Business and Financial Indicators: What are the Determinants of Default Probability Changes?

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ABSTRACT

This paper investigates some common determinants of default probability changes of individual firms using Standard & Poor’s ratings database. We analyze and quantify the responses of hazard rates to changes in various economic variables, namely financial markets, business cycle and credit indicators, and examine their persistency. First, we show that including non-financial information largely improves the poor explanatory power of financial-based factor models. More importantly, in comparison with market factors, business and credit factors become dominant as the issuer quality decreases. Second, we highlight the benefits of past information. Our results prove that both past shocks and subsequent economic trends are of prime importance in explaining probability changes. To draw these conclusions, we introduce a semi-parametric framework accommodating the continuous nature of probability changes and ageing effects. It allows us to recover default probabilities over any desired time horizon.

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Introduction

Understanding and modeling the common dynamics of default probabilities is a key concern for practitioners and academics. Probability changes have a significant impact on the prices of credit risky instruments but also on the way credit risk should be managed. Regulation issues especially through economic capital computations, as well as large variations in default rates over the last five years have renewed interest in this topic. Common dynamic changes in probabilities have mainly been investigated using historical rating migration matrices or, from pricing perspectives, by modeling credit quantities (in particular credit spreads). It turns out that significant differences in probability levels have been observed through expansion and recession periods (e.g. Nickell, Perraudin and Varotto 2000).

Researchers have concentrated on modeling and forecasting these changes either by filtering out unobservable common drivers (see e.g. Duffee 1999 or Driessen 2005 under the pricing measure, Koopman, Lucas and Monteiro 2008 or Wendin and McNeil 2006 under the historical measure) or using stock and bond markets factors (Janosi, Jarrow and Yildirim 2002). These approaches find their origins in structural models (firm-value based) and reduced-form models (intensity based). The former are closely linked to financial markets as they model default as the first time at which a firm asset value falls below its liabilities. But they regularly fail to provide accurate probability estimates. This is essentially due to structural models heavily relying on stock market components which are noisy and contain bubbles, as pointed out by Janosi et al. (2002). Another reason for this failure is that some parameters, such as the asset volatility, are difficult to observe and calibrate. The intensity-based methodology is more convenient for pricing and provides simple extensions to multiple assets (e.g. see Duffie and Singleton 2003).\footnote{We will abusively use indifferently the terms of hazard rates and intensities in this paper. We will only point out the differences when outlining our framework.} However, since dynamics are exogenously specified, this approach ignores what the default mechanisms are and leads to disappointing predictive power as well. It is worth noticing that firm-specific variations have also been documented. This last stream of studies relies on different modeling techniques, namely credit scoring or hazard rate models (see e.g. Duffie et al. 2007) which will be used in this paper.

A key issue lies in the identification of the systematic drivers of default probabilities. Some studies observed surprisingly low sensitivities of spreads to interest rate factors or to stock market variables (see e.g. Collin-Dufresne, Goldstein and Martin 2001), but explanations are lacking. This paper contributes to the existing literature by considering a new set of potential determinants that explicitly link the default process to economic fundamentals. This study shows that including all economic components provides more accurate measures than those
obtained from financial market factors. For the first time, the critical roles of business cycle related indicators as well as aggregate credit market factors are identified and quantified. An additional contribution lies in the use of past information, jointly with contemporaneous one. We show that large benefits can be obtained by taking into account lagged effects between the economy and the default cycle. The empirical part of the paper relies on a unique rating event database that captures more precisely the default probabilities, whereas the previous body of evidence is based on a noisy measure of these probabilities, namely the credit spreads.

We believe there are four main contributions of this paper. First, we analyze and quantify the impact of financial markets, business cycle indicators and credit market indicators on default probabilities of various rating classes. Second, we show that although all economic variables help explaining default likelihoods, their explanatory power differs greatly between investment- and noninvestment-grade classes. As issuer quality decreases, the dominant systematic factors change from financial markets to business cycle and endogenous default indicators. Relying solely on financial information to explain default probabilities leads to poor results for noninvestment-grade firms. Default peaks are underestimated whereas default probabilities predicted by the model overshoot realized probabilities during stable and low-default periods. Third, we establish the critical importance of combining past information with contemporaneous factors. We show that default probability changes are the joint effects of past shocks and subsequent economic trends. Resulting lead-lag effects between the economy and the default cycle partially explain the lower speed and higher persistence of the default cycle. Fourth, we introduce a semi-parametric framework which explicitly takes into account the term structure of hazard rates. Firm ageing effects as well as calendar-time effects are explicitly taken into account. Models are set up in continuous time using the natural time-to-default framework and taking care of censoring. In particular, default probabilities can consistently be computed at all horizons. A specification test is also introduced.

The paper is organized as follows. The first Section briefly present the default data. Section 2 presents our modeling framework. We then propose a list of potential default determinants which will be investigated in later sections. We extensively study sensitivities of the default cycle with respect to financial markets, business and endogenous indicators through conditional single factor hazard models in Section 4. Section 5 gathers main results and their implications looking at multifactor models. We check misspecifications of financial market based models, and assess the benefits of considering other economic components. Our results deliver clear empirical guidelines for modeling. They rely on analyzes of predictive variables of default probabilities exploiting the history of companies in the rating process. This study opens new avenues to investigate the intensity-based pricing and the management of default risk. It more
particularly sheds light on relevant systematic factors for the construction of realistic stress tests of credit portfolios in the Basel II framework.

1. Default Dataset

We extract information on times-to-default from the Standard & Poor’s Credit Pro 6.6 rating database. As the first goal of ratings consists of providing a cross-sectional ranking of firms with respect to their default likelihood, they allow to classify firms into “homogenous” classes of default risk. The database contains S&P’s rating histories for 10439 companies over the period January 1981 to December 2003. The Credit Pro database has already been used and extensively described by Bangia et al. (2002) and Couderc (2005). Overall 33044 rating migrations are recorded in CreditPro as well as 1386 defaults and default rate ranges from 3% to 29% across industries. Within our sample, firms are classified by industrial groups distributed among 93 countries. 6897 firms or 66% are US ones. Moreover, S&P attributes 25 distinct ratings plus the not-rated one (NR) one, but we aggregate the data coming from a grade and its plus/minus modifiers because of minimal population requirements. Besides, all grades below B- have been put in the CCC class.

Rating events require careful treatment as three sources of censoring are present in the database. The first type of right censoring is an inherent feature of any rating database as most companies survive after the end of the recordings. Another type of right censoring requires specific consideration. Some companies leave the rating process and fall into the NR category. Several reasons may explain this fact: the rated company may be acquired by another firm or may simply decide no longer to be rated by S&P. In the database we can identify firms that migrated to NR and subsequently defaulted. Therefore the NR class is not a complete loss of information: although there is no longer any indication of credit quality, a NR firm is a non defaulter. Finally, left censoring arises from 1371 issuers having already received a rating before they were included in the database (i.e. before January 1981). We do not have information about the attribution date of their first rating and therefore, for robustness checks, we run all estimations both on the full sample and on the reduced sample excluding left-censored data (the reduced sample contains 9068 companies and 25993 rating migrations). It is worthwhile to notice that these left censored issuers faced the 1991 and 2001 US recessions. The number of issuers tracked by the database increases linearly. Consequently, one third of the total number of issuers were active on average since 8 years in 1991 and 18 years in 2001, and another one third only faced the 2001 recession after an average of 4 years in the rating process.
The database allows to consider two types of durations, implying two different approaches to the behavior of default probabilities. On the one hand, we can look at times-to-default from entry in a risk class up to the last available observation. Such durations provide a picture of default riskiness over the whole life of the firms without any assumption on the rating process behavior. On the other hand, we can examine times-to-default conditional on staying in a given risk class up to the default time. In this case, it would require assumptions on rating migration dynamics so as to design the default riskiness of a firm over the long term.

2. Models of Default Probabilities

Hereafter we develop a powerful framework to analyze impacts of systematic factors. A straightforward practice to study the effects of structural factors on a given variable consists in performing regressions. Investigations of spreads follow this approach but it cannot be used for probability changes because hazard rates or equivalently instantaneous default probabilities are not directly observable. Since hazard rates fully characterize a default time distribution, modeling hazard rates leads to specifying the conditional distribution of times-to-default, and all parameters can be estimated in a single stage. We rely on the standard way to construct hazard models, with log-intensities that are linear in the factors. We briefly recall the basic parametric framework before turning to our more complete semi-parametric framework.

2.1. Factor Models of Intensities

For a firm $i$ in a given risk class, let $D_i$ denote the uncensored duration up to default, or time-to-default, and $C_i$ the censored duration. $U_i = \min(C_i, D_i)$ is the time at which the firm leaves the class either because of censoring ($C_i$) or default ($D_i$). The $U_i$ are therefore the true observations. We also let $Z$ and $u$ denote respectively a vector of explanatory variables and the time-to-default or ageing time, while $t_i$ denotes the date at which a firm $i$ enters into the class. Hence, $u + t_i$ represents calendar time. We consider intensities $\lambda^i$ as exponential affine functions which remain constant between two observations of the factors. Conditional on the realization of the factors, durations are exponentially distributed between factor updates:

$$
\lambda^i(u) = \lambda(u, t_i) = \exp(\gamma + \beta'Z(t_i, u + t_i)) \quad \forall i,
$$

where $Z$ can include a combination of time-dependent and time-independent covariates. The exponential assumption could be relaxed by replacing the constant $\exp(\gamma)$ by another formulation. For instance, one could impose a conditional Weibull hazard model. The survival
probability for a firm $i$ beyond the time $u + t_i$ can then be retrieved as $P(U > u | t_i) = \exp \left(- \int_0^u \lambda(u, t_i) \, du \right)$.

This parametric framework allows us to use maximum likelihood to efficiently estimate $\hat{\beta}$. Details of maximum likelihood estimation in this context are provided in Appendix. The estimation of such models is therefore straightforward and both censored and uncensored durations contribute to the likelihood.

The accuracy of the above estimator depends on the proper specification of the model, on the selection of explanatory variables and on time-to-default being conditionally exponentially distributed. However, some empirical studies have demonstrated that these simple models are misspecified even if they successfully take into account all systematic factors. Fledelius et al. (2004) and Couderc (2005) have shown that economic changes create bumps in hazard rates and have established that the distribution of times-to-default is not exponential (i.e. the hazard is not constant with $u$). In the remainder of this paper we propose to use a semi-parametric framework for hazard models which relaxes this exponential assumption. The nonparametric component of these models allows us to extract the baseline hazard, which is the true conditional distribution of times-to-default which characterizes the whole term structure of hazard rates. The parametric part of the models reflects shocks to the hazard rates due to changes in economic and financial factors. Once these shocks have been accounted for, the remaining bumps in the baseline hazard indicate the proportion of the default cycle which is not captured by the factors. If the factors were able to fully describe the patterns of historical times-to-defaults, then the baseline hazard would be constant and we would fall back on the class of simple models described above.

2.2. A Semi-Parametric Framework for Intensities

In our semi-parametric setting, calendar time effects, or factor impacts, can easily be separated from pure duration or "life cycle" effects on hazard rate deformations. To build our setup we start from a fully nonparametric estimator of hazard rates and add a multiplicative parametric component (Cox 1972, 1975 proportional hazard methodology). The nonparametric baseline hazard is estimated using the GRHE (Gamma Ramlau-Hansen Estimator) as the solution of the maximum likelihood objective function, while the parametric part is estimated by partial likelihood.
2.2.1. The Basic Estimator

The GRHE introduced by Couderc (2005) is based on a convenient Gamma kernel smoother of the hazard rate and allows to recover the term structure of hazard rates. Standard smooth estimators suffer from large bias and oversmoothing, which lead to incorrect inferences on hazard rates. The GRHE however is free of boundary bias and is able to capture changes in intensities in the short run (which may cover up to 5 years) as well as subsequent deformations. Not using an unbiased estimator would lead to strongly inaccurate estimations and assessments of hazard models.

The necessary conditions ensuring the consistency of the estimator are assumed to be met. Accordingly, all firms in a given risk class are assumed to be homogenous and conditionally independent. Censoring mechanisms which may prevent from observing firms up to their default time are random and independent from the default process. For a given firm, these mechanisms are reported through a process $Y^i (u)$, which equals one only if the firm was observed at least up to the time-to-default $u$. The most important building block of the GRHE then lies in the following assumption:

Assumption 2.1 The intensity of individual firms satisfies the Multiplicative Intensity Model:

$$\lambda^i (u) = \alpha (u) Y^i (u)$$

(2)

where $\alpha (u)$ is deterministic and called the hazard rate whereas $Y^i (u)$ is a predictable and observable process.

The difference between the intensity and the hazard rate resides in their observability. The estimator of the hazard rate is specified as:

Definition 2.1 The gamma kernel estimator $\hat{\alpha} (u)$ of the hazard rate (Gamma Ramlau-Hansen Estimator, GRHE) is defined by

$$\hat{\alpha} (u) = \int_{0}^{\infty} \frac{1}{Y (s)} \frac{s^{u/b} e^{-s/b}}{b^{u/b+1} \Gamma \left( \frac{u}{b} + 1 \right)} dN_s ,$$

(3)

where $dN_s$ counts the number of defaults occurring at time $s$ and $Y (u)$ is computed as the number of firms for which the last time of observation is greater than $u$.

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2This feature has already been widely documented in the case of density function estimation. Standard smoothers are biased because of the inadequacy between the domain of definition of the kernel and the one of the data. For instance, see Chen (2000).
Y (u) handles censoring and is the sum of the $Y^i (u)$ over $i$. It is usually called the risk set. $b$ is a smoothing parameter, the so-called bandwidth. The intuition behind the above nonparametric estimator is the following: probabilities of default in the very short run ($\alpha (u) \ du$), knowing survival up to time $u$, are estimated as a weighted average of past (if any), current and subsequent observed instantaneous default frequencies ($\frac{dN_s}{Y(u)}$). The weights are determined by the kernel, the bandwidth, as well as by the durations between $u$ and observed default events.

The restrictions imposed on the process $Y (u)$ are sufficiently weak to permit more complex specifications of this process. In what follows, we rely on this multiplicative intensity model, to characterize economic shocks on time-to-default distributions.

2.2.2. Factor Models for the Term Structure of Hazard Rates

Let $Z$ denote a vector of factors. We enrich standard factor models of hazard rates from section 2.1 by relaxing the conditional exponential assumption, or equivalently by not constraining the term structure of hazard rates. This is done by using a semi-parametric framework. As discussed previously the parametric part reflects the impact of economic and financial factors on default intensities, whereas the baseline hazard or conditional distribution of times-to-default is estimated through a slight modification of the GRHE.

Assumption 2.2 The intensities conditional on structural factors are proportional to a class baseline intensity $\lambda^0 (u)$ representing the common intensity shape:

$$
\lambda (u, t_i) = \lambda^0 (u) \ \exp \left( \beta'Z (t_i, u + t_i) \right) \ \forall i \tag{4}
$$

where $Z (t_i, u + t_i)$ is the set of structural variables taken at the date of entry $t_i$ of the firm $i$ in the class or at calendar time $u + t_i$, and $\beta$ is the vector of sensitivities associated with a given risk class.

Provided that structural variables dynamics are not explosive, the Gamma kernel estimator of the baseline hazard $\alpha^0 (u)$ becomes:
Corollary 1 Under assumptions 2.1 and 2.2, a semi-parametric estimator of the baseline hazard function is given by

$$
\hat{\alpha}^\circ (u) = \int_0^\infty \frac{1}{\hat{Y}(s)} \frac{s^{u/b} e^{-s/b}}{b^{u/b+1} \Gamma(\frac{t}{b} + 1)} dN_s
$$

(5)

$$
\hat{Y}(s) = \sum_i Y^i(s) \exp(\hat{\beta}'Z(t_i, u + t_i))
$$

(6)

A convenient feature of this model is that an estimate \(\hat{\beta}\) of the sensitivities \(\beta\) can be derived separately from the baseline intensity through Cox partial likelihood:

$$
\hat{L} = \prod_{k=1}^n \frac{Y^k(u_k) \exp(\beta'Z(t_k, u_k + t_k))}{\sum_i Y^i(u_k) \exp(\beta'Z(t_i, u_k + t_i))}
$$

(7)

where \(u_k\) is the observed duration of firm \(k\) and \(t_k\) is its date of entry in the class. A complete survey of related models, estimation techniques and asymptotic can be found in Andersen and Gill (1982) or Andersen et al. (1997).

This two-stage estimation technique does not affect the nonparametric estimation of the baseline intensity as the speed of convergence of the partial likelihood estimator is of order \(\frac{1}{\sqrt{n}}\) and therefore higher than that of the kernel estimator. The parametric factor model from equation (1) corresponds to the case where the constraint \(\alpha^\circ(u) = \exp(\gamma)\) has been imposed. The estimators \(\hat{\beta}\) and \(\hat{\beta}\) converge to the same parameter \(\beta\), but \(\hat{\beta}\) has a slower rate of convergence to its asymptotic distribution since it does not use the full likelihood.\(^3\)

Corollary 1 is a direct consequence of Andersen et al. (1997) and Couderc (2005), \(\hat{\alpha}^\circ\) being the maximum likelihood estimator of the baseline hazard.

The relative performance of the various factors can be tested by comparing the hazard rate \(\hat{\alpha}_{NP}\) of the fully nonparametric specification (\(\beta = 0\)) and the baseline hazards \(\hat{\alpha}^\circ\) of semi-parametric models. The lower \(\hat{\alpha}^\circ\) is with respect to \(\hat{\alpha}_{NP}\), the higher the proportion of changes in default hazard rates captured by the factors. Moreover, the comparison between \(\hat{\alpha}^\circ\) and the constant hazard \(\exp(\gamma)\) of equivalent parametric models allows us to check misspecifications of standard hazard rate models. More precisely, we can observe deviations from the conditional exponential hypothesis and their distributions through the term structure of times-to-default.

\(^3\)We checked this last point on the subsequent multifactor models. We first found that all sensitivities keep the same signs at the same horizons when switching from a parametric factor model to a semi-parametric specification. Small variations in magnitude can be observed between \(\hat{\beta}\) and \(\hat{\beta}\). However, using bootstrap technique on the distance \(\|\hat{\beta} - \hat{\beta}\|_2\), we could never reject the null hypothesis (\(\hat{\beta} = \hat{\beta}\)) at a 95% confidence level.
3. Potential Determinants of Default

In this section, we discuss some potential common determinants of default intensities that will be tested in our model. Mainstream models that attempt to explain risk-neutral default probabilities are usually calibrated on financial variables: interest rates and equity information. Another stream of literature focuses on long-term economic and credit cycles and relies mainly on macroeconomic variables to explain default rates. The business cycle has for example been factored in time series analyzes of procyclicality with the bankruptcy cycle (e.g. see Koopman and Lucas 2004). To our knowledge, there exists no systematic study of the common determinants of default including both financial and non financial variables, and we will attempt to fill that gap here. Given that our sample is primarily American, we use US explanatory variables. Our economic data was extracted from the Federal Reserve of St. Louis web site and Bloomberg. All factors are annualized, deseasonalized and updated monthly or quarterly.

3.1. Financial Market Information

The stock and bond markets are sources of information used both by structural (i.e. firm-value based, la Merton) and reduced form models. Reduced form models typically require to design a stochastic process of interest rates. For instance, Duffee (1999) relies on a two factor interest rate model to extract default intensities. In a structural approach, the volatility and return of the firm asset determine how close a firm is to its liability barrier which represents the default threshold. Cremers et al. (2006) show that individual stock return and volatility are significant determinants of spreads. Using an aggregate measure, Janosi et al. (2002) and Collin-Dufresne et al. (2001) also establish the impact of respectively the S&P500 index and the VIX on credit spreads. The latter further claim that using individual stock volatility rather than index volatility does not modify their results. We consequently test the following factors on hazard rates:

- Annual return on S&P500: as a measure of asset levels, the higher the stock return, the higher the distance to default should be. An increase in equity prices tends to decrease firm leverage and therefore pushes down default probabilities. Moreover, from an economic standpoint, short- and mid-term economic performance should be positively correlated with S&P500’s returns. We expect a negative impact on default intensities (i.e. intensities should be a decreasing function of the factor).

- Volatility of S&P500 returns: in a traditional Merton (1974) -type model, the two drivers of default probabilities are leverage and the volatility of firms assets. The implied volatility
of equity returns is often used as a proxy for the latter. We use the realized annualized volatility computed over 60 trading days. We expect it to have a positive impact on default intensities.

- 10-year treasury yield: higher interest rate levels imply higher cost of borrowing. Hence, this variable could impact positively on default probabilities. However interest rates tend to be lower in contraction periods and higher in expansions. The ultimate impact on intensities is therefore uncertain and may depend on issuer quality.

- Slope of term structure (10-year rate minus 1-year rate): steep term structures of interest rates are usually associated with strong growth prospects. It can also reflect expectations of higher future spot rates. We expect this variable to impact negatively on mid- to long-term intensities.

3.2. Business Cycle

We believe that it is crucial to extract information from the business cycle. If stocks were available for all firms and markets were fully efficient, financial market and business cycle variables might be redundant. As mentioned, the return on the market index does not constitute a perfect proxy for the state of the economy. To complement financial variables, Fons (1991) regresses default rates on the GDP growth, and Helwege and Kleiman (1997) add the NBER economic indicator. These business cycle indicators explain 30% of the annual default probabilities. We will use these variables in our estimations and will also include the personal income growth as Duffie et al. (2007).

- Real GDP growth: as a signal of current macro-economic conditions this variable should be negatively correlated with short term probabilities.

- Industrial production growth: this is an alternative growth measure which should have a similar impact as that of GDP growth. Its advantage over GDP growth is that it offers more frequent updates (monthly vs. quarterly).

- Personal Income Growth: same expected impact as the previous two variables. This business factor is more volatile and should consequently be less persistent. This indicator could also convey some slightly lagged information on past business conditions.

- CPI growth: inflation is again a general indicator of economic conditions. We expect to observe a negative correlation with short term default probabilities, as high inflation has often been associated with growth.
3.3. Credit Market Information

In addition to general economic variables and financial information, more specific credit factors should prove valuable in explaining default intensities. Although corporate bond spreads do not only reflect changes in underlying default probabilities (see e.g. Ericsson and Renault 2006), they should still contain some forward-looking information on default probabilities. Moreover, when spread variations are due to changes in the default risk premium, they involve changes in expectations of future economic conditions. Spread factors may therefore be more persistent than other market factors.

- Spread of long term BBB bonds over treasuries: they reflect future default probabilities, expected recoveries as well as default and liquidity premia. It should therefore be positively correlated with default intensities.

- Spread of long term BBB bonds over AAA bonds: this variable factors in the risk aversion of investors and may be a measure of their risk forecast. It filters out mixed effects contained into the BBB spread. Furthermore, an increase in the relative spread may reflect an increase in firms asset volatilities (see Prigent et al. 2001). We therefore expect default intensities to increase with this variable.

- Net issues of Treasury securities: this indicator should positively impact short term probabilities of default as higher deficit and borrowing is an indicator of economic difficulties (it is at least negatively correlated with the business cycle). Furthermore, high public sector borrowing may crowd out private borrowers and lead to increased financial difficulties for firms. However, if borrowing is used for investments, an increase in Treasury issuance may be linked to stronger growth in the long term and decreasing probabilities of default.

- Money lending (M2-M1) and bank credit growth: these factors measure credit liquidity and should be associated with default intensities. It is well known that the information content of this indicator and more particularly of M2 has changed a lot over our sample period (series of adjustments have been done by the Federal Reserve). As a consequence, this indicator cannot be conclusive in the short run, but its implications in the long run turn out to be fairly stable. We thus expect clearer impacts when using lags.

3.4. Inner Dynamics of the Default Cycle

A striking feature of the default cycle might not be captured by the above variables. After the last two recessions strong persistence in default rates has been observed. The number of
defaults remained high even during economic recoveries. The default cycle seems to exhibit its own dynamics. We believe that the set of predictive variables should be expanded with default-endogenous variables. Kavvathas (2000) used the weighted log upgrade-downgrade ratio and the weighted average rating of new issuers as explanatory variables. He actually only took into account the first PCA factor of these variables. The average rating of financial institutions may be of primary interest in describing the short term trend of the global economy (in terms of credit crunch). This trend can also be captured by the ratio of downgrades over all non-stayer transitions. For instance, Jönsson and Fridson (1996) show that the credit quality of speculative issuers explains a large proportion of annual aggregate default rates. As representative of the default cycle trend we choose to include the following rating-based variables:

- IG and NIG upgrade rates: both variables should include information on economic health.\(^4\)
- IG and NIG downgrade rates: downgrades should be higher in bad conditions. Differences between upgrade and downgrade impacts should capture a potential asymmetry in the default cycle.

3.5. Statistics

Table 1 presents basic statistics on the set of retained factors. Obviously some of the above variables such as real GDP growth and industrial production growth are highly correlated, which would deteriorate statistical significance on the full set of variables. Many of these factors are redundant, and will be eliminated at the estimation stage in multifactor analysis. Taking a pragmatic approach, we will first identify the most relevant factors and then construct parsimonious multifactor models.

Although we do not explicitly include firm-specific factors, our approach does not fully rule out non systematic risk as we will examine rating classes separately. Ratings should indeed constitute stable and good proxies for firm-specific components and a fair alternative to specific variables, which are not always available or are not updated frequently. Leverage targeting, jointly with the way ratings are reviewed, justifies this point. From an accounting perspective, default cannot realistically be initiated by small changes in earnings, leverage or any balance sheet information but rather by negative trends or by unexpected large changes

\(^4\)The investment-grade class (IG) gathers the AAA, AA, A and BBB classes whereas BB, B and CCC classes are collected in the noninvestment-grade (NIG) class.
in cash flows. Any negative trend should have been incorporated in issuer ratings. One may argue that ratings do not react quickly enough to new information, but there is no consensus on this issue (see Altman and Rijken 2004 or Löffler 2004) and we will assume that ratings capture significant changes in firm-specific components.

4. Predictors and Indicators of the Default Cycle

In this section, we investigate time-dependent covariates which embed the impact of successive shocks of the economic environment on intensities: when the factor updates, hazard rates are shifted and the conditional distribution of times-to-default is updated too. We distinguish the impacts of current and past economic conditions on default time distributions. We examine the sensitivity of hazard rates to each factor in order to determine which factors are the most relevant. We also explore their persistency. Surprisingly, the issue of lagged information has been ignored in most papers on default probabilities, although Koopman and Lucas (2004) have reported lagged effects between the market and the bankruptcy cycle.

We analyze the explanatory power of each factor through maximum likelihood estimations. For each covariate we run distinct lagged estimations to examine the persistency of its effects. In all cases, we look at 95% and 99% confidence tests, and likelihood ratios. The alternative model of the likelihood ratio test corresponds to unconditional exponentiality, i.e. the case of constant intensity. We also break our dataset into several samples, namely investment-grades (IG), noninvestment-grades (NIG), AA, A, BBB, BB, B and CCC samples.

Tables 2, 3 and 4 present results on the broad IG and NIG samples respectively over financial, business and credit indicators. In addition to sensitivities, implied percentage changes in the hazard rates are provided when the factor changes by minus or plus one standard deviation. These changes are not symmetric as the factor impacts the hazard rates through the exponential function. Table 5 reports estimates of sensitivities with respect to upgrade and downgrade rates over rating classes. To test the stability of the results we have estimated the models on various sub-samples and found that the sample used makes little difference in most cases. For example, considering durations up to the first exit from a risk class (rather than default) keeps sensitivities almost unchanged and only lowers the significance of parameters. Focusing solely on the US sub-sample does not modify our estimates by more than 10% on average, and does not alter signs. Such robustness could be expected as risk classes are quite stable and the whole sample is made of 66% US firms. From a general perspective, all included variables are significant. We observe that most lagged factors are significant at all stages too. In addition, these findings do not change across ratings. Estimated intercepts ($\gamma$) lie around...
-13.1 for IG and -8.8 for NIG. Results on lags between two to five years are similar than the ones of the three-year lagged factors. These results are not reported here.

4.1. Influence of Financial Markets

Table 2 shows that financial markets impact default probabilities as predicted by structural models (lags up to three months). Increases in the market index decrease the probability of default while increases in volatility raise default probabilities. Increases in long term interest rates are good news because they reflect growth expectations. These findings are consistent with the results of Duffee (1998) and Collin-Dufresne et al. (2001) on spreads.

Unlike the above authors, we find that the slope of the term structure is significant and has a large effect on hazard rates. In particular, Collin-Dufresne et al. show a significant and negative relationship only with long maturity bonds with reasonable leverage, whereas it becomes positive for short maturities. Table 2 explains this result. Decreases in short term yield raise default probabilities as low rates are strongly correlated with recessions. A steep contemporaneous or recent slope of the term structure of riskless rates tends to be associated with higher intensities of default while past steep slopes (over one-year lags) tend to decrease intensities. Therefore, effects on bond spreads are the results of these conflicting phenomena, which dampens significance. Short maturity bonds are mainly affected by recent changes in slope, so that an increase in the slope leads to an increase in hazard rates and spreads. The only exception to this short term/long term interest rates split is for the CCC class, for which a steep slope is always associated with lower intensities, irrespective of the lags. This can be due to two reasons. First, low short-term interest rates can indicate a slowdown of economic activity and an increased competition in the corporate bond market. Second, increases in long term interest rates are often interpreted as expectation of higher growth. Future growth is the main determinant of survival for junk issuers, as these companies are highly levered and require strong business conditions to move up the rating ladder.

Some of the studies of corporate bond spreads mentioned above indicate a higher impact of the market index than interest rates. Looking at implied percentage changes on the hazard rate, our results are more contrasted. For investment grades, a decrease in the long term yield and an increase in the slope lead to shocks of the same order of magnitude than a decrease in the S&P500 return. Even if all market indicators have smaller effects on NIG, contemporaneous changes in the slope have lower impacts than changes in the S&P500 return, and impacts of the market index are a bit more persistent. It may explain why interest rates have usually been
found to be less economically significant. The results on the impact of volatility are interesting and challenging for the structural models a la Merton on an aggregated pool of firms. We indeed find that volatility has a lower impact on hazard rates for noninvestment-grade firms, which is the opposite of what is predicted by structural models. We will come back to this point later on.

4.2. Business Cycle Effects

The business cycle appears to have large effects on the default cycle. Of course, business expansion tends to decrease intensities, as found by Fons (1991) and Helwege and Kleiman (1997) for bankruptcy rates. Consistently with Duffie et al. (2007), the personal income growth (PIG) has larger impacts than real GDP growth. But Table 3 shows that GDP effects are more persistent and that the causality between the PIG and changes in the hazard rate is unclear. From likelihood ratios, the results suggest that decreases in hazard rates imply future increases in the PIG. The effects of the PIG and of the CPI growth diminish as the credit quality lowers. Conversely, the GDP becomes more important for NIG, indicating a closer link with the global economic health. The positive skewness of the PIG and the negative skewness of the GDP (see Table 1) reinforce the relevance of the GDP. Remark that the PIG is strongly correlated with the long term interest rate (74%) and with the slope of the term structure (-59%), and therefore with parallel shifts in the term structure.

Allowing for business cycle effects enables us to compare the relative usefulness of financial market information and business cycle information. Janosi et al. (2002) stress the difficulty to extract default information from stock markets. Our implied percentage changes in the hazard rates are in line with this claim and shows that business information is more reliable. The lower volatility of business indicators makes one-percent changes more economically significant: the sensitivity to the GDP is -22.66 for NIG while the sensitivity to the S&P500 return is -1.96. Taking into account the variability of these factors lowers the differences but still shows that the GDP is more important, especially for NIG firms: an increase by one standard deviation in the GDP reduces intensities by 34.98% for NIG whereas an increase by one standard deviation in the S&P500 return reduces intensities by 26.63%.

4.3. Impacts of Credit and Default Factors

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So far, we have not considered information from credit markets. Table 4 shows that credit information brings significant explanatory power, in particular through the BBB spread. Its effects are similar across ratings. Signs of the money lending variable confirm that a credit crunch amplifies defaults. Nevertheless, recent changes in net treasury issues and money lending have minor but statistically significant impacts on intensities. We argue that this is due to their weak short term informational content. Interestingly, the BBB yield and the investment-grade spread seem to carry little information on default.

Finally, rating trend indicators (upgrade/downgrade rates) appear as major explanatory components from Table 5. They express the persistency of the default cycle both in declines and recoveries. Studies of rating migrations (e.g. Nickell et al. 2000) have shown that the NIG downgrade ratio is highly correlated with increases in the number of defaults. Including such "endogenous" factors in multifactor models should be highly relevant as downgrade ratios are not much correlated with other factors (less than +/-25%).

4.4. Persistency and Importance of Past Conditions

Lagged information provides further insights on the explanatory power and the time-span of economic shocks over the default cycle. We only report sensitivities to three-year lagged factors in Tables 2 to 4 because the highest likelihood ratios have been found for this particular lag. Similar results are obtained with 2, 4 and 5-year lagged factors. One-year lagged factor bring intermediate results between short and long lags but are not statistically significant.

With three-year lags half of the non financial factors become insignificant for IG hazard rates, whereas all variables but the PIG remain significant for NIG. The three-year lagged changes in money lending have large effects on hazard rates: a one standard deviation increase induces an increase of 60.24% in IG hazard rates. As expected the money lending contains long-term information. The significance of the factors evidences the high degree of persistence of economic shocks on the firms likelihood of default. It bears major implications from a modeling standpoint as Markovian processes are unlikely to provide such features.

Besides, we find that some factors impact default probabilities differently in the long run. The S&P500, the term structure slope, the real GDP growth and net treasury issues appear to be leading indicators of future peaks of defaults. It supports the claim that the default cycle lags the economy (e.g. see Koopman and Lucas 2004). The interesting point lies in the signs of these factors which come as warnings: expansion peaks of the financial market or of the
business cycle seem to announce increases in the number of defaults three years later. It has to be taken with care as it could only represent the singularities of the global economy over the past 25 years and will not necessarily apply to the future. For example, early repayments and small levels of issues by US Treasury signaled the peak of the US cycle which was later followed by a major default crisis. However, we stress that these lagged effects could also be significant because migration from distress to default takes time as reported by Altman (1989). From that perspective, the default cycle has to remain high after economic recoveries, generating explanatory power for past conditions and persistency for economic shocks. We argue that lagged factors when used as supplementary information could at least help in capturing business and market trends, which constitute the essential information on future default probability. Section 5.4 examines this issue.

5. Efficient Hazard Models across Rating Classes

In the previous section we identified some major default determinants using univariate hazard models. We now turn to multifactor models. By doing so, we can first compare the pertinence and the complementarity of the various economic components. We show that explained variations in intensities could be severely underestimated because of inappropriate choices in the information set. For instance, leaving aside information provided by the business cycle can be damaging for the performance of the model. Second, we can identify relevant parametric conditional distribution and propose a specification test in finite samples. We finally show the critical importance of economic trends and past information.

5.1. Failure of Contemporaneous Financial Market Factors

Table 6 presents estimates of sensitivities $\hat{\beta}$ for different specifications on the basis of contemporaneous financial predictors over rating classes. It enables us to test whether models based solely on stock prices or interest rates or on both (joint model) are sufficient to explain historical default intensities.

5 Indeed, from time series cycle analysis between the GDP and bankruptcy rates, Koopman and Lucas (2004) observe differences in magnitude and lengths. The default cycle being much smoother and persistent than the financial market or the business cycles, transitory shocks should not represent the most relevant information.
The table presents three multifactor models. For robustness checks, we included a dummy indicating non-US firms. Sensitivities to this non-US indicator were not significant. Likelihood ratio tests identify the joint model as the best one, whereas interest rates alone provide the poorest fits. This last result confirms findings of Driessen (2005) or Janosi et al. (2002) on credit spreads. Interestingly, stock market volatility is not always significant and its relative impact on default is minor with respect to other factors. Such a finding is highly challenging for structural models as the volatility determines the dynamics of equities and as a consequence default probabilities. However BBB and BB classes are significantly affected by market volatility. This may be explained by the economic impact (in terms of increased funding cost) of a migration from investment-grade to noninvestment-grade. Some fund managers systematically rule out noninvestment-grade corporate bonds from their portfolios: at the time of a downgrade from BBB, numerous funds close their positions, resulting in a jump in credit spread and the cost of debt. The market volatility could be a good proxy for that kind of market segmentation behavior and consequently has to be a key indicator for these classes.

Figure 1 focuses on IG and NIG classes displaying baseline hazard rates for nonparametric (solid line), semi-parametric (solid bold line) and parametric counterparts (dashed lines). Unreported baseline hazards for the "Interest Rates" model show that our interest rates factors, when considered as a group, are unhelpful determinants of default across rating classes. Conversely, stock market information brings significant explanatory power on the IG class. In a Merton-like intensity model with additional stochastic liabilities, it could be interpreted as evidence of the level and higher variability of assets being the main determinants of the default probability changes. The baseline hazard is leveled down by 29% on Figure 1(a) thanks to stock indicators. But for each bump in Figures 1(a) and 1(c), deviations from the constant (bold lines) remain significant, implying that the S&P500 return and volatility do not succeed in capturing all shocks of the economy which affect the default riskiness. In particular, for the NIG category, the small difference between $\hat{\alpha}_{NP}(u)$ and $\hat{\alpha}(u)$ shows that these joint financial components perform poorly on NIG issuers as univariate results suggested.

The analysis of the "Financial Markets" model evidences how correlated factors can be damaging. We concentrate on IG and NIG classes on Table 6 and on Figures 1(b) and 1(d). Sensitivities to the long term yield are insignificant. From a statistical standpoint, it can be explained by the high correlation with the other factors. From an economic standpoint, the long term yield is less informative on current economic conditions than the market index, and on future growth opportunities than the term structure slope. Figure 1(d) proves that misspecified and correlated factors dramatically reduce the explanatory power. The light curve
(all financial factors) lies above the nonparametric model, meaning that the model introduces additional noise in hazard rates instead of improving the fit. The bold line, on the other hand, shows the baseline hazard when the sensitivity to the treasury yield is constrained to be null. For the IG (resp. NIG) category, the other coefficients become $-1.53^{**}$, $2.64^*$ and $27.12^{**}$ (resp. $1.67^{**}$, $0.62^*$ and $6.29^{**}$). Both Figure 1(b) and 1(d) prove in this constrained case the benefit to extract additional information from the term structure slope. The variations in hazard rates explained by this specification are about 43% for IG and 12% for NIG.

Finally, we observe that a constant hazard either unconditional or conditional on financial information does not represent the data correctly. Short term default probabilities are completely overstated by standard hazard models (the dashed line is higher than $\hat{\alpha}(u)$ up to 2 to 4 years) as well as long term default probabilities for NIG. Introducing financial factors leads to overestimating long term hazard rates on a larger part of the debt life. In an attempt to capture effects of the 2001 default peak, the model grants too much weight to covariations between the financial markets and defaults, yet still undershoot this peak. Notice that given our sample window, the 2001 recession is responsible to a large extent for the first hump of $\hat{\alpha}_{NP}(u)$ (among NIG observed times-to-default which range between 1 to 5 years approximately one fourth faced the 1991 recession at these horizons while one half faced the 2001 recession).

5.2. Models Based on Non Financial Information

Table 7 reports multifactor specifications using business and credit information. All factors enter the various models with the same signs. Unlike Duffie et al. (2007) who worked on bankruptcy rates, we observe that the real GDP and the Personal Income Growth are complementary explanatory variables for our dataset. The impact of the GDP increases as the credit quality decreases whereas that of the PIG decreases. This is in accordance with univariate results. All factors from the credit cycle are significant. In particular, the impact of the BBB spread is homogenous across rating classes, and so are the effects of the NIG downgrade rate. This homogeneity can indicate the influence of changes in the default risk premium. Higher premia increase future coupon rates as most of companies issue floating rate bonds, pushing some firms up to their limit. As one could expect, the IG downgrade rate brings default information for IG hazard rates. But looking at rating classes, we observe that the effects

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6We check this overfitting problem due to the 2001 recession by estimating the "Stock Market" model on the sub-sample constituted by firms entered in the process after the 03/01/1991. This basically cuts durations higher than 10 years, and thus the second hump on intensity graphs. On this reduced sample the model delivers higher sensitivities of -2.82 on the S&P500 return and of 3.14 on its volatility for IG, -1.99** and 0.88** respectively for NIG. At the same time, it also exhibits a higher overestimation of instantaneous probabilities from the 6 to 10 remaining years after the peak.
are starker around BBB and BB firms. This factor captures local demand and supply shocks whereas the NIG downgrade rate captures global health of issuers.

For each rating class, we report as the "Non Financial" model the best model using these five factors. The model is selected from likelihood ratio tests. Figures 2(a) and 2(c) plots corresponding baseline hazards for IG and NIG categories. It is striking to observe that the GDP does not add information on the IG likelihood of default whereas the situation reverses for NIG (actually, substituting the PIG by the GDP do not modify significantly the likelihood for the BBB class). As we previously mentioned, this split between the GDP and the PIG over rating classes can be explained by the effect of interest rates and their correlation with the PIG: Adding the term structure slope makes the PIG insignificant on investment-grade firms but does not change impacts of the GDP. Our results explain the findings of Duffie et al. (2007) since they do not include interest rates information. It could also reflect that their sample consists mainly of investment-grade firms.

Figures 1(b) and 2(a) show that IG firms are more closely linked to financial markets than to the other components of the economy. The semi-parametric baseline hazard is indeed higher for the "Non Financial" model than for the "Financial Markets" model. Conversely, considering NIG firms, the business cycle and credit specific information have much larger impacts on default hazard rates than stock and interest rates information. The "Non Financial" model explains 48% of the variation in NIG hazard rates, and 34% of the changes in IG hazard rates.

5.3. Benefits of Considering All Current Economic Indicators

In traditional structural or intensity-based models market factors are expected to integrate information on the business cycle or on credit specific phenomena. However these factors may merely supply a noisy signal of the business cycle and may not be the most efficient vehicle of financial information. We have shown above that financial information is of prime importance for explaining default probability changes of IG firms. Nonfinancial information is more relevant for NIG firms. We will now test whether financial and nonfinancial variables are complementary or substitute predictors of the default cycle. The top part of the Table 8 presents our best global models, over IG, NIG and BBB to CCC classes.
Both broad categories of factors (financial and nonfinancial) form part of the most relevant information set. However, some individual factors are no longer statistically significant. The return of the S&P500 is not relevant for all classes except the AA (unreported). Both its correlations with the business cycle (36% with the GDP) and with the BBB spread (-46%) explain this result. It confirms that the market index is too noisy to provide fully reliable information on changes in overall default risk. Because of the significant impact of the term structure slope, the PIG is also insignificant and does not contribute to the explanatory power of the model, according to likelihood ratio test. The GDP, on the other hand, is a key instrument. As before, the IG downgrade ratio has significant impact on the BBB and BB classes. No variable should be added from LR test outputs.

Figures 2(b) and 2(d) show the baseline hazards for the global models. The IG baseline is strongly shifted downwards by 62% compared to the previous specification. The NIG baseline is very similar to the baseline of the ”Non-Financial” model, explaining 49% of the variations in hazard rates. These results prove that financial and non financial factors are complementary and should be included in any model of default probabilities under the historical measure. Nevertheless, we remark that the informational content of stock and interest rates markets on the default likelihood of NIG issuers is highly limited. For that kind of issuers, it is crucial to rely on business factors. The proportions of explained hazard variations of our models are much higher than the reported 25% to 30% by Collin-Dufresne et al. (2001) on credit spreads. This evidence that the new factors are key determinants of the default regime changes even if the NIG model is not fully satisfactory. In particular the level of the first hump (Figure 2(d)) suggests that the default cycle is longer than the business cycle and that some persistency has still to be incorporated.

To verify our observations, we test whether the different models are misspecified or not. Well specified models could indeed be used in risk management and pricing applications. There exists no full specification test of semi-parametric hazard models a la Cox (test of the parametric part could be done), but Fernandes and Grammig (2005) have proposed an asymptotic specification test of parametric hazard models. We can therefore assume a functional form for the baseline hazard rate (i.e. specify a conditional distribution of times-to-default) and test the model using their approach. Figures 1 and 2 show that the conditional exponentiality is strongly misspecified. Other standard distribution of durations are the Weibull, the log-normal and the log-logistic distributions. Full maximum likelihood estimations show that for all baseline hazards across rating classes, similar fits are obtained with the log-logistic and the
log-normal distribution. Hereafter, we check misspecification of conditionally log-logistic factor models. Keeping previous notations, hazard models with log-logistic baseline are defined by:

\[ \lambda(u, t_i) = \delta p (\delta u)^{p-1} \frac{\exp(\beta' Z(t_i, u + t_i))}{1 + (\delta u)^p} \quad \forall i, \] (8)

\( \delta \) and \( p \) being the parameters of the baseline hazard. The asymptotic theory doubtfully applies to our limited samples. We rather propose a specification test in finite samples, starting from the Fernandes and Grammig statistic and using bootstrap:

1. Estimate \( \hat{\theta} (\theta = (\delta, p, \beta)) \) and \( \hat{\alpha}^{\circ}_{NP}(u) \) on the observed durations \( U = (U_1, \ldots, U_N) \).

2. Compute the statistic \( \Lambda(\hat{\theta}) = \int \left( \int_0^u (\alpha(u; t, \hat{\theta}) - \hat{\alpha}^{\circ}_{NP}(u; t)) du \right)^2 dF(u) \) where \( F \) is the true probability function of the \( U_i \)s.

3. Draw a new uncensored duration sample \( D^{(i)} = (D^{(i)}_1, \ldots, D^{(i)}_N) \) from the estimated distribution \( f(u; \hat{\theta}) \), called a bootstrap sample.

4. Apply a uniform right censoring scheme matching the censoring percentage of the observed sample. It creates simulated durations \( U^{(i)} = (U^{(i)}_1, \ldots, U^{(i)}_N) \).

5. Estimate \( \hat{\theta}^{(i)} \) and \( \hat{\alpha}^{\circ(i)}_{NP}(u) \) on the simulated sample \( U^{(i)} \), and compute the corresponding statistic \( \Lambda(\hat{\theta}^{(i)}) \).

6. Repeat steps 3 to 5 \( S \) times, and obtain the empirical distribution of the statistic \( \Lambda(\hat{\theta}^{(i)}) \), called the bootstrap distribution.

7. Reject the null hypothesis of correct specification at significance level 5% if \( \Lambda(\hat{\theta}) \) is larger than the 95% percentile of the bootstrap distribution.

This type of bootstrap procedures is known to work extremely well in finite samples. Notice that dates of entry into the risk classes are kept fixed (parametric bootstrap). The bottom part of Table 8 displays p-values based on 1000 bootstrap samples for the "Financial Markets", the "Non Financial" and the "Global" models, where non significant sensitivities have been constrained to zero. Results confirm previous graphical outputs and likelihood ratio tests. The "Non Financial" model performs quite well over classes from BBB to CCC and for the broader NIG category. As expected, in the case of NIG firms and junk issuers, modeling hazard rates by means of financial factors is strongly rejected. Nevertheless, stock and interest rates information is sufficient to capture changes in hazard rates of IG firms. Unreported results
corroborate the inability of conditionally exponential models to represent the data at a 99.9% confidence level. We also checked that simple unconditional log-logistic models are rejected. As a consequence, it is necessary both to rely on a complete set of economic factors and to take ageing effects (from the most recent rating review) into account so as to correctly capture and predict hazard rates. When using all sources of information, results are less conclusive. The ”Global” model is indeed rejected at 95% over CCC and IG classes. Correlation between the different factors may be at the origin of these misspecifications.

5.4. Trends and Persistency of Shocks

So far, all selected variables have been contemporaneous, but potential lead-lag effects between the economy and the default cycle should also be considered. Looking back at Table 2, we can argue that lagged variables bring information upon current economic conditions, and some variables lead the default cycle by an average of three years. In order to address the benefits of past information, we concentrate on the ”Financial Markets” model which is the most akin to structural Merton-type models. We add three-year lagged volatility and stock market return, 10-year treasury yield and term structure slope. Estimated parameters $\hat{\beta}$ are provided in Table 9 using past information only, and using both contemporaneous and past information.

[INSERT TABLE 9 HERE]

In the complete case, we also report for each factor two additional sensitivities which deliver another decomposition of the usefulness of information. The ”trend” is the sensitivity which can be attributed to the differential between the contemporaneous and the three-year lagged factor, whereas the ”persistency” is the total sensitivity to the lagged factor. Using past information only, results are in line with univariate analyzes. Past increases in volatility and 10-year treasury yield respectively increase and decrease the default likelihood but the effects are stronger than that of contemporaneous factors. It supports the presence of a significant lag between the financial cycle and the default cycle. The sensitivity to past changes in the term structure slope becomes negative. We discussed that point for the particular CCC class: three years later, the remaining informational content of past slopes lies in growth anticipation. The positive sign of the sensitivity to the S&P500 return is less obvious. We still advocate for anticipation of changes in the business cycle which is confirmed by the ”Full” model. Let us consider a concrete situation. Assume that we were at the top of an economic expansion three years earlier, i.e. the persistency factor is positive. The more the trend factor is negative (the smaller today returns are with respect to past ones), the higher the default likelihood is. Further assume that three years earlier we were at the very beginning of the expansion period.
The more the trend factor is positive, the lower the default likelihood is provided that the "growth" or the differential is higher than a given point depending on past conditions (e.g. from Table 9 \( \frac{-1.37t \times Z(t-3\text{yr.})}{(t-0.40)} \) for the NIG case).

From a global perspective, Table 9 shows that both trend and past information are determinants of default probabilities. The "Full" models are not rejected while the "Past" models are. This shows that it is not sufficient to consider the lag between the economy and the default cycle. Figures 3(a) and 3(b) confirm the poor explanatory power of past information alone but Figures 3(b) and 3(d) diagnose high explanatory power when it is used with contemporaneous factors in order to capture economic trends. Explained variations in hazard rates reach 78% for IG and 59% for NIG categories. The improvement is even more substantial than that achieved by additional business and credit indicators for the NIG class.

We can propose two explanations for such findings. First, it may reflect some cyclicality in equity returns, and an idiosyncrasy of the period we are studying. We have found that high current equity returns tend to be associated with low current default rates. If there is cyclicality in equity returns, with a peak-to-trough time of approximately 3 years, it is plausible that high returns will be associated with high default rates 3 years on. Nonetheless, we have found no evidence of such a cyclicality since "Past" models do not perform in explaining hazard rate variations. An alternative, more likely, explanation would be that in good times (when the equity market is performing well), companies can afford to raise large amounts of debt while preserving acceptable levels of leverage. Several years later, this level may become unsustainable for some firms, thereby raising the default rate. Table 9 and Figures 3(b) and 3(d) do not contradict such a hypothesis. If the stock market has an upward trend, the market appreciation induces a decrease in hazard rates. If the market depreciates or stagnates, hazard rates increase as some firms start experiencing difficulties.

Summary

In this paper, we study times-to-default in the Standard & Poor’s rated universe using hazard models. We rely on a tractable framework that enables us to analyze the behavior of default probabilities with respect to changes in various economic indicators. This is done under the historical measure to focus precisely on probabilities without any perturbations due to changes in the default risk premium. The paper concentrates on common default determinants and investigates default probabilities, rather than bankruptcy rates as considered in most previous
research. Our setting also highlights the distribution of explanatory errors through time. The models we propose can be operated to forecast default probabilities at any horizon, and as a consequence, as inputs to standard credit risk applications.

We examine different economic components which should alter the default cycle and quantify their explanatory power: financial market information, the business cycle and endogenous proxies from credit markets and the default cycle. We explore further the sensitivity of default probabilities to past economic conditions and show their persistency as well as impacts of subsequent trends. Specification tests for parametric hazard rate models are also proposed and applied to time-to-default modeling.

Our initial empirical results show that the business cycle and financial market factors offer comparable explanatory powers. Business cycle changes have larger influences on NIG issuers whereas stock and bond market indicators are key determinants of future default probabilities of IG companies. Leverage ratio-targeting and business diversification provide some intuition for this phenomenon. We show that selecting a set of factors from each economic components (financial, business, credit) outperforms other specifications in capturing movements in default probabilities.

Results on impacts of the market volatility are challenging for structural models as stock price volatility is only really significant for the BBB and BB classes which are more strongly impacted by supply and demand phenomena. However, a lag between the market and default rates can explain these results as lagged volatility is a significant determinant of default probabilities for all issuers.
References


Appendix

For parametric hazard rate models, the standard estimation procedure relies on the maximum likelihood technique and works in the following way. Assuming that structural variables dynamics are independent, the likelihood is separable into two terms, one related to the dynamics of factors and the other dealing with conditional durations. Therefore, if we are not interested in factors dynamics, we can ignore this part and focus purely on time-to-default. For a given firm $i$, the likelihood $l$ of observed duration $u_i$ can be written conditionally on factor realizations at firm’s ”death or exit” but the whole construction of the risk classes information set has to be known up to that calendar time:

$$l(u_i) = l_1(u_i | \mathcal{F}^Z_{t_i+u_i}) \times l_2(F^Z_{t_i+u_i})$$

where $l_1$ is the univariate likelihood of the conditional duration and $l_2$ the likelihood associated with the factor dynamics. From that point, letting $L_1$ and $L_2$ denote the multivariate counterparts of $l_1$ and $l_2$, the multivariate likelihood function for a sample of $n$ firms observed up to time $t = \max_i \{ t_i + u_i \}$ is defined by

$$L(u_1, .., u_n) = L_1(u_1, .., u_n | \mathcal{F}^Z_t) \times L_2(\mathcal{F}^Z_t)$$

with

$$L_1(u_1, .., u_n | \mathcal{F}^Z_t) = \prod_{i=1}^n \exp \left( - \int_0^{u_i} \lambda(s, t_i) \, ds \right) \left[ \mathbb{I}_{(d_i > c_i)} + \lambda(u_i, t_i) \mathbb{I}_{(d_i \leq c_i)} \right]$$

(9)

where $c_1, .., c_n$ (resp. $d_1, .., d_n$) are realizations of censoring variables $C_1, .., C_n$ (resp. default durations $D_1, .., D_n$).
Baseline Hazards of Multifactor Models Relying on Financial Information

Estimated nonparametric baseline hazard rates ($\hat{\alpha}_{NP}(u), \hat{\alpha}(u)$) and corresponding means over investment grades and noninvestment Grades. Thin lines denote the full nonparametric model ($\alpha(u, t_i) = \alpha^0(u)$) and bold lines show semi-parametric specifications ($\alpha(u, t_i) = \alpha^0(u) \exp(\beta'Z(u + t_i))$) including only significant factors whereas light lines keep all factors. Dashed lines represent averages of baselines - they are not statistically different from the estimated constants $\exp(\gamma)$ of parametric model counterparts ($\alpha(u, t_i) = \exp(\gamma + \beta'Z(u + t_i))$). The "Stock Market" model uses the contemporaneous return and volatility on the S&P500. The "Financial Markets" model includes in addition the US Long Term Yield and Term Structure Slope.
Figure 2
Baseline Hazards of Non Financial and Global Multifactor Models

Estimated nonparametric baseline hazard rates ($\hat{\alpha}^{\circ} (u)$ and $\hat{\alpha}^{\circ} (u)$) and corresponding means over investment grades and noninvestment grades. Thin lines denote the full non-parametric model ($\alpha (u, t_i) = \alpha^{\circ} (u)$) and bold lines show semi-parametric specifications ($\alpha (u, t_i) = \alpha^{\circ} (u) \exp (\beta Z (u + t_i))$). Dashed lines represent averages of baselines - they are not statistically different from the estimated constants $\exp(\gamma)$ of parametric model counterparts ($\alpha (u, t_i) = \exp (\gamma + \beta' Z (u + t_i))$). The "Non Financial" model uses the BBB Spread, the IG and NIG downgrade rates. In addition, the IG model relies on the Personal Income Growth and the NIG model on the Real Gross Domestic Product Growth. The "Best Global" model relies on the S&P500 volatility, the Term Structure Slope, the GDP, the BBB Spread and both IG and NIG downgrade rates.
Figure 3
Baseline Hazards of MultiFactor Models with Past Information

Estimated nonparametric baseline hazard rates ($\hat{\alpha}_{NP}^\circ (u), \hat{\alpha}^\circ (u)$) and corresponding means over investment grades and noninvestment grades. Thin lines denote the full non-parametric model ($\alpha (u, t_i) = \alpha^\circ (u)$) and bold lines show semi-parametric specifications ($\alpha (u, t_i) = \alpha^\circ (u) \exp (\beta'Z(u + t_i))$). Dashed lines represent averages of baselines - they are not statistically different from the estimated constants $\exp (\gamma)$ of parametric model counterparts ($\alpha (u, t_i) = \exp (\gamma + \beta'Z(u + t_i))$). The "Past" model uses three-year lagged factors (S&P500 return and volatility, 10 yr. treasury yield and term structure slope). The "Full" models add the same comparaneous factors.
Basic statistics on retained factors. Figures are given on an annual basis, and in percentages (except for the skewness). All variables but upgrade and downgrade rates are US indicators.

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<th>Max</th>
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Table 2

Sensitivities w.r.t Financial Markets Information

Estimations of log-linear intensities $\lambda(u, t_i)$ on IG and NIG with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities $\hat{\beta}$ from univariate specifications $\lambda(u, t_i) = \exp(\gamma + \beta'Z(u + t_i))$ where the default arrival is assumed to be piecewise exponential conditional on factor realizations. We consider lags from five years backward to two months forward - only main lags are reported. Constants $\hat{\gamma}$ are not reported. * (resp. **) stands for significance at 95% (resp. 99%) confidence level. For each factor the lag offering the highest likelihood ratio has been stressed in italics. Other figures display percentage changes in the hazard rate for minus/plus one standard deviation changes in the factor.

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<td>-24.50/32.45</td>
<td>38.96/-28.04</td>
<td>25.76/-20.49</td>
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Table 3
Sensitivities w.r.t. Business Cycle Information

Estimations of log-linear intensities $\lambda(u, t_i)$ on IG and NIG with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities $\hat{\beta}$ from univariate specifications $\lambda(u, t_i) = \exp (\gamma + \beta' Z (u + t_i))$ where the default arrival is assumed to be piecewise exponential conditional on factor realizations. We consider lags from five years backward to two months forward - only main lags are reported. Constants $\gamma$ are not reported. * (resp. **) stands for significance at 95% (resp. 99%) confidence level. For each factor the lag offering the highest likelihood ratio has been stressed in italics. Other figures display percentage changes in the hazard rate for minus/plus one standard deviation changes in the factor.

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35
Estimations of log-linear intensities $\lambda(u, t_i)$ on IG and NIG with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities $\hat{\beta}$ from univariate specifications $\lambda(u, t_i) = \exp(\gamma + \beta' Z(u + t_i))$ where the default arrival is assumed to be piecewise exponential conditional on factor realizations. We consider lags from five years backward to two months forward - only main lags are reported. Constants $\hat{\gamma}$ are not reported. * (resp. **) stands for significance at 95% (resp. 99%) confidence level. For each factor the lag offering the highest likelihood ratio has been stressed in italics. Other figures display percentage changes in the hazard rate for minus/plus one standard deviation changes in the factor.

### Investment Grades

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Table 5
Sensitivities to Aggregate Default Indicators

Estimations of log-linear intensities $\lambda(u, t_i)$ with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities $\beta$ from univariate specifications $\lambda(u, t_i) = \exp(\gamma + \beta'Z(u + t_i))$ where the default arrival is assumed to be piecewise exponential conditional on factor realizations. Constants $\gamma$ are not reported. * (resp. **) stands for significance at 95% (resp. 99%) confidence level. Other figures display percentage changes in the hazard rate for minus/plus one standard deviation changes in the factor.

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<th>Default Factors / Class</th>
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<td>-55.70**</td>
<td>-6.34</td>
<td>-16.69</td>
<td>-38.85</td>
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<td></td>
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<td>1.92/-1.88</td>
<td>5.13/-4.88</td>
<td>12.36/-11.00</td>
</tr>
<tr>
<td>NIG Upgrade Rate</td>
<td>-18.87**</td>
<td>-5.05</td>
<td>-7.65</td>
<td>-22.78</td>
</tr>
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<td></td>
<td>18.51/-15.62</td>
<td>4.65/-4.45</td>
<td>7.13/-6.65</td>
<td>22.75/-18.53</td>
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<tr>
<td>IG Downgrade Rate</td>
<td>72.91**</td>
<td>38.52</td>
<td>53.03**</td>
<td>64.62**</td>
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<td></td>
<td>-39.97/66.59</td>
<td>-23.63/30.95</td>
<td>-31.01/44.95</td>
<td>-36.39/57.20</td>
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<td>22.43</td>
<td>24.38**</td>
<td>23.82**</td>
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<td>BB</td>
<td>B</td>
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<td>-42.94**</td>
<td>-68.57**</td>
<td>-70.05**</td>
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<td>13.75/-12.09</td>
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<td>-34.02/51.56</td>
<td>-29.74/42.32</td>
<td>-21.59/27.54</td>
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<td>24.91**</td>
<td>25.41**</td>
<td>26.21**</td>
<td>23.99**</td>
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Table 6
Contemporaneous Financial Multifactor Models

Estimations of semi-parametric models of default hazard rates with time-varying factors over rating classes for durations up to the first exits and all countries. The table displays sensitivities $\hat{\beta}$ from multivariate specifications $\lambda(u, t_i) = \lambda^0(u) \exp(\beta'Z(u + t_i))$. We focus on financial market information. * (resp. **) stands for significance at 95% (resp. 99%) confidence level.

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<td><strong>IG</strong></td>
<td><strong>NIG</strong></td>
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<td>-1.87**</td>
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<td>2.53**</td>
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<td>0.33</td>
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<td>-7.97**</td>
<td>-4.21**</td>
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<td>24.87**</td>
<td></td>
<td>15.52**</td>
<td>4.09**</td>
<td></td>
</tr>
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<td><strong>BB</strong></td>
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<td>S&amp;P500 Return</td>
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<td>-3.03*</td>
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<td>-1.44**</td>
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<td>1.15**</td>
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<td>-5.70</td>
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<td>-3.05*</td>
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<td>-1.96**</td>
<td>-1.69**</td>
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<td>0.35</td>
<td>0.33</td>
<td>0.17</td>
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<td>Term. Str. Slope</td>
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<td>19.68**</td>
<td></td>
<td>16.93**</td>
<td>4.91**</td>
<td></td>
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<tr>
<td><strong>Factors / Class</strong></td>
<td><strong>BBB</strong></td>
<td><strong>CCC</strong></td>
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<tr>
<td>S&amp;P500 Return</td>
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<td>-1.44**</td>
<td>-1.27**</td>
<td>-1.27**</td>
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<td>S&amp;P500 Vol.</td>
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<td>2.55**</td>
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<td>Treas. Yield</td>
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<td>-14.61**</td>
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<td>-12.85*</td>
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Estimations of semi-parametric models of default hazard rates with time-varying factors over rating classes for durations up to the first exits and all countries. The table displays sensitivities $\hat{\beta}$ from multivariate specifications $\lambda(u, t_i) = \lambda^c(u) \exp(\beta'Z(u + t_i))$. We focus on business and credit information. The "Non Financial" model displays the best model from this set of factors, on the basis of likelihood ratio tests. * (resp. **) stands for significance at 95% (resp. 99%) confidence level.

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<th>Factors / Class</th>
<th>Business Cycle</th>
<th>Credit Cycle</th>
<th>Non Financial</th>
<th>Business Cycle</th>
<th>Credit Cycle</th>
<th>Non Financial</th>
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<td>-13.90**</td>
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<td>14.54**</td>
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<td>18.42**</td>
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<tr>
<td>IG Down. Rate</td>
<td>39.34**</td>
<td>30.77**</td>
<td>17.21**</td>
<td>15.13**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NIG Down. Rate</td>
<td>15.53**</td>
<td>14.23**</td>
<td>18.76**</td>
<td>15.04**</td>
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<table>
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<th>BB</th>
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<td>Real GDP Growth</td>
<td>-7.87</td>
<td>-16.13**</td>
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<td>BBB Spread</td>
<td>25.31*</td>
<td>14.11</td>
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<td>IG Down. Rate</td>
<td>9.93</td>
<td>6.26</td>
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<tr>
<td>NIG Down. Rate</td>
<td>17.67*</td>
<td>17.73*</td>
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<table>
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<td>19.11**</td>
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<td>15.96**</td>
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<td>IG Down. Rate</td>
<td>33.39**</td>
<td>28.54**</td>
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<tr>
<td>NIG Down. Rate</td>
<td>13.77**</td>
<td>11.81**</td>
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Estimations of semi-parametric models of default intensities with time-varying covariates over IG and NIG classes for durations up to the first exits and all countries. The first part of the table displays sensitivities \( \hat{\beta} \) from multivariate specifications \( \lambda(u, t_i) = \lambda(u) \exp(\beta'Z(u + t_i)) \). * (resp. **) stands for significance at 95% (resp. 99%) confidence level. The second part presents p-values of specification tests on conditionally log-logistic counterparts.

### Table 8
Global Multivariate Proportional Hazard Models and Specification Tests

<table>
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<tr>
<th>Factors / Class</th>
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<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
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<tr>
<td>S&amp;P500 Vol.</td>
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<td>0.61**</td>
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<tr>
<td>Term. Str. Slope</td>
<td>26.24**</td>
<td>3.22*</td>
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<td>10.91**</td>
<td>4.18*</td>
<td>-10.67**</td>
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<td>BBB Spread</td>
<td>8.82*</td>
<td>12.21**</td>
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<td>12.44**</td>
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<td>14.92**</td>
<td>8.74**</td>
<td>14.36**</td>
<td>16.42**</td>
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Table 9

Multivariate Proportional Hazard Models with Past Information

Estimations of semi-parametric models of default intensities with time-varying covariates over rating classes for durations up to the first exits and all countries. The table displays sensitivities $\hat{\beta}$ from multivariate specifications $\lambda(u, t_i) = \lambda^c(u) \exp(\beta^r Z(u + t_i))$. * (resp. **) stands for significance at 95% (resp. 99%) confidence level. If $\beta_F^c$ is the sensitivity to the contemporaneous factor $F$ and $\beta_F^p$ to its three-year lag, the sensitivity to the trend component is given by $\beta_F^t$ and the one to the persistency component by $\beta_F^c + \beta_F^p$.

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<th>Past</th>
<th>Full</th>
<th>Contemp.</th>
<th>Past</th>
<th>Full</th>
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