 + r)P_1 \mid \tilde{v}_1 \right] = \frac{y_i^1}{\frac{1}{\sigma_1^i} + y_i^1 + \frac{1}{\sigma_1^i}}^2 + d_1^2 \sigma_{x1}^2. \]

(55)

Hence, the period-1 unconditional variance is

\[ \left( \sigma_1^i \right)^2 \equiv Var_1 \left[ \hat{\mu}_1^i - (1 + r)P_1 \right] = \frac{1}{\frac{1}{\sigma_1^i} + y_i^1 + \frac{1}{\sigma_1^i}} + d_1^2 \sigma_{x1}^2. \]

(56)
Analogously, we have that

$$E_1 \left[ \hat{\mu}_2 - (1 + r)P_2 \right] = q_2^j - \delta_0 - q_2^j = -\delta_0, \quad (57)$$

and

$$\left( \sigma_2^j \right)^2 \equiv \text{Var}_1 \left[ \hat{\mu}_2 - (1 + r)P_2 \right] = \frac{1}{\sigma_2^j} + y_2^j + \frac{1}{\delta_1^2 \sigma_2^j} + \delta_1^2 \sigma_2^j. \quad (58)$$

We can thus rewrite the period-1 problem as

$$\max_{y^j \in \mathbb{R}_+^2} \left\{ A \left( W_0^j - C_{11}(y_1^j) - C_{12}(y_2^j) \right) (1 + r) + \frac{\left( \sigma_1^j \right)^2}{2 \left( \sigma_1^j \right)^2} E_1 \left[ \left( \hat{\mu}_1^j - (1 + r)P_1 \right)^2 \right] + \frac{\left( \sigma_2^j \right)^2}{2 \left( \sigma_2^j \right)^2} E_1 \left[ \left( \hat{\mu}_2 - (1 + r)P_2 \right)^2 \right] \right\}. \quad (59)$$

Here, \( (\hat{\mu}_1^j - (1 + r)P_1)^2 / \left( \sigma_1^j \right)^2 \) has a noncentral chi-square distribution with one degree of freedom and a noncentrality parameter of \( d_0^j / \left( \sigma_1^j \right)^2 \). Similarly, \( (\hat{\mu}_2^j - (1 + r)P_2)^2 / \left( \sigma_2^j \right)^2 \) has a noncentral chi-square distribution with one degree of freedom and a noncentrality parameter of \( \delta_0^2 / \left( \sigma_2^j \right)^2 \). Therefore,

$$\frac{\left( \sigma_1^j \right)^2}{2 \left( \sigma_1^j \right)^2} E_1 \left[ \left( \hat{\mu}_1^j - (1 + r)P_1 \right)^2 \right] = \frac{\left( \sigma_1^j \right)^2}{2 \left( \sigma_1^j \right)^2} \left( 1 + \frac{d_0^2}{\left( \sigma_1^j \right)^2} \right) =$$

$$1 + \frac{1}{2} \left( \frac{d_0^2}{d_1^2 \sigma_{x_1}^2} + (d_1^2 \sigma_{x_1}^2 + d_0^2) \frac{1}{\sigma_1^2} \right) + \frac{1}{2} \left( d_1^2 \sigma_{x_1}^2 + d_0^2 \right) y_1^j, \quad (60)$$

and

$$\frac{\left( \sigma_2^j \right)^2}{2 \left( \sigma_2^j \right)^2} E_1 \left[ \left( \hat{\mu}_2^j - (1 + r)P_2 \right)^2 \right] =$$

$$1 + \frac{1}{2} \left( \frac{\delta_0^2}{\delta_1^2 \sigma_{x_2}^2} + (\delta_1^2 \sigma_{x_2}^2 + \delta_0^2) \frac{1}{\sigma_2^2} \right) + \frac{1}{2} \left( \delta_1^2 \sigma_{x_2}^2 + \delta_0^2 \right) y_2^j. \quad (61)$$

Hence, the period-1 optimization problem can be further rewritten as

$$\max_{y^j \in \mathbb{R}_+^2} \left\{ A \left( W_0^j - a_1(y_1^j)^c - b_1(y_2^j)^c \right) (1 + r) + \right.$$
\[ + \frac{1}{2} \left( \frac{d_0}{d_1^2 \sigma^2_{x_1}} + \left( d_1^2 \sigma^2_{x_1} + d_0^2 \right) \frac{1}{\sigma^2_1} \right) + \frac{1}{2} \left( \frac{\delta_0^2}{\delta_1^2 \sigma^2_{x_2}} + \left( \delta_1^2 \sigma^2_{x_2} + \delta_0^2 \right) \frac{1}{\sigma^2_2} \right) + \\
+ \frac{1}{2} \left( \frac{d_1^2}{d_2^2 \sigma^2_{x_1}} + d_2^0 \right) y_1^j + \frac{1}{2} \left( \delta_1^2 \sigma^2_{x_2} + \delta_0^2 \right) y_2^j \right]. \] (62)

Just as in the case where prices are exogenous, the objective function \( V(y_1^j, y_2^j) \) is separable in \( y_1^j \) and \( y_2^j \). Provided that \( c \neq 1 \), the extremum point of the objective function on the domain \( \{ (y_1^j, y_2^j) \mid y_1^j \geq 0, y_2^j \geq 0 \} \) is

\[
\hat{y}_1^j = \left( \frac{d_1^2 \sigma^2_{x_1} + d_0^2}{2A(1 + r)a_1 c} \right)^{-\frac{1}{r}} \quad \text{(63)}
\]

\[
\hat{y}_2^j = \left( \frac{\delta_1^2 \sigma^2_{x_2} + \delta_0^2}{2A(1 + r)b_1 c} \right)^{-\frac{1}{r}}. \quad \text{(64)}
\]

As in the case of exogenous asset prices: If \( 0 < c < 1 \), then \( \left( \hat{y}_1^j, \hat{y}_2^j \right) \) is a global minimum on the domain \( \{ (y_1^j, y_2^j) \mid y_1^j \geq 0, y_2^j \geq 0 \} \); whereas, if \( c > 1 \), then \( \left( \hat{y}_1^j, \hat{y}_2^j \right) \) constitutes a global maximum on the domain \( \{ (y_1^j, y_2^j) \mid y_1^j \geq 0, y_2^j \geq 0 \} \). We thus obtain the following proposition:

**Proposition 5** If cost functions are convex \( (c > 1) \), then a solution to the Home investor’s information acquisition problem (his reaction function) is given by equations (63) and (64).

**Proof.** See the above discussion. ■

**B.4 Equity Home Bias**

Now, we can turn our attention to the main issue of whether we can expect there to be a so-called equity home bias also in equilibrium. To simplify the analysis, we focus on the case in which cost functions are convex \( (c > 1) \).

Notice first that if investment opportunities are symmetric, so that \( \sigma_1 = \sigma_2 = \sigma, \sigma_{x_1} = \sigma_{x_2} = \sigma_x, \bar{\tau}_1 = \bar{\tau}_2 = \bar{\tau}, a_1 = b_2 \), and \( b_1 = a_2 \), then an equilibrium occurs when
investors in the two countries acquire equal amounts of information regarding their
domestic and foreign assets ($y^H_1 = y^F_2$ and $y^H_2 = y^F_1$, where $H$ denotes an investor
in the Home country, and $F$ denotes an investor in the Foreign country). We refer
to such an equilibrium as a "symmetric equilibrium." In such an equilibrium, the
average precisions are going to be identical, $\bar{y}_1 = \bar{y}_2$.

Second, notice that in equilibrium, the period-1 expected demand for the Home
asset for a Home investor is

$$E_1^j \left[ D_1^{*j} \right] = \bar{x}_1 + \frac{1}{A} \left( y^j_1 - \bar{y}_1 \right) (-d_0) = \bar{x}_1 + \frac{\left( y^j_1 - \bar{y}_1 \right)}{\left( \frac{1}{\sigma_1^2} + \bar{y}_1 + \frac{\left( \frac{1}{\sigma_1^2} + \bar{y}_1 \right)^2}{A^2 \sigma_1^2} \right)} \bar{x}_1. \quad (65)$$

Similarly, for a Home investor, the period-1 expected demand for the Foreign asset is

$$E_1^j \left[ D_2^{*j} \right] = \bar{x}_2 + \frac{1}{A} \left( y^j_2 - \bar{y}_2 \right) (-d_0) = \bar{x}_2 + \frac{\left( y^j_2 - \bar{y}_2 \right)}{\left( \frac{1}{\sigma_2^2} + \bar{y}_2 + \frac{\left( \frac{1}{\sigma_2^2} + \bar{y}_2 \right)^2}{A^2 \sigma_2^2} \right)} \bar{x}_2. \quad (66)$$

The intuition is that demands are tilted toward the assets that the investor knows
more about than the average investor.

If there are no private signals available for purchase, the period-1 expected equi-
librium demand is simply equal to the period-1 expected per-capita supply. That is,
in that case, the period-1 expected equilibrium demands are $\bar{x}_1$ and $\bar{x}_2$, respectively.

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To see this, first conjecture $y^H_1 = y^F_2$, $y^H_2 = y^F_1$ in the second step and then confirm that it is
consistent with investors’ choices in the first step and determine investors’ information acquisition.
Finally, confirm that this fulfills the definition of an equilibrium (Definition 3). Notice that since
the information acquisition problem is identical for all Home investors and all Foreign investors,
respectively, the Home investors’ information choices are going to be identical and the Foreign
investors’ information choices are going to be identical as well.

The result follows from comparing the expectations of equations (35) and (39).
Thus, using that case as a benchmark, we say that we can expect an equity home bias if \( E_1[D_i^j] > \pi_1 \) for \( j \in h_1 \). By equation (65), provided that the expected supply is strictly positive, this occurs whenever the demand for information regarding the Home asset’s payoff is above average. That is, provided that \( \pi_1 > 0 \), \( E_1[D_i^j] > \pi_1 \) for \( j \in h_1 \) whenever \( y_i^j > y_1 \). If the expected supply is zero (\( \pi_1 = 0 \)), there cannot be an equity home bias, even if the Home investor’s demand for information regarding the Home equity is above average (\( y_i^j > y_1 \)).

Taking the ratio between optimal information demands, we obtain

\[
\frac{y_1}{y_2} = \left( \frac{b_1}{a_1} \right)^{\frac{1}{2}}.
\] (67)

This ratio is greater than one since \( 0 < a_1 < b_1 \) and \( c > 1 \). It follows that the Home investor’s demand for information is above average (\( y_i^j - y_1 > 0 \)), and therefore, we can expect an equity home bias, provided that the expected supply is strictly positive.

What is the effect of the investors’ information acquisition on prices? In order to answer that question, consider expected prices in the beginning of the first period. These are given by

\[
E_1^j [P_1] = \frac{1}{1 + r} \left( q_i^j + d_0 \right) = \frac{1}{1 + r} \left( q_i^j - \frac{Ap_i^j}{\left( \frac{1}{\sigma_1^2} + y_1 + \frac{(1 + y_1)}{A^2 \sigma_2^2} \right)^2} \right)
\] (68)

and

\[
E_1^j [P_2] = \frac{1}{1 + r} \left( q_i^j + d_0 \right) = \frac{1}{1 + r} \left( q_i^j - \frac{Ap_i^j}{\left( \frac{1}{\sigma_1^2} + y_1 + \frac{(1 + y_1)}{A^2 \sigma_2^2} \right)^2} \right).
\] (69)
It follows that the effect of information acquisition on expected prices in the beginning of the first period is positive: the period-1 expected prices are at least as high when investors can learn as when they cannot, where the equality occurs when comparing to a situation when they can learn but choose not to. The intuition behind the result is that information acquisition lowers the perceived risk, both directly and indirectly. The direct effect is simply that as agents acquire more precise information regarding future payoffs through their signals, they perceive the payoffs as being less risky, while the indirect effect is due to the fact that as investors acquire more information, prices become more informative.
References


