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(Un)anticipated Technological Change in an Endogenous Growth Model

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(Un)anticipated Technological Change in an Endogenous Growth Model*

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Abstract

This paper examines numerically the impact of a *negative* exogenous shock to marginal productivity (such as ecological government regulation that becomes effective at some point in time) in an endogenous finite time growth model with sluggish reallocation of human capital. The policy can be anticipated or unanticipated by the economic agents, and it can also be announced but not implemented. It turns out that these frictions have very strong long-run effects on consumption and output, and on the optimal allocation of capital and labor in particular. The qualitative properties are closely related to those found in homogenous labor models with *positive* productivity shocks. The numerical optimization method employed here proved very successful in qualitatively similar problems in engineering but has not yet found its way into macroeconomic models of growth.

JEL-Classification: C61, E32.
Keywords: Two-sector endogenous growth model, technology shock, frictions, Runge-Kutta parallel-shooting algorithm.

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1 Introduction

Our goal in this paper is to study the announcement and implementation effects of a *negative* exogenous change to consumption good productivity when the allocation of labor is subject to frictions. Physical capital, however, is completely mobile and may partly offset the effect of this friction.[1] To this end we analyze numerically the optimal policy in a standard endogenous growth model with production and education sectors. The analysis focuses on a technology with a finite lifetime. The current physical production technology (and with it all specialized human capital) becomes obsolete at a certain point in time due, for instance, to a structural break.

The exogenous change to the physical capital sector can be anticipated or unanticipated by firms. Moreover, we allow for the possibility that it is announced but not implemented. One can think of a change in productivity caused by government regulation such as anti-pollution laws, or any other measure that lowers marginal productivity of firms.

The main feature of the model considered here is that frictions in the reallocation of labor between the two sectors rule out an instantaneous adjustment, for instance in response to an unanticipated (or non-enacted) change. This may lead to an imbalance between the levels of physical and human capital in both sectors. On the optimal path, allocation of physical capital is thus indirectly affected by the friction. We show that the sluggish reallocation of labor has severe long-run effects on consumption, output and labor- and capital-allocation. None of these qualitative properties (except for the interest rate in the pre-change time when the constraint is not binding) are observed for the balanced growth path of the corresponding infinite horizon model without frictions.

Our findings are related to insights from homogenous labor models in which the impact of a *positive* production shock on employment is analyzed. For a brief summary see, e.g., Trehan (2003). For instance, Phelps and Zoega (2001) argue that the news of a productivity jump that will materialize at some point in the future leads to an expansion (and to more employment in particular) before the change actually occurs; the effect will dissipate once the change happens. We find that the news of a *negative* productivity shock (and its materialization) leads to the same pattern of employment in the sector in which technology changes.

In the presence of frictions and (anticipated) exogenous changes to the technology the application of necessary first-order conditions for optimality is problematic. Instead, in this paper the optimal path is found numerically,

[1] The cost of factor reallocation is studied, e.g., in Beckham Gramm (2005).

using a method that has had considerable success in qualitatively similar problems in engineering. The method is referred to as the Runge-Kutta parallel-shooting algorithm. While this method has not yet found its way into macroeconomic modeling, the Runge-Kutta reverse-shooting method has been applied to study economic dynamics, e.g. Heer (2003). In the Runge-Kutta parallel-shooting method, the continuous problem is discretized and converted into a nonlinear programming problem. The system-governing equations are enforced through the use of nonlinear constraint equations that are implicit integration rules. The method is direct in that the system Lagrange multipliers (i.e. adjoint variables) are not required.

The next section introduces the model. The numerical technique is explained in detail in Section 3. The results follow in Section 4 before summarizing and concluding the paper. The appendix contains the graphical output of the simulations.

2 Model

The model is a finite time horizon version of the two-sector model of endogenous growth with different technologies for production and education, cf. Barro and Sala-i-Martin (1995, Chap. 5.2.1), and Rebelo (1991). Education is labor-augmenting in both sectors. There are additional constraints on labor mobility and the availability of information with respect to exogenous changes to the marginal productivity. The technology's lifetime is finite and the human skills associated with this technology eventually become worthless. Only the terminal stock of a consumption good is assigned a value as a consumption good.

Production functions are Cobb-Douglas in both sectors. The output of physical goods is given by

$$Y_t = C_t + \dot{K}_t + \delta_K\, K_t = A_t\, (\phi_t\, K_t)^{\alpha}\, (\psi_t\, H_t)^{1-\alpha} \tag{1}$$

and human capital growth is given by

$$\dot{H}_t + \delta_H\, H_t = B\, ([1-\phi_t]\, K_t)^{\eta}\, ([1-\psi_t]\, H_t)^{1-\eta}. \tag{2}$$

The exogenous technology parameter $A_t > 0$ is time-dependent. Changes in A_t are Hicks neutral, i.e. marginal productivity of both inputs is affected to the same extent.

The control variables $0 \leq \phi_t \leq 1$ and $0 \leq \psi_t \leq 1$ are the fractions of physical and human capital respectively used in production of physical capital, the remainder being employed in education. The physical good can be used for consumption or investment. δ_K and δ_H denote the depreciation rate of physical and human capital respectively. We assume a constant population size, normalized to one.

In contrast to the standard assumption of frictionless reallocation of labor and capital between the two sectors, the model assumes that only physical capital is completely mobile while adjustment in human capital is subject to frictions. These frictions originate e.g. from the requirement of different skills in each sector. The friction in the reallocation of labor between the two sectors is modelled as a constraint:

$$-b_\psi \leq \frac{d\psi_t}{dt} \leq +b_\psi. \tag{3}$$

Human capital is perfectly mobile for $b_\psi = \infty$. An alternative specification of (3) is obtained by introducing a (convex) cost of reallocation of labor for instance. This does not, however, allow for an analytical study, though it does complicate the numerics considerably.

The central planner's maximization problem is given by

$$\max \int_0^T e^{-\rho t} \frac{C_t^{1-\sigma} - 1}{1-\sigma} \, dt + e^{-\rho T} \frac{1}{\rho} \frac{(\rho K_T)^{1-\sigma} - 1}{1-\sigma} \tag{4}$$

$\rho > 0$, $0 < \sigma \neq 1$ subject to (1), (2), (3), and the no-borrowing condition $K_t \geq 0$. The term on the far right is the present value of the remaining stock of consumption good K_T when production ceases.

A change in the technology parameter A_t (which determines the return on inputs in the production of the consumption good) can be anticipated or unanticipated by the economic agents. A policy can also be announced but not enacted. Four cases are studied in this paper. They are delineated in terms of the optimization problem faced by the central planner in response to the revelation of information over time:

No Action [N] The optimization problem is solved subject to $A_t = 1$ for all $t \in [0, T]$.

Anticipated [A] The optimization problem is solved subject to $A_t = 1$ for all $t \in [0, T/2]$, and $A_t = 1/2$ for all $t \in (T/2, T]$.

Not Anticipated [NA] The optimization problem is solved subject to $A_t = 1$ for all $t \in [0, T]$, but at time $t = T/2$ it is revealed that $A_t = 1/2$ for all $t > T/2$. At time $t = T/2$ the optimization problem is newly solved for the remaining time horizon subject to $A_t = 1/2$ for all $t > T/2$.

Not Enacted [NE] The optimization problem is solved subject to $A_t = 1$ for all $t \leq T/2$, and $A_t = 1/2$ for all $t > T/2$. At time $t = T/2$ it is revealed that A_t will remain equal to 1 for all $t > T/2$. The optimization problem is newly solved for the remaining time horizon at time $t = T/2$ subject to $A_t = 1$ for all $t > T/2$.

3 Numerical Method

The numerical method used in this paper is explained in some detail because it has not yet been used in the study of economic problems. The need to apply this approach (which has had considerable success in engineering) stems from the fact that the optimal paths in our model are not (unconstraint) saddle paths and thus reverse shooting does not work.

The approach to solving optimal control problems employed here is based on the discretization of the original problem. One transforms the problem into a parameter optimization problem, which is then solved using mathematical programming, see Enright and Conway (1991, 1992). This transcription method is essentially an adaptation of the parallel-shooting method for boundary-value problems to the discretization of the equation of motion constraints in the direct approach to the optimal control problem. An explicit four-stage Runge-Kutta integration rule is used to propagate the governing equations across the discretized segments of the trajectory. Requiring continuity at the nodes of the segment boundaries generates a set of discrete nonlinear algebraic constraints involving the states and controls on the boundaries of the segment.

In the Runge-Kutta parallel-shooting algorithm the previously defined optimal control problem is first discretized into a sequence of stages. The partition $[t_0, t_1, ..., t_N]$ is introduced, with $t_0 = 0$, $t_N = t_f$, where $t_0 < t_1 < ... < t_N$, and let $h_i = t_i - t_{i-1}$ for $i = 1, 2, ..., N$. The h_i's may or may not be uniform. The mesh points t_i are referred to as *nodes*, whereas the intervals $[t_{i-1}, t_i]$ are referred to as *segments*. The state variables $x_i = x(t_i)$ are approximated by values at the nodes, for $i = 0, 1, ..., N$. Control variables are provided at the nodes t_i as well as the segment center points, $t_i + h/2$, by $u_i = u(t_i)$ for $i = 0, 1, ..., N$ and $v_i = u(t_{i-1} + h/2)$ for $i = 1, 2, ..., N$. From a given node, t_{i-1}, the equations of motion are integrated forward from initial condition x_{i-1} to the next node t_i using the controls u_{i-1}, v_i, and u_i by a step of a four-stage Runge-Kutta formula:

$$y_i^1 = x_{i-1} + \frac{h}{2} f(x_{i-1}, u_{i-1}) \tag{5}$$

$$y_i^2 = x_{i-1} + \frac{h}{2} f(y_i^1, v_i) \quad (6)$$

$$y_i^3 = x_{i-1} + h f(y_i^2, v_i) \quad (7)$$

$$y_i^4 = x_{i-1} + \frac{h}{6} f(x_{i-1}, u_{i-1}) + 2f(y_i^1, v_i) + 2f(y_i^2, v_i) + f(y_i^3, u_i). \quad (8)$$

The state at the next node is estimated by y_i^4, so, for continuity it is necessary that the "Runge-Kutta defects"

$$\Delta_i = y_i^4 - x_i = 0 \quad (9)$$

for $i = 1, 2, ..., N$. The Runge-Kutta procedure has order h^5 local truncation errors.

Perhaps the biggest advantage of this method is that since it is explicit, it can be incorporated into a parallel-shooting approach, as illustrated in Figure 1. The single step of the Runge-Kutta procedure previously described is replaced with multiple steps. This allows the use of larger intervals, resulting in smaller nonlinear programming (NLP) problems. However, in each segment additional control variables must be introduced to accommodate the multiple integration steps.

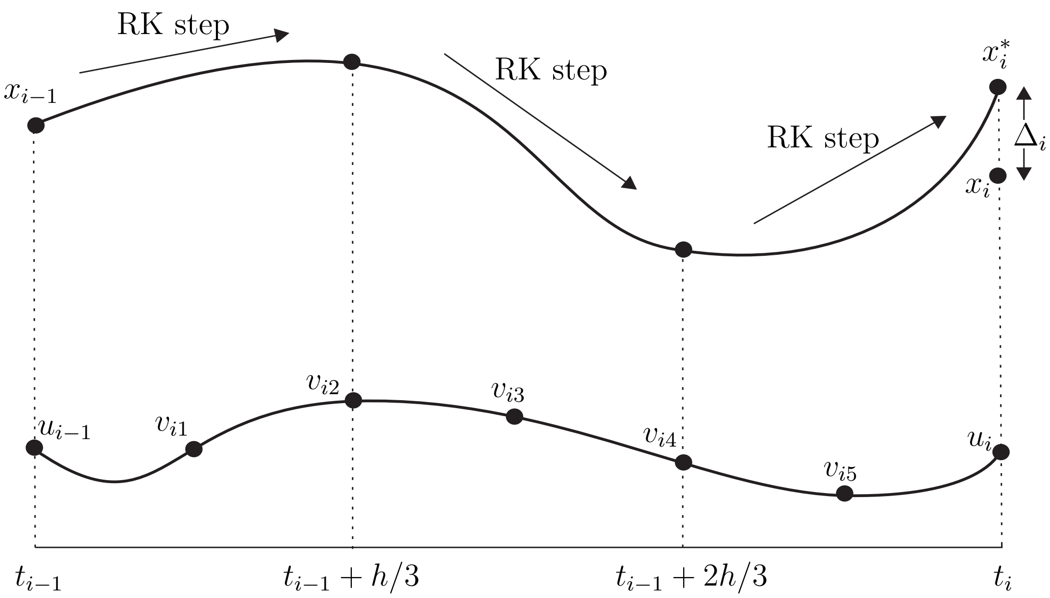

Figure 1: Chart illustrating the Runge-Kutta parallel-shooting discretization of the continuous optimal control problem. Only one of many segments is shown.

Let p be the number of integration steps per segment, $[t_{i-1}, t_i]$. In the usual manner, states and controls are provided at the nodes, $x_i = x(t_i)$ and $u_i = u(t_i)$. Now, the "center" controls $v_{ij} = u(t_{i-1} + jh/2p)$ for $j = 1, 2, ..., 2p - 1$ and for $i = 1, 2, ..., N$ must be provided, as shown in Figure 1 for $p = 3$. Then, using equations (5)–(8) the states are integrated from t_{i-1} forward one step to $t_{i-1}+h/p$ with controls u_{i-1}, v_{i1}, and v_{i2}. Using the resulting estimate of the state at $t_{i-1} + h/p$, the state integration is continued forward using the four-stage Runge-Kutta procedure and the controls yielding the estimate of the state at $t_{i-1} + 2h/p$. The process is repeated once more using controls, resulting in an estimate of the state, x_i^*, at node t_i which then replaces y_i^4 in the defect formula, equation (9). Note that the $p - 1$ intermediate estimates of the state vector do not appear explicitly in the NLP problem.

The parallel-shooting method allows for the use of larger intervals, and results in smaller NLP problems than collocation methods. Although additional control variables must be introduced in each segment to accommodate the multiple integration steps, the intermediate state variables (resulting from the forward propagation) are used to reinitialize values for the next step, and never appear explicitly in the NLP problem. Since most optimal control problems have many more state variables than control variables, the tradeoff of introducing more control variables and reducing the number of state variables is a favorable one.

The Runge-Kutta defects constitute a set of nonlinear "defect" equations, i.e. nonlinear equality constraints. These defect equations become the nonlinear constraints in the NLP problem. Collecting all the independent variables into a single vector P defined as:

$$P^T = (x_0, u_0, x_1, u_1, ..., x_N, u_N, t_f) \tag{10}$$

where, in this problem, the cost function is the final time t_f (the last parameter in the vector P^T), and similarly collecting all the nonlinear constraint equations into a vector C^T, the optimal control problem can then be restated as an NLP problem of the form:

$$\text{minimize } \varphi(P)$$

subject to:

$$b_L \leq \left\{ \begin{array}{c} P \\ AP \\ C(P) \end{array} \right\} \leq b_U \tag{11}$$

where AP contains all the linear relationships of the stated problem, and b_L and b_U are the lower and upper bounds on the variables and constraints, Hargraves and Paris (1987). The vast majority of the nonlinear constraint equations comprising vector C^T are the defect equations (9) for which upper and lower bounds will both be zero. Another attractive feature of this method, in comparison to the Euler-Lagrange necessary conditions, is that known discontinuities in either the state or control variables are easily accommodated, for example through the linear equations AP in (11) or directly using the upper and lower bounds of the parameter vector P containing the state and control variables.

4 Results

4.1 Overview

We present the optimal solution to the model for each of the four cases detailed in Section 2 using the numerical procedure described above. A graphical representation of the results on the optimal paths are collected in Appendix A. The figures show optimal time paths of consumption, physical and human capital in production, net interest rate of physical capital in production, fraction of human capital in production as well as its change, physical capital–human capital ratio, consumption–physical capital ratio, total human capital, and total output.

Parameter settings, which are taken from Barro and Sala-i-Martin (1995, Chap. 5) where possible, are fixed throughout the numerical analysis as follows:

$$\rho = 0.02,\ \sigma = 3,\ \alpha = 0.4,\ \eta = 0.2,\ B = 0.136, \text{ and } \delta_K = \delta_H = 0.05. \quad (12)$$

In this model the education sector is relatively intensive in human capital ($\eta < \alpha$). We further set

$$b_\psi = 0.05,\ T = 50,\ K_0 = 1,\ H_0 = 1, \text{ and } \psi_0 = 0.5. \quad (13)$$

The friction in the reallocation of labor is significant; that is, the constraint is frequently effective, as observed in the simulations. With the discount rate set to $\rho = 0.02$, one unit of time can be interpreted as a year; the lifetime of the technology thus being $T = 50$ years.

Denoting the overall utility, which is defined in eq. (4), by $U[\cdot]$, one obtains the following ranking: $U[N] = -13.49 > U[NE] = -23.58 > U[A] = -62.72 > U[NA] = -94.39$. This result is perfectly in line with intuition: (a) no decrease in productivity is best, (b) if a change is anticipated it is better if it does not occur, (c) an unanticipated change is worst.

The optimal paths can be divided into two main phases: a pre-change and a post-change regime.

In the pre-change regime, the cases [N] and [NA] must show the same solutions, as the shock is not anticipated by the households and they therefore behave as if no shock is occurring at time $T/2$. For the same reason [NE] and [A] coincide. In both cases the household is expecting a change and adapts its behavior accordingly.

In the post-change phase, the true policy has been revealed and the four cases show different dynamics. Households that built wrong expectations ([NE] and [NA]) have to adapt to a new reality. As might be expected, the consumption paths for these households are rather volatile. The agent who was expecting a negative change in productivity refrained from consumption in the pre-shock phase to save up for the future. Realizing that the government decided not to enact the policy, the economic agents have to make up for too little consumption in the pre-shock phase. The opposite is true for the totally surprised household in the case [NA]. Since the households relatively overconsumed in the pre-shock regime, they have to face a cut-off in consumption when the shock occurs.

The households which had correct expectations show smooth consumption paths. There is no marked difference between the consumption path when the change is correctly anticipated ([A]) or when it does not happen at all ([N]). In both cases the consumption path increases at a roughly constant rate. Obviously, consumption is low in the case of an anticipated negative shock ([A]) in comparison to the case in which the government does not interfere at all ([N]). The household which anticipates correctly a cut in productivity will invest more for the future to realize a smooth consumption path. The corresponding physical capital stocks in production are in line with these observations. In cases [A] and [NE] a build-up of physical capital takes place in the pre-change regime as a response to the expected change in productivity. After the change occurs in case [A], most of the capital is used to smooth consumption. In our model physical capital is completely mobile. Therefore, the more surprising results are reflected in the allocation of human capital since it is subject to frictions.

4.2 Allocation of Human Capital

The optimal path of the change in the flow of human capital, $d\psi/dt$, depicted in Appendix A, allows several episodes to be distinguished. Recall that the friction in labor reallocation becomes binding when the rate of readjustment in either direction reaches $b_\psi = 5\%$.

In case [N], in which a shock is neither expected nor occurring, human capital is first allocated at maximum rate to physical production and then moved back to education as an investment to boost future production of the physical good. From period 11 to 33 the constraint is not binding. Thereafter, the human capital is moved back into the goods producing sector until the share of human capital in production equals 1 by the end of the time horizon. In all cases ψ_t has to equal 1 by the end of the time horizon, since the technology—and with it all specialized human capital—expires.

When the change of productivity occurs as a surprise ([NA]) the allocation of human capital has to coincide with the one in case [N] up to time $T/2$. Once the negative change in production is revealed in case [NA], human capital is, for a few periods, reallocated at maximum rate to education and, afterwards, moved back to production. The first move into education is to make up for the decline in productivity by investing in skills which become important in the future. With this strategy, consumption is falling dramatically right after the shock, but shows a smooth pattern thereafter.

In case [A], in which the change is correctly anticipated, the time-path of $d\psi/dt$ is governed by the conflicting goals of increasing productivity to ease the future transition to a low productivity regime and the need to produce more consumption good to smooth consumption over time. This causes a reallocation of labor to the production sector before the change to build up a stock of the consumption good. The accumulation of human capital in production peaks around $t = 20$ due to the friction in the reallocation of labor. Thereafter, for quite a while, human capital flows back to the education sector to improve future production of the consumption good. In case [NE], where a change is expected but does not finally occur, human capital is reallocated to the consumption good sector immediately after the information is released. The surge in consumption is a response to too little consumption and too much production of human capital before $T/2$. The most remarkable difference between these two cases ([A] and [NE]) occurs right after the true policy has been revealed. A build-up of the share of human capital in production ψ_t is already observed a few periods after the shock.

In the presence of an anticipated *negative* productivity shock ([A]), the time-pattern of employment in the consumption goods sector (which is directly hit by the shock) exhibits a dynamics that closely resembles the impact of a *positive* production shock on employment in Phelps and Zoega (2001), see also Phelps (1994, 1999). Phelps and Zoega argue that the news of a productivity jump that will materialize at some point in the future leads to an expansion (and to more employment in particular) before the change actually occurs; the effect will dissipate once the change happens. Our numerical results show that the news of a *negative* productivity shock (and its materialization) leads to the same pattern of employment in the sector in which technology changes and to an increase in output. In line with our observations of a negative correlation between an anticipated change in technology and employment are, e.g., Atkeson and Kehoe (2000), Greenwood and Yorukoglu (1997) and Manuelli (2000). They find that an anticipated positive change in technology leads to a temporary decrease in employment: An anticipated but not-yet-implemented positive change in technology decreases the market value of existing firms using the old technology. Subsequently, firms will cut back on investment and employment. Our results suggest that the surge in employment in the consumption good sector in the pre-change phase reflects the need to produce more output and to build up capital in order to smooth consumption over the entire time-horizon.

4.3 Effectiveness of Cheap Talk

This subsection focuses on the case [NE], in which the government announces a policy change but, eventually, does not implement it. In this sense the government is ‘cheating’ on the economic agents. The deviation from its earlier announcement—the cheap talk (see, e.g., Farell and Rabin (1996))—is costless for the government: There is no punishing mechanism or loss of reputation since the “game” is only played once. This view allows for an interesting interpretation of the dynamics. There has been much research on the credibility of monetary policy e.g. Kydland and Prescott (1977), Barro and Gordon (1983) and Stein (1989). These papers show that if there is no credible commitment device, the government or the central bank may have an incentive to deviate from earlier policy announcements or plans.

A government’s cheap talk has strong effects on the optimal paths for consumption C_t and output Y_t, see Appendix A. Both consumption and output increase significantly just after the true policy of cheap talk has been revealed. The rise in consumption and output is the reaction of households to the non-enactment of the announced plan. Households, who were wrongly expecting a negative change in productivity, saved in the pre-change phase for the anticipated bad times. As the news is revealed, these savings are used to increase consumption as well as output.

If the performance of the government (being in place at the time the policy is due to be implemented) is measured solely by output and consumption growth, non-enactment will obviously be the government’s preferred strategy.

4.4 Comparison with the Balanced Growth Path

We finally discuss how the qualitative behavior of the optimal solution relates to that of the balanced growth path of the corresponding unconstrained infinite horizon economy. In the latter model the long-run characteristics of the economy are as follows (the numbers are obtained by following Barro and Sala-i-Martin (1995, Chap. 5)). For $A = 1.0$, $\psi^\star = 0.167$, $(K/H)^\star = 3.11$, $(C/K)^\star = 0.0433$, and the net interest rate is 8.0%. For $A = 0.5$, $\psi^\star = 0.318$, $(K/H)^\star = 1.57$, and $(C/K)^\star = 0.0886$, and the interest rate is 5.9%.

Some guidance about the difference between our model and the infinite horizon model without frictions may be extracted in particular from the human capital share in production and the interest rate path. The other characteristics do not carry over.

The share of labor in the consumption good sector in our model is almost always higher than in the infinite horizon model. Only around the period in which productivity is cut by half does this fraction fall below the optimal share in the infinite horizon model. What appears to be counterintuitive at first glance is actually an optimal reaction to the reduction of productivity: The high initial share of human capital in production contributes to the build-up of physical capital to smooth consumption. While this "saved" good is used for consumption, human capital is moved to education to increase the total amount of human capital, which partly offsets the drop in productivity for the remaining period of time.

As to the interest rate, the first few periods are an adjustment phase in all four cases. After a few periods the interest rate falls to about 8%, i.e. the interest rate that would prevail along the balanced growth path of the corresponding unconstrained infinite horizon economy. This convergence is quicker for the cases in which a change in productivity is anticipated ([A] and [NE]) and takes longer if no change in policy is expected ([N] and [NA]). The changes in the interest rate cannot be explained by the friction in labor allocation alone. For instance, in cases [A] and [NE] the flow of human capital away from the education sector in periods 16 to 20, which happens at a maximum rate, is not reflected in any instant interest rate change. When a cut in productivity is anticipated, physical capital is accumulated. This build-up of physical capital is disproportionately high in comparison to human capital (see Appendix A for a graph of the capital-labor ratio), which lowers the interest rate. The interest rate falls to approximately 5% at the expected productivity cut date. This is roughly the interest rate of the balanced growth path for $A = 0.5$. In the post-change regime the interest rate first increases in all cases as the capital-labor ratio is decreasing. Subsequently, the pattern of the interest rate is caused by the finite time horizon of the technology.

5 Conclusion

We examined an endogenous growth model of finite time horizon with frictions in the allocation of human capital. In this model a negative shock to production could or could not occur and this event could be anticipated or not. The corresponding four cases were examined. To solve the model numerically, we applied the Runge-Kutta parallel-shooting algorithm. This technique has been successfully applied in engineering but has not been introduced into the numerical analysis of macroeconomic models of growth. We feel the method has potential for the analysis of problems similar to the one studied here.

The overall utility, unsurprisingly, is maximized if there is no shock at all. If a shock is enacted, it is best if the economic agents anticipate the event and can adapt their behavior accordingly. We showed that if the government has an interest in boosting output and consumption at a certain point in time, cheap talk and hence the element of surprise may be a successful strategy. The introduction of frictions in the accumulation of labor, while capital is completely mobile, has a pronounced effect on the long-run dynamics of output, consumption and the allocation of capital and labor in particular. As mentioned, the literature finds contrasting observations on the correlation of an anticipated technology shock and employment. We suggest that in a finite time horizon model with frictions in the allocation of labor an anticipated negative technology shock can generate a temporary rise in employment in the consumption goods sector. While these results per se do not challenge the findings of Phelps and Zoega, the paper provides further evidence that the relation between technological change and employment is neither obvious nor trivial. There appears to be scope for further research on this issue.

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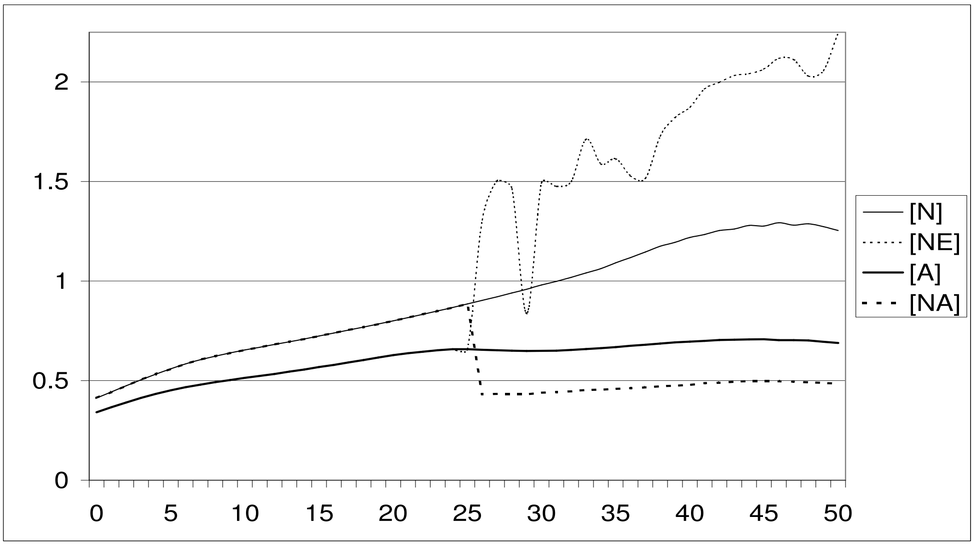
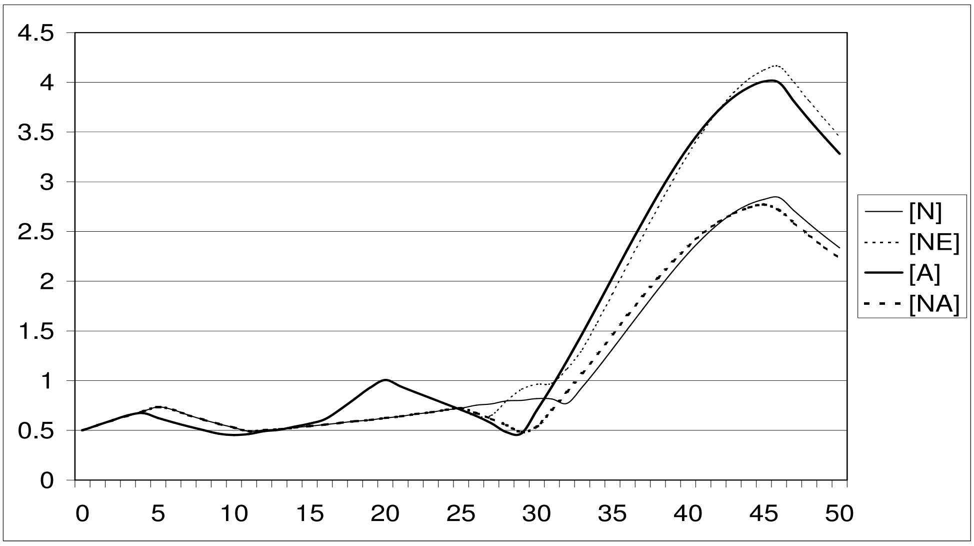
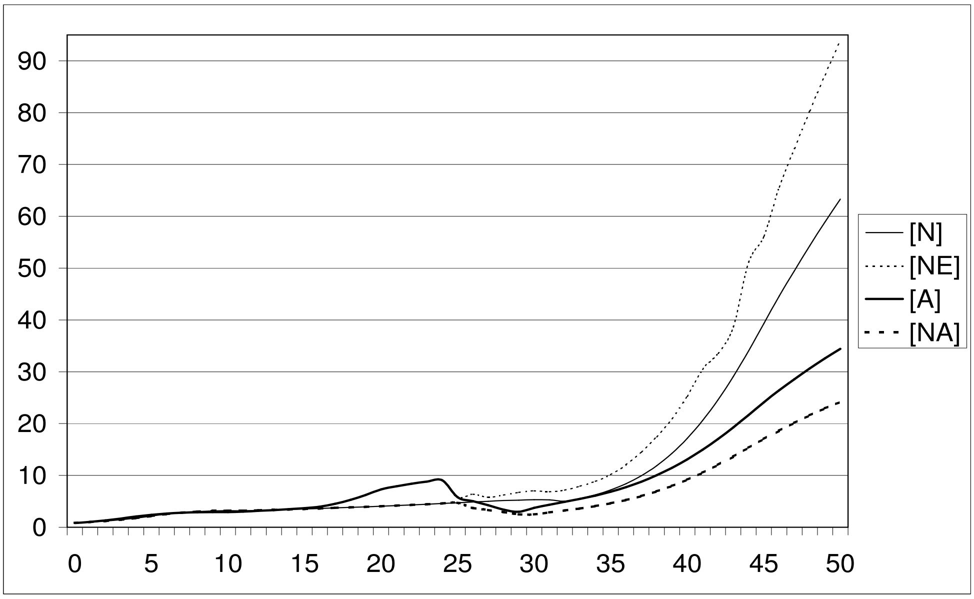
A Graphs of Optimal Paths and Controls



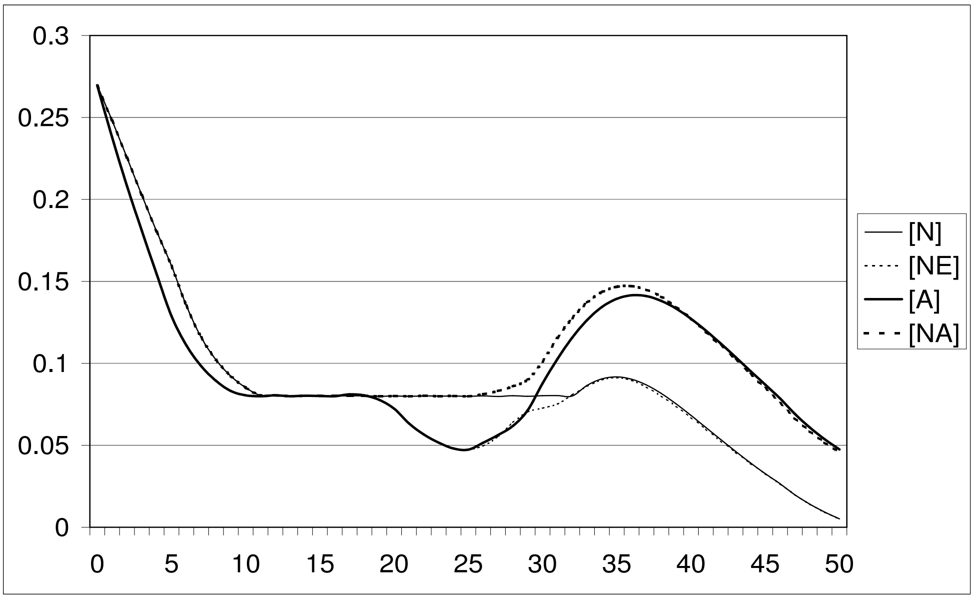
Consumption C_t



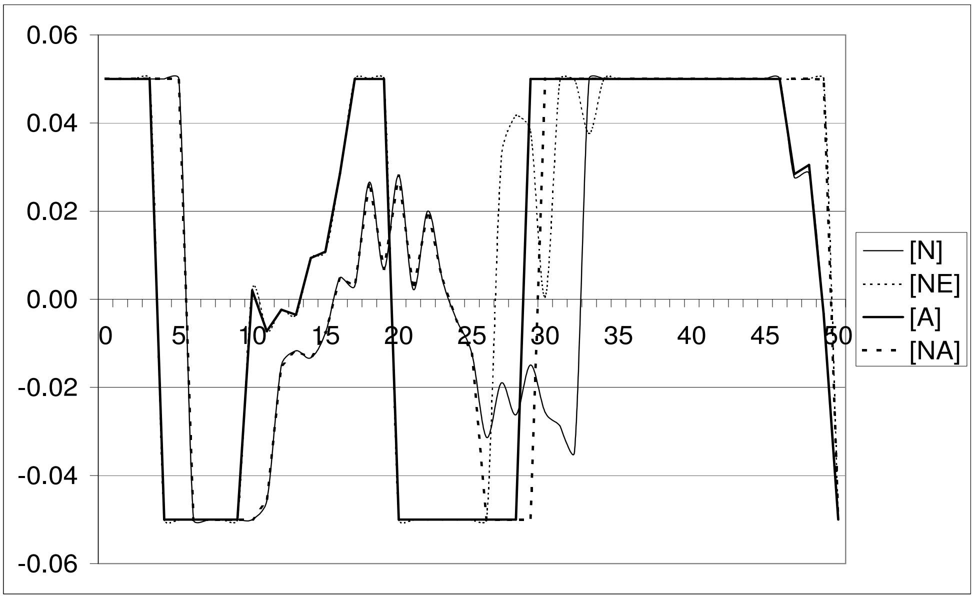
Human capital in production $\psi_t\, H_t$



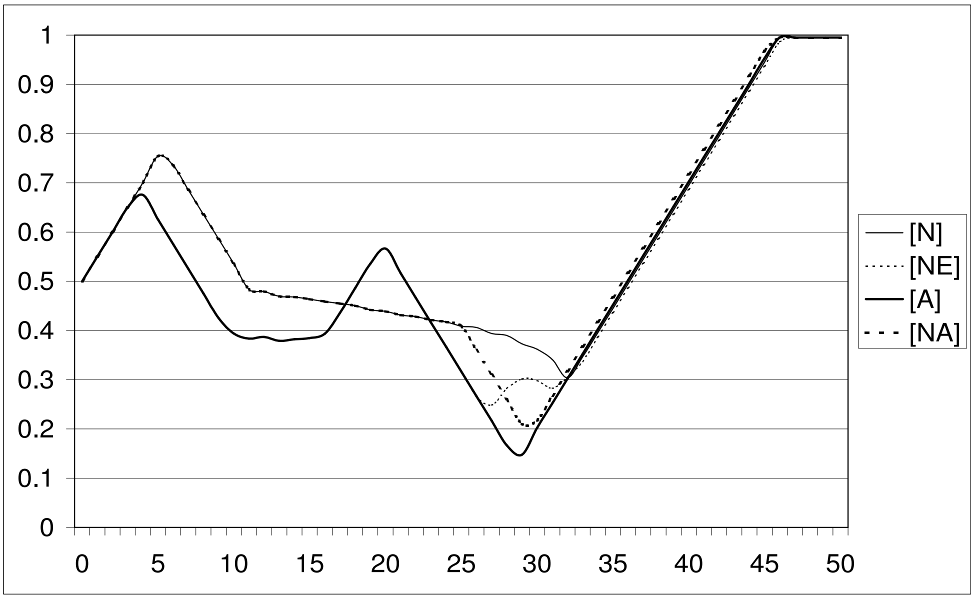
Physical capital in production $\phi_t \, K_t$



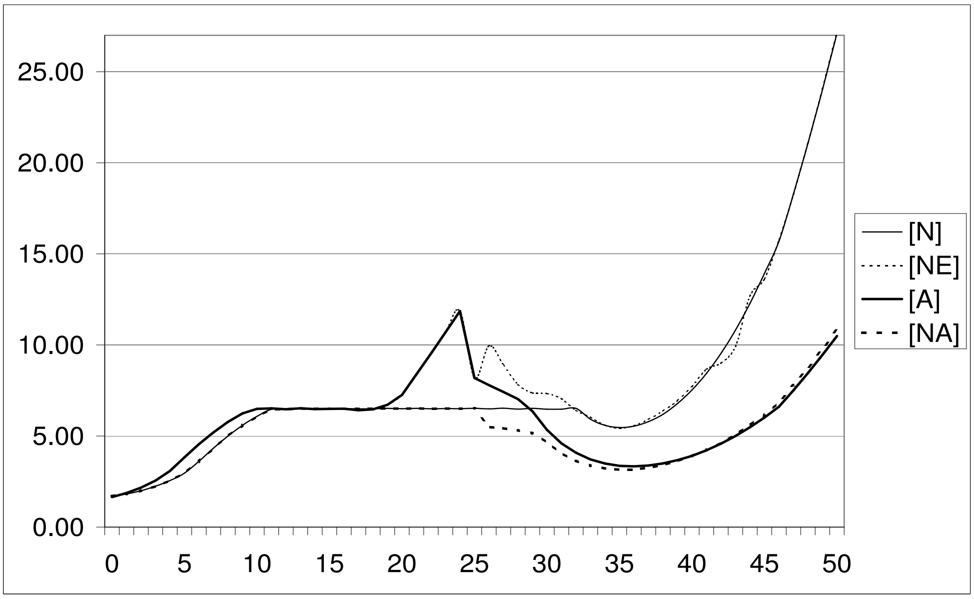
Net interest rate of physical capital in production



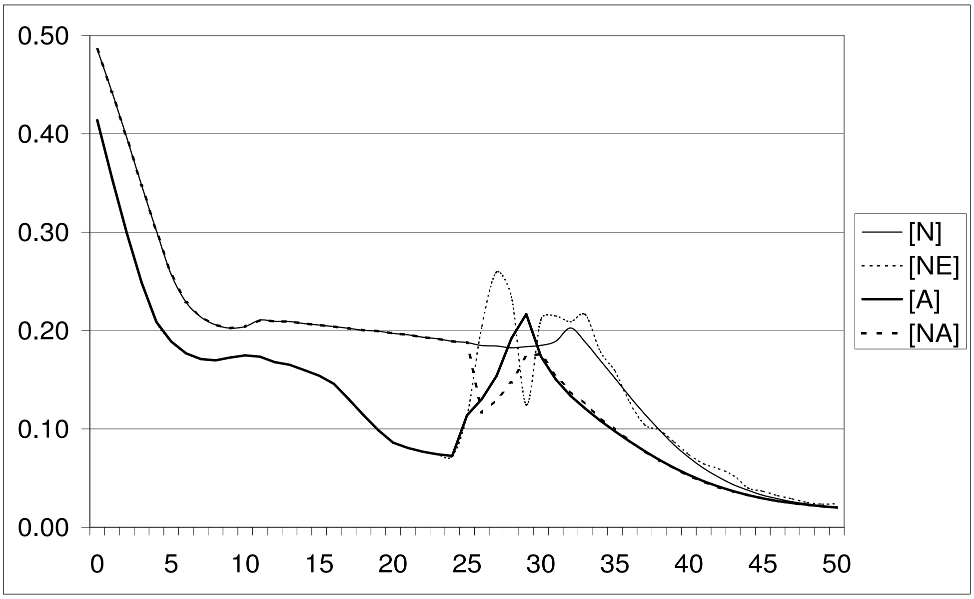
Change of human capital share in production $d\psi_t/dt$



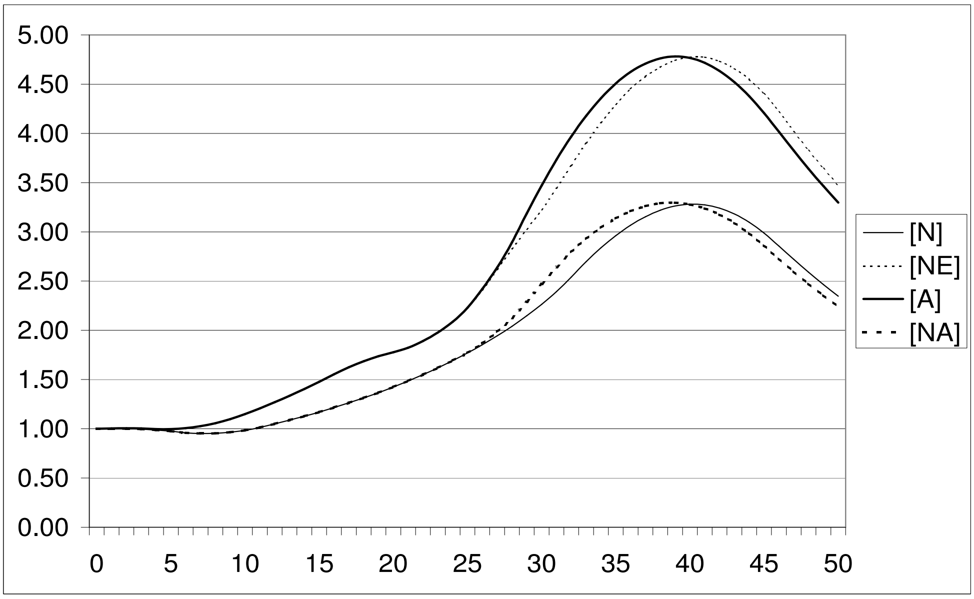
Human capital share in production ψ_t



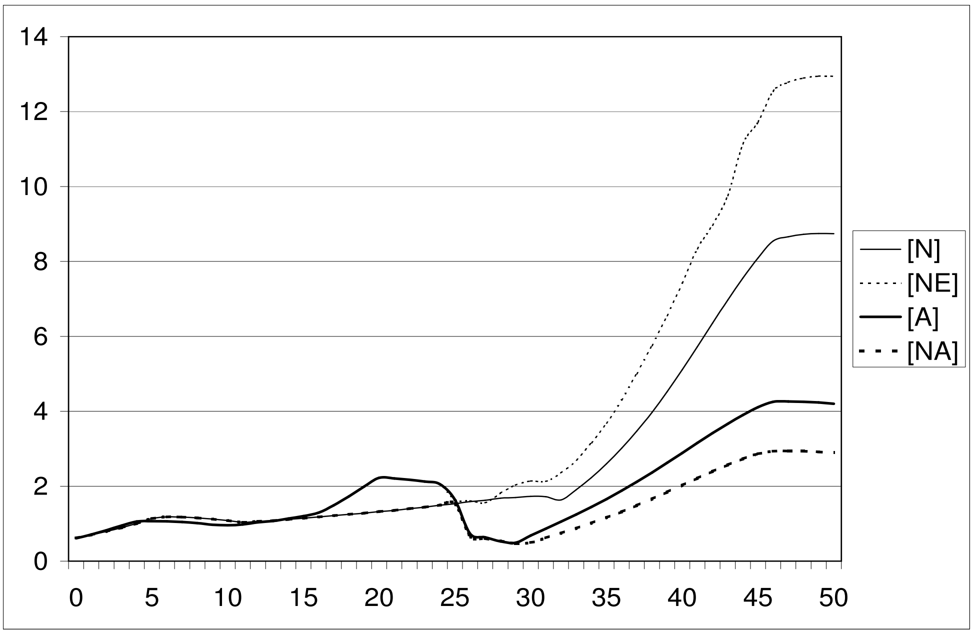
Capital-labor ratio K_t/H_t



Consumption-capital ratio C_t/K_t



Human capital H_t



Total output of physical good Y_t