House Prices, Real Estate Returns and the Business Cycle

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House Prices, Real Estate Returns, and the Business Cycle∗

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Abstract

The main objective of this work is to develop a general equilibrium business cycle model linking financial and real estate markets to the macroeconomy. The ability of a production economy to account simultaneously for asset pricing, business cycle and real estate market facts is then evaluated by comparing the model predictions to the empirical facts. The observed high volatility of house prices, the equity premium and the difference between equity and real estate excess returns can be explained without giving rise to excessive risk-free rate variation.

• Keywords: house prices, real estate returns, equity premium, business cycles, production economies.

• JEL: E30, E22, G12

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1 Introduction

1.1 Motivation and Overview

According to the IMF (2002), the probability of a house price boom being followed by a bust is about 40 percent, and housing price busts are associated on average with output loss of about 8 percent. However, while the recent worldwide surge in house prices has been identified as a potential threat to economic stability, the relationship, and the channels through which asset prices and the real economy interact remain poorly understood. Real estate being by far the largest component of household total wealth, understanding the determinants of the high volatility of house prices is yet an important issue. Is this volatility a normal phenomenon, or is it a symptom reflecting that real estate markets are not working properly? The answer to this question has important policy implications. If the high volatility of house prices is a signal that markets are failing to allocate resources efficiently, action from the monetary or the fiscal authorities may well be needed.

From a business cycle perspective, the fact that, in most economies, real estate represents around two-thirds of the tangible capital stock\(^1\) illustrates that, when it comes to understanding the determinants of aggregate variables, the housing market is likely to play a key role. In spite of that, while capital accumulation is the centerpiece of business cycle theory [see King and Rebelo (2000)], in most cases, studies aiming at explaining aggregate fluctuations ignore residential investment. Finally, while the case of equity has been extensively treated, very few studies have attempted to explain simultaneously real estate and equity returns in a model able to account for other dimensions of the real estate market.

As recently documented by Cochrane (2006), the lack of interactions between the finance and the macroeconomic literatures might explain why few studies have attempted to derive a unified framework linking financial and housing markets to the real economy. As far as the traditional literature on finance is concerned, the fact that in a large number of studies, the endowment economy assumption [see Lucas (1978)] is still the starting point of the analysis, constitute a major obstacle. By relying on models where quantities are fixed, and where the consumption and investment decision is omitted, it seems difficult to extend the scope of these studies beyond

the analysis of prices. Moreover, while this literature has been successful at explaining many pricing puzzles, the fact that in equilibrium, in standard endowment economy models consumption is exogenously determined, constitutes another important limitation. Finally, in the case of real estate, as illustrated by several studies that have followed this approach [Piazzesi, Schneider and Tuzel (2006); Davis and Heathcote (2006)], explaining the high observed equity premium while generating plausible housing market implications remains difficult.

When it comes to studies that have followed a more macroeconomic based approach, in most cases, linking asset prices to the real economy is found to constitute a challenging task. While successful at explaining quantities, as reported by Cochrane (2006), standard macroeconomic frameworks are often unable to deliver plausible asset pricing implications. In the case of the housing market, the difficulty to reproduce simultaneously the behavior of quantities and prices has been illustrated by a study of Davis and Heathcote (2005). While successful at explaining important dimensions of the housing market, modified versions of the neoclassical model fail however to explain key asset pricing facts such as the high volatility of house prices.

When it comes to reconciling macroeconomics with asset pricing, as shown by the literature initiated by Jermann (1998), general equilibrium business cycle models are promising candidates. In contrast to endowment economy models, the production approach has demonstrated that the equity premium puzzle could be reproduced within frameworks allowing consumption and dividends to be endogenously determined [see also Boldrin, Christiano and Fisher (2001)]. However, while housing is the largest component of household total wealth, this literature ignores the real estate market.

Given the success of this literature in explaining prices and quantities, following Jermann (1998), this work proposes to adopt a similar approach in order to develop a framework linking financial and housing markets to the real economy. Compared to existing studies, the aim of this work is to develop a model that could explain simultaneously real estate market, asset pricing, and business cycle facts within a unified general equilibrium environment. To evaluate the framework that is proposed, the model prediction are then confronted to a set of 11 stylized facts characterizing this link.
1.2 Questions and Methodology

As pointed out by several studies, the ability of business cycle models to generate high risk premiums often comes at the cost of generating large fluctuations in the risk-free rate. As argued by Cochrane (2006), this is a central point that needs to be addressed since such variations are clearly not observed in the data. When it comes to explaining asset pricing facts, the first objective of this study is to propose a solution to this problem by adopting preferences allowing to add persistence in the marginal utility of consumption. Second, by presenting empirical evidence on real estate returns, this work proposes to add an additional dimension to the equity premium puzzle [see Mehra and Prescott (1985)]. As documented in section 2, since data on real estate trust indices are available, the risk premium on real estate returns has been higher than the equity premium. This work proposes to develop a framework that could be used to explain this additional challenging stylized fact.

When it comes to the real estate market, the aim of this study is to determine whether the high volatility of house prices could be explained in a model compatible with the view that markets are working efficiently. As noted by Iacoviello (2005), while it is accepted that capital market imperfections matter for economic activity, there is still controversy about the quantitative relevance of these frictions at the aggregate level. In a recent study discussing the main forces that are the most likely to have caused the Great Depression, Chari, Kehoe and McGrattan (2006) reach the conclusion that, to the extent that monetary shocks drove the Great Depression, the mechanisms advocated by Bernanke and Gertler (1989) could explain neither the important fall in output from 1929 to 1933 nor the recovery after 1933 that have been documented. While the debate is still ongoing, according to the results of Chari, Kehoe and McGrattan (2006), it seems that building models based on credit market imperfections is not likely to yield a very high payoff.

The final objective of this study is to assess whether these asset pricing puzzles could be explained in a model also able to reproduce the main empirical regularities of the business cycle. Compared to the business cycle literature, we ask whether, in addition to the main macroeconomic aggregates, the behavior of key real estate market variables, such as residential real estate services and residential investment could be reproduced. As regards quantities, the main challenges consist of explaining, firstly, that residential investment is more than twice as volatile as non-residential investment, and
secondly, the low volatility of residential real estate services.

To address these questions, we develop a two-sector model composed of a representative agent, a numeraire or aggregate sector and a real estate sector. Dividends in the numeraire sector can be used to study equity returns, and similarly dividends in the real estate sector are used to derive real estate returns. While introducing sector specific productivity shocks is often convenient to account for differences in dynamics, in this work, it will be assumed that the same technology is available for firms in both sectors, and that aggregate technology shocks are the only source of business cycle fluctuations. In this economy, the differences in dynamics across sectors arise from the structure of the model, and can be summarized by 5 structural parameters. These 5 parameters are then calibrated to maximize the model’s ability to explain a set of 11 stylized facts.

1.3 Findings

The main contribution of this study is to develop a general equilibrium business cycle model able to reproduce a large set of asset pricing, real estate market and business cycle facts. The observed high volatility of house prices, the equity premium and the difference between equity and real estate excess returns can be explained without giving rise to excessive risk-free rate variation.

Compared to other studies, the advantage of this approach is that these asset pricing facts can be explained in a model also able to reproduce the main empirical regularities characterizing the link between real estate markets and the business cycle. The main ingredient that allows the model to explain these facts is the introduction of heterogeneity across sectors that is due to the cost of adjusting capital and to differences in the formation of consumption and real estate habits.

2 Stylized Facts

As proposed by Prescott (1986), one way of evaluating the predictions of a theoretical framework is to compare moments that summarize the actual experience of an economy with similar moments from the model. In our context, the key moments to replicate are presented in Table 1, where $\hat{y}_t^T, \hat{c}_t^a, \hat{c}_t^h, \hat{\pi}_t^h, \hat{\pi}_t^c$ and $\hat{\pi}_t^i$ denote respectively output, aggregate consumption, resident-
tial real estate services, residential investment, non-residential investment and house prices. Quarterly data on residential real estate services, are only available from 1975, and are taken from the study of Davis and Heathcote (2005). House price data are provided by the Bank for International Settlements. The remaining series can be found on the web site of the Federal Reserve Bank of St-Louis. All series are available quarterly. To generate the empirical moments, the variables are, firstly, expressed in logarithm. The cyclical component is then computed by subtracting the HP filtered series from the actual series. We denote variables that have been subject to this transformation by a hat.

Second, following McCue and Kling (1994), we use Real Estate Investment Trust (REIT) data as a proxy for real estate returns. The NAREIT equity index, includes all real estate investment trust currently trading on the New York Stock Exchange, the NASDAQ and the American Stock Exchange that owns and operates income-producing real estate. Firms included in the NAREIT equity index earn revenues from operating residential and commercial real estate. Activities in which REITs are involved include leasing, development of real property and tenant services.

One major distinction between REITs and other real estate companies is that a REIT must acquire and develop its properties primarily to operate them as part of its own portfolio rather than to resell them once they are developed. Variations in commercial and residential real estate prices therefore do not have a direct effect on REIT’s profit. Profitability of firms composing the NAREIT equity index depends essentially on market conditions affecting the rental price of commercial and residential real estate.

Total return statistics for the NAREIT equity index can be found on Datastream and are available at a quarterly frequency. As regards equity returns, the MSCI equity index is used. Total return statistics are also available from Datastream. The mean and standard deviation of the risk free rate are computed using the three month treasury bill rate and CPI inflation.4

Finally, the NAREIT equity price index, is used as a proxy for equity prices in the real estate industry. The SP 500 composite index is used to capture the evolution of equity prices in the rest of the economy. To study

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2Their data set is publicly available at: http://www.marginalq.com/morris/
3Collected from national sources
4Source: Federal Reserve Bank of St-Louis for the three month treasury bill rate and CPI inflation
the evolution of equity prices at business cycle frequency, the cyclical component of both price indices is extracted using a HP filter. The price index corresponding to real estate returns is denoted \( \hat{p}^r_t \) and the equity price index corresponding to equity returns is denoted \( \hat{p}^e_t \). These price indices can be found on datastream and have been deflated using the CPI index.

Table 1: Empirical Moments USA (1970-2002)

<table>
<thead>
<tr>
<th>Empirical Moments</th>
<th>( \sigma_{\hat{x}} )</th>
<th>( \sigma_{\hat{x}} / \sigma_{yT} )</th>
<th>corr(( \hat{x}_t, \hat{y}_T ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{y}_T )</td>
<td>1.58</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \hat{\sigma}^c_h )</td>
<td>1.28</td>
<td>0.81</td>
<td>0.85</td>
</tr>
<tr>
<td>( \hat{\sigma}^c_l )</td>
<td>0.24</td>
<td>0.16</td>
<td>0.42</td>
</tr>
<tr>
<td>( \hat{\sigma}^i_h )</td>
<td>10.52</td>
<td>6.66</td>
<td>0.73</td>
</tr>
<tr>
<td>( \hat{\sigma}^i_l )</td>
<td>5.03</td>
<td>3.18</td>
<td>0.80</td>
</tr>
<tr>
<td>( \hat{\sigma}^p_h )</td>
<td>2.16</td>
<td>1.37</td>
<td>0.65</td>
</tr>
<tr>
<td>( \hat{\sigma}^p_e )</td>
<td>11.53</td>
<td>7.3</td>
<td>0.44</td>
</tr>
<tr>
<td>( \hat{\sigma}^p_r )</td>
<td>10.40</td>
<td>6.59</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 2: Risk Free Rate, Real Returns and Risk Premiums USA (1970-2002)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Free Rate</td>
<td>1.45%</td>
<td>2.38%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Returns</th>
<th>Risk Premiums</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>7.41%</td>
</tr>
<tr>
<td>Real Estate</td>
<td>8.9%</td>
</tr>
</tbody>
</table>

As illustrated by the study of Davis and Heathcote (2005), the first challenge consists of developing a general equilibrium framework that could explain simultaneously the high volatility of residential investment, the high volatility of house prices and capture the main regularities of the business

\[^5\] \( \sigma_{\hat{x}} \) denotes the standard deviation of the cyclical component of the serie, and \( \sigma_{\hat{x}} / \sigma_{yT} \) the relative standard deviation of the serie with respect to output.

\[^6\] All series have been deflated using the CPI index and are expressed in annualized percents.
cycle. As regards investment, as shown in Table 1, residential investment, $\hat{\imath}_h$, is not only more than 6 times as volatile as output but also twice as volatile as non-residential investment, $\hat{\imath}_n$. The low volatility of residential real estate services, $\hat{\imath}_c$, is also striking. Explaining both residential and non-residential investment dynamics together with the fact that house prices, $\hat{p}_h$, are more volatile than output, and the low volatility of residential real estate services, are the first objectives of this study.

Following the literature on the equity premium puzzle [see Mehra and Prescott (1985)], the second objective of this study is to explain the high risk premium on equity returns, the low mean risk-free rate and the low volatility of the risk-free rate. In our sample, the mean risk-free rate is 1.45%, the standard deviation of the risk-free rate is 2.38% and the equity premium is about 6%.

Compared to the standard literature, we propose to add another dimension to the puzzle by introducing real estate returns. As shown in Table 2, since data on REITs are available, real estate returns have been higher than equity returns. In our sample, the risk premium premium on real estate returns, as measured by excess returns on the NAREIT equity index, is 7.45%. Which implies a spread between the equity premium and the real estate premium of about 1.5%. The fact that durable good industries tend to have higher excess returns has also been documented in a study by Gomes, Kogan and Yogo (2006).

Finally, as pointed out by Jermann (1998), while general equilibrium business cycle models have been able to account for the high equity premium and the low mean risk-free rate, this empirical success often comes at the cost of generating a counterfactually high volatility for the risk-free rate. The relevance of the model that is developed will thus also be evaluated in terms of its ability to improve the performances of existing approaches on this particular dimension. In our sample, the volatility of the risk-free rate is 2.38%.$^7$

3 The Environment

The economy is composed of a representative agent, a numeraire or aggregate sector, denoted by the subscript $n$, and a real estate sector, denoted by $h$.

$^7$Jermann (1998) and Bodrin, Christiano and Fisher (2001), using longer samples report a value of about 5%.
The numeraire sector produces a standard numeraire consumption good, $c^n_t$, using labor, $N^n_t$, business capital, $k_t$, and commercial real estate, $q_t$. Increases in the stock of business capital are financed through retained earnings.

As for the real estate sector, the industry produces a composite real estate output good, $y^h_t$. This final good is purchased by the household at the relative price $z_t$, and can be divided into a residential investment good and residential real estate services. The real estate sector uses labor, $N^h_t$, residential structures, $h_t$, and land, $l_t$, to produce its final good. In this economy, the initial endowment of land is assumed to be fixed and is owned by the firms in the real estate industry.

Firms in the real estate sector allocate a fixed fraction of land to the production of the real estate final good. The remaining fraction of land that is not used to produce the final output good is rented to the firms in the numeraire sector under the form of commercial real estate. Residential structures can be rented from the representative household.

The representative household derives utility from consuming the standard consumption good and residential real estate services. Households in this economy hold stocks from both the numeraire and the real estate firms, and receive a dividend payment from owning the firms. Each period, household have to decide how to allocate the real estate output good, $y^h_t$, between consumption of residential real estate services, $c^h_t$, and residential investment, $i^h_t$. The residential investment good is used to accumulate residential structures. House prices, $p^h_t$, are given by the price of residential structures, $h_t$.

To ensure steady state growth, technical progress, $\Gamma_t$, is introduced and is assumed to take a labor augmenting form [see King, Plosser and Rebelo (2002)]. The model is expressed in terms of stationary variables where $\gamma$ denotes the steady state growth rate of the economy$^8$.

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$^8$ The law of motion for $\Gamma_t$ which denotes the deterministic component of growth evolves according to:

$$\frac{\Gamma_{t+1}}{\Gamma_t} = \gamma$$
3.1 The Consumers

The representative consumer derives utility from consuming the numeraire consumption good, $c^n_t$ and residential real estate services, $c^h_t$. Habit formation in consumption and in residential real estate services is assumed. Following Abel (1998), and Campbell and Cochrane (1999), incomplete depreciation in the stock of habit formation is introduced. The stocks of real estate and consumption habit evolve according to the following laws of motion:

$$\gamma x^n_{t+1} = (1 - a^n)x^n_t + b^n c^n_t$$  \hspace{1cm} (1)
$$\gamma x^h_{t+1} = (1 - a^h)x^h_t + b^h c^h_t$$  \hspace{1cm} (2)

where $c^n_t$ and $c^h_t$ denote respectively, aggregate consumption and the service flow of a composite real estate good, and where $\gamma$ is the quarterly growth trend rate. The stock of residential real estate or housing habit is $x^h_t$. The stock of housing habit in period $t+1$ is given by the sum of the stock of the current period, $x^h_t$, adjusted for depreciation, $1 - a^h$, and of the service flow of the real estate good of the current period, $c^h_t$. The parameter $b^h$, where $0 \leq b^h \leq 1$, captures the sensitivity of the stock of real estate habit with respect to the amount of services consumed during the current period. Similarly, $x^n_{t+1}$, denotes the stock of consumption habit accumulated by the agent.

The representative agent maximizes expected lifetime utility given by:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^* \left[ (c^n_t - x^n_t)^\kappa (c^h_t - x^h_t)^{1-\kappa} \right]^{1-\sigma} \right\}$$

where $\beta^*$ is the modified subjective rate of time discount\(^{10}\), and $\sigma$ the coefficient of risk aversion. The utility weight of numeraire consumption is denoted by $\kappa$, where $0 \leq \kappa \leq 1$.

At this stage, the introduction of financial assets into the model requires some additional notation. The subscript $e$ will be used to characterize financial variables related to equity returns of the numeraire sector, and the subscript $r$ will denote financial variables related to real estate returns.

\(^9\)In the stationary economy, detrended variables are denoted by lower case letters. For example, detrended consumption is expressed as: $c^n_t = C^n_t / \Gamma_t$. Capital letters are used to denote stationary variables that are not growing in the steady state.

\(^{10}\)See King, Plosser and Rebelo (2002) for a detailed discussion of the derivation of the stationary economy.
The budget constraint of the representative household is described by equation (3). As far as expenses are concerned, consumers firstly decide how much of the consumption good, \( c^c_n \), to purchase from the firms in the numeraire sector. The final real estate output good \( y^h_t \) has then to be allocated between consumption of residential services, \( c^h_t \), and residential investment, \( i^h_t \). The relative price of the real estate final good is denoted by \( z_t \). Finally, households decide how many equities of the real estate firm, \( S^r_{t+1} \), and of the numeraire firm, \( S^e_{t+1} \), to hold. Equity prices of the real estate and of the numeraire firm are denoted by respectively \( p^r_t \) and \( p^e_t \).

When it comes to revenues, agents firstly receive a labor income, \( W^n_t N^n_t \) and \( W^h_t N^h_t \), from working in the numeraire and in the real estate sector.\(^{11}\) In this economy, households can only choose to work in one sector at a time, and working time is divided between time that can be allocated to the numeraire and to the real estate sector. Time that can be allocated to the real estate sector, \( T^h \), and to the numeraire sector, \( T^n \), is then divided between leisure activities and hours worked, respectively, \( L^h_t \) and \( N^h_t \) for the real estate sector, and; \( L^n_t \) and \( N^n_t \) for the numeraire sector.

Secondly, a dividend income from owning the real estate and the numeraire firm, \( d^r_t \) and \( d^e_t \), is received each period. The total capital income from being a shareholder of the real estate and of the numeraire firm is given by the number of stocks held in each sector time the market value of the asset plus dividends, \( S^r_t (p^r_t + d^r_t) \) and \( S^e_t (p^e_t + d^e_t) \).

Finally, a revenue from renting residential structures to the firm in the real estate sector, is received and is denoted by \( r^h_t h_t \), where \( h_t \) denotes residential capital or residential structures, and \( r^h_t \), the rental rate paid by the real estate firm to households.

\[
c^c_t + z c^h_t + z i^h_t + p^r_t S^r_{t+1} + p^e_t S^e_{t+1} = W^n_t N^n_t + W^h_t N^h_t + S^e_t (d^e_t + p^e_t) + S^r_t (d^r_t + p^r_t) + r^h_t h_t
\]

The accumulation of residential capital is subject to the kind of adjustment costs proposed by Baxter and Crucini (1993), and is governed by the following law of motion:

\[
(1 - \delta^h) h_t + \Phi^h \left( \frac{r^h_t}{h_t} \right) h_t = \gamma_{t+1}
\]

\(^{11}\)\(W^h_t \) and \( W^n_t \) denote the wage rate paid by the real estate and the numeraire firm. \( N^h_t \) and \( N^n_t \) denote hours worked in the real estate and in the numeraire firm.
where $\delta^h$ denote the depreciation rate of residential structure. The parameters of the capital adjustment costs function $\Phi^h(\frac{\dot{h}}{h})$ are set so that the model with adjustment costs has the same steady state as the model without adjustment costs and it is assumed that near the steady state point: $\Phi^h > 0$, $\Phi^{hh} > 0$ and $\Phi^{hh} < 0$. This captures the idea that increasing the capital stock rapidly is more costly than changing it slowly.

3.1.1 Equity Prices

The dynamics of equity prices in the numeraire and in the real estate sector can be derived using the first-order conditions with respect to $S^e_{t+1}$ and $S^r_{t+1}$:

$$S^e_{t+1} : \quad \lambda_t p^e_t = \beta^* E_t \lambda_{t+1} [d^e_{t+1} + p^e_{t+1}]$$

$$S^h_{t+1} : \quad \lambda_t p^r_t = \beta^* E_t \lambda_{t+1} [d^r_{t+1} + p^r_{t+1}]$$

These two conditions state that, in this economy, the behavior of equity prices are described by the central asset pricing formula, where $\lambda_t$ is the Lagrange multiplier associated with the budget constraint (3). Given the payoff and given the investor’s consumption choice $\lambda_t$, this condition tells us what market price to expect.

The left hand side is the loss in utility if the agent buys another unit of the asset, and the right hand side denotes the increase in expected utility he or she obtains from the extra payoff at $t + 1$. The agent continues to buy or sell the asset until the marginal loss equals the marginal gain $^{12}$.

3.1.2 House Prices

In this economy, house prices, $p^h_t$, which are the price of residential structures, $h_t$, are determined by the intertemporal trade-off governing the allocation of the final real estate good, $y^h_t$, between residential real estate services, $c^h_t$, and residential investment, $i^h_t$. If households decide to allocate a larger proportion of the real estate output good, $y^h_t$, to the consumption of real estate services, they perfectly understand that this will come at the cost of having less investment in residential structures. Structures being a key input

$^{12}$See Cochrane (2001) for a more detailed interpretation of this condition.
to the production of residential real estate services, less investment today will imply fewer residential services in the future.

The first-order condition with respect to residential investment illustrates this trade-off. Defining house prices, $p^h_t$, as the ratio of the marginal gain from investing in residential structures, which is given by the Lagrange multiplier on the accumulation equations (4), $\mu_t$, and the marginal cost of renouncing to enjoy residential real estate services, given by the marginal utility of residential real estate services, $z^h_t \lambda_t$:

$$ p^h_t = \frac{\mu_t}{z^h_t \lambda_t} \quad (7) $$

house prices can firstly be expressed as:

$$ p^h_t = \frac{1}{\Phi^h_t(h^h_t/h_t)} \quad (8) $$

The second derivative of the adjustment function being negative, the expression states that in equilibrium house prices are increasing in the investment to structure ratio, $i^h_t/h_t$. This formula illustrates why standard models have troubles explaining simultaneously the high volatility of residential investment and the high volatility of house prices. On the one hand, being able to account for the high volatility of residential investment requires low adjustment costs. However, on the other hand, the lower the adjustment cost parameter, the less volatile will prices be\textsuperscript{13}.

In a general equilibrium model where prices and quantities are endogenously determined, prices can also be characterized by an Euler equation relating current prices to future payoffs. From the first-order condition with respect to $h_{t+1}$, after some manipulations, house prices can be expressed as:

$$ p^h_t = \bar{\beta}^* E_{t-1} \frac{\lambda_{t+1} z_{t+1}}{\lambda_t z_t} \left[ p^h_{t+1} f(h_{t+1}, i^h_{t+1}, \delta^h) + i^h_{t+1} \right] \quad (9) $$

where $\bar{\beta}^* = \beta \gamma^{-\sigma}\textsuperscript{14}$. Compared to the usual standard asset pricing formulae, (5) and (6), an additional term, $f(h_{t+1}, i^h_{t+1}, \delta^h)$, enters the capital gain

\textsuperscript{13}In the limiting case with no adjustment costs, $\Phi^h_t(h^h_t/h_t) = 1$, and house prices are always constant and equal to 1.

\textsuperscript{14}The modified discount factor is adjusted to reflect that the stock of residential capital is growing in the steady state.
component of the payoff. This term, which depends on $h_{t+1}, i_{t+1}^h$ and $\delta^h$, is due to the presence of adjustment costs and illustrates that agents, when investing in real estate, integrate that adjustment costs have an impact on the valuation.

The second component of expected payoff is given by the rental rate associated with investing in real estate structures, $r_t^h$. Finally, compared to the case of equity, as far as investing in real estate is concerned, the rate at which future payoffs are discounted is given by the marginal utility of residential real estate services, $z_t \lambda_t$.

### 3.2 The Numéraire Sector

Each period, the numéraire good producer has to decide how much labor to hire, $N^n_t$, how many units of commercial real estate, $q_t$, to rent from the real estate industry and how much to invest in business capital, $i^n_t$. Managers maximize the value of the firm to its owners, the representative agent, which is given by the present discounted value of all current and expected cash flows $d^e_t$ [see Jermann (1998)]:

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^* \frac{\lambda_t}{\lambda_0} d^e_t$$

where:

$$d^e_t = y^n_t - W^n_t N^n_t - r^n_t q_t - i^n_t$$

and where $\beta^* \lambda_t$ is the marginal rate of substitution of the owner. Total output in the numéraire sector is denoted by $y^n_t$, the wage rate paid to the workers is $W^n_t$, and $r^n_t$ is the rental cost of commercial real estate. The firm’s capital stock follows an intertemporal accumulation equation with adjustment costs:

$$(1 - \delta^n)k_t^n + \Phi^n \left( \frac{i^n_t}{k_t^n} \right) k_t^n = \gamma k_{t+1}^n$$

As in Baxter and Crucini (1993), the parameters of the capital adjustment costs function $\Phi^n \left( \frac{i^n_t}{k_t^n} \right)$ are set so that the model with adjustments costs has the same steady state as the model without adjustment costs and it is assumed that near the steady state point: $\Phi^n > 0$, $\Phi'' > 0$ and $\Phi'''' < 0$. 

14
Production of the final numeraire good, $y_t^n$, requires the use of labor, $N_t^n$, capital, $k_t$, and commercial real estate, $q_t$. The good is produced via Cobb-Douglas production function:

$$y_t^n = A_t k_t^\xi N_t^n q_t^{1-\alpha-\xi}$$  \hspace{1cm} (12)

The share of business capital in the production function is $\xi$. To simplify notation the subscript $n$ on labor is omitted. The labor share is $\alpha$, and $1 - \alpha - \xi$ is the share of commercial real estate. Since in equilibrium commercial real estate, $q$, is a fixed factor, the time subscript is dropped. $A_t$ is the standard random productivity shock variable that can be interpreted as a temporary displacement to total factor productivity, and which is identical across sectors.

3.3 The Real Estate Sector

To produce the final real estate good, $y_t^h$, the real estate firm uses residential structures, $h_t$, labor, $N_t^h$, and land, $l_t$. Residential structures are rented from the households, at the rental rate $r_t^h$. In this economy, the real estate industry owns an initial fixed endowment of land. A fixed fraction of this endowment of land, $l$, is allocated to the production of the final good, $y_t^h$. The remaining fraction, $q$, is rented to the firms in the numeraire sector\(^{15}\), at the rental price $r_t^q$, under the form of commercial real estate.

Renting commercial real estate and producing a final output good, that can be used as residential services or as an investment good, constitute two distinct sources of profits for the firms in the real estate industry. Dividends in the real estate sector are given by:

$$d_t^r = z_t y_t^h + r_t^q q - W_t^h N_t^h - r_t^h h_t$$  \hspace{1cm} (13)

The Cobb-Douglas production function is:

$$y_t^h = A_t h_t^\theta N_t^h l_t^{1-\theta-\rho}$$  \hspace{1cm} (14)

where the subscript $h$ on labor has been omitted to simplify notation.

\(^{15}\)To keep the analysis tractable, $q$ and $l$ do not vary overtime. This assumption is not key and could easily be relaxed.
3.4 Market Clearing Conditions

The quantity of equity supplied by each sector being constant and normalizing it to one, the market clearing conditions in the equity market are:

\[ S^e_t = S^e_{t+1} = 1 \]  

(15)

and:

\[ S^e_t = S^e_{t+1} = 1 \]

(16)

for the real estate sector and for the numeraire sector.

As far as the labor market is concerned, since utility is strictly increasing in the consumption good and in the real estate good and since leisure does not yield utility, agents will find optimal to work full-time in each sector\textsuperscript{16} and labor will be supplied inelastically:

\[ N^h_t = \overline{N}^h \]

(17)

and:

\[ N^n_t = \overline{N}^n \]

(18)

Equilibrium on the market for residential structures implies that the amount of residential structures supplied by households, \( h^S_t \), equals the amount demanded by firms, \( h^D_t \):

\[ h^D_t = h^S_t \]

Finally market clearing in each sector implies the two following equilibrium conditions:

\[ y^n_t = c^n_t + i^n_t \]  

(19)

\[ y^h_t = c^h_t + i^h_t \]  

(20)

4 Solution Method

In terms of optimal control theory [see King, Plosser and Rebelo (2002)], \( c^n_t, c^h_t, i^n_t \) and \( i^h_t \) are control variables; \( x_t^n, x_t^h, h_{t+1} \) and \( k_{t+1} \) are control-state variables; and the lagrange multipliers associated to the market clearing condition in the numeraire sector (19), the real estate sector (20), the...

\textsuperscript{16} We could think about this economy as one where agents work in the real estate sector in the morning and in the numeraire sector in the afternoon.
accumulation of residential capital equation (4), the business capital accumulation equation (11), the accumulation of habit equations, (1) and (2); denoted respectively, $\lambda_t$, $\lambda_t z_t$, $\mu_t$, $\phi^0_t$, $\phi^1_t$ and $\mu_t$, are co-state variables.

Following the resolution methods developed by King, Plosser and Rebelo (2002), the model can be solved and represented in a state-space form using around steady state linearization technics. The competitive equilibrium can equivalently be stated as a centralized problem. The centralized problem is presented in the annex.

### 4.1 Risk Premiums

As far as the risk premiums are concerned, as shown by Jermann (1998)\(^{17}\), the conditional expected return of a claim to a dividend that will be paid in $k$ periods can be expressed as:

\[
E_t(R_{t+1} | D_{t+k}) = R_{t,t+1} [1_{t+1}] \times \exp \left[ -\text{cov}_t(\lambda_{t+1}, E_{t+1} \lambda_{t+k} - \lambda_{t+1}) \right] \\
\times \exp \left[ -\text{cov}_t(\lambda_{t+1}, E_{t+1} d_{t+k}) \right]
\]

(21)

where $D_{t+k}$ denotes the payoff associated with an asset offering a gross return of $R_{t,t+1} [D_{t+k}]$, $\lambda_{t+k}$ and $d_{t+k}$ the marginal utility of consumption and dividends in logs. The first term represents the return to holding a one-period bond until maturity, *i.e.*, the risk free rate. The second term can be thought of as a holding or term premium for a $k$-periods discount bond that depends on the term structure of interest rates. If one expects capital gains as a consequence of lower interest rates when consumption is high, then this unfavorable correlation has to be compensated by a positive risk premium. The last element, the payout uncertainty premium, is linked to a possible capital gain or loss at time $t + 1$. If capital gains are negatively correlated with the valuation, then a risk premium is needed to compensate the investor for the undesirable cyclical property of this asset. The above formula can be rewritten as:

\[
E_t(R_{t+1} [D_{t+k}]) = R_{t,t+1} [1_{t+1}] \exp(\eta_h(k) + \eta_p(k))
\]

(22)

where $R_{t,t+1} [1_{t+1}]$ is the risk free rate, $\eta_h(k)$ the term premium and $\eta_p(k)$ the payout uncertainty premium.

\(^{17}\)See also Jermann (1994)
Using this formulation, and considering an asset which represents a claim to an infinite sequence of dividends:

\[ V_t [\{D_{t+k}\}_{k=1}^\infty] = \sum_{k=1}^\infty V_t [D_{t+k}] \] (23)

the expected return of a common stock can be derived and is given by:

\[ E_t (R_{t,t+1} [\{D_{t+k}\}_{k=1}^\infty]) = R_{t,t+1} [1_{t+1}] \sum_{k=1}^\infty w_t [D_{t+k}] \exp(\eta_h(k) + \eta_p(k)) \] (24)

with \( w_t [D_{t+k}] \) being the portfolio weight attached to the date \( t + k \) dividend strip return. The unconditional risk premium can then be computed by simulating data for the artificial economy described in the previous section.

This methodology can be applied to derive the equity premium and the real estate premium corresponding to dividends in the numeraire and in the real estate sector.

5 Calibration

The objective of the quantitative evaluation is to derive the theoretical moments implied by the model and to compare them to the set of stylized facts presented earlier. The calibration is carried out in two steps. A first set of parameters is chosen based on National Income Account data and on estimations available from previous studies, when available. The remaining five parameters describing habit formation and adjustment costs, for which precise a priori knowledge is weak, are chosen to maximize the model’s ability to replicate the set of business cycle and asset price moments.

- Long-run behavior

A first set of parameters is chosen to match long-run model behavior. The quarterly trend growth rate \( \gamma \) is 1.005, and the capital depreciation rate \( \delta^n \) is 0.025, the discount factor \( \beta \) is set to 0.997. The constant labor share in the Cobb-Douglas production function of the numeraire good producer \( \alpha \) is 0.65, the capital share \( \xi \) is 1/3 which implies a share of commercial real estate, \( 1 - \alpha - \xi \), of 0.01.
As for the production function of the real estate firm, the residential structures share, $\rho$, is fixed to $1/3$, the labor share, $\theta$, is 0.56, and the share of land, $1 - \rho - \theta$, is 0.106. The rate of depreciation of residential structure, $\delta^h$ is 0.004. The value use to calibrate the share of land and the depreciation parameter of residential structures are the one suggested by Davis and Heathcote (2005). The share of consumption in the utility function, $\kappa$, is set to $3/4$.

- **Aggregate output**

  Aggregate output is defined as the weighted average of output in both sectors. Using data on the component of value added by industry, we set the share of the real estate sector to 0.15.

- **Technology shocks**

  The parameters describing the technology shocks are chosen such as to replicate U.S. postwar quarterly output growth volatility. The standard deviation of the shock innovation $\sigma^2_\varepsilon$ is 1.12 percent. The persistence parameter $\rho_A$ is 0.99.

- **Coefficient of risk aversion**

  In Mehra and Prescott (1985), it is argued that values for this parameter ranging from 1 to 10 could be considered as reasonable. Drawing on Kocherlakota (1996)’s review, we use $\sigma = 3$.

- **Habit Formation**

  To limit the number of free parameters, we start by assuming that the stocks of consumption and real estate habit depreciate at the same rate and set $a = a^h = a^n$. This parameter is chosen so as to match the low volatility of the risk free rate and is set to 0.006. Given the capital adjustment costs parameter, $\nu^n$, the coefficient capturing the impact of current consumption on the stock of habit, $b^n$, is chosen so as to match the equity premium and the mean risk free rate, and is set to 0.05. The parameter capturing the impact of residential services on the stock of real estate habit, $b^h$, is chosen such as to match the volatility of residential real estate prices, and is set to $1/3$. 

19
Capital adjustment costs

As in Baxter and Crucini (1993), the parameters of the capital adjustment cost function are set so that the model with adjustments costs has the same steady state as the model without adjustment costs. This captures the idea that increasing the capital stock is costly. As a result, to reach a given level of capital stock, compared to the situation without adjustment costs, more investment will be necessary as the adjustment cost function imply that part of the increase in investment is wasted.

The elasticity parameters $\nu^n$ and $\nu^h$ captures the curvature of the adjustment costs function. Thus, the higher it is, the higher is the marginal cost of increasing the investment to capital ratio. The residential capital adjustment cost parameter, $\nu^h$, is chosen such as to allow the model to account for the high volatility of residential investment. The business capital adjustment cost parameter, $\nu^n$, is chosen such as to maximize the model ability to reproduce the high equity premium and the low mean risk-free rate. We set $\nu^n$ to 2.63 and $\nu^h$ to 0.2.

5.1 Comparing Parameter Values

When possible, the calibration that is used to generate the results follows the one adopted in existing studies. Except for habit formation, the parameter values that are chosen for the non-residential sector and for the driving process are similar to the one used in Jermann (1998).

The non-residential sector

<table>
<thead>
<tr>
<th>Habit formation</th>
<th>$a^n$</th>
<th>$b^n$</th>
<th>$\nu^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jermann (1998)</td>
<td>1</td>
<td>0.82</td>
<td>1/0.23</td>
</tr>
<tr>
<td>Real estate model</td>
<td>0.006</td>
<td>0.05</td>
<td>1/0.38</td>
</tr>
</tbody>
</table>

$^{18}$where $a^n$, $b^n$ and $\nu^n$ denote respectively the depreciation rate of the habit stock, the persistence of consumption habit and the elasticity of the capital adjustment costs function.
Long-run behavior and risk aversion

<table>
<thead>
<tr>
<th></th>
<th>$\beta^{1-\sigma}$</th>
<th>$\delta^n$</th>
<th>$\gamma$</th>
<th>$\xi$</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jermann (1998)</td>
<td>0.99</td>
<td>0.025</td>
<td>1.005</td>
<td>1/3</td>
<td>2/3</td>
<td>5</td>
</tr>
<tr>
<td>Real estate model</td>
<td>0.987</td>
<td>0.025</td>
<td>1.005</td>
<td>1/3</td>
<td>0.65</td>
<td>3</td>
</tr>
</tbody>
</table>

- The driving process

<table>
<thead>
<tr>
<th>Technology Shocks</th>
<th>$\sigma^2$</th>
<th>$\rho_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jermann (1998)</td>
<td>0.01</td>
<td>0.99</td>
</tr>
<tr>
<td>Real estate model</td>
<td>0.0112</td>
<td>0.99</td>
</tr>
</tbody>
</table>

As for the real estate sector, when available, we adopt the calibration suggested by estimated values taken from Davis and Heathcote (2005), and set the depreciation rate of capital structures, $\delta^h$ to 0.004, and the share of land to 0.106.

6 Simulations and Results

To evaluate the model performance, the theoretical predictions are now confronted to the empirical facts. Given a driving process that is calibrated such as to match the actual volatility of output, the first objective is to reproduce the empirical volatilities of the remaining 7 variables presented in Table 1. The second objective is to explain the asset pricing facts presented in Table 2, including the mean and volatility of the risk free rate, the equity premium and the real estate premium. As discussed in the previous section, the model structure implies that 5 parameters, $a, b^n, b^h, \nu^n$ and $\nu^h$, describing habit formation and capital adjustment costs can be used to maximize the model’s ability to account for these 11 stylized facts.

19 where $\beta^{1-\sigma}, \delta^n, \gamma, \xi, \alpha$ and $\sigma$ denote the modified discount factor, the rate of capital depreciation, the quarterly trend growth rate, the capital and the labor share in the numeraire sector and risk aversion.

20 where $\sigma^2$ is standard deviation of the shock innovation and $\rho_A$ the persistence parameter.
Table 3: Theoretical Moments\textsuperscript{21} vs Empirical Moments

<table>
<thead>
<tr>
<th>Data</th>
<th>$\sigma_{\bar{x}}$</th>
<th>$\frac{\sigma_{\bar{x}}}{\sigma_{\bar{y}}}^T$</th>
<th>$\text{corr}(\hat{x}_t, \hat{y}_t^T)$</th>
<th>Model</th>
<th>$\sigma_{\bar{x}}$</th>
<th>$\frac{\sigma_{\bar{x}}}{\sigma_{\bar{y}}}^T$</th>
<th>$\text{corr}(\hat{x}_t, \hat{y}_t^T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{y}_t^T$</td>
<td>1.58</td>
<td>1</td>
<td>1</td>
<td>$\bar{y}_t^T$</td>
<td>1.56</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{c}_t^n$</td>
<td>1.28</td>
<td>0.81</td>
<td>0.85</td>
<td>$\bar{c}_t^n$</td>
<td>0.94</td>
<td>0.60</td>
<td>0.99</td>
</tr>
<tr>
<td>$\bar{c}_t^h$</td>
<td>0.24</td>
<td>0.16</td>
<td>0.42</td>
<td>$\bar{c}_t^h$</td>
<td>0.28</td>
<td>0.18</td>
<td>0.61</td>
</tr>
<tr>
<td>$\bar{i}_t^h$</td>
<td>10.52</td>
<td>6.66</td>
<td>0.73</td>
<td>$\bar{i}_t^h$</td>
<td>10.53</td>
<td>6.74</td>
<td>0.99</td>
</tr>
<tr>
<td>$\bar{i}_t^n$</td>
<td>5.03</td>
<td>3.18</td>
<td>0.80</td>
<td>$\bar{i}_t^n$</td>
<td>3.63</td>
<td>2.32</td>
<td>0.99</td>
</tr>
<tr>
<td>$\bar{p}_t^h$</td>
<td>2.16</td>
<td>1.37</td>
<td>0.65</td>
<td>$\bar{p}_t^h$</td>
<td>2.11</td>
<td>1.35</td>
<td>0.99</td>
</tr>
<tr>
<td>$\bar{p}_t^n$</td>
<td>11.53</td>
<td>7.3</td>
<td>0.44</td>
<td>$\bar{p}_t^n$</td>
<td>9.70</td>
<td>6.21</td>
<td>0.99</td>
</tr>
<tr>
<td>$\bar{p}_t^r$</td>
<td>10.40</td>
<td>6.59</td>
<td>0.48</td>
<td>$\bar{p}_t^r$</td>
<td>15.32</td>
<td>9.81</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 3 reports summary statistics on HP-filtered cyclical components of the model predictions that can be compared with the stylized facts presented earlier, which are repeated to facilitate comparisons. The model implications regarding asset pricing facts are presented in Table 4 and 5.

Table 4: Theoretical vs Empirical Risk Premiums\textsuperscript{22}

<table>
<thead>
<tr>
<th>Equity Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td>Model</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Real Estate Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td>Model</td>
</tr>
</tbody>
</table>

Table 5: Theoretical vs Empirical Risk Free Rate: Mean and Standard Deviation

<table>
<thead>
<tr>
<th>Mean Risk Free Rate</th>
<th>St. Dev. Risk Free Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.45%</td>
</tr>
<tr>
<td>Model</td>
<td>1.30%</td>
</tr>
</tbody>
</table>

\textsuperscript{21}Log linear business cycle moments are the average over 5000 simulations.

\textsuperscript{22}The risk premiums are the average over 5000 simulations, each 1000 periods long.
6.1 Asset Pricing Facts

The key parameters allowing to match the 4 asset pricing moments presented in Table 4 and 5 are the depreciation rate of habit formation, $a$, the consumption habit parameter, $b^n$, and the business capital adjustment cost parameter, $\nu^n$.

As illustrated by equations (5) and (6), prices of financial assets are determined by two components, the stochastic discount factor, which is common to both assets, and dividends. As in Jermann (1998), the combination of high adjustment costs and habit formation in the numeraire sector is the key mechanism that gives rise to the high equity premiums on financial assets. The introduction of internal habit formation leads agents to be reluctant to bear large variations in their consumption streams. This increases their willingness to smooth consumption over time. By causing, a reduction in the potential for intertemporal substitution, capital adjustment costs prevent consumers from smoothing consumption easily. Following a positive productivity shock, this mechanism leads to a pronounced decline in the marginal utility of consumption, $\lambda_t$, illustrating that agents are willing to transfer wealth across time and that they find difficult to do it.

Next, the inverse of the risk-free rate being determined by expected changes in marginal utility, $1/R_t^f = \beta^*E_t(\lambda_{t+1}/\lambda_t)$, this pronounced decline in the marginal utility causes the risk-free rate to decline in response to a positive technology shock. This decline in the risk-free rate, which illustrates that a lower interest rate is needed to induce savings, leads to a significant increase in the risk premiums, by generating a higher term premium. This increase in the term premium reflects that, in this economy, in periods of economic booms, when consumption is high, agents expect capital gains as a result of lower interest rates. This undesirable cyclical property implying capital gains in good time and capital losses in bad time, has therefore to be compensated by a high term premium. This effect due to the term premium is common to both assets.

Compared to models that rely on a similar mechanism to generate high risk premiums, it is interesting to note that, in this study, this result does not come at the cost of giving rise to excessive risk-free rate variation. In Jermann (1998), the standard deviation of the risk-free rate that is generated by the benchmark model is 11.46%. In Boldrin, Christiano and Fisher (2001), the standard deviation of the risk-free rate is 24.6%. Using a longer sample,

---

23 See section 4.1
both studies report an empirical value for the standard deviation of the risk-free rate around 5.5%. This apparent contradiction has been identified as a potential serious problem [see Cochrane (2006)]. The concern being that in this class of models, the high risk premiums may be due to large variations in the risk-free rate, that are not observed in the data.

As illustrated by the results shown in Table 5, in this economy, the introduction of a stock of habit that does not depreciate completely within the period is the key ingredient that allows to overcome this difficulty. Figure 1 (see annex p. 35) compares the result obtained with the specification of habit adopted in this study:

$$\gamma x^n_{t+1} = 0.994x^n_t + 0.05c^n_t$$  \hspace{1cm} (25)

with the specification adopted in Jermann (1998), where the following specification is used:

$$\gamma x^n_{t+1} = 0.82c^n_t$$  \hspace{1cm} (26)

Figure 1 shows the response to a standard 1% technology shock of marginal utility implied by both specifications. The green line is the response of marginal utility in a model using specification (26), that explains the equity premium, the low risk free rate and generate a standard deviation for the risk-free rate of about 11%. As illustrated by the blue dotted line, using the specification with incomplete depreciation (25), allows to generate more persistence in marginal utility. On impact, the decline in marginal utility implied by both specifications is similar. However, the risk-free rate being determined by expected changes in marginal utility, adding persistence allows to generate a significant decline in marginal utility without implying a counterfactually high volatility for the risk-free rate.

As shown in Table 4, the model is also able to generate a difference between the real estate premium and the equity premium that is in line with the empirical facts. The main ingredient that helps generating a higher real estate premium is the introduction of heterogeneity among sectors which gives rise to fluctuations in the relative price of the real estate output good, \(z_t\). These variations in \(z_t\) reflect that agents in this economy find optimal to allocate the final numeraire and output goods, \(y^n_t\) and \(y^h_t\), differently. Differences in the way agents allocate the two final goods between consumption and investment can alternatively be interpreted using changes in the marginal utilities of the two consumption goods. The relative price \(z_t\) being determined by the marginal utility of residential real estate to consumption
ratio\textsuperscript{24}, these variations are mostly driven by differences in adjustment costs and habits.

In the numeraire sector, the combination of high adjustment costs and habit formation induce large changes in the marginal utility of numeraire consumption, $\lambda_t$, in response to shocks. In contrast, low adjustment costs in residential capital implies that transferring consumption of residential services, $c^h_t$, across time is easier. This translates into less volatility in the marginal utility of residential real estate, $\lambda_t z_t$.

In response to a positive technology shock, compared to the response of the marginal utility of consumption, the more moderate decline in the marginal utility of residential real estate gives rise to an increase in $z_t$. This reflects that, due to the difficulty and the importance of transferring numeraire consumption over time, allocating the numeraire output good to business investment is found to constitute a priority. In comparison, the fact that consumption smoothing of residential real estate services, while important, can easily be achieved, implies that allocating the real estate output good to investment is not as crucially needed.

The cyclical property of the relative price, $z_t$, which increases in periods of economic booms, when consumption is high, leads to an increase in the real estate premium. This increase comes from the payout uncertainty premium and reflects that agents buying equity of the real estate firm need to be compensated for holding an asset whose payoff is positively correlated with consumption. By making dividends in the real estate sector riskier, these variations in the relative price generates a difference in excess returns of about 1%. As illustrated in Table 3, the higher real estate premium is however obtained at the cost of generating too much volatility for equity prices in the real estate sector, $\hat{p}_t$.

\textbf{6.2 Residential and Non-residential Investment}

As shown in Table 3, the model is able to account for the high volatility of residential investment, $\hat{i}_t^h$, and the fact that it is more than twice as volatile as non-residential investment, $\hat{i}_t^n$. Introducing different depreciation rates for non-residential capital and residential structures contributes to account for these differences in volatility. By implying that, in the steady state, a smaller fraction of output needs to be invested to replace existing structures, slower

\textsuperscript{24} z_t = \lambda_t z_t / \lambda_t
depreciation in the real estate sector leads residential investment to be more volatile. Intuitively, given that slower depreciation implies a reduction in the steady state share of residential investment, larger variations are needed to allow households to smooth their consumption of housing services over time.

The residential capital adjustment costs parameter, $\nu^h$, is chosen to match the volatility of residential investment. While the business capital adjustment costs parameter $\nu^n$ is chosen to match the asset pricing facts presented in Table 4 and 5, the model is still able to correctly account for the fact that business investment is more than twice as volatile as output. The responses of residential and non-residential investment to a 1% technology shock are shown in figure 4 (top panels).

### 6.3 House Prices

The reason why standard models are not able to generate simultaneously the high volatility of residential investment and the fact that house prices are more volatile than output can be illustrated by equation (8). In the limiting case with no adjustment costs, $\nu^h = 0$, the first-derivative of the adjustment costs function, $\Phi^h(t^h)_{n_h}$, is equal to unity, which implies that house prices, $p^h_t$, are constant and equal to 1. This tension between having high adjustment costs, high volatility in prices but low volatility in investment, and having low adjustment costs, high volatility in investment but low volatility in prices can be overcome by adding habit formation. This is due to the fact that habit formation, by inducing agents to be more willing to transfer consumption of residential services over time, induces more investment in residential structures. With this formulation of adjustment costs, the second derivative of the adjustment costs function, $\Phi^{hn}(t^h)_{n_h}$, being negative, an increase in agent’s willingness to invest in residential structure translates into higher house price volatility.

The impact of habit can also be seen from equation (7). Increasing habit formation in residential services, by inducing the marginal utility of residential services, $\lambda_t z_t$, to be more volatile, affects directly house prices. House prices being determined by the ratio of the marginal benefit of investing in structures, $\mu_t$, and of the marginal cost of doing so, $\lambda_t z_t$, a calibration implying that agents are more reluctant to bear variations in their consumption of residential services, translates into more volatility in house prices.

Intuitively, in response to a positive shock, to facilitate consumption
smoothing of residential services, habit formation leads agents to become more willing to invest in residential structures. Following a positive shock, the more pronounced decline in $\lambda_t z_t$ that is induced by an increase in habit formation, leads house prices to increase even more. The response of house prices to a 1% technology shock is shown in figure 5 (bottom right panel). The real estate habit parameter, $b^h$, is calibrated such as to match the volatility of house prices.

6.4 Consumption and Residential Real Estate Services

Finally, as shown in Table 3, the model is able to correctly predict that consumption, $\hat{c}^n_t$, is less volatile than output, and considerably more volatile than residential real estate services, $\hat{c}^h_t$. In the model, this difference in volatility arise as a consequence of the calibration adopted to reproduce the high risk premiums, the volatility of house prices and the volatility of residential investment.

To match the high volatility of house prices, as discussed above, habit formation in real estate is more pronounced than habit formation in the standard consumption good. Second, higher adjustment costs in business capital implies that smoothing consumption of the numeraire consumption good is more difficult than smoothing residential real estate services. The combination of a higher degree of habit formation and lower adjustment costs in real estate allows to explain the low volatility of residential services observed in the data. As shown in picture 2 (bottom right panel), the fact that, following a positive shock, the response of residential services is very gradual and hump shaped allows to lower the output and residential services correlation down to 0.61.

7 Conclusion

The main contribution of this study is to have developed a theoretical framework linking real estate and financial markets to the real economy, where key real estate and asset market facts can be explained within a general equilibrium business cycle model. When it comes to asset pricing facts, it has been shown that introducing incomplete depreciation in the stock of habit formation allows to generate high risk premiums without giving rise to excessive risk-free rate variation. While the volatility of the risk-free rate is often
thought to constitute the Achille heel of this class of models [see Jermann (1998); Boldrin, Christiano and Fisher (2001)], this study illustrates that this is however not a central problem. High risk premiums can be explained in general equilibrium models with production without relying on implausibly large variations in the risk-free rate.

When it comes to explaining housing facts, the first contribution of this paper is to have documented that, since data on REITs are available, excess returns on real estate have been higher than excess returns on equity. The key ingredient allowing the model implications to be robust to this additional dimension of the puzzle is the introduction of heterogeneity among sectors. Differences in dynamics arising from habit formation and the cost of adjusting capital lead to variations in the relative price of the real estate final good. By increasing the riskiness of dividends in the real estate sector, these procyclical variations in the relative price allow the model to account for the observed difference in excess returns.

As far as the real estate market is concerned, the second finding of this paper is that introducing habit formation in residential real estate services is key to explain the high volatilities of residential investment and house prices. In contrast to existing studies, it has been possible to accurately explain these two challenging facts in a model able to account for a broader set of asset pricing and real estate market facts, including the equity premium puzzle and the dynamics of residential real estate services.

It is also interesting to note that this successful characterization of the real estate market does not rely on the introduction of market failures. According to the results presented in this study, it is therefore not clear that, in the United States, the observed high volatility of house prices is a symptom of poorly functioning real estate markets.
8 References


Aoki K., Proudman J. and Vlieghe V. (2001) "House as a collateral: Has the Link between House Prices and Consumption Changed", mimeo, Bank of England


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9 Annex

In this economy without market failures, the first welfare theorem implies that the competitive equilibrium can be centralized and expressed as the following problem:

\[
L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ (c^n_t - x^n_t)^{\kappa} (c^h_t - x^h_t)^{1-\kappa} \right]^{1-\sigma} \right. \\
+ \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ A_t k_t^\xi N_t^{\alpha} q^{1-\alpha-\xi} - c^n_t - i^n_t \right] \\
+ \sum_{t=0}^{\infty} \beta^t z_t \lambda_t \left[ A_t h_t^\rho N_t^{\theta} t^{1-\rho-\theta} - c^h_t - i^h_t \right] \\
\left. \sum_{t=0}^{\infty} \beta^t \phi_t \left[ (1 - \delta^n) k_t + \Phi^n \left( \frac{i^n_t}{k_t} \right) k_t - \gamma k_{t+1} \right] \right. \\
\sum_{t=0}^{\infty} \beta^t \mu_t \left[ (1 - \delta^h) h_t + \Phi^h \left( \frac{i^h_t}{h_t} \right) h_t - \gamma h_{t+1} \right] \\
\sum_{t=0}^{\infty} \beta^t \phi_t^h \left[ b^h c^h_t + a^h x^h_t - \gamma x^h_{t+1} \right] + \sum_{t=0}^{\infty} \beta^t \phi_t^c \left[ a^n x^n_t + b^n c^n_t - \gamma x^n_{t+1} \right] \left. \right\}
\]

where \( \lambda_t, z_t \lambda_t, \phi_t^h, \phi_t^c, \varphi_t \) and \( \mu_t \) are the Lagrange multipliers attached to the six constraints. Using this more convenient representation of the economy, quantities and Lagrange multipliers can be solved using the technics developed by King, Plosser and Rebelo (2002). Dividends in each sector can then be expressed in terms of quantities, and house prices as a ratio of Lagrange multipliers. Equity prices can then be computed by expressing prices as the infinite discounted sum of future dividends.

\[^{25}\text{A detailed technical appendix is available on request.}\]
Figure 1: Marginal Utility and Habit Persistence

Figure 2: Shock ($\hat{A}_t$), Output ($\hat{y}_t^T$), Consumption ($\hat{c}_t^n$) and Residential Services ($\hat{c}_t^b$)
Figure 4: Residential Investment ($\hat{i}_t^h$), Non-residential Investment ($\hat{i}_t^n$), Capital Stock ($\hat{k}_t$) and Residential Structures ($\hat{h}_t$)

Figure 5: Shock ($\hat{A}_t$), Equity Prices ($\hat{p}_t^e$), Equity Price Real Estate Sector ($\hat{p}_t^r$) and House Prices ($\hat{p}_t^h$)