The Dynamics of Investment and Financing under Asymmetric Information

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Abstract

This paper develops a tractable real options framework to analyze the effects of asymmetric information on investment and financing decisions when firms require external funds to finance investment. Our analysis shows that corporate insiders can signal their private information to outside investors using the timing of investment and/or the firm’s debt-equity mix. Several important contributions follow from this result. First, we show that firms’ equilibrium investment strategies differ significantly from those implied by standard real options models with perfect information. In particular, informational asymmetries erode the value of the option of waiting to invest and induce firms with good prospects to speed up investment. Second, we demonstrate that informational asymmetries may not translate into a financing hierarchy. Most notably, we find that equity issues can be more attractive than debt issues even for firms with ample debt capacity, thereby providing a rationale for the fact that high-growth firms do not behave according to the pecking order theory.

Keywords: asymmetric information; financing decisions; investment timing.

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I. Introduction

Investment and financing decisions are among the most important faced by management, and among the most studied by academics. In a seminal contribution, Myers and Majluf (1984), showed that informational asymmetries between firm insiders and outside investors could increase the cost of outside financing, distort investment decisions, and lead to a financing hierarchy – or a pecking order – among securities. In particular, they showed that when financing investment, firms would prefer debt to equity because of the lower information costs associated with debt issues. This notion of a pecking order among securities is the central focus of many empirical tests (see, e.g., Shyam-Sunder and Myers, 1999; Fama and French, 2002; Frank and Goyal (2003); Leary and Roberts, 2007). However, to date empiricists have struggled identifying the effects on asymmetric information on financing decisions and the empirical evidence on the pecking order theory is, at best, mixed.

An important characteristic of the Myers and Majluf (1984) framework is that firms cannot delay investment decisions and can only signal their private information to investors using the type of securitites they issue. In this paper, we extend the basic Myers and Majluf setup by allowing firms to choose not only the securities that they will use to finance investment but also the timing of investment. Specifically, we explore the possibility that the timing of investment (or hurdle rate for investment) may be used as a communication device that reveals the private information of corporate insiders to outside investors.

To make the intuition as clear as possible, we use a simple generalization of the standard Myers and Majluf (1984) framework in which the timing of investment and the firm’s financing strategy are jointly and endogenously determined. Specifically, we consider a firm that has a valuable real investment opportunity. However, it has to issue securities to undertake the investment project. The firm has flexibility in the timing of its investment and financing decisions and can choose its debt-equity mix. The investment project, once completed, will produce a continuous stream of cash flows forever. The level of cash flows depends on firm

\footnote{In most real options models, agents formulate optimal policies under the assumption that the firm has enough funds to finance the capital outlay or that the capital market has unlimited access to information about the firm. However, as documented in Frank and Goyal (2003), external finance is much more significant than is usually recognized in that it often exceeds investment. In addition, in most cases, firms need to raise external funds in situations where they have more information than outside investors about their growth prospects.}
There are two types of firms in the economy: high type (high cash flows) firms and low type (low cash flows) firms. Firm types are private information, so that insiders know more about the value of the firm’s investment projects than potential investors. When making investment and financing decisions, management acts in the old stockholders’ interest.

The model demonstrates that while under perfect information the two types of firms choose a different investment policy and issue fairly priced claims, this is not the case when outside investors are imperfectly informed about the firm’s growth prospects. With asymmetric information, the low type has incentives to mimic the good high to sell overpriced securities. Hence, in the pooling equilibrium in which all firms raise funds and invest at the same time, asymmetric information reduces (increases) the value of high type (low type) firms and increases (reduces) their cost of investment. This in turn causes good firms to delay investment (or, equivalently, raise their hurdle rate for investment) and bad firms to speed up investment compared to the perfect information benchmark. Because asymmetric information imposes costs on good firms, these firms may try to separate by imposing mimicking costs on bad firms. As we show in the paper, they can do so by changing their investment and/or their financing policies. In particular, our analysis demonstrates that firms can signal their private information to outside investors using either the timing of investment or their debt-equity mix. Several contributions follow from this result.

Our first contribution is to show that informational asymmetries imply investment behavior that differs substantially from that of the standard real options model with perfect information. In particular, we demonstrate that by investing early — i.e. by reducing the hurdle rate for investment and the value of the project at the time of investment — firms with good prospects can reduce the benefits of pooling for other firms and signal their positive information to outside investors. Although these distortions in investment policy have a cost, they allow good firms to obtain a better price for the claims they issue. As a result, asymmetric information provides such firms with an incentive to invest early, as manifested by their decision to select an investment threshold (minimum NPV or hurdle rate) that is too low relative to the threshold that maximizes project value. Since adverse selection problems are more severe for young, high-growth firms, an immediate implication of the model is that these firms will invest sooner so that their investment projects will have a greater likelihood of turning out poorly. Another implication of the model is that the information released at the time of investment will trigger a positive jump in the value of the good type, consistent
with the finding of McConnell and Muscarella (1985) that unexpected increases in investment lead to increases in stock prices.

The second key contribution of this paper relates to firms’ financing decisions. According to the pecking order theory, when outside funds are necessary to finance investment, firms prefer to issue debt rather than equity in an attempt to minimize adverse selection costs. While the theory should perform best among firms that face particularly severe adverse selection problems, Frank and Goyal (2003) have reported that small high-growth firms do not behave according to the pecking order hypothesis (see also Helwege and Liang, 1996, and Leary and Roberts, 2007). Our model reveals that firms can signal their private information to outside investors using the timing of investment and/or their debt-equity mix. As a result, asymmetric information may not translate into a preference ranking over securities. In particular, one implication of our analysis is that equity issues can be more attractive than debt issues even for firms with ample debt capacity. It is therefore not surprising that even as a descriptor of investment financing, the pecking order theory seems to struggle. This result is also consistent with the evidence reported in Leary and Roberts (2007) that “firms that issue equity against the pecking order’s prediction have an average leverage ratio that is almost half of that of their debt issuing counterparts.”

Our analysis, while shedding some light on the some important issues in corporate finance, also presents some new challenges to empirical tests that aim at determining which theory describes best observed financing decisions. Specifically, our modified pecking order model generates testable implications that could also result from a trade-off theory or a model of financing decisions based on agency conflicts within the firm. For example, we find that firms’ leverage ratios should decrease with the volatility of cash flows and with bankruptcy costs, consistent with a trade-off model (see e.g. Leland, 1994). We also show that operating leverage facilitates separation through investment timing and hence reduces the need to issue debt, also consistent with a trade-off model. Finally, we find that an improvement in investment opportunities also facilitates separation through timing and reduces leverage ratios, consistent with agency models based on shareholder-debtholder conflicts (see e.g. Myers, 1977) or manager-shareholder conflicts (see e.g. Morellec, 2004). These results suggest that it will be difficult, if not impossible, to determine whether asymmetric information affects firms’ policy choices by looking at financing decisions only.
The present paper relates to several articles in the literature.\footnote{Our paper also relates to the literature that studies the magnitude of dynamic investment and financing distortions due to conflicts of interest between debt- and equityholders (see e.g. Sundaresan and Wang (2007), Lobanov and Strebufalov (2007), Gomes and Schmid (2008), and Morellec and Schuerhoff (2008)). None of these papers have examined the impact of adverse selection on firm’s investment and financing strategies.} Myers and Majluf (1984) are the first to analyze the effects of asymmetric information on firms’ investment and financing decisions. One key assumption in their model is that “the project evaporates if the firm does not go ahead at time $t = 0$. (Myers and Majluf, pp. 190)” The current paper considers instead that the firm has flexibility in the timing of investment. Grenadier (1999), Lambrecht and Perraudin (2003), and Morellec and Zhdanov (2005) develop models of investment timing where agents infer the private information of other agents through their observed investment strategies. In these models investment is financed internally and there is no interaction between asymmetric information and the timing of investment. Finally, Grenadier and Wang (2005) examine the impact of moral hazard on the timing of investment. Our paper differs from theirs in two important dimensions. First, we consider that the firm has to raise funds to invest in the project. Second, we abstract from owner-manager conflicts and focus instead on insider-outsider conflicts (signaling vs. screening), as in Myers and Majluf (1984). As shown in the paper, these differences have important implications for equilibrium investment strategies. Notably, while in Grenadier and Wang firms invest later than in the full information benchmark, in our model they invest sooner.

The remainder of paper is organized as follows. Section two describes the model and the selected investment policy in a pooling equilibrium. Section three demonstrates that firms can separate by timing appropriately their investment decisions. Section four introduces debt financing. Section five concludes. Technical developments are gathered in the Appendix.

II. Model and assumptions

This paper considers a firm that must issue securities to undertake a valuable investment opportunity. Management knows more about the project value than potential investors. The firm has flexibility in the timing of investment as well as in the timing of the security issuance. Investors interpret the firm’s actions rationally and use Bayes’ rule to update their beliefs. An equilibrium model of the issue-invest decision is developed under these assumptions.
A. Setup

The model is an adaptation of Myers and Majluf (1984). Throughout the paper, financial markets are frictionless and competitive. Agents are risk neutral and discount cash flows at a constant rate $r$. We consider a set of infinitely-lived firms, each of which has monopoly rights to an investment project. The cost of investment is constant, denoted by $I$, and investment is irreversible. The project, once completed, produces a continuous stream of cash flows. We assume that the level of cash flows depends on firm type, which is indexed by $k$. Specifically, at any time $t$ after investment, a firm of type $k$ has profit flow given by $\Lambda_k X_t - f$ where $\Lambda_k$ is a positive constant that is known to corporate insiders only, $f$ represents constant operating expenses, and $X_t$ is an observable cash flow shock that evolves according to:

$$dX_t = \mu X_t \, dt + \sigma X_t \, dZ_t, \quad X_0 > 0.$$  \hspace{1cm} (1)

In this equation, the growth rate $\mu < r$ and volatility $\sigma > 0$ of the cash flow shock are constant and $(Z_t)_{t \geq 0}$ is a standard Brownian motion. We consider that there are two types of firms – high growth (type) and low growth (type) firms – so that $\Lambda_k$ has discrete sample space $\{\Lambda_b, \Lambda_g\}$, with $\Lambda_g > \Lambda_b > 0$ and $\Pr(\Lambda_k = \Lambda_g) = p$. Before investment, firm types are private information (i.e. are known to corporate insiders only).

The initial capital structure of each firm consists of $n_k^- = 1$ share of common equity. To fund the investment project the firm sells risky debt and/or new equity. Following Myers and Majluf (1984), we assume that when making investment and financing decisions, management acts in the old stockholders’ interest by maximizing the intrinsic value of existing shares, which equals the selling price of the shares when investors have full information. We also assume that when the firm issues shares, old stockholders are passive so that the issue goes to a different group of investors. We denote by $n_k^+ = 1 + \Delta n_k$ the number of shares outstanding after the round of financing and by $c$ the selected coupon payment. When the capital outlay is financed with risky debt, the decision to default is endogenous and chosen by shareholders. In default, bankruptcy costs consume a fraction $\alpha \in (0, 1]$ of the firm’s revenue stream.

B. Investment timing under symmetric information

Before analyzing the effects of asymmetric information on equilibrium investment strategies, we start by reviewing the benchmark case in which all agents have full information about the
firms’ investment projects. Since debt financing induces deadweight costs of bankruptcy and claims are fairly priced, it is optimal for firms to finance the capital expenditure by raising common equity in this full information benchmark.

Denote by $V_k^-$ the value of type $k$’s project before investment and by $\Pi(x)$ the present value of a perpetual stream of cash flows $X$ starting at $X_0 = x$:

$$\Pi(x) = \mathbb{E} \left[ \int_0^\infty e^{-rt} X_t \, dt \mid X_0 = x \right] = \frac{x}{r - \mu}. \quad (2)$$

Similarly, denote by $F$ the present value of operating expenses, i.e. $F = \int_0^\infty e^{-rt} f \, dt = \frac{f}{r}$. Because the firm does not produce any cash flows before investment, old shareholders only receive capital gains of $\mathbb{E}[dV_k^-]$ over each time interval $dt$. The required rate of return for investing in the firm’s equity is $r$. Applying Itô’s lemma, it is then immediate to show that the value of equity before investment satisfies:

$$rV_k^- = \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 V_k^-}{\partial X^2} + \mu X \frac{\partial V_k^-}{\partial X}, \quad \text{for } k = g, b. \quad (3)$$

This equation is solved subject to the following boundary conditions. First, the value of equity at the time of investment is equal to the payoff from investment (value-matching):

$$V_k^- (X) \big|_{X = X_k} = \Lambda_k \Pi(X_k) - F - I,$$

where $X_k$ is the threshold selected by type $k = g, b$.\(^3\) In addition, to ensure that investment occurs along the optimal path, the value of equity satisfies the smooth pasting condition: $\frac{\partial V_k^-}{\partial X} \big|_{X = X_k} = \Lambda_k \frac{\partial \Pi(X)}{\partial X} \big|_{X = X_k}$ at the endogenous investment threshold (see Dixit and Pindyck, 1994). Finally, as the value of the cash flow shock tends to zero, the option to invest becomes worthless so that $\lim_{X \to 0} V_k^- (X) = 0$.

Solving this optimization problem yields the following expression for equity value under perfect information (all proofs are relegated to the Appendix):

$$V_k^- (X) = [\Lambda_k \Pi(X_k) - F - I] \left( \frac{X}{X_k} \right)^\xi, \quad \text{for } k = g, b. \quad (4)$$

where $\xi > 1$ is the positive root of the quadratic equation $\frac{1}{2} \sigma^2 y(y-1) + \mu y - r = 0$. Equation (4) shows that equity value can be written as the product of the surplus created by investment (term in brackets) and the present value of $\$1$ contingent on investment (given by $(X/X_k)^\xi$). In the model, the timing of investment is endogenous and investment occurs the first time

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\(^3\)For now we ignore the option to abandon assets to keep the analysis tractable. We will examine the effects of exit/default on equilibrium investment and financing policies when we introduce debt.
the cash flow process reaches $X_k$. The value-maximizing investment threshold $X_k$ is given by:

$$X_k = \frac{\xi}{\xi-1} \frac{r - \mu}{\Lambda_k} (F + I), \text{ for } k = g, b. \quad (5)$$

This investment threshold reflects the option value of waiting through the factor $\xi/(\xi - 1)$. If this option had no value, shareholders would follow the simple NPV rule, according to which one should invest as soon as the investment surplus is positive, i.e., as soon as $X \geq (r - \mu)(F + I)/\Lambda$. Importantly, since $\Lambda_g > \Lambda_b$ implies $X_g < X_b$, equation (5) reveals that high-type firms invest before low-type firms in the full information benchmark.

C. Investment timing and signaling

While under perfect information different types of firms choose different investment policies and issue fairly priced claims, this is not the case when outside investors are imperfectly informed about the firm’s growth prospects. Indeed, since $\Lambda_g > \Lambda_b$, we have $V^-_g > V^-_b$ and there is an incentive for the bad type to mimic the good type (to sell overpriced securities). In a pooling equilibrium in which all firms invest at the same time and have the same value, asymmetric information imposes some costs on high type firms – they need to dilute their stake more than they would otherwise. As a result, good type firms may try to separate by imposing some mimicking costs on bad type firms.

As shown by equation (5), the value-maximizing investment threshold in the perfect information benchmark depends on firm type. This implies that the timing of investment can be used as a signal since the marginal cost of distorting investment depends on firm type. In particular, the high type may find it worthwhile to speed up investment and still realize a positive NPV on the project, while the bad type may face a negative NPV at the same investment threshold. This can be seen as follows. Suppose that the firm invests at date $t$ for a value of the cash flow shock of $X_t$. Let the firm type perceived by investors be $\Lambda$, $\Lambda_b \leq \Lambda \leq \Lambda_g$. The budget constraint requires that the capital raised equals the capital expenditure: $\triangle n(X_t; \Lambda)[\Pi(X_t) - F]/n^+(X_t; \Lambda) = I$. The firm then needs to issue a number of shares given by

$$\triangle n(X_t; \Lambda) = \frac{I}{\Pi(X_t) - F - I}. \quad (6)$$

Equation (6) reveals that the higher the type $\Lambda$ and the larger the investment trigger $X$, the lower the ownership dilution.
The single-crossing property captures the joint effect on valuations of investment distortions and ownership dilution. Proposition 1 shows that the first effect dominates the second so that the single-crossing condition holds (see the Appendix):

**Proposition 1 (Single Crossing)** The elasticity of substitution between project type $\Lambda$ and investment threshold $X$, $\frac{\partial \Lambda}{\partial X}|_{dV_k^k=0}$, depends positively on the type $k$ so long as $f > 0$, such that the single-crossing property holds:

$$\frac{\partial}{\partial \Lambda_k} \left( \frac{\partial}{\partial X} V_k(X; \overline{X}, \Lambda) \right) > 0. \hspace{1cm} (7)$$

Proposition 1 implies that high-type firms find it less costly to distort investment than low-type firms. Financial markets can therefore reasonably view the timing of investment or, equivalently, the hurdle rate of return as a signal of project quality. In the remainder of the paper, we will restrict attention to the case that operating expenses $f$ are positive and investment timing represents a valid signal.

III. Signaling through investment timing

A. Investment timing in the separating equilibrium

Assume for now that the capital outlay is financed by issuing equity. Our objective in this section is to show that there exists a timing of investment (for the good type) that makes it possible to sustain a separating equilibrium in which the two types of firms choose a different investment threshold and issue fairly priced claims. The mechanism underlying the equilibrium is a simple one. When deciding whether to pool or not, the bad type firm makes a trade-off between the overpricing of the shares (positive effect) and the reduction in intrinsic value due to the change in investment policy (negative effect). By speeding up investment (i.e. by lowering the hurdle rate for investment), the good type reduces the value of the project at the time of investment and, hence, the benefits of pooling for the bad type. The question we want to address is whether there exists an investment threshold (or hurdle rate for investment) such that the good type finds it profitable to invest and the bad type does not find it profitable to mimic.
To determine whether there exists such a separating equilibrium, we first need to check the incentive compatibility constraint for the bad type. Suppose that the good type invests at date \( t\) for a value of the cash flow shock \( X_t\). If the bad type mimics the investment behavior of the good type, the intrinsic value of the old shareholders’ claim in the bad firm after investment is given by:

\[
\Lambda_b \Pi(X_t) - F = \frac{\Lambda_b \Pi(X_t) - F}{\Lambda_g \Pi(X_t) - F} \left[ \Lambda_g \Pi(X_t) - F - I \right],
\]

since by pooling the bad type only needs to issue a number of shares equal to \( \triangle n(X_t; \Lambda_g) \) (given by equation (6)) to finance the capital expenditure. This equation shows that pooling with the good type reduces the investment cost for the bad type but alters investment. Instead of mimicking the good type, the bad type can follow its first-best strategy under perfect information, i.e., raise equity and invest when the cash flow shock reaches \( X_b\).

The bad type firm is indifferent between mimicking the good type at \( X \leq X_b\) and waiting to follow its first-best strategy under perfect information if:

\[
\frac{\Lambda_b \Pi(X) - F}{\Lambda_g \Pi(X) - F} \left[ \Lambda_g \Pi(X) - F - I \right] = \frac{\Lambda_b \Pi(X_b) - F - I}{\Lambda_g \Pi(X_b) - F} \left( \frac{X}{X_b} \right)^\xi.
\]

At the 0-NPV threshold for the good type, \( X^0_g = (F + I) / \Pi(\Lambda_g)\), the left-hand side of equation (8) is equal to zero whereas the right-hand side is positive (being an option value). As a result it is better for the bad firm to wait and not mimic the good firm. At the value-maximizing investment threshold of the bad type, \( X_b\), the left-hand side of equation (8) is larger than the right-hand side (since \( I > 0\)). In this case it is better for the bad firm to mimic the good firm. These observations, combined with the strict monotonicity of \( V_k\), imply there exists a unique value \( X^*\) for the cash flow shock, with \( X^0_g < X^* < X_b\), such that good firms can separate from bad firms by raising funds and investing before the cash flow shock exceeds this value. The critical threshold \( X^*\) is given by the solution to equation (8).\(^4\)

To ensure that investing at or below \( X^*\) is an equilibrium strategy, we need to verify incentive compatibility for the good type. The following incentive compatibility constraint

\(^4\)We can restrict attention to values of the cash flow shock \( X \leq X_b\) since all equilibria with \( X > X_b\) are pareto-dominated by corresponding equilibria with \( X \leq X_b\).
(ICC) is a necessary condition for the good type to separate from the bad type at \( X \leq X_b \):

\[
\frac{\Lambda_g \Pi(X) - F - I}{1 + \triangle n(X_b; \Lambda_b)} \geq \frac{\Lambda_g \Pi(X_b) - F}{X_b} \xi.
\]  

(9)

Value in separating equilibrium \hspace{1cm} Value under pooling > 0

The threshold \( X_{\text{min}} \) for which the incentive compatibility constraint (9) is binding represents the lowest value of the cash flow shock such that the good type prefers separation over pooling with the bad type. Expression (9) shows that since the value of the good type under pooling is strictly positive, the separating investment threshold cannot be too close to the 0-NPV threshold. A separating equilibrium exists if and only if \( X_{\text{min}} \leq X^\star \). By the optimality of \( X_{g} \) in the absence of information asymmetry, we also have \( X_{\text{min}} \leq X_{g} \).

We then have the following result:

**Proposition 2** (i) There exists a separating equilibrium in which growth firms separate by issuing equity and investing the first time the cash flow shock reaches \( X \) satisfying \( X_{\text{min}} \leq X \leq X^\star \) so long as \( f > 0 \). (ii) In the unique least-cost separating equilibrium good firms invest at the lower of the thresholds \( X^\star \) and \( X_g \), while bad firms invest at their first-best investment threshold \( X_b \). The market value of each firm before investment is independent of project quality and satisfies for \( X < X^\star \wedge X_g \):

\[
V_{\text{lcs}}(X) = \begin{cases} 
\frac{\Lambda_g \Pi(X^\star) - F}{\Lambda_g \Pi(X_b) - F} V_b^-(X), & \text{if } X^\star < X_g, \\
\frac{p \Lambda_g + (1-p) \Lambda_b}{\Lambda_b} V_b^-(X), & \text{otherwise},
\end{cases}
\]  

(10)

where \( \Lambda_{\text{pool}} = p \Lambda_g + (1-p) \Lambda_b \) and the market value under perfect information equals

\[
V_k^-(X) = \frac{F + I}{\xi - 1} \left( \frac{X}{X_k} \right)^\xi, \quad k = g, b.
\]  

(11)

The intrinsic value of the bad and, respectively, good firm before investment equal \( V_b^-(X) \) and

\[
V_{\text{lcs,g}}(X) = \begin{cases} 
\frac{\Lambda_g \Pi(X^\star) - F}{\Lambda_g \Pi(X_b) - F} V_b^-(X), & \text{if } X^\star < X_g, \\
\left( \frac{\Lambda_g}{\Lambda_b} \right)^\xi V_b^-(X), & \text{otherwise}.
\end{cases}
\]  

(12)

(iii) Good firms invest more aggressively than first-best (\( X^\star < X_g \), i.e. overinvest) whenever

\[
\frac{\xi}{\xi - 1} \left[ \Lambda_b - \Lambda_g \left( \frac{\Lambda_b}{\Lambda_g} \right)^\xi \right] > \frac{F}{F + I} \left[ 1 - \left( \frac{\Lambda_b}{\Lambda_g} \right)^\xi \right].
\]  

(13)
Proposition 2 shows that there exists an investment threshold $X^*$ solving equation (8) such that good types can separate from bad types by issuing equity and investing at or below that threshold. The good type will want to follow this strategy only if the cost of separating (i.e., the cost of investing early) is not too high compared to the underpricing of the shares. Since the value of the good type decreases with the selected investment threshold (for any threshold below $X_g$), this is equivalent to saying that there exists a lower bound $X_{\text{min}} \leq X^*$ on the separating investment threshold. Finally, since investing early is costly for good firms, in the least-cost separating equilibrium the good firm will want to raise equity and invest the first time the cash flow shock reaches the lower of $X^*$ and $X_g$.

B. Implications of the separating equilibrium

B.1. Adverse selection and investment timing

One of the major contributions of the real options literature is to show that with uncertainty and irreversibility, there exists a value of waiting to invest so that firms should only invest when the asset value exceeds the investment cost by a potentially large option premium. This effect is well summarized in the survey by Dixit and Pindyck (1994). These authors write:

“We find that for plausible ranges of parameters, the option value [of waiting] is quantitatively very important. Waiting remains optimal even though the expected rate of return on immediate investment is substantially above the interest rate or the normal rate of return on capital. Return multiples as much as two or three times the normal rate are typically needed before the firm will exercise its option and make the investment.”

Although this investment policy is consistent with what firms would do in the perfect information benchmark, it is not consistent with what they will do when taking into account informational asymmetries. As shown in Proposition 2, with asymmetric information the good type has a strong incentive to invest early in the project, as manifested by its decision to select an investment threshold $X^*$ (minimum NPV) that is lower than the threshold $X_g$ that maximizes project value. The intuition underlying this result is that when choosing the investment threshold the good type balances the value of waiting to invest with the underpricing associated with a pooling equilibrium. By investing early, the good type reduces
the intrinsic value of the bad type at the selected investment threshold and, hence, the benefits of pooling for the bad type. At the separating threshold, the bad type no longer wants to pool. This eliminates the underpricing, triggers a reduction in the cost of investment, and makes it optimal for the good type to exercise its real option and invest (provided that the incentive compatibility constraint of the good type is satisfied).

To gain more insights into the determinants of investment policy, Figure 1, Panel A, plots the value-maximizing investment threshold (solid blue line), the separating threshold (bold dotted line), and the 0-NPV threshold (dashed green line) as a function of the growth potential of the high type $\Lambda_g$, the volatility of the cash flow shock $\sigma$, and operating leverage $F$.\(^5\) In this figure, we use the following parameter values: the risk-free rate $r = 6\%$, the volatility and growth rate of cash flow shock: $\sigma = 20\%$ and $\mu = 1\%$, the firm’s operating leverage $F = 5/r$, the size of the growth option for the good and the bad firms: $\Lambda_g = 1$ and $\Lambda_b = 0.8$ (in the separating equilibrium, $p$ has no bearing on $X^\ast$).

When deciding whether to pool or to separate bad firms are balancing the value of investing now with the value of their option to invest. Since the value of the option to invest increases with the volatility of the cash flow shock, an increase in volatility decreases the bad firms’ incentives to mimic and allows good firms to invest later (i.e. to increase their investment threshold). By contrast, an increase in the size of the growth option for good firms $\Lambda_g$ leads to an increase in the benefits of mimicking and in a decrease in the separating investment threshold. In other words, investment is more distorted for higher (lower) values of $\Lambda_b$ ($\Lambda_g$). As operating leverage increases, the cost of mimicking for the bad type increases. Thus, for larger values of operating leverage $f$, the investment distortion decreases.

Importantly, while Grenadier and Wang (2005) show that moral hazard leads to late investment, our analysis demonstrates that adverse selection leads to early investment. This difference in equilibrium investment strategies is not surprising as, in the presence of owner-manager conflicts, the good type wants to hide its positive information about project values.

\(^5\)The risk free rate is taken from the yield curve on Treasury bonds. The growth rate of cash flows has been selected to generate a payout ratio consistent with observed payout ratios. Similarly, the value of the volatility parameter is chosen to match the (leverage-adjusted) asset return volatility of an average S&P 500 firm’s equity return volatility.
– and pool with the bad type – to extract more (informational) rents from the principal. As a result, the objective of the principal is to offer a contract to the agent that will induce truthful revelation. This can be achieved by making it more costly for the good type to pool with the bad type, i.e. by delaying investment for the bad type. By contrast, in the presence of adverse selection, the good type wants to reveal its positive private information and the bad type wants to pool with the good type. It is therefore optimal for the good type to speed up investment, to make it more costly for the bad type to mimic its investment behavior.

B.2. Hurdle rates for investment

Another way to look at the investment distortions induced by adverse selection is to examine the hurdle rate for investment implied by equilibrium investment strategies. Given the dynamics of the cash flows from investment in equation (1) and a value $X_\tau$ of the cash flow shock at the time $\tau$ of investment, the internal rate of return on the project, denoted $HR_k$, satisfies

$$\frac{\Lambda_k X_\tau}{HR_k - \mu} - F = I, \quad k = g, b.$$  

The left-hand side of this equation gives the present value of the cash flows from the project at the time of investment (similar to the Gordon growth formula). The right-hand side gives the cost of investment. This expression can alternatively be written as

$$HR_k = \frac{\Lambda_k X_\tau}{F + I} + \mu. \quad (14)$$

By changing the investment threshold, adverse selection alters the good firm’s hurdle rate for investment, $HR_g$. In our base case environment, the hurdle rate for investment is 11% in the perfect information benchmark and 8.6% in the presence of adverse selection (the 0-NPV rate is 6%). Again, by reducing the hurdle rate for investment, the good type reduces the value of the project at the time of investment and makes it less profitable for the bad type to mimic. Figure 1, Panel B, plots the value-maximizing hurdle rate (solid blue line), the hurdle rate under adverse selection (bold dotted line), and the 0-NPV rate of return (dashed green line) for the good firm as a function of the parameters. As in Panel A, the comparative statics are on the parameters $\Lambda_g$, $\sigma$, and $F$.

Figure 1, Panel B, shows that an increase in the size of the growth option for good firms, $\Lambda_g$, leads first to an increase in the benefits of mimicking and hence in a decrease of the hurdle
rate for investment. After some critical value, however, it becomes punitively expensive to mimic (since the investment distortions for bad type become larger) so that the hurdle rate becomes increasing in $\Lambda_g$. By contrast, an increase in the volatility of the cash flow shock leads to a monotonic increase in the hurdle rate for investment. The hurdle rate for investment in the perfect information benchmark increases more than the hurdle rate with adverse selection since the vega of an option increases with its moneyness (and the good type’s real option, determining $\overline{X}_g$, is more in the money than the bad type’s real option, determining $X^*$).

**B.3. External financing costs and ex-post losses**

By distorting the firm’s investment policy, asymmetric information reduces the value of the good firm. This reduction in value is equal to the difference between intrinsic firm (option) values under selected and value-maximizing policies. Figure 1, Panel C, plots this drop in value, defined by $\text{Cost} = (V^-(X) - V^-_{\text{cs},g}(X))/V^-(X)$, as a function of the relative growth potential of the low type $\Lambda_b/\Lambda_g$, the volatility of the cash flow shock $\sigma$ and operating leverage $F$. The figure reveals that the reduction in project value due to asymmetric information can be substantial, ranging up to 15% in our base case. The comparative statics for the drop in firm value mirror those in Panels A and B since greater investment distortions imply a larger reduction in firm value.

Using the results in Proposition 2, we can also examine the likelihood that the firm’s project will turn out poorly after investment. The probability that, given a value $X_\tau$ of the cash flow shock at the time of investment, the asset value falls below the investment cost at some time over the next $T$ years is given by [see Harrison (1985), pp. 15]:

$$
\Pr\left[\inf_{t\in[\tau,\tau+T]} \Lambda_k \Pi(X_t) - F \leq I\right] = \left(\frac{a_k}{X_\tau}\right)^{2\pi / \sigma^2} \mathcal{N}\left[\frac{\ln\left(\frac{a_k}{X_\tau}\right) + mT}{\sigma\sqrt{T}}\right] + \mathcal{N}\left[\frac{\ln\left(\frac{a_k}{X_\tau}\right) - mT}{\sigma\sqrt{T}}\right],
$$

(15)

where $a_k \equiv (r-\mu)(F+I)/\lambda_k$, $m \equiv \mu - \sigma^2/2$, $\mathcal{N}$ is the normal cumulative distribution function, $X_\tau = \overline{X}_g$ under perfect information, and $X_\tau = X^* \wedge \overline{X}_g$ under asymmetric information.

Figure 1, Panel D, plots the loss probability given by expression (15) over a 5 year horizon under adverse selection (bold dotted line) in comparison with the loss probability under the first-best (solid blue line) investment policy. Across panels, comparative statics vary on the parameters $\Lambda_b/\Lambda_g$, $\sigma$, and $F$ (from left to right). In the base case environment, asymmetric
information increases the probability of ex-post losses over a 5 year horizon from 15% to 40%. Thus, adverse selection has a significant impact on the likelihood of ex post losses. Because an increase in the quality of the good type’s project leads to a drop in the separating threshold, an increase in this factor implies an increase in the likelihood of ex post losses. Finally, the impact of volatility on the likelihood of ex post losses results from two opposite effects. First, as volatility increases, the separating investment threshold increases (see Panel A). This reduces the probability of bad outcomes. Second, for any given threshold, an increase in volatility increases the probability that the cash flow shock will hit the threshold over a given horizon. In the base case environment, the second effect dominates, leading to a positive relation between volatility and the probability of ex post losses.

B.4. Change in stock price at the time of investment

In the separating equilibrium, outside investors have incomplete information regarding the quality of the firms’ investment projects before investment (assuming that the initial value of the cash flow shock is less than $X^*$. However, at the time of investment, firm types become public information and this uncertainty is resolved. Importantly, because outside investors cannot predict when investment will arise, the information released at the time of investment triggers a positive (negative) jump in the stock price of the good (bad) type. This is consistent with the empirical finding of McConnell and Muscarella (1985) that unexpected increases in investment lead to increases in stock prices (and vice versa for unexpected decreases).

In the present model, we can compute the announcement returns associated with the good type’s decision to invest. Denote by $AR_k(X) = (V_k^+(X) - V_{ls}^-(X))/V_{ls}^-(X)$ the jump in the value of type $k$ when the value of the cash flow shock is $X$. At the time of investment (consider the case $X^* > X_g$), these abnormal returns are given by

$$AR_g(X^*) = \frac{\Lambda_g \Pi(X^*) - F}{\Lambda_{pool} \Pi(X^*) - F} - 1 > 0,$$

and

$$AR_b(X^*) = \frac{\Lambda_b \Pi(X^*) - F}{\Lambda_{pool} \Pi(X^*) - F} - 1 < 0. \quad (16)$$

Equation (16) shows that abnormal returns are equal to the unexpected component of the surplus accruing to old shareholders as a fraction of equity value at the time of investment. Within the present model, this unexpected component arises because market participants have incomplete information regarding the characteristics of the firm’s growth prospects. However, at the time of investment, this uncertainty is resolved by observing the value of the
trigger threshold $X^*$. The jump in the value of the good type at the time of investment is positive as investment at $X^*$ signals good news about project quality. By contrast, the jump in the value of the bad type is negative.

Figure 1, Panel E, plots the abnormal announcement returns of the good type (solid blue line) and bad type (dashed red line) as a function of $\Lambda_b/\Lambda_g$, $\sigma$, and $F$. Because the timing of investment depends on size and volatility of the cash flows generated by the firms’ investment project and on the firm’s operating leverage, abnormal returns to shareholders depend on these factors as well. In particular, the model predicts that abnormal returns should decrease with the size of the low-type’s project $\Lambda_b$ and the project’s cash flow volatility $\sigma$. By contrast, abnormal announcement returns should increase with the size of the high-type’s project $\Lambda_g$ and operating leverage $F$ (since mimicking is more costly).

C. Pooling in equity: The underpricing-overinvestment trade-off

Consider next a situation in which financial markets are not able to distinguish among firm types, leading to a pooling equilibrium. In a pooling equilibrium, all firms invest at the same time and issue common equity to finance the capital outlay. The pooled value of the firms at the time of investment is given by

$$V_{pool}^+(X) = \sum_{k=b,g} \Pr(\Lambda = \Lambda_k) \Lambda_k \Pi(X) - F = \Lambda_{pool} \Pi(X) - F,$$

where $\Lambda_{pool} = p \Lambda_g + (1-p) \Lambda_b$. The constraint $V_{pool}^+(X) \triangle n(X; \Lambda_{pool})/[1+\triangle n(X; \Lambda_{pool})] = I$ determines the number of shares $\triangle n(X_{pool}; \Lambda_{pool})$ that have to be issued at the time of investment given the value of the cash flow shock $X_{pool}$ at that time. Solving this budget constraint yields $\triangle n(X_{pool}; \Lambda_{pool}) = I/\Lambda_{pool} \Pi(X_{pool}) - F - I$. This equation shows that asymmetric information leads to a dilution of the good type’s equity stake and to an increase in the cost of investment for the good type ($\Lambda_{pool} < \Lambda_g$). Ex post outside investors make money on the good type and lose money on the bad type: There is cross-subsidization.

To determine whether a pooling equilibrium exists, we first need to verify that pooling with the good type is an optimal strategy for the bad type. The ICC of the bad type writes

$$\frac{\Lambda_b \Pi(X_{pool}) - F}{1 + \triangle n(X_{pool}; \Lambda_{pool})} \geq \frac{[\Lambda_b \Pi(X_{b}) - F - I] \left(\frac{X_{pool}}{X_{b}}\right)^\xi}{\left(\frac{X_{pool}}{X_{b}}\right)},$$

Value in pooling equilibrium Real option value $> 0$
Since $\Lambda_{pool} < \Lambda_g$, the threshold at which condition (18) is binding lies between $X^*$ and the first-best threshold $X_b$. For smaller values of the cash flow shock condition (18) is violated and investment at such a threshold does not constitute a pooling equilibrium.

As is standard in signaling games, we face multiplicity of equilibria. Maskin and Tirole (1992), however, show that in the augmented game in which the firm’s insiders ex ante offer contracts to investors in the capital market, only those pooling equilibria survive that (weakly) pareto-dominate the least-cost separating equilibrium characterized in Proposition 2. In any pooling equilibrium all firm types invest at the same time and issue common equity. The incentive compatibility constraint (18) for the bad type puts a perfect Bayesian best-response restriction on the set of pareto-dominant pooling equilibria. The remaining restriction is that the value of the good type in the pooling equilibrium is larger than the corresponding value in the least-cost separating equilibrium:

$$\Lambda_g \Pi(X_{pool}) - F \left( \frac{X}{\Lambda_{pool}} \right)^\xi \geq 1_{X_g \leq X^*} V_g(X) + 1_{X_g > X^*} \left( \frac{\Lambda_g \Pi(X^*) - F}{\Lambda_b \Pi(X^*) - F} \right) V_g(X).$$

Value in pooling equilibrium Value in separating equilibrium

(19)

Pooling equilibria exist if and only if there is a threshold $X_{pool}$ for which conditions (18) and (19) hold. We can check these conditions as follows. Whenever $X_g \leq X^*$, condition (19) is violated since the good type cannot do better than the first-best value $V_g$. Now consider the reverse situation. The objective of management in a pooling equilibrium is to select the investment threshold $X_{pool,k}$ that maximizes the present value $V_{pool,k}(X)$ of the cash flows accruing to incumbent shareholders. Solving the optimization problem for the good type yields a different answer as for the bad type. Specifically, we show in the Appendix that

$$X_{pool,k} = \frac{\chi_k}{\chi_k - 1} \frac{r - \mu}{\Lambda_{pool}} (F + I), \quad k = g, b$$

(20)

where $\chi_k > 1$ is defined in Appendix D. There will therefore not be a single pareto-optimal pooling equilibrium. Notice, however, that condition (18) holds whenever condition (19) is satisfied. A pooling equilibrium therefore exists if and only if $X^* < X_g$ and condition (19) holds at $X_{pool,g}$. We then have the following existence result (see the Appendix):

**Proposition 3** There exists a pareto-dominant pooling equilibrium in which all firms raise equity and invest the first time the cash flow shock reaches a threshold $X_{pool}$ satisfying (18)
and (19) with $X^* \leq X_{pool} \leq X_b$ if and only if condition (13) holds (so that $X^* < X_g$) and the fraction of good projects in the economy, $p$, exceeds the threshold $\bar{p}$ defined in the Appendix.

The next proposition characterizes the pareto-dominant pooling equilibria:

**Proposition 4** Good firms invest more conservatively than first-best (underinvest) and bad firms invest more aggressively than first-best (overinvest) in the pooling equilibrium, that is $X_g \leq X_{pool} \leq X_b$. The market value in the pooling equilibrium is independent of project type, and market and intrinsic values are equal to

$$V_{pool}(X) = [\Lambda_{pool} \Pi(X_{pool}) - F - I] \left(\frac{X}{X_{pool}}\right)^{\xi},$$

and

$$V_{pool,k}(X) = \left(\frac{\Lambda_k \Pi(X_{pool}) - F}{\Lambda_{pool} \Pi(X_{pool}) - F}\right) V_{pool}(X).$$

Several results follow from Propositions 3 and 4. First, the investment policy under pooling does not necessarily coincide with the first-best policy of a firm with average type. Second, asymmetric information raises the investment threshold selected by the good type compared to the perfect information benchmark depending on beliefs and firm characteristics. As a result, asymmetric information causes good firms to delay investment in the pooling equilibrium. Hence they underinvest relative to first best. Third, asymmetric information reduces the value of good firms and increases the value of bad firms. Hence, although the good type can raise the funds to finance its project, it is hurt by the presence of the bad one.

Figure 2 plots the least-cost equilibrium (Panel A), the equilibrium investment threshold (Panel B), and the external financing costs due to investment distortions and underpricing (Panel C) for the high-type firm as a function of the growth potential of the high type $\Lambda_g$, the volatility of the cash flow shock $\sigma$, operating leverage $F$, and investors’ belief about the fraction of high-type firms $p$. Input parameter values for the base case environment are set as in Figure 1. We assume that the probability of high-type firms is $p = 50\%$. The figure shows that for low values of the firms’ operating leverage $F$, cash flow volatility $\sigma$, or growth differential $\Lambda_g/\Lambda_b$, the cost of separating are too high for the good type so that pooling equilibria pareto-dominate the least-cost separating equilibrium (i.e. external financing is cheaper when pooling). When this is the case, the good type underinvests relative to first best and there is no announcement return at the time of investment.
D. Investment timing under product market competition

One essential difference between the analysis in this paper and the analysis in Myers and Majluf (MM, 1984) is that MM assume that “the investment opportunity evaporates if the firm does not go ahead at time $t = 0$. (pp. 190)” In the current paper we make the opposite assumption and consider that each firm has monopoly rights on its investment project and can delay the investment decision as much as it desires.

To consider intermediate cases, suppose that if the firm does not exercise its investment opportunity promptly, the project can evaporate. Specifically, consider that with some probability $\lambda \, dt$ over the time interval $dt$ the project can disappear because, e.g., another company has invested in a similar project. Under this assumption, the expected time before the project evaporates is given by

$$E[T] = \int_0^\infty \lambda t e^{-\lambda t} \, dt = \frac{1}{\lambda}.$$ 

This equation shows that when $\lambda = 0$, the firm does not face any timing constraint. As $\lambda$ tends to infinity, the firm can no longer delay investment.

As before, denote by $V_k^-$ the value of type $k$’s investment project and by $\Pi(x)$ the present value of a perpetual stream of cash flows $X$ starting at $X_0 = x$. Because the firm does not produce any cash flows before investment, the value of the growth option satisfies the following ODE (for $X < X_k$):

$$(r + \lambda) V_k^- = \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 V_k^-}{\partial X^2} + \mu X \frac{\partial V_k^-}{\partial X}.$$ 

This ordinary differential equation is similar to the one obtained above and incorporates an additional term that reflects the impact of the timing constraint on the value of the project. This term equals $\lambda(0 - V_k^-)$, since with probability $\lambda \, dt$ the value of the investment opportunity will drop from $V_k^-$ to zero. In the perfect information benchmark, this equation is solved subject to the same no-bubbles, value-matching, and smooth-pasting conditions as before.

Solving this optimization problem yields the value of equity in the perfect information benchmark as:

$$V_k^- (X) = \left[ \Lambda_k \Pi(X_k (\lambda)) - F - I \right] \left( \frac{X}{X_k (\lambda)} \right)^{\beta(\lambda)}.$$ 

In this equation, the value-maximizing investment threshold $X_k (\lambda)$ is defined by:

$$X_k (\lambda) = \frac{\beta(\lambda)}{\beta(\lambda) - 1} \left( \frac{r - \mu}{\Lambda_k} \right) (F + I),$$ 

19
where
\[ \beta(\lambda) = \frac{(\sigma^2 - \mu)}{\sigma^2} + \sqrt{\left[ \frac{(\mu - \sigma^2)}{\sigma^2} \right]^2 + 2(\mu + \lambda)/\sigma^2}. \]

These expressions are identical to those reported above except that the elasticity \( \beta(\lambda) \) reflects (through its dependence on the parameter \( \lambda \)) the time constraint imposed on the investment decision. The value of the project is again the product of the surplus created by investment and a stochastic discount factor. The investment threshold reflects the option value of waiting to invest through the factor \( \beta(\lambda)/(\beta(\lambda) - 1) \). As the hazard rate \( \lambda \) increases, \( \beta(\lambda) \) increases and therefore \( \beta(\lambda)/(\beta(\lambda) - 1) \) decreases. That is, firms speed up investment as \( \lambda \) increases. In the limit as \( \lambda \to \infty \), the factor \( \beta(\lambda)/(\beta(\lambda) - 1) \) capturing the delay in investment converges to one and all the investment thresholds converge to the Marshallian thresholds.

Consider now the limiting case in which \( \lambda \to \infty \) and \( F = 0 \). As long as the pooled value exceeds the investment cost (i.e. as long as the initial value of the cash flow shock is such that \( \Lambda_{\text{pool}}\Pi(X) > I \)), incumbent shareholders can finance the project and will keep a positive fraction of the firm’s equity after investment. In particular, using the expression for the number of shares \( \Delta n(X) \) that have to be issued at the time of investment (given by equation (6)), we have that the value of the claims of the incumbents in the pooling equilibrium is given by:
\[ \frac{\Lambda_k\Pi(X)}{1 + \Delta n(X)} = \frac{\Lambda_k\Pi(X)}{\Lambda_{\text{pool}}\Pi(X)} \left[ \Lambda_{\text{pool}}\Pi(X) - I \right] \text{ for } k = g, b. \]

This expression is positive as long as the pooled value exceeds the cost of investment. As a result, investment creates value for the old shareholders of the two types of firms. By contrast if the firms do not invest at time 0, the value of the incumbent’s claims falls to zero as the investment opportunity evaporates.\(^6\) We then have the following result:

**Proposition 5** In the limit as \( \lambda \to \infty \) and \( F = 0 \), both types of firms find it profitable to invest as long as the initial value of the cash flow shock satisfies
\[ X_0 > \frac{I}{\Pi(\Lambda_{\text{pool}})}. \]

In this case it is no longer possible to have a separating equilibrium with equity issuance. Good type reject positive NPV projects for initial values of the cash flow shock satisfying
\[ \frac{I}{\Pi(\Lambda_g)} \leq X_0 \leq \frac{I}{\Pi(\Lambda_{\text{pool}})}. \]

\(^6\)In other words, the right-hand side of the incentive compatibility constraint of the bad type (8) goes to zero as \( \lambda \) tends to infinity (i.e. as \( \beta \) tends to infinity), making immediate investment optimal.
This proposition shows that the model of Myers and Majluf (1984) is nested in ours. In the limit as the firm cannot postpone investment, the option value of waiting to invest vanishes and firms face a now-or-never investment decision. As long as the pooled net present value of the project is positive, both types of firms will want to invest now. For initial values of the cash flow shock of \( X_g \leq X_0 < X_{pool} \), the good type will find it profitable to invest while the bad type will want to mimic. The pooled value of the firm is negative, however, and no investor would be willing to provide sufficient funds for investment. This is the standard lemon’s problem in markets with asymmetric information.

IV. Signalling through debt financing

We have thus far not explored the possibility that firms might issue debt to finance the capital expenditure. In this section we relax this assumption and examine the effects of debt financing on firms’ equilibrium investment strategies. We show that while good types can separate from bad types by issuing equity and investing sooner than first-best, they can also separate by issuing debt (as in Leland and Pyle, 1977). The least-cost financing choice depends on investors’ prior beliefs as well as on the characteristics of the project.

A. Debt Issuance and Firm Valuation

Suppose that the investment outlay \( I \) is funded with risky debt. Denote the selected coupon payment by \( c \) and the default threshold of type \( k \) by \( X_k(c) \). In the perfect information benchmark, the total value of the firm \( V^+_k \) and the value of debt \( D_k \) after investment are given by the following expressions for \( k = g, b \) (see Appendix E):

\[
V^+_k(X, c) = \frac{X}{X_k(c)} \left( \frac{X}{X_k(c)} \right)^\nu,
\]

\[
D_k(X, c) = \frac{c}{r} - \left[ \frac{c}{r} - (1 - \alpha) \frac{X}{X_k(c)} + F \right] \left( \frac{X}{X_k(c)} \right)^\nu,
\]

where \( \nu < 0 \) is the negative root of the equation \( \frac{1}{2} \sigma^2 y(y - 1) + \mu y - r = 0 \). Equation (22) shows that for any value of the cash flow shock, debt financing reduces firm value by inducing bankruptcy costs (third term on the right-hand side of this equation). The first term on the right-hand side of equation (23) is the value of risk-free debt. The second term captures the
impact of default risk on the value of corporate debt. The intrinsic value of an equity claim, in turn, is given by

\[ V_k^+ (X, c) - D_k (X, c), \quad k = g, b, \]

and the endogenous default threshold satisfies (see the Appendix):

\[ 
X_k (c) = \frac{\nu}{\nu - 1} \frac{r - \mu}{\Lambda_k} \left( F + \frac{c}{r} \right), \quad k = g, b. \tag{24} 
\]

Since \( \Lambda_g > \Lambda_b \) implies \( X_g (c) < X_b (c) \), equation (24) shows that for a given coupon payment \( c \) the default threshold of the good firm is lower than that of the bad firm.\(^7\)

In order to simplify the exposition of results, we define the coefficients \( \eta \) and \( \eta_k \), for \( k = g, b \), as follows:

\[ 
\eta = \frac{\alpha \nu}{\nu - 1 - (1 - \alpha) \nu}, \quad \text{and} \quad \eta_k = 1 - \frac{\Lambda_k ^\nu (1 - \eta)}{p \Lambda_g ^\nu + (1 - p) \Lambda_b ^\nu}. \tag{25} 
\]

Under symmetric information the budget constraint \( D_k (X, c_k (X)) = I \) implies that the coupon \( c_k (X) \) selected by type \( k \) at the time of investment is given by the solution to:

\[ 
\left( \frac{c_k}{r} + F \right) ^{\nu - 1} \left( \frac{c_k}{r} - I \right) = \alpha \left( \frac{\nu}{\nu - 1} \right) ^{1 - \nu} \left[ \Lambda_k \Pi(X) \right] ^\nu. \tag{26} 
\]

The credit spread on the debt contract satisfies \( \rho_k (X) = \frac{c_k (X)}{r}/I - 1 \) at the date of issuance, and firm value at the investment date amounts to \( V_k^+ (X, c_k (X)) = \Lambda_k \Pi(X) - F - \eta \rho_k (X) I \). The third term in this expression captures the discount due to deadweight costs of default. In the perfect information benchmark, debt issuance (and investment) at \( X \) yields the following valuation for type \( k \)'s equity at the issuance date:

\[ 
V_k^+ (X, c_k (X)) - D_k (X, c_k (X)) = \Lambda_k \Pi(X) - F - I [1 + \eta \rho_k (X)]. \tag{27} 
\]

For \( k = g \), this equation represents the equity value of the good type in the separating equilibrium. Due to the deadweight losses in default, the value of the firm is reduced by more than the direct cost of investment \( I \) when issuing corporate debt. This effect is captured by the term in square brackets on the right-hand side of equation (27) (since \( \rho_k > 0 \)).

Last, in the perfect information benchmark, a firm of type \( k \) optimally invests when the cash flow shock reaches the investment threshold \( \overline{X}_{k,D} \) defined by

\[ 
\overline{X}_{k,D} = \overline{X}_k \left[ 1 + \frac{\eta \rho_k I}{I + F} \left( 1 - \frac{\nu \left[ F + \left( 1 + \rho_k \right) I \right]}{\xi \left[ F + (1 + \nu \rho_k) I \right]} \right) \right] \geq \overline{X}_k. \tag{28} 
\]

\(^7\)This result is robust to alternative bankruptcy policies. For instance, if the firm is constrained from raising additional funds, it defaults when \( \underline{X}_k (c) = \frac{c_k}{\Lambda_k}, \quad k = g, b, \) and we again have \( \underline{X}_g (c) < \underline{X}_b (c) \).
Equation (28) shows that debt financing leads to a delay in investment beyond the first-best threshold $X_k$ (the term in brackets is larger than unity). This is due to the well known debt overhang effect of Myers (1977). The value function of firm type $k$ at any time before investment is then given by

$$V_{k,D}^-(X) = \frac{F + (1 + \nu \frac{1-\alpha}{\alpha} \rho_k) I}{F + (1 + \nu \rho_k) I} \left( \frac{X_k}{X_{k,D}} \right)^\xi V_k^-(X).$$

This expression reveals that debt financing reduces firm value through two channels in the perfect information benchmark. First, for a given investment policy, debt financing induces bankruptcy costs (first factor on the right-hand side of this equation). Second, debt financing distorts investment policy (second factor).

### B. Separation through debt issuance

Under symmetric information, equity issuance maximizes firm value because of the dead-weight costs of default and investment distortions due to debt financing (i.e. $X_{g,D} > X_g$). However, in the presence of informational asymmetries, issuing equity may not be in the best interests of incumbent shareholders as it may lead to underpricing in a pooling equilibrium or to large investment distortions in a separating equilibrium.

To determine whether there exists a separating equilibrium in which the good type issues debt, we first need to check the incentive compatibility constraint of the bad type. The budget constraint $D_g(X, c_g) = I$ implies that the bad type is indifferent between mimicking the good type by issuing debt at the threshold $X_D^*$ and waiting to follow its first-best strategy if the following incentive compatibility constraint is satisfied:

$$V_b^+(X_D^*, c_g(X_D^*)) - I + \left[ \left( \frac{\Lambda_b}{\Lambda_g} \right)^\nu - 1 \right] \rho_g(X_D^*) I = V_b^-(X_D^*).$$

In this equation, $V_b^-$ is the value of the bad type in the perfect information benchmark defined in Proposition 2. This incentive compatibility constraint is similar to that derived under equity financing and reflects the fact that cross-subsidization reduces the cost of debt financing for the bad type ($D_b(X, c_g) = I \left[ 1 - (\Lambda_b \Lambda_g^\nu - 1) \rho_g(X) \right] < I$). The positive effect of selling overpriced debt is counter-balanced by the investment distortions ($X_D^* \neq X_b$).
and the negative effect of bankruptcy costs on the value of the bad type after investment (captured in the term $V_b^+$). When the value of the cash flow shock is less than $X^*_D$ (solving equation (30)), the bad type prefers to separate and invest at its first-best trigger under perfect information. Otherwise, the bad type prefers to pool with the good type.

To check whether issuing debt and investing below the threshold $X^*_D$ is an equilibrium strategy for the good firm, we also need to check its incentive compatibility constraint:

$$V_g^+(X, c_g(X)) - I \geq \frac{\Lambda g \Pi(X_b) - F}{1 + \beta n(X_b; \Lambda_b)} \left( \frac{X}{X_b} \right)^\xi,$$

where $V_g^+(X, c_g(X))$ is given in equation (22). The threshold $X_{\text{min},D}$ for which the incentive compatibility constraint (31) is binding represents the lowest value of the cash flow shock such that the good type prefers separation with debt over pooling with the bad type. A separating equilibrium exists if and only if $X_{\text{min},D} \leq X^*_D$. By the optimality of $X^*_{g,D}$ in the absence of information asymmetry, we also have $X_{\text{min},D} \leq X^*_{g,D}$.

Using the incentive compatibility constraints of the good and bad types, it is immediate to establish the following result:

**Proposition 6**

(i) There exists a separating equilibrium in which growth firms separate by issuing equity and investing the first time the cash flow shock reaches $X$ satisfying $X_{\text{min},D} \leq X \leq X^*_D$. (ii) In the unique least-cost separating equilibrium with debt, good firms invest at the lower of the thresholds $X^*_D$ and $X^*_{g,D}$, given by (28), while bad firms invest at their first-best investment threshold $X_b$. The selected coupon payment $c_g$ and the credit spread $\rho_g$ are determined by condition (26). Before investment firm value is independent of project quality and satisfies:

$$V_{\text{los},D}(X) = pV_{g,D}(X) + (1 - p)V_b(X).$$

(iii) Separation in debt is least-cost if and only if

$$\frac{F + (1 + \nu \frac{1 - \epsilon}{1 - \nu} \rho_g) I}{F + (1 + \nu \rho_g) I} \left( \frac{X_b}{X^*_{g,D}} \right)^\xi \geq \text{Max} \left[ 1_{X_g = X^*} \left( \frac{\Lambda g}{\Lambda_b} \right)^\xi + 1_{X_g > X^*} \frac{\Lambda g \Pi(X^*) - F}{\Lambda g \Pi(X^*) - F} \cdot \frac{V_{\text{pool},g}(X)}{V_g(X)} \left( \frac{\Lambda g}{\Lambda_b} \right)^\xi \right],$$

where $V_{\text{pool},g}(X)$ is defined in Proposition 4.
Proposition 6 shows that there exists an investment threshold $X_D^*$ solving equation (30) such that good types can separate from bad types by issuing debt and investing at or below that threshold. The good type will want to follow this strategy only if the cost of separating (i.e., the cost of issuing debt and distorting investment) is not too high compared to the underpricing of the shares. Since the value of the good type increases with the selected investment threshold (for any threshold below $X_{g,D}$), this is equivalent to saying that there exists a lower bound $X_{\min,D} \leq X_D^*$ on the separating investment threshold. Finally, since distorting investment is costly for good firms, in the least-cost separating equilibrium the good firm will want to issue debt and invest the first time the cash flow shock reaches the lower of $X_D^*$ and $X_{g,D}$.

### C. Implications of the separating equilibrium with debt

Given that several investment and financing strategies are available, one question that naturally arises is what is the value-maximizing strategy for the good type? The traditional answer to this question is that firms should first issue the securities with the lowest information costs, i.e. that informational asymmetries make conventional equity issues unattractive (see Myers and Majluf, 1984, or Stein, 1992). Proposition 6 shows that in our model the financing choice of the good type is determined by a trade-off between investment distortions, the more severe underpricing of equity, and the deadweight costs of default associated with risky debt claims. The good type wants to use debt financing only if the cost of debt is not too high.

To gain more insights into the determinants of equilibrium investment and financing policies, Figure 3, Panel A, represents the least-cost separating equilibrium (as characterized by condition (33) in Proposition 6) as a function of the various inputs of the model. The figure shows that when $\Lambda_g/\Lambda_b$ is high, the good type finds it optimal to separate from the bad type by issuing equity, as it is too costly for the bad type to distort its investment strategy in that case. By contrast, when $\Lambda_g/\Lambda_b$ is small both types pool in equity. These results are due to the fact the the value of the option to invest is increasing and concave in the selected investment threshold so that the cost to the bad type of deviating from its first best threshold is increasing and convex. Debt issuance is the optimal strategy to minimize external
financing costs for intermediate values of $\frac{\Lambda_g}{\Lambda_b}$. When deciding which type of security to issue, the good type balances the investment distortions associated with equity issues with the expected bankruptcy costs of debt. Figure 3 reveals that high bankruptcy costs $\alpha$, high cash flow volatility $\sigma$, a large operating leverage, or a large growth differential between types makes it more likely for the good type to issue equity. This result suggests that, consistent with the evidence reported by Frank and Goyal (2003), small high-growth firms may not find it optimal to behave according to the static pecking order theory. Hence our model can explain observed departures from the static pecking order theory such as equity issuance by firms with ample debt capacity.

One interesting feature of our model is that it generates predictions that could also arise from a standard trade-off model or from a model based on agency costs within the firm. Specifically, our model predicts that the use of debt should decrease with the quality of the investment opportunities of the good type, the volatility of the cash flow shock, bankruptcy costs, and operating leverage. The first prediction could also be generated by a model based on shareholder-debtholder conflicts (see e.g. Myers, 1977) or based on manager-shareholder conflicts (see e.g. Morellec, 2004). The last three predictions could be generated by a trade-off model in which optimal capital structure balances taxes with deadweight costs of bankruptcy (see e.g. Leland, 1994). These results suggest that it will be difficult, if not impossible, to determine whether asymmetric information affects firms’ policy choices by looking at financing decisions only. One essential difference between our model and these competing theories is that investment and financing decisions trigger abnormal announcement returns, consistent with the empirical evidence.

As in the case of equity financing, we can examine the implications of debt financing for investment distortions (Panel B), external financing costs (Panel C), and abnormal announcement returns. Figure 3 plots all of these quantities as a function of the relative size of the growth option of bad firms, $\frac{\Lambda_b}{\Lambda_g}$, the volatility of the cash flow shock $\sigma$ and operating leverage $F$. Input parameter values for the base case environment are set as in Figure 1. The figure shows that when separation in debt is least-cost, good firms underinvest relative to first best (i.e. $X_{k,D} \geq X_k$; see equation (28)). However, the cost associated with these distortions can be much lower than those that would arise with equity financing (Panel C).

The figure also shows that because the timing of the investment depends on size and volatility of the cash flows generated by the firms’ investment project and on the firm’s
operating leverage, abnormal returns to shareholders depend on these factors as well. In particular, the model predicts that abnormal returns should decrease with volatility of the project cash flows $\sigma$ and the fraction $p$ of good firms, and it should increase with the growth differential $\Lambda_g/\Lambda_b$ and operating leverage $F$. Interestingly, Figure 3 reveals that abnormal returns are higher with debt financing than with equity financing (compare the solid line with the dashed line). This is consistent with the positive return documented in debt for equity exchanges (see e.g. Masulis, 1983), even though the effect predicted by the model is small (consistent with Eckbo, 1986).

V. Conclusion

This paper develops a real options model to examine the impact of asymmetric information on investment and financing decisions when external funds are needed to finance investment. In the model, the firm’s financing and investment strategies are jointly determined and result from value-maximizing decisions. We show that by timing their decisions corporate insiders can communicate their private information about the firm prospects to outside investors. The model generates implications that are consistent with the available empirical evidence and yields a number of new predictions. In particular, we demonstrate that in the presence of informational asymmetries firms with valuable investment opportunities will speed up investment, leading to a significant erosion of the value of the option to wait. Second, we show that in a dynamic environment asymmetric information may not translate into a preference ranking over securities and that securities with the lowest information costs might not be issued first. Finally, we provide a rationale for the fact that small, high-growth firms do not behave according to the pecking order theory and relate the firm’s optimal investment and financing strategies to various firm and industry characteristics such as bankruptcy costs, operating leverage, or cash flow volatility.
Appendix

A Investment timing with symmetric information

In the region for the cash flow shock where there is no investment \( X < \overline{X}_k \), the value of the firm’s growth option satisfies:

\[
rV_k^- = \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 V_k^-}{\partial X^2} + \mu X \frac{\partial V_k^-}{\partial X}. \tag{A.1}
\]

The solution of (A.1) is

\[
V_k^-(X) = AX^\xi + BX^\nu, \tag{A.2}
\]

where \( \xi \) and \( \nu \) are the positive and negative roots of the equation \( \frac{1}{2} \sigma^2 y(y - 1) + \mu y - r = 0 \). This ODE is solved subject to the following three boundary conditions:

\[
V_k^- (X) \bigg|_{X=\overline{X}_k} = \Lambda_k \Pi(\overline{X}_k) - I, \tag{A.3}
\]

\[
\frac{\partial V_k^-}{\partial X} \bigg|_{X=\overline{X}_k} = \Lambda_k \frac{\partial \Pi(X)}{\partial X} \bigg|_{X=\overline{X}_k}, \tag{A.4}
\]

\[
\lim_{X \to 0} \frac{\partial V_k^-}{\partial X} (X) = 0. \tag{A.5}
\]

Condition (A.5) implies \( B = 0 \). Condition (A.3) implies \( A = [\Lambda_k \Pi(\overline{X}_k) - I](\overline{X}_k)^{-\xi} \). Simple manipulations of (A.3) and (A.4) yield the expression for \( \overline{X}_k \).

B Single-Crossing Property

The valuation of type \( k \) when signaling by investing at \( \overline{X} \) and when the perceived type is \( \Lambda \) equals

\[
V_k(X; \overline{X}, \Lambda) = \frac{\Lambda_k \Pi(X) - F}{1 + \triangle n(X; \Lambda)} \left( \frac{X}{\overline{X}} \right)^\xi = \frac{\Lambda_k \Pi(X) - F}{\Lambda \Pi(X) - F - I} \left( \frac{X}{\overline{X}} \right)^\xi. \tag{B.1}
\]

This implies that we have

\[
\frac{\partial}{\partial X} V_k(X; \overline{X}, \Lambda) = \left[ \frac{\Lambda_k \Pi(X)}{\Lambda_k \Pi(X) - F} - \frac{\Lambda \Pi(X)}{\Lambda \Pi(X) - F - I} + \frac{\Lambda \Pi(X)}{\Lambda \Pi(X) - F - I} - \xi \right] \frac{1}{X} V_k(X; \overline{X}, \Lambda),
\]

\[
\frac{\partial}{\partial X} V_k(X; \overline{X}, \Lambda) = \left[ \frac{\Pi(X)}{\Lambda \Pi(X) - F - I} - \frac{\Pi(\overline{X})}{\Lambda \Pi(\overline{X}) - F} \right] V_k(X; \overline{X}, \Lambda).
\]
Single-crossing can be checked as follows. Along any iso-value curve, \( \frac{\partial}{\partial X} V_k(X; X, \Lambda) + \frac{\partial}{\partial X} V_k(X; X, \Lambda) \frac{\partial A}{\partial X} = 0 \). The elasticity of substitution between perceived quality \( \Lambda \) and investment signal \( X \) equals

\[
\frac{\partial \Lambda}{\partial X} \Lambda = -\frac{\frac{\partial}{\partial X} V_k(X; X, \Lambda)}{\frac{\partial}{\partial X} V_k(X; X, \Lambda)} = \frac{\Lambda_g \Pi(X) - \xi}{\Lambda_b \Pi(X) - \xi} - 1. \tag{B.2}
\]

Expression (B.2) depends positively (and, hence, the elasticity between the competitively required ownership dilution \( \Delta n(X; \Lambda) \) and investment threshold \( X \) depends negatively) on the type \( k \) as long as \( f \geq 0 \). That is, the single-crossing property holds:

\[
\frac{\partial}{\partial \Lambda_k} \left( \frac{\partial}{\partial X} V_k(X; X, \Lambda) \right) > 0. \tag{B.3}
\]

### C Investment timing in the separating equilibrium

The bad type firm is indifferent between mimicking the good type at \( X^* \leq X_b \) and waiting to follow its first-best strategy under perfect information if the incentive compatibility constraint (8) holds. After simplifications, this equation can be written as:

\[
\left( \frac{X^*}{X_b} \right) \xi \left[ 1 - \left( 1 - \frac{\Lambda_g}{\Lambda_b} \right) \frac{\xi (F + I) X^*}{\xi (F + I) X^* - (\xi - 1) FX_b} \right] = (1 - \xi) + \xi \left( \frac{\Lambda_g}{\Lambda_b} \right) \left( \frac{X^*}{X_b} \right). \tag{C.1}
\]

The condition for \( X_{\text{min}} \) can be derived analogously. The threshold \( X_{\text{min}} \) for which the incentive compatibility constraint (9) is binding represents the lowest value of the cash flow shock such that the good type prefers separation over pooling with the bad type. The threshold \( X_{\text{min}} \) satisfies:

\[
\Lambda_g \Pi(X_{\text{min}}) - F - I = \frac{\Lambda_g \Pi(X_b) - F}{\Lambda_b \Pi(X_b)} \left[ \Lambda_b \Pi(X_{\text{min}}) - F - I \right] \left( \frac{X_{\text{min}}}{X_b} \right) \xi,
\]

or

\[
\left( \frac{X_{\text{min}}}{X_b} \right) \xi \left[ 1 - \left( 1 - \frac{\Lambda_g}{\Lambda_b} \right) \frac{\xi (F + I)}{\xi (F + I) - (\xi - 1) F} \right] = (1 - \xi) + \xi \left( \frac{\Lambda_g}{\Lambda_b} \right) \left( \frac{X_{\text{min}}}{X_b} \right). \tag{C.2}
\]

Investment is distorted in the separating equilibrium if at \( X_g \) the left-hand side in (8) is larger than the right-hand side, i.e.:

\[
\frac{\Lambda_g \Pi(X_g) - F}{\Lambda_g \Pi(X_g)} \left[ \Lambda_g \Pi(X_g) - F - I \right] > \left[ \Lambda_b \Pi(X_b) - F - I \right] \left( \frac{X_g}{X_b} \right) \xi.
\]
or, equivalently, condition (13). From the bad type’s incentive compatibility (8) we have

\[ \Lambda_g \Pi(X^*) - F - I = \left( \frac{\Lambda_g \Pi(X^*) - F}{\Lambda_b \Pi(X^*) - F} \right) \frac{F + I}{\xi - 1} \left( \frac{X}{X_b} \right)^\xi. \]

Hence, the market value of each firm before investment satisfies condition (10) for \( X < X^* \), and the intrinsic values of the high type and, respectively, low type firm before investment are given by:

\[
V_{ics,g}(X) = \begin{cases} 
[\Lambda_g \Pi(X^*) - F - I] \left( \frac{X}{X_g} \right)^\xi = \left( \frac{\Lambda_g \Pi(X^*) - F}{\Lambda_b \Pi(X^*) - F} \right) \frac{F + I}{\xi - 1} \left( \frac{X}{X_b} \right)^\xi, & \text{if } X^* < X_g, \\
[\Lambda_g \Pi(X_g) - F - I] \left( \frac{X}{X_g} \right)^\xi = \left( \frac{\Lambda_g}{\Lambda_b} \right) \frac{F + I}{\xi - 1} \left( \frac{X}{X_b} \right)^\xi, & \text{otherwise.}
\end{cases}
\]

and

\[
V_{ics,b}(X) = [\Lambda_b \Pi(X_b) - F - I] \left( \frac{X}{X_b} \right)^\xi = F + I \frac{X - X_b}{\xi - 1} = V_b^-(X).
\]

**D Investment timing in the pooling equilibrium**

Conditions (18) and (19) can be respectively rewritten as

\[
\frac{\Lambda_b \Pi(X_{pool}) - F}{\Lambda_{pool} \Pi(X_{pool}) - F} [\Lambda_{pool} \Pi(X_{pool}) - F - I] \geq \frac{F + I}{\xi - 1} \left( \frac{X_{pool}}{X_b} \right)^\xi,
\]

and

\[
\frac{\Lambda_g \Pi(X_{pool}) - F}{\Lambda_{pool} \Pi(X_{pool}) - F} [\Lambda_{pool} \Pi(X_{pool}) - F - I] \geq \begin{cases} 
\left( \frac{\Lambda_g \Pi(X^*) - F}{\Lambda_b \Pi(X^*) - F} \right) \frac{F + I}{\xi - 1} \left( \frac{X_{pool}}{X_b} \right)^\xi, & \text{if } X^* < X_g, \\
\left( \frac{\Lambda_g}{\Lambda_b} \right) \frac{F + I}{\xi - 1} \left( \frac{X_{pool}}{X_b} \right)^\xi, & \text{otherwise.}
\end{cases}
\]

Since

\[ \Lambda_b \Pi(X_{pool}) - F \geq \left( \frac{\Lambda_b \Pi(X^*) - F}{\Lambda_g \Pi(X^*) - F} \right) [\Lambda_g \Pi(X_{pool}) - F], \]

condition (18) holds whenever condition (19) is satisfied so long as \( X_{pool} \geq X^* \) (and \( X^* < X_g \)). The objective of management in firm \( k \) is to select the investment threshold \( X_{pool,k} \) that maximizes the present value of the cash flows accruing to the incumbent shareholders:

\[
\max_{X_{pool,k}} \left\{ \frac{\Lambda_k \Pi(X_{pool,k}) - F}{\Lambda_{pool} \Pi(X_{pool,k}) - F} [\Lambda_{pool} \Pi(X_{pool,k}) - F - I] \left( \frac{X}{X_{pool,k}} \right)^\xi \right\}.
\]
The solution to firm $k$’s problem is given by the smooth-pasting condition

$$\frac{\Lambda_k \Pi(X_{\text{pool},k})}{\Lambda_k \Pi(X_{\text{pool},k}) - F} + \frac{\Lambda_{\text{pool}} \Pi(X_{\text{pool},k})}{\Lambda_{\text{pool}} \Pi(X_{\text{pool},k}) - F - I} - \frac{\Lambda_{\text{pool}} \Pi(X_{\text{pool},k})}{\Lambda_{\text{pool}} \Pi(X_{\text{pool},k}) - F} = \xi. \quad (D.1)$$

The solution to firm $g$’s problem is given by

$$X_{\text{pool},g} = \frac{\chi_g}{\chi_g - 1} \frac{r - \mu}{\Lambda_{\text{pool}}} (F + I), \quad (D.2)$$

with $\chi_g > 0$ as the solution to

$$\chi_g + \frac{F + I}{I + \left[1 - \frac{\Lambda_{\text{pool}}}{\chi_g} \left(\frac{\chi_g - 1}{\chi_g}\right)\right] F} - \frac{F + I}{I \left(1 + \frac{F}{\chi_g}\right)} = \xi$$

or, equivalently

$$\frac{\chi_g - \xi}{\chi_g (\chi_g - 1)} \left(\chi_g + \frac{F}{I}\right)^2 + \left[\left(\frac{\chi_g - \xi}{\chi_g}\right) \left(\chi_g + \frac{F}{I}\right) - \left(1 + \frac{F}{I}\right)\right] \left(1 - \frac{\Lambda_{\text{pool}}}{\Lambda_k}\right) \frac{F}{I} = 0. \quad (D.3)$$

A pooling equilibrium, hence, exists if and only if $X_\star < X_g$ and condition (19) holds at $X_{\text{pool},g}$ or, equivalently, if the fraction of good projects in the economy, $p$, is large enough that the positive root $\chi_g$ of equation (D.3) satisfies

$$\frac{I + \left[1 - \frac{\Lambda_{\text{pool}}}{\chi_g} \left(\frac{\chi_g - 1}{\chi_g}\right)\right] F}{(I\chi_g + F) \left(\frac{\chi_g - 1}{\chi_g}\right)^{1-\xi}} \geq \frac{1}{\chi_g} \left(\frac{\chi_g - \xi}{\chi_g}\right)^{1-\xi} \left(\frac{\chi_g - \xi}{\chi_g}\right)$$

Finally, the market value in the pooling equilibrium is related to the intrinsic value of each type as follows:

$$V_{\text{pool}}^-(X) = p \frac{\Lambda_g \Pi(X_{\text{pool}}) - F}{\Lambda_{\text{pool}} \Pi(X_{\text{pool}}) - F} \left[\Lambda_{\text{pool}} \Pi(X_{\text{pool}}) - F - I\right] \left(\frac{X}{X_{\text{pool}}}\right)^\xi$$

$$+ (1 - p) \frac{\Lambda_k \Pi(X_{\text{pool}}) - F}{\Lambda_{\text{pool}} \Pi(X_{\text{pool}}) - F} \left[\Lambda_{\text{pool}} \Pi(X_{\text{pool}}) - F - I\right] \left(\frac{X}{X_{\text{pool}}}\right)^\xi$$

$$= \left[\Lambda_{\text{pool}} \Pi(X_{\text{pool}}) - F - I\right] \left(\frac{X}{X_{\text{pool}}}\right)^\xi. \quad (D.5)$$

### E Separation through debt issuance

We denote the values of equity and corporate debt after investment by $E_k^+(X, c)$ and $D_k^+(X, c)$ respectively. Assuming that the firm has issued debt with coupon payment $c$, the cash flow
accruing to shareholders after investment over each interval of time of length $dt$ is given by:

$$(\Lambda k X - f - c)dt.$$  

In addition to this cash flow, shareholders receive capital gains of $E_d$ over each time interval. The required rate of return for investing in the firm’s equity is $r$. Applying Itô’s lemma, it is then immediate to show that the value of equity after investment satisfies the following ODE in the region for the cash flow shock where there is no default $(X > X_k(c))$:

$$rE_k^+ = \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 E_k^+}{\partial X^2} + \mu X \frac{\partial E_k^+}{\partial X} + \Lambda k X - f - c. \quad (E.1)$$

The solution of (E.1) is

$$E_k^-(X, c) = AX^\xi + BX^\nu + \Lambda k \Pi(X) - F - \frac{c}{r} \quad (E.2)$$

where $\xi$ and $\nu$ are the positive and negative roots of the equation $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r = 0$, and $F = f/r$. This ordinary differential equation is solved subject to the following two boundary conditions:

$$\lim_{X \to \infty} \left[ \frac{E_k^+(X, c)}{X} \right] < \infty, \quad (E.3)$$

$$E_k^+(X, c) |_{X=X_k(c)} = 0. \quad (E.4)$$

The first condition is a standard no-bubble condition implying $A = 0$. The second condition states that equity is worthless in default implying $B = [\Lambda k \Pi(X(c)) - F - \frac{c}{r}]^{(X(c))^{-\nu}}$. In addition to these two conditions, the value of equity satisfies the smooth-pasting condition:

$$\frac{\partial E_k^+}{\partial X} |_{X=X_k(c)} = 0$$

at the endogenous default threshold (see e.g. Mello and Parsons, 1992, or Leland, 1994).

Solving this optimization problem yields the following expression for equity value:

$$E_k^+(X, c) = \Lambda k \Pi(X) - F - \frac{c}{r} - \left[ \Lambda k \Pi(X) - F - \frac{c}{r} \right] \left( \frac{X}{X_k} \right)^\nu, \quad (E.5)$$

where the selected default threshold $X_k$ is given by:

$$X_k(c) = \frac{\nu}{\nu - 1} \frac{r - \mu}{\Lambda k} \left( F + \frac{c}{r} \right). \quad (E.6)$$

Taking the trigger strategy $X_k(c)$ as given, the value of corporate debt satisfies in the region for the cash flow shock where there is no default $(X > X_k(c))$:

$$rD_k^+ = \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 D_k^+}{\partial X^2} + \mu X \frac{\partial D_k^+}{\partial X} + c. \quad (E.7)$$
This equation is solved subject to the standard no-bubbles condition \( \lim_{X \to \infty} D_k^+(X, c) = c/r \) and the value-matching condition: \( D_k^+(X, c)|_{X=X_k(c)} = (1 - \alpha) \Pi(X_k(c)) - F \). This condition states that when the firm defaults, the value of corporate debt is equal to the abandonment value of the firm net of default costs. Solving this valuation problem gives the value of corporate debt as:

\[
D_k^+(X, c) = \frac{c}{r} - \left( \frac{c}{r} - (1 - \alpha) \Lambda_k \Pi(X_k(c)) + F \right) \left( \frac{X}{X_k(c)} \right)^\nu \\
= \frac{c}{r} - \Lambda_k^\nu \left[ 1 - (1 - \alpha) \frac{\nu}{\nu - 1} \right] \left( \frac{c_{pool}}{r} + F \right)^{1-\nu} \left( \frac{\Pi(X)}{\nu} \right)^{\nu}. \tag{E.8}
\]

Expression (E.8) yields the following useful relation between the debt values: \( \Lambda_b^\nu [D_b(X, c) - \xi] = \Lambda_b^\nu [D_g(X, c) - \xi] \). The value of the firm after investment is now given by \( V_k^+(X, c) = D_k^+(X, c) + E_k^+(X, c) \), or

\[
V_k^+(X, c) = \Lambda_k \left[ \Pi(X) - \alpha \Pi(X_k(c)) \left( \frac{X}{X_k(c)} \right)^\nu \right] - F \\
= \Lambda_k \Pi(X) - F - \eta \left( \frac{c}{r} - D_k(X, c) \right), \tag{E.9}
\]

where the coefficient \( \eta \) is given by (25). The second line in (E.8) and (E.9), respectively, follow from (E.6).

### E.1. Investment timing with debt under symmetric information

Using the budget constraint \( D_k(X, c_k) = I \), one can rewrite (E.9) as \( V_k^+(X, c_k(X)) = \Lambda_k \Pi(X) - F - \eta I \rho_k(X) \). The value function of a type \( k \) firm at any time before investment at threshold \( \overline{X}_{k,D} \) (see also equation (27)) then equals

\[
V_{k,D}^-(X) = [\Lambda_k \Pi(\overline{X}_{k,D}) - F - I(1 + \eta \rho_k(\overline{X}_{k,D}))] \left( \frac{X}{\overline{X}_{k,D}} \right)^\xi. \tag{E.10}
\]

The smooth-pasting condition for the optimal investment threshold \( \overline{X}_{k,D} \) requires:

\[
\xi [\Lambda_k \Pi(\overline{X}_{k,D}) - F - I(1 + \eta \rho_k(\overline{X}_{k,D}))] = \Lambda_k \Pi(\overline{X}_{k,D}) - \eta I \frac{\partial \rho_k(\overline{X}_{k,D})}{\partial \overline{X}_{k,D}} \overline{X}_{k,D}. \tag{E.11}
\]

In equation (E.11), applying the Implicit Function theorem to (26) yields

\[
\frac{\partial \rho_k(\overline{X}_{k,D})}{\partial \overline{X}_{k,D}} = \left( \frac{\nu}{1 + (\nu - 1) \frac{\rho_k(\overline{X}_{k,D})}{\overline{X}_{k,D}}} \right) \frac{\rho_k(\overline{X}_{k,D})}{\overline{X}_{k,D}}.
\]
Solving for the investment threshold yields equation (28). The value function then equals

\[ V_{k,D}(X) = \left[ \Lambda_k \Pi(\overline{X}_{k,D}) - \eta I \frac{\partial \rho_k(\overline{X}_{k,D})}{\partial X_{k,D}} \overline{X}_{k,D} \right] \frac{1}{\xi} \left( \frac{X_{k,D}}{\overline{X}_{k,D}} \right)^\xi \left( \frac{\overline{X}_{k,D}}{X_{k,D}} \right)^\xi \]

\[ = \left[ 1 + \frac{(1 - \nu)\eta \rho_k(\overline{X}_{k,D})}{1 + F + \nu \rho_k(\overline{X}_{k,D})} \right] \left( \frac{X_{k,D}}{\overline{X}_{k,D}} \right)^\xi \frac{F + I}{\xi - 1} \left( \frac{X_{k,D}}{X_k} \right)^\xi. \]  
(E.12)

### E.2. Incentive compatibility

The critical threshold \( \overline{X}_D^* \) at which the inventive compatibility constraint (30) of the bad type binds is given by the solution to:

\[ V_b^+ (\overline{X}_D^*, c_g(\overline{X}_D^*)) - D_b(\overline{X}_D^*, c_g(\overline{X}_D^*)) = V_b^- (\overline{X}_D^*). \]

Using the expressions for \( V_b^+ (X, c_g), D_b(X, c_g), \) and \( V_b^- (\overline{X}_D^*) \) given in the main text, we can rewrite this equation as

\[ \left( \frac{\overline{X}_D^*}{X_b} \right)^\xi - \xi \left( \frac{\overline{X}_D^*}{X_b} \right) = (1 - \xi) \left[ 1 + \rho_g(\overline{X}_D^*) I \frac{1 - (1 - \eta)(\Lambda_b^{\xi})^\nu}{1 + F} \right]. \]

The IC condition (31) holds with equality at some threshold \( \overline{X}_{\min,D} < \overline{X}_{g,D}. \) For all \( X < \overline{X}_{\min,D} \) separation is not a best-response for the good type since the investment distortions required to separate from the bad type are too large compared to the underpricing in a pooling equilibrium. A separating equilibrium in debt exists only if \( \overline{X}_{\min,D} \leq \overline{X}_D^* \). The critical threshold \( \overline{X}_{\min,D} \) at which the inventive compatibility constraint (31) of the good type binds is given by the solution to:

\[ V_g^+ (\overline{X}_{\min,D}, c_g(\overline{X}_{\min,D})) - D_g (\overline{X}_{\min,D}, c_g(\overline{X}_{\min,D})) = \frac{\Lambda_g \Pi(\overline{X}_b) - F}{1 + \triangle_n(\overline{X}_b; \Lambda_b) \left( \frac{\overline{X}_{\min,D}}{X_b} \right)^\xi}, \]

which reduces to

\[ \left( \frac{\overline{X}_{\min,D}}{X_b} \right)^\xi \left[ 1 - \left( 1 - \frac{\Lambda_b}{\Lambda_g} \right) \frac{(1 - \xi)F}{\xi I + F} \right] - \xi \left( \frac{\overline{X}_{\min,D}}{X_b} \right)^\xi \left[ 1 + \rho_g(\overline{X}_{\min,D}) \frac{\eta I}{I + F} \right]. \]
E.3. When is debt financing the least-cost separating equilibrium?

The valuations of the good type when separating in debt, separating in equity or pooling in equity are:

**Separation in Debt:**
\[
\frac{F + (1 + \nu \frac{1-\alpha}{1-\nu} \rho_k) I}{F + (1 + \nu \rho_k) I} \left( \frac{X_g}{X_{g,D}} \right) \xi V_g^{-} (X) = \frac{F + (1 + \nu \frac{1-\alpha}{1-\nu} \rho_k) I}{F + (1 + \nu \rho_k) I} \left( \frac{X_b}{X_{g,D}} \right) \xi V_b^{-} (X).
\]

**Separation in Equity:**
\[
[\Lambda_g \Pi(X^*) - F - I] \left( \frac{X}{X^*} \right) \xi = \left( \frac{\Lambda_g \Pi(X^*) - F}{\Lambda_b \Pi(X^*) - F} \right) V_b^{-} (X) \text{ if } X^* < X_g.
\]
\[
[\Lambda_g \Pi(X_g) - F - I] \left( \frac{X}{X_g} \right) \xi = \left( \frac{\Lambda_g}{\Lambda_b} \right) \xi V_b^{-} (X) \text{ if } X^* \geq X_g.
\]

**Pooling in equity**
\[
V_{pool,g}^{-} (X)
\]
References


Figure 1: Equity Separating Equilibrium.

Panel A plots the investment threshold under equity financing and separation for different parameter values. Depicted are the investment threshold in the equity separating equilibrium (bold dotted line), the first-best investment threshold (solid blue line), and the 0-NPV threshold (dashed green line) for the high-type firm. Panel B plots the hurdle rate for investment under equity financing and separation for different parameter values. Depicted are the hurdle rate in the equity separating equilibrium (bold dotted line), the first-best hurdle rate (solid blue line), and the 0-NPV rate of return (dashed green line) for the high-type firm. Panel C plots the external financing costs under equity financing and separation for different parameter values. The value loss is measured by the drop in firm value due to investment distortions in per cent of first-best (option) value. Panel D plots the loss probability under equity financing and separation for different parameter values. Depicted are the loss probabilities in the equity separating equilibrium (bold dotted line), under the first-best investment policy (solid blue line), and the 0-NPV investment policy (dashed green line) for the high-type firm. The loss probability is measured by the likelihood that the asset value falls below the investment cost at some time over the next 5 years and is given by expression (15). Panel E plots the abnormal announcement returns under equity financing and separation for different parameter values. Depicted is the rise in the stock price for high-type firms (solid blue line) and the drop for low-type firms (dashed red line) at the separating investment threshold. The base parametrization is $p = .5$, $\Lambda_g = 1.25$, $\Lambda_b = 1$, $r = .06$, $\mu = .01$, $\sigma = .2$, $I = 100$, $F = 5/r$.

Panel A: Investment threshold.

Panel B: Hurdle rate for investment.
Figure 1: EQUITY SEPARATING EQUILIBRIUM—CONTINUED.

Panel C: External financing costs \( (V_g^- - V_{lcs,g}^-) / V_g^- \).

Panel D: Loss probability.

Panel E: Announcement returns \( (V_k^+ - V_{lcs}^-) / V_{lcs}^- \).
Figure 2: LEAST-COST EQUITY EQUILIBRIUM.

The different panels plot the least-cost equity equilibrium, investment threshold, and external financing costs under equity financing for different parameter values. Panel A depicts the least-cost equilibrium as a function of the parameters $\Lambda_g$, $\Lambda_b$, $\mu$, $\sigma$, $F$, and $r$ on the $x$-axis and of investors’ beliefs about the fraction of high-type firms $p$ on the $y$-axis. Panel B depicts the investment threshold in the least-cost equity equilibrium (bold dotted line), the first-best investment threshold (solid blue line), and the 0-NPV threshold (dashed green line). Panel C depicts the drop in firm value due to investment distortions in the least-cost equity equilibrium for different parameter values. The value loss is evaluated at the 0-NPV threshold, measured in per cent of first-best (option) value, and given by Cost (%) = $(V_g^- - \max(V_{pool,g}, V_{lcs,g}^-))/V_g^-$. Panel D depicts the underpricing of the shares issued in the pareto-dominant pooling equilibrium for different parameter values. The underpricing is measured in per cent and given by Underpricing (%) = $(V_g^+ - V_{pool})/V_{pool}$ at the issuance date. The base parametrization is $p = .5$, $\Lambda_g = 1$, $\Lambda_b = .8$, $r = .06$, $\mu = .01$, $\sigma = .2$, $I = 100$, $F = 5/r$.

Panel A: Least-cost equity equilibrium.
Figure 2: Least-Cost Equity Equilibrium—continued.

Panel B: Investment threshold.

Panel C: External financing costs.
Figure 2: Least-Cost Equity Equilibrium—continued.

Panel D: Underpricing.
Figure 3: Least-Cost Equilibrium.

The different panels plot the least-cost equilibrium, investment threshold, external financing costs, and abnormal announcement returns in the least-cost equilibrium for different parameter values. Panel A depicts the least-cost equilibrium as a function of the parameters $\Lambda_g$, $\Lambda_b$, $\mu$, $\sigma$, $F$, and $\alpha$ on the x-axis and of investors' beliefs about the fraction of high-type firms $p$ on the y-axis. Panel B depicts the investment threshold in the least-cost equilibrium (bold dotted line), the first-best investment threshold (solid blue line), and the 0-NPV threshold (dashed green line). Panel C depicts the drop in firm value due to investment distortions in the least-cost equilibrium for different parameter values. The value loss is evaluated at the 0-NPV threshold, measured in per cent of first-best (option) value, and given by $\text{Cost} (\%) = (V_g^- - \max (V_{pool,g}^-, V_{lcs,g}^-, V_{g,D}^-)) / V_g^-$. Panel D plots the announcement returns in the least-cost equilibrium for different parameter values. Depicted is the rise in the stock price for high-type firms (solid blue line) and the drop for low-type firms (dashed red line) at the (equity or debt) separating investment threshold. Abnormal announcement returns are given by $\text{AR}_k (\%) = (V_k^+(X) - D_k(X)) / \max (V_{pool,k}^+(X), V_{lcs,k}^+(X)) - 1$ for $k = g,b$ at $X = \overline{X} \lor \overline{X}_g$ if equity separation is least-cost and at $X = \overline{X}_g \lor \overline{X}_D$ if debt separation is least-cost. The base parametrization is $p = .5$, $\Lambda_g = 1$, $\Lambda_b = .8$, $r = .06$, $\mu = .01$, $\sigma = .2$, $I = 100$, $F = 5/r$.

Panel A: Least-cost equilibrium.
Figure 3: Least-Cost Equilibrium—continued.

Panel B: Investment threshold.

Panel C: External financing costs.
Figure 3: Least-Cost Equilibrium—continued.

Panel D: Announcement returns.