Credit Supply and Corporate Policies

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Abstract

This paper develops a model of corporate investment and financing decisions that differs from previous contributions by recognizing that firms face uncertainty regarding their future access to credit markets and may have to search for creditors when raising debt financing. We show that accounting for credit supply uncertainty is critical in understanding corporate policy choices and use the model to explain a key set of stylized facts in corporate finance. Notably, our model provides an explanation for the conservative debt policy puzzle. The model also explains why firms may appear to time the market when issuing common stock. Finally, the model explains why negative shocks to the supply of credit may hamper investment even if firms have enough financial slack to fund all profitable investment opportunities internally.

Keywords: Corporate investment; capital structure; search; credit supply; credit lines.

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1 Introduction

Since the famous irrelevance theorem of Modigliani and Miller (1958), financial economists have devoted much effort to understanding the effects of frictions, such as corporate taxes or bankruptcy costs, on corporate investment and financing decisions. Although we have learned much from this work, virtually all of the models implicitly assume that firms are always able to secure funding for positive net present value projects and that a firm’s capital structure is entirely determined by a firm’s demand for debt or equity. That is, the supply of capital is frictionless in these models so that corporate behavior and capital availability depend solely on firm characteristics.

This demand-driven approach to corporate finance has recently been called into question by a number of large-sample empirical studies. These studies show that firms often face uncertainty regarding their future access to credit markets. They also find that credit supply conditions are very important in determining firms’ capital structure decisions, the structure of debt issues, and the level of corporate investment. Using a different research approach, several surveys of corporate managers of public and private firms from around the globe have confirmed the inferences of the large-sample studies. These surveys indicate that financing decisions are generally governed by the preferences of the suppliers of capital rather than by the demands of the users of capital (see Graham and Harvey, 2001). They also reveal that the inability to borrow externally causes firms to bypass attractive investment opportunities or to postpone investment (see Campello, Graham, and Harvey, 2010).

The purpose of this paper is therefore twofold. First, we seek to understand whether and when credit supply uncertainty affects real investment. Second, we are interested in determining the effects of supply conditions in credit markets on corporate financing, i.e. on firms’ debt-equity mix, the cost of external finance, and the level of credit lines. To this end, we build a dynamic model of investment and financing decisions that differs from previous contributions by recognizing that firms sometimes face difficulty in raising capital.

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for positive net present value (NPV) projects. We then use this model to understand whether the supply of capital corresponds to a separate channel through which market imperfections affect corporate behavior.

We consider as in Morellec and Schürhoff (2010a) a firm with assets in place and a growth option to expand operations. The firm is initially financed with common equity and risky debt and has the possibility to exercise its growth option at any time. A rich feature of the model is that the firm can finance the capital expenditure using any combination of common stock, debt, and credit lines. The model follows previous contributions by assuming shareholders have deep pockets and can finance the capital outlay when optimal for the firm to use equity financing (see e.g. Mello and Parsons, 1992, Manso, 2008, or Gomes and Schmid, 2010). It departs from prior work by considering that firms face uncertainty regarding their future access to credit markets and may have to search for creditors when raising debt financing. Based on these assumptions, the model yields an explicit characterization of the value-maximizing investment and financing policies for a firm acting in the best interests of incumbent shareholders and shows that credit market frictions have first-order effects on corporate policy choices.

In order to aid in the intuition of the model, consider the specific case of financing decisions, in which capital markets supply frictions are likely to be especially important. If there are no supply frictions in credit markets, then firms can borrow as much as they want and any firm’s observed debt level will correspond to its demanded level. This is the traditional assumption of the theoretical literature. In such environments, only demand factors explain variation in the firm’s debt level, where demand factors are any firm characteristic – such as the corporate tax rate, bankruptcy costs, or cash flow growth and volatility – that raises or reduces the net benefit of debt. However, if the supply of capital is uncertain or limited in credit markets, then the firm’s debt level will depend on both demand and supply factors and the observed debt level will most often differ from the demanded debt level.

More generally, this paper shows that while supply effects can be dismissed in the benchmark MM setting, they can become of paramount importance in the presence of capital market frictions. In particular, one of the main contributions of the paper is to show that credit supply uncertainty has important consequences not only for current capital structure but
also for corporate investment and for the dynamic evolution of financial contracts. Consider first investment policy. In the model, the firm has perpetual rights to a project and seeks to choose the investment date that maximizes the value of the project. Because the surplus from investment is uncertain and investment is irreversible, the value-maximizing policy for shareholders is to invest only when the project’s NPV exceeds an endogenously determined threshold. We show in the paper that credit market access has two opposite effects on the selected investment threshold and, hence, on the timing of investment. First, credit market access loosens the restrictions on current investment. Second, it makes waiting less risky and thus increases the opportunity cost of current investment. The first effect encourages investment but the second effect does the opposite. We show that generally the second effect dominates, so that negative shocks to the supply of credit may hamper investment even if firms have enough financial slack to fund investment opportunities internally.

Another interesting and novel result of the paper that relates to the feedback from future to current financing decisions is the critical role played by lines of credit in hedging credit supply uncertainty. We show that credit lines come with benefits and costs. On the one hand, firms may draw on their credit lines to finance capital expenditures, thereby eliminating the risk of not being able to secure funding for profitable projects. On the other hand, credit lines impose commitment fees and lead to investments distortions because of the predetermined borrowing rate. The analysis in the paper allows us to determine in which economic environments this financial contracting solution will maximize shareholder wealth. It also shows that when credit markets are frictionless, firms are less aggressive with their initial financing policy – initial leverage choice and level of credit lines.

The theory put forth in this paper also sheds new light on many important issues in corporate finance. For example, Graham (1999) and others have argued that most firms appeared to be underlevered and, as a result, were leaving money on the table by choosing not to increase their leverage. An alternative explanation suggested in this paper for the conservative debt policy puzzle is that firms may sometimes find it difficult to raise

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2 While they have received little attention in the theoretical literature, credit lines are an important component of firm financing. Between 1994 and 2008, credit lines accounted for 60% (by dollar volume) of all USD denominated corporate debt (Data source: Loan Pricing Corporation).
additional debt because of frictions in credit markets. The model also explains why firms may appear to time the market when issuing equity. In traditional capital structure models, the supply of different sources of capital is infinitely elastic at the correct price. As a result, the costs of different forms of capital do not vary independently. Our model shows that with credit market frictions there can be gain of switching from debt financing to equity financing so that market timing may benefit incumbent shareholders.

The present paper relates to several contributions in the literature. Mello and Parsons (1992) are the first to examine the interactions of investment and financing decisions in a dynamic setting. They show that capital structure can have a significant impact on firms’ operating decisions. Sundaresan and Wang (2007) and Tserukevich (2008) propose dynamic models in which firms can issue debt to exercise a sequence of growth options. Chen, Miao, and Wang (2010) derive the utility-maximizing investment and financing policies for an entrepreneurial firm that is run by a risk-averse manager. Gomes and Schmid (2010) examine the relation between financial leverage and stock returns in a dynamic model in which both investment and financing decisions are endogenous. Hackbarth and Mauer (2010) study the relation between the priority structure of corporate debt and firms’ investment and financing decisions. Morellec and Schürhoff (2010a, 2010b) examine respectively the effects of personal taxation and asymmetric information on corporate investment and financing. None of these models examine the effects of credit supply uncertainty on corporate behavior. That is, although the MM irrelevance is assumed not to hold on the demand side of the market in these models, it is assumed to hold on the supply side.3

Before proceeding, it should be noted that a central message arising from the analysis is that accounting for both demand and supply factors is critical to understanding firms’ policy choices. In that respect, our paper is also related to the study of Lemmon, Roberts and Zender (2008), who find that traditional determinants of leverage account for little of the variation in capital structure. Our analysis helps explain this finding by illustrating the role of capital market frictions in corporate policy choices. Since real life decisions by corporations will reflect both demand- and supply-side frictions, the paper also suggests

3Other recent contributions of this literature include Miao and Wang (2007), Manso (2008), He (2009), He and Xiong (2009), Gorbenko and Strebulaev (2009), and Manso, Strulovici, and Tchistyi (2009).
that empirical studies that fail to control for the role of both type of frictions in corporate behavior are unlikely to be very informative. Similarly, our analysis raises doubts about the usefulness of models of corporate decisions that focus exclusively on demand factors in several real-world applications. This is particularly true when the supply of capital becomes limited, as was the case during the global financial crisis of 2008.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the value-maximizing financing and investment policies in the presence of credit supply uncertainty. Sections 4 and 5 extend the basic model to analyze the effects of credit lines and of time-varying credit supply on corporate policy choices. Section 6 concludes.

2 Model and assumptions

2.1 Assumptions

Throughout the paper, agents are risk neutral and discount cash flows at a constant interest rate $r$. Corporate taxes are paid at a constant rate $\tau$ on operating cash flows and full offsets of corporate losses are allowed.

**Assets in place and growth option.** We consider an infinitely-lived firm with assets in place and a growth option. Assets in place generate a continuous flow of operating income $f(X_t) = (1 - \tau) X_t$, where $(X_t)_{t \geq 0}$ is a cash flow shock governed by the process:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 = x > 0.$$  

(1)

In this equation, $\mu < r$ and $\sigma > 0$ are constant parameters and $W = (W_t)_{t \geq 0}$ is a standard Brownian motion. This assumption implies that the growth rate of cash flows is Normally distributed with mean $\mu \Delta t$ and variance $\sigma^2 \Delta t$ over the time interval $\Delta t$. At any point in time, the firm can exercise its growth option to increase capacity and operating income from $X$ to $\pi X$, where $\pi \geq 1$ is a constant factor that determines the growth potential of the firm. The cost of investment is constant and denoted by $I$. Investment is irreversible.

**Financing decisions.** The firm is initially financed with common equity (the set-up can be extended to incorporate a mix of debt and equity; see section 4 below). To fund
the investment project the firm can issue a mixture of debt and equity at the investment
date. To stay in a simple time-homogeneous setting, we consider debt contracts that are
characterized by a perpetual flow of coupon payments $c$ and a principal $P$ that shareholders
have to repay in default (as in Leland, 1994, Duffie and Lando, 2001, or Morellec, 2004).
After investment, the equity owners’ only choice is when to liquidate the firm. At the chosen
liquidation time, a fraction $\alpha \in (0, 1]$ of the firm’s capital stock is lost as a frictional cost,
leading to a reduction in operating cash flows. That is, we consider that if the instant of
default is $T$, then $X_T = (1-\alpha)X_{T-}$. In default, absolute priority is enforced and debtholders
assume ownership of the firm’s assets. We also consider that the proceeds from the debt
issue are paid out as a cash distribution to shareholders at the time of flotation.

**Capital supply.** We are interested in building a model in which corporate behavior de-
pends not only on firm characteristics but also on the supply of capital in credit markets.
Indeed, as documented by a series of recent papers, credit supply conditions are very im-
portant in determining firms’ capital structure decisions, the structure of debt issues, as
well as the level of corporate investment. For example, Gan (2007) shows that following a
shock to their financial health caused by a decline in real estate markets, banks cut back
lending leading to a significant erosion in corporate investment. Becker (2007) finds that
local deposit supply affects local loan supply which in turn determines local investment.
Three studies by Faulkender and Petersen (2006), Leary (2009), and Sufi (2009) provide
evidence suggesting an important role for the supply of credit in determining leverage ratios.
Lemmon and Roberts (2007) find that negative shocks to the supply of credit lead to large
declines in debt issuance and investment. Kashyap, Stein and Wilcox (1993) and Kashyap,
Lamont, and Stein (1994) show in related work that monetary policy affects the supply of
bank loans and corporate investment.

In more recent studies, Ivashina and Scharfstein (2008), Duchin, Ozbas, and Sensoy
(2009), and Almeida, Campello, Laranjeira, and Weisbenner (2010) confirm these findings
by showing that the financial crisis of 2007-2008 led to a significant contraction in credit
supply and that corporate investment declined significantly following the onset of the crisis
(controlling for firm fixed effects and time-varying measures of investment opportunities).
Massa, Yasuda, and Zhang (2009) show that the capital supply uncertainty of institutional
bond investors significantly affect firms’ financing decisions while Massa and Zhang (2009) find that the relative availability of bond and bank financing affect the firm’s ability to borrow and to use its leverage to buffer shocks. Choi, Gemantsky, Henderson, and Tookes (2010) show that the issuance of convertible bonds is positively related to a number of capital supply measures. Finally, in a recent survey by Campello, Graham, and Harvey (2009), CFOs indicate that their ability to invest in positive NPV projects is tied to the supply of capital in credit markets.

These studies suggest that credit supply is a key determinant of firms’ policy choices and that firms often face uncertainty regarding their access to credit markets. To capture this important feature of capital markets, we consider in the model that it takes time to secure debt financing and that credit supply is uncertain. In particular, if the firm decides to finance the capital expenditure by issuing debt (so as to take advantage of the debt tax shield), then it has to search for debt investors. In the analysis below, we assume that creditors appear to the firm with Poisson arrival rate $\delta_t$. That is, the mean arrival rate of debt investors given all information available at time $t$ is $\delta_t$ and the probability of getting financing over each time interval $[t, t + dt]$ is $\delta_t dt$.\(^4\) In addition, to highlight the effects of credit supply uncertainty on corporate behavior, we follow previous contributions in the literature on corporate investment and financing decisions by assuming that shareholders have deep pockets and can finance the capital outlay when optimal for the firm to use equity financing. Therefore, our model does not have a role for retained earnings.\(^5\)

Our assumptions imply that in the model each firm has a debt level that is a function of the supply of debt, governed by $\delta_t$, and of its demand for debt, governed by traditional

\(^4\)A growing body of literature argues that assets prices may be more sensitive to supply shocks than standard asset pricing theory would predict. Search theory has played a key role in the formulation of models capturing this idea (see e.g. Duffie, Garleanu, and Perdersen, 2005, Duffie, Manso, and Malamud, 2009, or Vayanos and Weill, 2008). Duffie (2009) provides an early survey of this literature.

\(^5\)See for example Sundaresan and Wang, 2007, Manso, 2008, Gomes and Schmid, 2010, or Morellec and Schürhoff, 2010a. In a more general model, the supply of equity capital could also be uncertain. This would lead the firm to save part of its cash flows in order to facilitate investment and reduce default risk. Intuitively, the effects on financing and investment decisions would be similar to those obtained in the current setup, with the firm switching from debt financing to (internal) equity financing at an endogenously determined threshold. We leave this extension for future research. One interesting feature of the present model is that firms time their equity issues even though the supply of equity capital is infinitely elastic at the correct price.
factors such as the corporate tax rate $\tau$, bankruptcy costs $\alpha$, or cash flow volatility $\sigma$. The paper does not attach any particular interpretation to the uncertainty in the supply of credit. It may be related to shocks to banks health (as in Gan, 2007), to regulatory changes (as in Lemmon and Roberts, 2007, or Leary, 2009), to the limited ability of financial intermediaries to verify the viability of projects (as in Faulkender and Petersen, 2006), to variations in monetary policy (as in Kashyap, Stein and Wilcox, 1993), or to geographical segmentation (as in Becker, 2007). In the following, we start by analyzing a model in which the arrival rate of creditors is constant, given by $\delta > 0$ (so that the expected financing lag is $1/\delta$). In section 5, we consider an environment with time-varying credit supply.

Management’s objective. Throughout the paper, management acts in the best interest of shareholders and seeks to maximize shareholder wealth when making policy choices. For doing so, management makes three types of decisions: (i) select the firm’s investment policy; (ii) issue debt at the investment date in order to shield the firm’s operating profits from taxation when optimal to do so; and (iii) select the firm’s default policy. Because the decision to invest is irreversible, the firm’s initial asset structure remains fixed until the cash flows rise to a sufficiently high level and the manager invests. Similarly, the firm cash flows need to reach a low level for the firm to default on its debt obligations after investment. We can thus see the manager’s policy choices as determining the coupon payment $c$ at the time of investment, the threshold at which it is optimal for the firm to invest, and the default boundary after investment. In section 4, we additionally allow management to select the firm’s initial capital structure (and default policy) as well as the size of its credit lines.

2.2 Firm value after investment

We denote the values of equity, corporate debt and the firm when the firm has issued debt with coupon payment $c$ by $e_i(x,c)$, $d_i(x,c)$, and $v_i(x,c)$, with $i = 1$ before investment and $i = 2$ after investment. In our setup, the value of the firm before investment equals the sum of the present value of the cash flows accruing to shareholders until the time of investment and the change in this present value that arises at the time of investment. Since the latter depends on the total value of the firm after investment, we start by deriving this value.

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Footnote 6: In general, $\delta$ may reflect both the firm’s credit market access and the supply of credit in capital markets.
Assume first that the capital expenditure is financed with equity at the investment date. In this case, the value of the firm after investment is equal to the value of equity. Since assets in place generate a continuous stream of cash flows \((1 - \tau)\pi X_t\) at each time \(t\) after the exercise of the growth option, the value of the firm after investment is simply given by

\[
v_2(x,0) = E \left[ \int_0^\infty e^{-rt} (1 - \tau) \pi X_t \, dt \middle| X_0 = x \right] = \pi \Lambda x \text{ where } \Lambda = \frac{1 - \tau}{r - \mu}, \quad (2)
\]

This expression is similar to the Gordon growth formula with the constant growth rate \(\mu\) for operating cash flows.

Assume next that the capital expenditure is financed with debt and equity. In that case, the value of the firm after the exercise of the growth option is given by the sum of the cash flows accruing to claimholders until the default time, i.e. the after-tax operating cash flow \((1 - \tau)\pi X_t\) plus the tax savings \(\tau C\), and the present value of the cash flows accruing after default, i.e. \((1 - \tau)(1 - \alpha)\pi X_t\). Denoting the default threshold selected by shareholders by \(x_D\) and the first time to reach this threshold by \(T(x_D)\), we can then write firm value after investment for \(x > x_D\) as:

\[
v_2(x,c) = E \left[ \int_0^{T(x_D)} e^{-rt} [(1 - \tau) \pi X_t + \tau C] \, dt + \int_{T(x_D)}^\infty e^{-rt} (1 - \tau) (1 - \alpha)\pi X_t \, dt \middle| X_0 = x \right]. \quad (3)
\]

The first term on the right hand side of this equation represents the present value of the cash flows accruing to claimholders before default. The second term accounts for the firm cash flows after default. Solving this equation yields (see Appendix A):

\[
v_2(x,c) = \pi \Lambda x + \tau C \left[ 1 - \left( \frac{x}{x_D} \right) \nu \right] - \alpha \pi \Lambda x_D \left( \frac{x}{x_D} \right) \nu, \quad (4)
\]

where \(\nu < 0\) is the negative root of the quadratic equation \(\frac{1}{2} \sigma^2 y(y-1) + \mu y - r = 0\). This equation shows that the value of the levered firm, \(v_2(x,c)\), is equal to the value of the unlevered firm (first term on the right hand side) plus the present value of the tax savings (second term on the right hand side) minus expected bankruptcy costs (third term on the right hand side). In this equation, the default threshold that maximizes equity value satisfies:

\[
x_D^* = \frac{\nu}{\nu - 1} \frac{c \tau - \mu}{r}, \quad (5)
\]
and the value-maximizing coupon payment is given by:

$$c^* = \pi x \frac{r}{\nu} \left(1 - \nu - \alpha \frac{\nu(1 - \tau)}{r\tau}\right)^{1/\nu}. \quad (6)$$

Plugging these expressions in equation (4), we finally get the value of the levered firm at optimal leverage as

$$v_2(x, c^*) = \Phi x$$

where $$\Phi = (1 + \Gamma) \pi \Lambda$$ with $$\Gamma > 0$$ defined in Appendix A. \quad (7)

In equation (7), the constant $$\Phi$$ is the price per dollar of the cash flow shock in a levered firm. The constant factor $$\Gamma$$ reflects both the tax benefits of debt and expected bankruptcy costs. At optimal leverage, debt financing increases firm value and we have $$\Gamma > 0$$.

3 Investment, financing, and uncertain credit supply

In this section we solve for the value-maximizing investment and financing policies in the presence of credit supply uncertainty. We first consider a simple environment in which the arrival rate of investors is constant, given by $$\delta > 0$$, and the firm has no outstanding debt. In this context, the firm has assets in place that generates cash flows $$(1 - \tau) X_t$$ at each time $$t$$ and the value of equity before the exercise of the growth option is equal to firm value. In the following sections, we will examine the effects of initial debt, credit lines, and time-varying credit supply on firms’ investment and financing decisions.

3.1 Optimal investment with debt financing

Suppose first that the capital expenditure is financed by issuing risky debt at the time of investment and denote the value of equity before investment by $$e_{1,D}(x, 0) \equiv v_{1,D}(x)$$. The total surplus generated by investment is then given by: $$v_2(x, c^*) - I - v_{1,D}(x) = \Phi x - I - v_{1,D}(x)$$. This total surplus is shared between incumbent shareholders and debtholders through the cost of debt financing $$p(x)$$.\footnote{Since the supply of capital is limited in credit markets, the pricing of corporate debt need not be competitive so that debtholders may be able to capture part of the investment surplus.} That is, once management and debt investors meet, they must bargain to determine the cost of outside funds or, equivalently, the proceeds from the debt issue. The price $$p(x)$$ determines the allocation of the surplus created by investment.
between incumbent shareholders and debt investors for any value \( x \) of the cash flow shock. Specifically, shareholders get \( \Phi x - p(x) - v_{1,D}(x) \) while debtholders get \( p(x) - I \).

We consider below the Nash cooperative bargaining solution whereby shareholders receive a fraction \( \theta = \delta/(\rho + \delta) \) of the surplus generated by investment, where \( \rho \geq 0 \) is a positive constant. When \( \rho = 0 \), we have \( \theta = 1 \) and shareholders capture all the investment surplus. This is the traditional assumption of the literature and this will constitute our base case scenario. When \( \rho > 0 \), the fraction of the surplus captured by shareholders increases with the arrival rate of creditors \( \delta \). The Nash bargaining solution determines uniquely the cost of outside financing to shareholders as \( p(x) = \theta I + (1 - \theta) [\Phi x - v_{1,D}(x)] \), implying that the surplus from investment to shareholders satisfies

\[
\Phi x - p(x) - v_{1,D}(x) = \theta [\Phi x - I - v_{1,D}(x)], \quad \text{for all } x. \tag{8}
\]

Because the surplus from investment is uncertain and investment is irreversible, operating cash flows must raise to a sufficiently high level for shareholders to invest and expand the size of operations. In the analysis below, we denote by \( x_{\delta,D}^* \) the level of the cash flow shock that warrants investment. For all values of the cash flow shock above (respectively below) the investment threshold \( x_{\delta,D}^* \), it is optimal for the firm to (respectively not to) invest in the project. In contrast to standard contingent claims models however, investment may no occur at \( x_{\delta,D}^* \) since the firm needs to find debt investors.

Using this notation, we can now derive equity value before investment as follows. Before investment, the firm delivers a cash flow stream \((1 - \tau)x\). In addition to this cash flow stream, investors also get capital gains \( \mathbb{E}[dv_{1,D}] \) over each interval \( dt \). Using Itô’s lemma, it is then immediate to show that firm value solves the following system of ordinary differential equations before investment (see Appendix B.1):

\[
rv_{1,D}(x) = \mu xv_{1,D}'(x) + \frac{\sigma^2}{2}x^2 v_{1,D}''(x) + (1 - \tau)x, \quad \text{for } x < x_{\delta,D}^*,
\]

\[
rv_{1,D}(x) = \mu xv_{1,D}'(x) + \frac{\sigma^2}{2}x^2 v_{1,D}''(x) + (1 - \tau)x + \delta \theta [\Phi x - I - v_{1,D}(x)], \quad \text{for } x \geq x_{\delta,D}^*. \tag{9}
\]

Within the current framework, investors are risk neutral and the risk-free rate is given by \( r \). Thus, the left hand side of these equations represents the required rate of return for
investing in the firm’s equity per unit of time. The right hand side of these equations is the sum of the cash flow generated by the firm’s assets and the expected change in the asset value (i.e. the realized rate of return). These expressions are similar to those derived in standard contingent claims models (see Leland, 1994, or Morellec and Schürhoff, 2010a). However, the second one contains the additional term \( \delta \theta \left[ \Phi - I - v_{1,D}(x) \right] \) that reflects the effects of credit supply uncertainty on firm value in the region where it is optimal to invest. This term is the product of the arrival of an investor \( \delta \) (i.e. the probability of obtaining the required funds) and the surplus that shareholders can extract from investment.

This system of equations is solved subject to the following boundary conditions. First, since \( 0 \) is an absorbing barrier for the cash flow shock, it must be that \( v_{1,D}(0) = 0 \). In that case, assets in place do not produce any cash flows and the option to expand is worthless. In addition, since the firm cash flows grow linearly with the cash flow shock, there exists some constant \( \psi > 0 \) such that \( v_{1,D}(x) \leq \psi x \), for all \( x \geq x^*_\delta \). Also, since cash flow to claimholders are given by a (piecewise) continuous Borel-bounded function, the value function \( v_1(\cdot) \) is piecewise \( C^2 \) (see Theorem 4.9 pp. 271 in Karatzas and Shreve, 1991). Therefore, equity value satisfies the continuity and smoothness conditions: \( \lim_{x \uparrow x^*_\delta} v_{1,D}(x) = \lim_{x \downarrow x^*_\delta} v_{1,D}(x) \), and \( \lim_{x \uparrow x^*_\delta} v'_{1,D}(x) = \lim_{x \downarrow x^*_\delta} v'_{1,D}(x) \), where derivatives are taken with respect to \( x \). Finally, the value-maximizing threshold for the investment region satisfies the value-matching condition: \( v_{1,D}(x^*_\delta) = \Phi x^*_\delta - I \).\(^8\)

Solving the optimization problem of shareholders yields the following proposition (see Appendix B.2 for a proof):

**Proposition 1** Consider a firm with assets in place that generate a continuous flow of income \( f(X_t) = (1 - \tau) X_t \). Suppose that the firm has the opportunity to increase capacity and cash flows by a factor \( \pi > 1 \) by paying a lump sum \( I \). When the firm issues debt to finance the capital expenditure and the instantaneous arrival rate of debt investors is given by \( \delta > 0 \), the value of equity is given by

\[
v_{1,D}(x) = \begin{cases} 
Ax^2 + \Lambda x, & \text{for } x < x^*_\delta, \\
Bx^n + \left( \frac{1 - \tau + \delta \phi \theta}{\tau + \delta \theta} - \frac{\delta \theta I}{\tau + \delta \theta} \right), & \text{for } x \geq x^*_\delta,
\end{cases}
\]

\(^8\)This condition follows from the value matching condition of shareholders at \( x^*_\delta \). Optimality is ensured by the continuity and smoothness conditions. One could also use a first order condition to determine \( x^*_\delta \).
where the value-maximizing investment threshold \( x_\delta^* \) satisfies

\[
x_\delta^*,D = \frac{\xi - \eta + \eta \frac{\delta \theta}{r + \delta \theta} I}{(\xi - \eta) \Phi - (\xi - 1) \Lambda - (1 - \eta) \frac{1 - r + \delta \theta}{r + \delta \theta - \mu}} ,
\]

the constant parameters \( A \) and \( B \) are defined by

\[
A = \left( \frac{1}{x_{\delta,D}^*} \right)^\xi \left[ \frac{1 - \eta}{\xi - \eta} \left( \frac{1 - \tau + \delta \theta \Phi}{r + \delta \theta - \mu} - \Lambda \right) x_{\delta,D}^* - \frac{\eta}{\xi - \eta} \frac{\delta \theta I}{r + \delta \theta} \right],
\]

\[
B = \left( \frac{1}{x_{\delta,D}^*} \right)^\eta \left[ \frac{\xi - 1}{\xi - \eta} \left( \Lambda - \frac{1 - \tau + \delta \theta \Phi}{r + \delta \theta - \mu} \right) x_{\delta,D}^* - \frac{\xi}{\eta - \xi} \frac{\delta \theta I}{r + \delta \theta} \right],
\]

and the constant elasticities \( \xi \) and \( \eta \) are given by

\[
\xi = \frac{(\sigma^2 - \mu)/\sigma^2 + \sqrt{[(\mu - \sigma^2)/\sigma^2]^2 + 2 r/\sigma^2}}{2 r/\sigma^2},
\]

\[
\eta = \frac{(\sigma^2 - \mu)/\sigma^2 - \sqrt{[(\mu - \sigma^2)/\sigma^2]^2 + 2 (r + \delta \theta)/\sigma^2}}{2 (r + \delta \theta)/\sigma^2}.
\]

The expressions for the value of equity in Proposition 1 can be interpreted as follows. The first term on the right hand side of equity (firm) value in the no-investment region \( x < x_{\delta,D}^* \) represents the option value of investing in the project and restructuring the firm’s capital structure. The second term represents the value of a perpetual claim to the current flow of income. Similarly the first term on the right hand side of equity value in the investment region \( x \geq x_{\delta,D}^* \) represents the change in the value of the firm if no debt investor can be found before the cash flow shock returns to the no-investment region. The second term represents the sum of the present value of current cash flows from assets in place and the increase in equity value due to investment. Since debt investors arrive at the rate \( \delta \), the value created by investment increases with the probability of investment \( \delta \).

In the model, the timing of investment is endogenous and investment occurs the first time the cash flow process reaches the threshold \( x_{\delta,D}^* \) (defined in equation (11)) and the firm can find debt investors. Figure 1 plots the value-maximizing investment trigger as a function of the arrival rate of investors \( \delta \), the share \( \theta \) of the investment surplus captured by shareholders, the corporate tax rate \( \tau \), and bankruptcy costs \( \alpha \).
In this figure, we use parameter values that roughly reflect a typical S&P 500 firm. The risk free rate is taken from the yield curve on Treasury bonds and set to $r = 5\%$. The growth rate of cash flows $\mu = 0.5\%$ has been selected to generate a payout ratio consistent with observed payout ratios.\(^9\) Similarly, the value of the volatility parameter is chosen to match the (leverage-adjusted) asset return volatility of an average S&P 500 firm’s equity return volatility and set to $\sigma = 20\%$. The tax advantage of debt captures corporate and personal taxes and is set equal to $\tau = 0.15$. Liquidation costs are defined as the firm’s going concern value minus its liquidation value, divided by its going concern value, which is measured by $\alpha$ in our model. Using this definition, Alderson and Betker (1995) and Gilson (1997) report liquidation costs of 36.5% and 45.5% for the median firm in their samples. We simply take the average, which is about 40%. The size of the growth option is set to $\pi = 1.2$.

Figure 1 shows that, consistent with economic intuition, the selected investment threshold increases with the arrival rate of debt investors $\delta$ and decreases with the share of the surplus from investment that shareholders can keep $\theta$. In particular, as the arrival rate of investors decreases, the opportunity cost of waiting to invest increases (as the likelihood of finding investors in the future decreases), so that the selected investment threshold decreases. In other words, when the firm has to find investors to finance the project, it balances the opportunity cost of early investment (i.e. the traditional option of waiting to invest) with the opportunity cost of waiting (the risk of not finding capital to finance the project). As in standard real options models, the selected investment trigger decreases with the size of the growth option (as measured by $\pi$) and increases with the volatility of the cash flow shock $\sigma$ (i.e. with the level of uncertainty over the future project cash flows). In addition, the value of equity increases with the size of the firm’s growth option $\pi$, cash flow volatility $\sigma$ (the value of the growth option being larger), the bargaining power of shareholders $\theta$, and the arrival rate of creditors $\delta$ (the selected investment policy being closer to first best).

When the expected delay associated with debt financing (as measured by $1/\delta$) tends to zero (i.e. as credit markets become frictionless), the value-maximizing investment threshold

\(^9\)The firm’s payout ratio reflects the sum of the payments to both bondholders and shareholders. Following Morellec and Schuerhoff (2010b), we take the weighted averages between the average dividend yields (4% according to Ibbotson and Associates) and the average historical coupon rate (close to 9%), with weights given by the median leverage ratio of S&P 500 firms (approximately 20%).
converges to the usual investment trigger, defined by:
\[
\lim_{\delta \to \infty} x_{\delta, D}^* = x_*^\infty = \frac{\xi}{\xi - 1} \frac{I}{\Phi - \Lambda}.
\]
Equation (16) for \( x_*^\infty \) can also be written as:
\[
[(\pi - 1) + \pi \Gamma] \Lambda x_*^\infty = \frac{\xi}{\xi - 1} I.
\]
The left-hand side of this equation represents the benefit from investment. At the time of investment, the firm (i) increases the cash flows its productive assets (first term in the square bracket) and (ii) rebalances its capital structure (second term in the square bracket). The right hand side of this equation is the adjusted cost of investment. This cost reflects the option value of waiting through the factor \( \xi/(\xi - 1) \). If this option had no value, shareholders would follow the simple NPV rule, according to which one should invest as soon as the investment surplus is positive (i.e. as soon as \( X > I/(\Phi - \Lambda) \)).

### 3.2 Optimal financing and firm value

Given that several financing strategies are available to finance the capital expenditure, one question that naturally arises is what is the value-maximizing strategy for incumbent shareholders. To answer this question, we need to derive the value of equity before investment assuming that the growth option is financed by issuing new equity, denoted by \( v_{1,E} (x) \). Using the same steps as before, it can be shown that the value of equity before investment satisfies the ordinary differential equation
\[
rv_{1,E} (x) = \mu xv'_{1,E} (x) + \frac{\sigma^2}{2} x^2 v''_{1,E} (x) + (1 - \tau)x, \quad \text{for } x \leq x_*^E.
\]
where \( x_*^E \) denotes the the value-maximizing investment threshold when the firm finances the capital expenditure by issuing new equity. As in the case of debt financing, the left hand side of this equation represents investors’ required rate of return whereas the right hand side represents the expected return from investing in the firm’s equity. Since the firm does not have to search for equity investors, management’s optimization problem is similar to that described in Dixit and Pyndick (1994). That is, when the cash flow shock is below \( x_*^E \) the firm does not invest; when the cash flow shock reaches \( x_*^E \), the firm invests.
This equation is solved subject to the following boundary conditions. First, the value of equity at the time of investment is equal to the payoff from investment (value-matching):
\[ v_{1,E}(x)|_{x=x^*_E} = \pi \Lambda x_E^* - I. \]
In addition, to ensure that investment occurs along the optimal path, the value of equity satisfies the smooth pasting condition:
\[ \frac{\partial v_{1,E}(x)}{\partial x}|_{x=x^*_E} = \pi \Lambda \]
at the endogenous investment threshold. Finally, as the value of the cash flow shock tends to zero, assets in place and the option to invest become worthless so that
\[ \lim_{x \to 0} v_{1,E}(x) = 0. \]
Solving this optimization problem yields the following expression for equity value:

**Proposition 2** When the capital expenditure is financed by shareholders, the value of equity satisfies
\[
v_{1,E}(x) = \begin{cases} 
\Lambda x + \frac{I}{\xi-1} \left( \frac{x}{x_E^*} \right)^\xi, & x < x_E^*, \\
\Lambda \pi x - I, & x \geq x_E^*, 
\end{cases}
\]
where \( \xi > 1 \) is defined in Proposition 1 and the value-maximizing investment threshold \( x_E^* \) is given by:
\[
x_E^* = \frac{\xi}{\xi - 1} \frac{I}{\Lambda (\pi - 1)} > x_E^*_{\infty}. 
\]

Proposition 2 shows that the value of the firm before investment is the sum of the value of assets in place and the value of the growth option. It also demonstrates that when the supply of credit is infinite (as in previous contributions), firms always finance investment with debt and invest earlier than if they were financing the cost of investment with equity (since \( \Phi > \pi \Lambda \)). One direct implication of this result is that negative shocks to the supply of credit may hamper investment even if firms have enough financial slack to fund all profitable investment opportunities internally.

Using this result, we can now complete the model solution by endogenizing shareholders’ financing decision. The value of equity before investment under the value-maximizing financing strategy for the firm’s growth option is defined by:
\[
v_1(x) = \max \{ v_{1,E}(x), v_{1,D}(x) \},
\]
where \( v_{1,D}(x) \) is defined in Proposition 1. Using equation (10) in Proposition 1, it is immediate to show that as \( \delta \) tends to infinity, the value of equity when financing the capital expenditure with debt tends to
\[
\lim_{\delta \to \infty} v_{1,D}(x) = \Lambda x + \frac{I}{\xi - 1} \left( \frac{x}{x_E^{\infty}} \right)^\xi > \Lambda x + \frac{I}{\xi - 1} \left( \frac{x}{x_E^*} \right)^\xi \equiv v_{1,E}(x).
\]
That is, debt financing always dominates equity financing for high arrival rate of debt investors, because of the tax savings associated with a debt issue. Also as \( \delta \) tends to zero, the left hand side of this equation tends to

\[
\lim_{{\delta \to 0}} v_{1,D}(x) = \Lambda x < \Lambda x + \frac{I}{\xi - 1} \left( \frac{x}{x_{E}^{*}} \right)^{\xi} \equiv v_{1,E}(x).
\]

That is, equity financing always dominates debt financing for low arrival rate of debt investors since the probability of exercise of the firm’s growth option tends to zero as \( \delta \) tends to zero. Since the payoff from investment is monotonically increasing in \( \delta \), so is the value of equity under debt financing \( v_{1,D}(x) \). Using this property of equity value, we can establish the following result.

**Proposition 3** When the firm has no debt outstanding, it is always optimal for the firm to finance the capital expenditure using debt if the rate of debt investors is greater than \( \delta^{*} \) defined by

\[
\delta^{*} \equiv (r - \mu) \frac{\pi - 1}{\theta \pi \Gamma}.
\]

When the rate of debt investors is lower than \( \delta^{*} \), there exists a cutoff level \( x^{*}(\delta) \) with \( \frac{\partial x^{*}(\delta)}{\partial \delta} > 0 \) such that the firm will find it optimal to finance the investment project with equity for \( x \in [x^{*}(\delta), \infty) \) and with debt for \( x \in [x_{\delta,D}^{*}, x^{*}(\delta)) \). This cutoff level for the cash flow shock solves the non-linear equation

\[
\left( 1 - \tau \right) \left( \pi - 1 \right) - \delta \theta \pi \Lambda x^{*}(\delta) - \frac{rI}{r + \delta \theta} = \left( \frac{x^{*}(\delta)}{x_{\delta,D}^{*}} \right)^{\eta} \left[ \xi - \eta \left( \Lambda - \frac{1 - \tau + \delta \theta \Phi}{r + \delta \theta - \mu} \right) x_{\delta,D}^{*} - \frac{\xi}{\eta - \xi} \frac{\delta \theta I}{r + \delta \theta} \right].
\]

We may interpret the model’s implications on capital structure as a generalized trade-off model that takes into account the limited supply of credit and its effects on valuations. In particular, several results follow from Proposition 3. First, since debt carries a tax benefit, it is always optimal for the firm to finance the capital expenditure by issuing debt if the arrival rate of investor is large enough (i.e. if \( \delta \geq \delta^{*} \)). In that case, the cost of waiting for new investors (the expected time to find an investor is \( 1/\delta \)) is smaller than the potential tax benefits associated with a debt issue.\(^{10}\)

\(^{10}\)Since the firm has no debt outstanding, there is a large benefit of issuing debt at the time of investment
Second, when the arrival rate of outside investors is lower than $\delta^*$, the model calls for some notion of **rational market timing.** That is, it will be optimal for firms to finance investment projects with equity issues only if the shock to cash flow is large enough, i.e. following a run-up in the firm’s stock price (i.e. for $x \geq x^* (\delta)$). The size of the run-up necessary to trigger an equity issue depends on all the parameters of the model, including the arrival rate of debt investors, the firm’s corporate tax rate, the volatility of the firm cash flow (and hence the volatility of stock returns), and bankruptcy costs.

To better understand the economic determinants of firms’ financing decisions, Figure 2 plots the separating trigger for financing decisions $x^* (\delta)$ as a function of the the arrival rate of debtholders $\delta$, the bargaining power of shareholders $\rho$, the corporate tax rate $\tau$, and direct bankruptcy costs $\alpha$. Input parameter values are set as in our base case environment. Consistent with economic intuition, Figure 2 shows that as the arrival rate of debt investors increases, the run-up in equity prices necessary to observe an equity issue becomes more important. Indeed, as $\delta$ increases, the present value of potential tax savings increases and it becomes relatively less interesting to finance the capital expenditure by issuing new equity. In addition, the figure reveals that, the region of equity financing decreases with the corporate tax rate $\tau$ (since tax benefits are greater) and increases with bankruptcy costs $\alpha$ (since the cost of debt is greater). Finally, the figure shows that an increase in the bargaining power of creditors makes equity financing more attractive.

**Credit supply and the low-leverage puzzle.** The theory put forth in this paper also sheds new light on other important issues in corporate finance. For example, Graham (1999) and others argue that most firms appear to be underlevered and, as a result, that these firms are leaving money on the table by choosing not to increase their leverage. An alternative explanation proposed in this paper is that firms may not be able to issue additional debt because of the limited supply of credit. That is, while standard demand-based models would predict that firms should always issue debt to finance (part of) the cost of investment (see in our basic model. This implies that we need to pick a large value for $\rho$ (i.e. to increase the cost of debt financing) to make it optimal for the firm to issue equity for some combination of parameter values. This problem does not arise in section 4 since the firm already has outstanding debt at the time of investment.
e.g. Gomes and Schmid, 2010, Morellec and Schürhoff, 2010a, or Hackbarth and Mauer, 2010), our theory predicts that some firms will choose to finance the capital expenditure with equity even if there is a tax advantage of debt. This prediction is in sharp contrast with existing theories of corporate financing.

Rational market timing. The model also explains why firms may appear to time the market when issuing equity. In traditional capital structure models, the costs of different forms of capital do not vary independently. As a result, there is no gain of switching from debt to equity. Our model shows that when credit supply is uncertain, market timing benefits incumbent shareholders (see Baker and Wurgler, 2002, and the references therein for empirical support). In particular, Proposition 3 demonstrates that there exists a minimum cutoff level for the firm cash flows – or for the firm stock price – such that it will be optimal for the firm to finance capital expenditures by issuing equity only if the firm cash flow goes beyond that level. That is, firms will only issue equity following a significant run-up in their stock market valuation. Our study allows us to make specific predictions regarding market timing. Notably it predicts that the size of the run-up that will trigger an equity issue depends positively on the marginal corporate tax rate and on the supply of credit in the capital markets and negatively on bankruptcy costs, cash flow volatility, or the size of the firm’s growth option. We show in section 4 below that the size of the run-up should also depend (negatively) on the firm’s current indebtedness.

Credit supply and investment. Another important implication of our model is that negative shocks to the supply of credit may hamper investment even if firms have enough financial slack to fund all profitable investment opportunities internally. To illustrate this idea, Figure 3 plots the probability of investment over a 3-year and a 5-year horizon as a function of the arrival rate of investors $\delta$, assuming that the capital expenditure is financed with debt (Appendix B.3 provides a closed-form expression for this probability).

Figure 3 shows that credit supply uncertainty has a significant impact on the probability of investment for any given horizon. For example, it shows that this probability increases from
57% to 77% over a 3-year horizon as the arrival rate of creditors increases from $\delta = 2$ to $\delta = 12$ (i.e. as the expected financing lag decreases from 6 months to 1 month). Interestingly, this increase in the probability of investment occurs even though the investment threshold goes up with the arrival rate of creditors, $\delta$. Two mechanisms drive this result. First, because of the tax advantage of debt, the investment trigger with debt financing is lower than the investment trigger with equity financing. Second, when the supply of credit is limited, investment occurs if the cash flow shock is in the investment region and the firm finds creditors. Accordingly, the effect of the limited supply of capital on the probability of corporate investment decreases with $\delta$, $\sigma$, and $\alpha$ and increases with $\tau$.

Our results on the relation between credit supply and corporate investment are consistent with the findings in Kashyap, Stein and Wilcox (1993) and Lemmon and Roberts (2007) that contractions in the credit supply lead to declines in investment. They are also consistent with the recent survey evidence in Campello, Graham, and Harvey (2009). In a survey of 1,050 CFOs in the U.S., Europe, and Asia following the financial crisis of 2008, they find that the negative shock to the supply of credit caused many firms to bypass attractive investment opportunities, with 86% of constrained U.S. CFOs saying their investment in attractive projects was restricted during the credit crisis of 2008. They also find that more than half of the respondents said they canceled or postponed their planned investments. The predictions of our theory regarding the effects of the various parameters are novel and provide grounds for further empirical work on corporate policy choices.

4 Initial debt, credit lines, and marginal financing decisions

4.1 Initial debt

We now turn to a richer setting in which the firm already has some debt outstanding before the exercise of the growth option and has to decide on both the value-maximizing default and investment decisions. In this setting, shareholders may choose to default on the firms’ debt obligations and give up the growth option, should the firms’ revenue stream drop to a sufficiently low level. Alternatively, they may decide to invest and issue additional debt, should the firm’s fortunes improve. As in the previous section, we respectively denote by $x^*_\delta,D$ and $x^*_E$ the investment thresholds with debt- and equity-financing. In addition, we
denote by \( x^*_{1,D} \) the default threshold that maximize equity value before investment. (Whenever the cash flow shock reaches \( x^*_{1,D} \), management defaults on the firm’s debt obligations before investment.) In this more general setting, we can thus see the manager’s policy choices as determining the change in the coupon payment \( c \) at the time of investment, the investment thresholds \( x^*_{\delta,D} \) and \( x^*_{E} \), and the default boundary \( x^*_{1,D} \).

To make the analysis precise, assume that the firm is initially capitalized with debt and common equity. The initial debt issue has infinite maturity and has a coupon payment of \( c_1 \) (endogenized below). The firm may issue additional debt when exercising the growth option. We assume that this new debt issue has the same priority as the old debt and has infinite maturity. We also consider that the total coupon payment after investment is \( c^* \) given by equation (6) (we show below that the value-maximizing financing policies imply that \( c_1 < c^* \) because of the smaller asset base before investment). Alternatively, the firm may finance the cost of investment by exclusively issuing new equity, in which case the coupon payment remains at its initial level \( c_1 \). As will become clear below, the financing decision at the time of investment is here again driven by a trade-off between tax benefits, bankruptcy costs, and supply side effects.

Suppose first that the capital expenditure is financed by issuing risky debt at the time of investment. To specify the boundary conditions at the investment trigger \( x^*_{\delta,D} \), we need to determine the value of the initial debt at the time of investment. We show in Appendix A that this value satisfies \( d_1(x^*_{\delta,D}, c_1) = \Delta c_1 \) under the value-maximizing debt policy, where \( \Delta \) is a positive constant. The surplus from investment to shareholders when exercising the growth option is then given by:

\[
v(x^*_{\delta,D}, c^*) - d_1(x^*_{\delta,D}, c_1) - I - p(x^*_{\delta,D}) - e_1(x^*_{\delta,D}) = \theta \left[ \Phi x^*_{\delta,D} - \Delta c_1 - I - e_1(x^*_{\delta,D}) \right],
\]

where the right hand side of this equation is the total surplus generated by investment multiplied by the share \( \theta \) of the surplus captured by shareholders.

Using the boundary condition (19) and derivations similar to those reported in section 3.1, it is possible to establish the following result (see Appendix C.1 for a proof).
Proposition 4 Assume that there exists a solution \( h < 1 \) to the non-linear equation

\[
\frac{(\nu - \eta)(I + \Delta c_1) - \nu(1 - \tau) \frac{\mu}{\nu} (1 - h^{-\xi}) + \eta \frac{\delta (I + \Delta c_1) + (1 - \tau) c_1}{r + \delta \theta - \mu}}{(\nu - \eta) \Phi + (1 - \nu) \Lambda (1 - h^{1 - \xi}) - (1 - \eta) \frac{1 - \tau + \delta \theta}{r + \delta \theta - \mu}} = \frac{(\xi - \eta)(I + \Delta c_1) - \xi(1 - \tau) \frac{\mu}{\nu} (1 - h^{-\nu}) + \eta \frac{\delta (I + \Delta c_1) + (1 - \tau) c_1}{r + \delta \theta}}{(\xi - \eta) \Phi + (1 - \xi) \Lambda (1 - h^{1 - \nu}) - (1 - \eta) \frac{1 - \tau + \delta \theta}{r + \delta \theta - \mu}}
\]

and that the cost of investment is (partly) financed by the issuance of new debt. Then the value of equity before investment is given by

\[
e_1(x) = \begin{cases} 
0 & \text{if } x \leq x_{1,D}^* \\
Ax^\xi + Bx^\nu + \Lambda x - (1 - \tau) \frac{\mu}{\nu}, & \text{if } x_{1,D}^* < x < x_{\delta,D}^*, \\
Cx^\eta + \frac{1 - \tau + \delta \theta}{r + \delta \theta - \mu}x - \frac{\delta (I + \Delta c_1) + (1 - \tau) c_1}{r + \delta \theta}, & \text{if } x \geq x_{\delta,D}^*,
\end{cases}
\]

where the default and investment thresholds respectively satisfy

\[
x_{1,D}^* = hx_{\delta,D}^*
\]

and

\[
x_{\delta,D}^* = \frac{(\xi - \eta)(I + \Delta c_1) - \xi(1 - \tau) \frac{\mu}{\nu} (1 - h^{-\nu}) + \eta \frac{\delta (I + \Delta c_1) + (1 - \tau) c_1}{r + \delta \theta}}{(\xi - \eta) \Phi + (1 - \xi) \Lambda (1 - h^{1 - \nu}) - (1 - \eta) \frac{1 - \tau + \delta \theta}{r + \delta \theta - \mu}}.
\]

In these equations, the constants \( A, B, \) and \( C \) satisfy

\[
A = \left( \frac{1}{x_{1,D}^*} \right)^\xi \left[ \frac{\nu - 1}{\xi - \nu} \Lambda x_{1,D}^* - \frac{\nu}{\xi - \nu} (1 - \tau) \frac{c_1}{r} \right],
\]

\[
B = \left( \frac{1}{x_{1,D}^*} \right)^\nu \left[ \frac{1 - \xi}{\xi - \nu} \Lambda x_{1,D}^* + \frac{\xi}{\xi - \nu} (1 - \tau) \frac{c_1}{r} \right],
\]

\[
C = \left( \frac{h}{x_{1,D}^*} \right)^\eta \left[ \left( \Phi - \frac{1 - \tau + \delta \theta}{r + \delta \theta - \mu} \right) x_{\delta,D}^* + \frac{(1 - \tau) c_1 - r (I + \Delta c_1)}{r + \delta \theta} \right],
\]

In addition, \( \xi \) and \( \eta \) are defined in Proposition 1 and \( \nu \) is defined by

\[
\nu = (\sigma^2 - \mu)/\sigma^2 - \sqrt{(\mu - \sigma^2)/\sigma^2)^2 + 2r/\sigma^2} < 0.
\]

The expressions for equity value in Proposition 4 can be interpreted as follows. The first term on the right hand side of equity value in the inaction region \( (x_{1,D}^* < x < x_{\delta,D}^*) \) represents the option value of investing in the project and restructuring the firm’s capital structure. The second term represents the value of the option to default. The third and
fourth terms represent the value of a perpetual claim on the current flow of income. Similarly the first term on the right hand side of equity value in the investment region \((x \geq x_{k,D}^*)\) represents the change in the value of equity if no debt investor can be found before the cash flow shock returns to the inaction region. The second and third terms represent the sum of the present value of current cash flows from assets in place and the increase in equity value at the time of investment.

Interestingly, equation (22) shows that shareholders’ investment and default decisions are interrelated. That is, the possibility for shareholders to default on the firm’s debt obligations (respectively to invest in the growth option) affects the optimal investment (respectively default) trigger. In particular, the analysis reveals that when the firm has risky debt outstanding shareholders underinvest in the firm’s growth option. This is illustrated by the change in the selected investment threshold due to the presence of risky debt, which increases from \(x_{E}^* = 1.65\) to \(x_{E}^* = 1.75\) in our base case environment as \(c_1\) moves from 0.3 to 0.7 (optimal leverage is \(c_1^* = 0.5\)). Since the selected investment trigger increases with \(c_1\), the probability of investment decreases and underinvestment gets more severe as the amount of debt in the firm’s capital structure increases. As first shown by Myers (1977), this is due to the fact that when the firm has risky debt outstanding, shareholders pay the full cost of investment but have to share the benefits with debtholders.

**Marginal financing decision.** Using the same steps as above, it is possible to derive the value of equity before investment assuming that the firm finances the capital expenditure by issuing new equity. The expressions for the value of equity, the value of corporate debt, and the value-maximizing investment and default triggers under this financing assumption are reported in Appendix C.3. Using these results, we can analyze the marginal financing decision made by shareholders at the time of investment when the firm has risky debt outstanding. Figure 4 plots the separating financing trigger \(x^*(\delta)\) as a function of the arrival rate of creditors, the bargaining power of shareholders, the corporate tax rate, and bankruptcy costs. Input parameter values are set as in the base case environment.

Insert Figure 4 Here
Consistent with economic intuition, Figure 4 shows that as the arrival rate of debt investors increases, the run-up in equity prices necessary to observe an equity issue becomes more important. Indeed, as $\delta$ increases, the present value of potential tax savings increases and it becomes relatively more interesting to finance the capital expenditure with debt. In addition, the figure reveals that the region of debt financing increases with the corporate tax rate $\tau$ (which makes debt more attractive) and decreases with bankruptcy costs $\alpha$. Importantly, the separating financing trigger is lower than in the benchmark case without initial debt. This is due to the fact that the marginal benefit of adding debt in the firm’s capital structure is lower, while the marginal cost (expected bankruptcy costs) is higher. A direct implication of this result is that in model with search, the firm’s capital structure will contain less debt than in a standard model without search.

**Initial financing decision.** The firm’s initial financing decisions reflects a trade-off between tax benefits, bankruptcy costs, and investment distortions. Bankruptcy costs include the loss of the interest tax shields, the loss of the growth option (assuming that default occurs before investment), and the fraction of asset value $\alpha$ lost in default. Appendix C.2 provides the value of corporate debt and the value of the firm assuming that the capital expenditure will be financed with debt financing. Based on the resulting values, Figure 5 plots the value maximizing initial coupon payment $c_1^*$ as a function of the arrival rate of debtholders $\delta$ and the bargaining power of shareholders $\rho$. Input parameter values are set as in our base case environment.

Figure 5 reveals that as the arrival rate of debt investors increases, the initial coupon payment of the firm decreases. This last result is due to the fact that the likelihood of finding debt investors in the future increases with $\delta$. As a result, the firm does not need to be as aggressive with respect to its financing policy when choosing the initial capital structure. Consistent with economic intuition, the figure also shows that $c_1$ decreases with the bargaining power of shareholders (i.e. increases with $\rho$). Finally, it is also possible to show that, as in standard contingent claims models, the initial coupon payment decreases with direct and indirect (i.e. underinvestment) bankruptcy costs – that is $c_1$ decreases with $\alpha$ and $\pi$ – and increases with the corporate tax rate $\tau$. 
4.2 Credit supply and credit lines

So far we have ignored the role of credit lines in liquidity or credit supply management. A firm that obtains a line of credit receives a nominal amount of debt capacity against which it can draw funds. Lines of credit are almost always provided by banks or financing companies. The used portion of a line of credit is a debt obligation, whereas the unused portion remains off the balance sheet. The empirical literature on lines of credit argues that they are motivated primarily by capital market frictions and that a committed line of credit overcomes these frictions by ensuring that funds are available for valuable investment projects. The extant literature on dynamic investment and financing is largely silent on why firms may use credit lines in liquidity management. Indeed, in these models firms can always raise debt and there is no role for credit lines (if anything, they are suboptimal). This section shows that credit supply uncertainty may lead firms to use such funding facilities.

The pricing of credit lines is generally characterized by two components: a commitment fee and a precommitted interest rate (see Sufi, 2009). The commitment fee represents a percentage of the unused portion of the line of credit. The precommitted interest rate is the rate at which firms can draw funds from the credit line. In the analysis below, we denote the size of the credit line by \( f I \) where \( f \in [0, 1] \), the commitment fee by \( \phi \), and the precommitted interest rate by \( r^f \). We assume that the firm has outstanding debt with contractual coupon payment \( c_1 \) before investment. The total payment to debtholders before investment is therefore \( c_1 + \phi f I \). In addition, at the time of investment, the capital expenditure is financed by using the line of credit and common stock (if \( f < 1 \)) so that total payment to debtholders after investment is \( c_1 + r^f f I \). Under these assumptions, it is possible to establish the following result (see Appendix D for a proof).

**Proposition 5** Assume that there exists a solution \( y > 1 \) to the non-linear equation

\[
\frac{\xi}{\xi - \nu} \left[ (c_1 + \phi f I) y^\nu + (r^f - \phi) f I + \frac{r (1 - f) I}{1 - \tau} \right]
\]

\[
= \frac{y^\nu + (\pi - 1) y^\nu (\xi - 1) [ (1 - \tau) (c_1 + \phi f I) y^\xi + (1 - \tau) (r^f - \phi) f I + r (1 - f) I ]}{y^\xi + (\pi - 1) y^\xi (1 - \tau) (\xi - \nu) (\nu - 1)}
\]

\[
+ \frac{(c_1 + r^f I)}{(1 - \nu)} \left( \frac{\nu y^\nu (1 - \tau) (c_1 + \phi f I) y^\xi + (1 - \tau) (r^f - \phi) f I + r (1 - f) I}{r \Lambda [y^\xi + (\pi - 1) y^\xi] x_{2,f}} \right)^\nu
\]
and that the cost of investment is financed by using the credit line and new equity. Then the value of equity before investment is given by

\[ e_1(x, c_1) = \Lambda x - \frac{1 - \tau}{r} (c_1 + \phi f I) + \Sigma(x) M + \Sigma(x) N, \]

where the investment and default thresholds before investment respectively satisfy

\[ x^*_1, E = y x^*_1, D \]

and

\[ x^*_1, D = \frac{\nu}{\nu - 1} \frac{(1 - \tau)(c_1 + \phi f I) y^k + (1 - \tau) (r^f - \phi) f I + r (1 - f) I}{r \Lambda [y^k + (\pi - 1) y]} \].

In these equations, \( \xi, \eta, \) and \( \nu \) are defined in Proposition 2, the constants \( M \) and \( N \) satisfy

\[ M = \frac{1 - \tau}{r} c_1 + \frac{(1 - \tau) \phi f I}{r} - \Lambda x^*_1, D \]

\[ N = (\pi - 1) \Lambda x^*_{1,E} - (1 - \tau) \left[ \frac{(r^f - \phi) f I}{r} + \frac{(c_1 + r^f f I)}{r (\nu - 1)} \left( \frac{x^*_{1,E}}{x^*_{2,f}} \right)^\nu \right] - (1 - f) I \]

and the stochastic discount factors \( \Sigma(x) \) and \( \Sigma(x) \) are defined in Appendix C.

The value of equity in Proposition 5 has a similar functional form as in the case of debt financing (reported in Proposition 4). The value of equity is the sum of the value of a perpetual claim to the current flow of income (first two terms in equation (29)) and the change in value that occurs in default (third term) and the change in value that occurs at the time of investment (fourth term).

Using Propositions 4 and 5, we can analyze the effects of credit lines of firms’ investment and financing decisions. In particular, the firm can either search for new creditors to finance the capital expenditure or it can use its credit line and issue some common stock (if such a facility has been negotiated initially). The benefit of using the credit line is that the firm has immediate access to outside funding. The costs of using the credit line are represented by the commitment fee and investment distortions induced by the precommitted interest rate. That is, because the precommitted rate of interest on the credit line does not depend on the timing of investment, debt will be either overpriced or underpriced leading to distortions in investment policy.
Figure 6 plots initial shareholder wealth when using a credit line and the value of equity when the firm has to search for outside investors, as a function of the arrival rate $\delta$ of creditors. In this figure we assume that the terms of the credit line are fixed at time $t = 0$ when the firm has found the initial creditor and has to select $c_1$. That is, we assume that the firm picks $f^* \in [0, 1]$ to maximize shareholder wealth at time $t = 0$, given $\phi$ and $r^f$. Input parameter values are set as in the base case environment. The parameters for the line of credit are based on a recent empirical study by Sufi (2009). In this study, Sufi finds that the median commitment fee $\phi$ is 25 basis points in a sample of 11,578 credit lines obtained by 4,011 public firms between 1996 and 2003. He also finds that the median precommitted interest rate is 150 basis points above LIBOR, i.e. $r^f = r + 1.5\%$. (Campello et al., 2010, report similar numbers in a recent survey of 800 CFOs from North America, Europe, and Asia.) The left panel represents an environment in which shareholders have high bargaining power (i.e. we set $\rho = 0$) and the right panel represents an environment in which they have low bargaining power (i.e. we set $\rho = 10$).

Figure 6 reveals that in our base case environment, the value of the firm with a credit line is always dominated by the value without a credit line for values of the arrival rate of creditors above $\delta = 3$ (i.e. for financing lags below 4 months). That is, the commitment fee and the costs induced by the pre-determined interest rate seem to impose larger costs than the search for outside investors in most economic environments. This is no longer true when the arrival rate of investors is very low, suggesting that firms may use credit lines as a risk management instrument (i.e. as a hedge against a contraction in the supply of credit).

This result is consistent with the evidence in Campello et al. (2010). In particular, they find that firms with more positive investment projects only use credit lines when immediate access to external funds is difficult (i.e. when $\delta$ is low). These authors write “the high proportion of funds being drawn from outstanding lines of credit during the crisis is consistent with a story where alternative sources of liquidity were unavailable.” Another implication of figure 6 is that credit lines become more attractive as the bargaining power of shareholders decreases, i.e. as the cost of debt in subsequent issues increases (due to the limited competition or supply of capital in the debt markets).
An important result that emerges from the analysis so far is that the corporate response to a limited supply of credit depends on the severity of the shortage in debt investors. In the next section, we investigate the properties of the model for the more general case in which the supply of capital in debt markets changes over time and characterize the value-maximizing investment and financing policies in this context.

5 Time-varying credit supply

So far, we have ignored the possibility that the arrival rate of creditors, and hence that corporate policy choices, could change over time. The financial crisis that began in August 2007 has shown however that the supply of external finance for non-financial firms could be subject to significant shocks. This section examines the effects of shocks to the supply of credit on firms’ investment and financing decisions. To do so, we go back to the initial model with no initial debt and consider an environment in which the arrival rate of debt investors \((\delta_t)_{t \geq 0}\) can take two values: \(\delta_L\) and \(\delta_H\) with \(\delta_H > \delta_L \geq 0\). In addition, assume that \(\delta_t\) is observable and that its transition probability follows a Poisson law, such that \((\delta_t)_{t \geq 0}\) is a two-state Markov chain. Let \(\lambda_i > 0\) denote the rate of leaving state \(i\) and \(\ell_i\) denote the time to leave state \(i\). Within the present model, the exponential law holds:

\[
\Pr(\ell_i > t) = e^{-\lambda_i t}, \quad i = H, L,
\]  

and there is a probability \(\lambda_i dt\) that the value of the shock \((\delta_t)_{t \geq 0}\) changes from \(\delta_i\) to \(\delta_j\) during an infinitesimal time interval \(dt\).

Assume that the capital expenditure is finance by issuing debt and denote by \(v_{i,D}(x)\) the value of the firm in supply regime \(i = L, H\) before investment. The total surplus generated by investment is given by:

\[
v_2(x, e^*) - I - v_{i,D}(x) = \Phi x - I - v_{i,D}(x).
\]

As before, this total surplus is shared between shareholders and debtholders through the cost of debt financing \(p_i(x)\). The Nash bargaining solution uniquely determines the surplus from investment to shareholders in regime \(i\) as

\[
\Phi x - p_i(x) - v_{i,D}(x) = \theta_i [\Phi x - I - v_{i,D}(x)], \quad \text{for all } x.
\]  

Let \(x_{i,D}^*\) denote the investment threshold that maximizes equity value in credit supply regime \(i = H, L\), when the firm finances the capital expenditure by issuing new debt. That
is, for all values of the cash flow shock above (respectively below) the investment threshold \( x_{i,D}^* \), \( i = H, L \), it is optimal for the firm to (respectively not to) invest in the project. Since the payoff from investment is strictly increasing in the value of the cash flow shock \( x \) and \( \delta_L < \delta_H \), we have that \( x_{H,D}^* > x_{L,D}^* \). That is, the firm speeds up investment as the arrival rate of outside investors decreases (making the opportunity cost of waiting for investors more important).

Proposition 9 in Appendix E characterizes the investment policy that maximizes equity value when the dynamics of the credit supply are governed by equation (34) and the firm finances the capital expenditure with debt. This investment policy takes the form of a trigger policy and there exists one investment trigger per level of credit supply. Since the two regimes (high and low credit supply) are related to one another through the \( \lambda_i \)'s, the investment threshold in each regime reflects the possibility for the firm to invest in the other regime. In other words, the firm has to determine an exercise strategy for its growth option in each credit supply regime, while taking into account the optimal investment strategy in the other regime. Because the persistence in regimes reflects the opportunity cost of investing in one regime vs. the other, the ratio of the two investment thresholds depends on the \( \lambda_i \)'s. Specifically, a lower persistence of regime \( i \) (i.e. a higher \( \lambda_i \)) reduces the opportunity cost of investing in regime \( i \), and hence narrows the gap between the investment thresholds in the two credit supply regimes.

In the model, the timing of investment is endogenous and investment occurs the first time the cash flow process reaches the threshold \( x_{i,D}^* \), \( i = H, L \), and the firm can find debt investors. Figure 7 plots the value-maximizing investment triggers (left panel) as well the financing separating trigger (right panel) as a function of the arrival rate of investors \( \delta \) and the share \( \theta \) of the investment surplus captured by shareholders. In this figure, we set the arrival rate of investors in the high credit supply regime to \( \delta_H = 3 \) (so that the expected financing lag is 6-month) and let the arrival rate of investors in the low credit supply regime vary in the interval \([1, 2.5]\). The blue (top) line in each panel represents the relevant threshold in the high state while the red (bottom) line represents the relevant threshold in the low state. The financing separating trigger is computed using the values of equity reported in Proposition 2 (for the case of equity financing) and in Proposition 9.
in Appendix E (for the case of debt financing).

Figure 7 shows that, consistent with economic intuition, the selected investment threshold increases with the arrival rate of debt investors \( \delta \). In particular, as the arrival rate of investors decreases, the opportunity cost of waiting to invest increases (as the likelihood of finding investors in the future decreases), leading to a decrease in the selected investment threshold. The figure also shows that the separating financing trigger is always above the investment trigger (as otherwise it would not be optimal to invest) and that the separating financing trigger in the high credit supply regime is above the separating financing trigger in the low credit supply regime. This last property is due to the fact that the value of equity when financing the capital expenditure with debt increases with the arrival rate of investors so that \( v_{H,D}(x) > v_{L,D}(x) \). This in turn implies that \( x_{H,D}^* > x_{L,D}^* \). That is, negative shocks to the supply of credit imply that the firm will optimally change its policy choices towards a greater use of alternative financing sources. As shown in the figure, the effect of a credit supply shock on the separating financing trigger can be important quantitatively.

6 Conclusion

Following Modigliani and Miller (1958), extant theoretical research in corporate finance generally assumes that capital markets are frictionless so that corporate behavior and capital availability depend solely on firm characteristics. This demand-driven approach has recently been called into question by a large number of empirical studies. These studies document the central role of supply conditions in credit markets in explaining corporate policy choices and highlight the need for an improved understanding of the precise role of supply in firms’ investment and financing decisions.

This paper takes a first step in constructing a dynamic model of corporate investment and financing decisions with capital supply effects by considering a setup in which firms face uncertainty regarding their future access to credit markets. The model provides an explicit characterization of the optimal investment and financing policies for a firm acting in the best interests of incumbent shareholders and shows that credit market frictions have first-order effects on corporate policy choices. The analysis in the paper yields a wide range of
empirical implications relating supply conditions in the credit markets to firms’ investment and financing policies, the timing of security issues, and the role of firm characteristics in shaping corporate policies. Some of these predictions shed light on existing findings. Others are novel and provide grounds for further empirical work on corporate investment and financing decisions. Overall, the analysis demonstrates that accounting for both demand and supply factors is critical to understanding firms’ policy choices.
Appendix

A. Firm value after investment

Firm value after investment for \( x > x_D \) as:

\[
v_2(x, c) = \mathbb{E} \left[ \int_0^{T(x_D)} e^{-rt} [(1 - \tau) \pi X_t + \tau c] dt + \int_{T(x_D)}^{\infty} e^{-rt} (1 - \tau) (1 - \alpha) \pi X_t dt \mid X_0 = x \right].
\]  

(A.1)

Using standard calculations (see e.g. Morellec, 2004), one can show that solving this equation yields:

\[
v_2(x, c) = \pi \Lambda x + \frac{c r}{\nu} \left[ 1 - \left( \frac{x}{x_D} \right)^{\nu} \right] - \alpha \pi \Lambda x_D \left( \frac{x}{x_D} \right)^{\nu},
\]  

(A.2)

where \( \nu < 0 \) is the negative root of the quadratic equation

\[
\frac{1}{2} \sigma^2 y(y-1) + \mu y - r = 0
\]

and the default threshold that maximizes equity value satisfies:

\[
x^*_D = \frac{\nu}{\nu - 1} \frac{c r - \mu}{r}.
\]  

(A.3)

The first order condition with respect to \( c \) is given by

\[
\frac{\partial v_2(x, c)}{\partial c} = 0,
\]  

(A.4)

the solution to which is given by (one can easily check that the second order condition for this optimization problem is negative, ensuring optimality):

\[
c^* = \pi x \frac{r (\nu - 1)}{\nu (r - \mu)} \left[ 1 - \nu - \alpha \frac{\nu (1 - \tau)}{\tau} \right]^{1/\nu}.
\]  

(A.5)

Plugging the expressions for \( c^* \) and \( x^*_D \) in equation (A.2) and simplifying yields

\[
v_2(x, c) = \pi \Lambda x \left\{ 1 + \frac{\tau}{1 - \tau} \left[ 1 - \nu - \alpha \frac{\nu (1 - \tau)}{\tau} \right]^{1/\nu} \right\}.
\]  

(A.6)

Is it also possible to show that the value of corporate debt satisfies

\[
d(x, c) = \frac{c}{r} \left[ 1 - \left( \frac{x}{x_D} \right)^{\nu} \right] + (1 - \alpha) \pi \Lambda x_D \left( \frac{x}{x_D} \right)^{\nu},
\]  

(A.7)

which, for \( c = c^* \), simplifies to \( d(x, c^*) = x \Theta \) with

\[
\Theta = \frac{\pi (\nu - 1)}{\nu (r - \mu)} \left[ 1 - \nu - \alpha \frac{\nu (1 - \tau)}{\tau} \right]^{1/\nu} \left[ \frac{(1 - \alpha)(1 - \tau) \nu - (\nu - 1) [\nu \tau + \alpha \nu (1 - \tau)]}{(\nu - 1) [(1 - \nu) \tau - \alpha \nu (1 - \tau)]} \right].
\]  

(A.8)

The value of the initial debt at the time of investment is then given by \( d(x, c_1) = \Delta c_1 \) with

\[
\Delta = \frac{\tau (1 - \alpha)(1 - \tau) \nu + (1 - \nu) [\nu \tau + \alpha \nu (1 - \tau)]}{r (\nu - 1) [(1 - \nu) \tau - \alpha \nu (1 - \tau)]}.
\]  

(A.9)
B. Value before investment

B.1 System of ODEs

In the model, credit supply is governed by a Poisson process \(N\) with intensity \(\delta\). The cash flow shock and the Poisson process are independent and the firm can only invest at jump times of the Poisson process (i.e. when creditors arrive). The \(n\)th jump time of the Poisson process is denoted by \(T_n\) with the conventions \(T_0 \equiv 0\) and \(T_\infty \equiv \infty\). In the following, we will use \(\mathcal{L}\) to denote the infinitesimal generator of the geometric Brownian motion \(X\). We thus have

\[
(\mathcal{L}u)(x) = \frac{1}{2}x^2 \sigma^2 u''(x) + \mu x u'(x), \quad \text{for all } x.
\] (B.1)

It is natural to guess that the optimal strategy takes the following form

\[
T^* = \inf \{ T_n : n \geq 1, X_{T_n} \geq x_{\delta,D}^* \},
\] (B.2)

where \(x_{\delta,D}^* > I\) is a constant threshold. Since it is optimal to continue when the cash flow shock is below \(x_{\delta,D}^*\), we have

\[
rv_{1,D}(x) + \mathcal{L}v_{1,D}(x) + (1 - \tau)x x_{\delta,D}^* = 0, \quad \forall x \in (0, x_{\delta,D}^*). \] (B.3)

When \(x \geq x_{\delta,D}^*\), the firm cannot invest unless it finds debt investors. In the small interval of length \(dt\), the firm has a probability \(\delta dt\) of finding debt investors. As a result, we have for \(x \geq x_{\delta,D}^*\):

\[
v_{1,D}(x) = \delta dt \{v_{1,D}(x) + \theta [\Phi x - I - v_{1,D}(x)]\} + (1 - \delta dt) \mathbb{E} \left[ e^{-rdt} v_{1,D}(X_{dt}) | X_0 = x \right],
\] (B.4)

or

\[
v_{1,D}(x) = \delta \{v_{1,D}(x) + \theta [\Phi x - I - v_{1,D}(x)]\} dt + (1 - \delta dt) \{v_{1,D}(x) + [-rv_{1,D}(x) + \mathcal{L}v_{1,D}(x) + (1 - \tau)x] dt\},
\] (B.5)

which yields

\[
r v_{1,D}(x) + \mathcal{L}v_{1,D}(x) + (1 - \tau)x + \delta \theta [\Phi x - I - v_{1,D}(x)] = 0, \quad \forall x > x_{\delta,D}^*.
\] (B.6)

Combining these results, we get the desired system of ODEs.

B.2 Value of equity

The total value of the firm before investment solves the following system of ordinary differential equations:

\[
r v_{1,D}(x) = \mu x v'_{1,D}(x) + \frac{\sigma^2}{2} x^2 v''_{1,D}(x) + (1 - \tau)x, \quad \text{for } x \leq x_{\delta,D}^*.
\]

\[
r v_{1,D}(x) = \mu x v'_{1,D}(x) + \frac{\sigma^2}{2} x^2 v''_{1,D}(x) + (1 - \tau)x + \delta \theta [\Phi x - I - v_{1,D}(x)], \quad \text{for } x \geq x_{\delta,D}^*.
\] (B.7)
The general solution to the set of equations (B.1) is

\[ v_{1,D}(x) = \begin{cases} A x^\xi + C x^\nu + \Lambda x, & \text{for } x < x_D^*, \\ B x^\nu + D x^\xi + \frac{1 - r - \delta \Phi}{r + \delta} x - \frac{\delta I}{r + \delta}, & \text{for } x \geq x_D^*, \end{cases} \] (B.8)

where \( A, B, C, D \) are constant parameters, where \( \xi > 1 \) and \( \eta < 0 \) are defined in Proposition 1, \( \nu < 0 \) is defined in Proposition 2 and \( \zeta > 1 \). The condition \( v_{1,D}(0) = 0 \) implies that \( C = 0 \). The condition \( v_{1,D}(x) \leq \psi x \) implies that \( D = 0 \). The last three conditions write

\[ A (x_D^*)^\xi + \Lambda x_D^* = \Phi x_D^* - I, \] \( A (x_D^*)^\xi + \Lambda x_D^* = B (x_D^*)^\nu + \frac{1 - r - \delta \Phi}{r + \delta} x_D^* - \frac{\delta I}{r + \delta}, \] \( \xi A (x_D^*)^\xi + \Lambda x_D^* = \nu B (x_D^*)^\nu + \frac{1 - r - \delta \Phi}{r + \delta} x_D^* \). \] (B.9) \( (B.10) \) \( (B.11) \)

Solving this system of equations yields the desired result.

**B.3 Probability of investment**

When financing the capital expenditure with debt, the firms invest if the cash flow shock is above the investment trigger and it can find debt investors. As a result, the probability of investing before time \( T \) is given by

\[ \Pr [\tau \leq T] = 1 - \mathbb{E} \left[ \exp \left\{-\delta \int_0^T 1_{[x_D^*,\infty)}(X_s) \, ds \right\} \right] \equiv 1 - \mathbb{E} [f(T)]. \] (B.12)

While this quantity cannot be computed in closed-form, we can get an expression for its Laplace transform. In particular, let \( \tau \sim \text{Exp}(\lambda) \) be an exponentially distributed stopping time independent of \( B^m \) where \( B^{m_0} = B_t + m_t \) and \( m = \frac{1}{2} (\mu - \sigma^2/2) \). We have (see Borodin and Salminen, 2002, pp279):

\[ \mathbb{E}_0 \left[ \exp \left\{ q \int_0^T 1_{(r,\infty)}(B^m_s) \, ds \right\} ; \sup_{0 \leq s \leq \tau} B^m_s < b \right] \] \( (B.13) \)

\[ = 1 - e^{(a-r)\tau_0} q e^{m(r-a)} \frac{\{m + \Upsilon q \coth ((b-r)\Upsilon q)\} \sinh ((b-r)\Upsilon q) + \lambda \Upsilon q e^{m(b-r)}}{(\lambda + q) \Upsilon q \sinh ((b-r)\Upsilon q) + \Upsilon q \coth ((b-r)\Upsilon q)}, \]

where \( \Upsilon_s = \sqrt{2(\lambda + s) + m^2} \) and

\[ \sinh (y) = \frac{1}{2} (e^y - e^{-y}) \quad \text{and} \quad \coth (y) = \frac{e^y + e^{-y}}{e^y - e^{-y}}. \] (B.14)

In our setup, we are interested in computing this quantity when \( b \) tends to infinity, \( q = -\delta \), \( m = \frac{1}{2} (\mu - \sigma^2/2) \), and \( r = \frac{1}{2} \ln(x_{3,D}^* / x_0) \). The last step involves the inversion of the Laplace transform for \( f(T) \) using standard numerical methods.
C. Initial financing

C.1. Proof of Proposition 4

In the region for the cash flow shock in which it is not optimal to default (i.e. for \( x > x_D \)), the value of equity solves the following system of ordinary differential equations:

\[
re_1 (x) = \mu x e_1'(x) + \frac{\sigma^2}{2} x^2 e_1''(x) + (1 - \tau) (x - c_1), \quad x < x^*_\delta, \\
re_1 (x) = \mu x e_1'(x) + \frac{\sigma^2}{2} x^2 e_1''(x) + (1 - \tau) (x - c_1) + \delta \theta [\Phi x - \Delta c_1 - I - e_1(x)], \quad x \geq x^*_\delta.
\]

These ODEs are solved subject to the following boundary conditions. First, at the endogenous default threshold, equity must be worthless so that \( e_1(x^*_\delta, D) = 0 \). Since the default threshold is chosen optimally by shareholders, it satisfies the smooth-pasting condition: \( e_1'(x^*_\delta, D) = 0 \). In addition, since cash flow grows linearly with the cash flow shock, it must be that there exists some constant \( \psi \) such that \( e_1(x) \leq \psi x \). Also, since cash flow to claimholders are given by a (piecewise) continuous Borel-bounded function, the value function \( e_1(\cdot) \) satisfies the continuity and smoothness conditions: \( \lim_{x \uparrow x^*_\delta, D} e_1(x) = \lim_{x \downarrow x^*_\delta, D} e_1(x) \), and \( \lim_{x \downarrow x^*_\delta, D} e_1'(x) = \lim_{x \uparrow x^*_\delta, D} e_1'(x) \), where derivatives are taken with respect to \( x \). Finally, the value-maximizing investment threshold satisfies: \( e_1(x^*_\delta, D) = \Phi x^*_\delta, D - \Delta c_1 - I \).

The general solution to the set of equations (C.1) is

\[
e_1(x, c_1) = \begin{cases} 
Ax^\xi + Bx^{\nu} + \lambda x - (1 - \tau) \frac{\sigma^2}{2} x^2 e_1''(x) \left( \frac{1}{\mu} \right) \left( \frac{1}{\xi(1 + \Delta c_1) + (1 - \tau) c_1} \right), & x^*_1 < x < x^*_\delta, D, \\
Cy^n + Dx^\nu + \frac{1 - r + \delta \theta}{r + \delta y} x - \frac{\delta \theta (1 + \Delta c_1) + (1 - \tau) c_1}{r + \delta y}, & x \geq x^*_\delta, D,
\end{cases}
\]

where \( \xi > 1 \) and \( \eta < 0 \) are defined in Proposition 1, \( \nu < 0 \) is defined in Proposition 2 and

\[
\zeta = (\sigma^2 - \mu)/\sigma^2 - \sqrt{[(\mu - \sigma^2)/\sigma^2]^2 + 2(r + \delta \theta)/\sigma^2} > 1.
\]

The linear growth condition implies that \( D = 0 \). Plugging the expressions in equation (C.2) in the other boundary conditions yields the desired result.

C.2. Value of corporate debt

The value of corporate debt satisfies the system of ODEs

\[
rd_1(x, c_1) = \mu x d_1'(x, c_1) + \frac{\sigma^2}{2} x^2 d_1''(x, c_1) + c_1, \quad x < x^*_\delta, D, \\
r d_1(x, c_1) = \mu x d_1'(x, c_1) + \frac{\sigma^2}{2} x^2 d_1''(x, c_1) + c_1 + \delta [\Delta c_1 - d_1(x, c_1)], \quad x \geq x^*_\delta, D
\]

which is solved subject to \( d_1(x^*_1, c_1) = (1 - \alpha) \Delta x^*_1, \). In addition, the value of corporate debt must the continuity and smoothness conditions: \( \lim_{x \uparrow x^*_1, D} d_1(x, c_1) = \lim_{x \downarrow x^*_1, D} d_1(x, c_1), \) and \( \lim_{x \downarrow x^*_\delta, D} d_1'(x, c_1) = \lim_{x \uparrow x^*_\delta, D} d_1'(x, c_1) \), where derivatives are taken with respect to \( x \).

The general solution to the set of equations (C.4) is

\[
d_1(x, c_1) = \begin{cases} 
Ex^\xi + Fx^{\nu} + \frac{\sigma^2}{2} x^2 + c_1, & x^*_1 < x < x^*_\delta, D, \\
Gx^n + Hx^\nu + \frac{c_1 + \delta \Delta c_1}{r + \delta}, & x \geq x^*_\delta, D,
\end{cases}
\]
Since the value of corporate debt must converge to the value of risk-free debt when the cash flow shocks grows without bounds, it must be that $H = 0$. Plugging the expressions in equation (C.5) in the other boundary conditions yields

$$E = \frac{\eta \left( \frac{c_1 + \delta \Delta c_1}{r} - \frac{c_1}{r} \right) + (\nu - \eta) \left[ (1 - \alpha) \Lambda x_{1,D}^* - \frac{c_1}{r} \right] \left( \frac{x_{1,D}^*}{x_{1,D}} \right)^\nu}{\left( \eta - \xi \right) \left( x_{\delta,D}^* \right) - \left( \eta - \nu \right) \left( x_{1,D}^* \right) \left( \frac{x_{1,D}^*}{x_{1,D}} \right)^\nu}, \quad (C.6)$$

$$F = \left[ (1 - \alpha) \Lambda x_{1,D}^* - \frac{c_1}{r} - E \left( x_{1,D}^* \right) \right] \left( \frac{1}{x_{1,D}^*} \right)^\nu, \quad (C.7)$$

$$G = \left( \frac{1}{x_{\delta,D}^*} \right)^\eta \left\{ \frac{\xi E \left( x_{\delta,D}^* \right) \xi + \nu \left( x_{\delta,D}^* \right)^\nu}{\eta \left( x_{1,D}^* \right)^\nu} \left[ (1 - \alpha) \Lambda x_{1,D}^* - \frac{c_1}{r} - E \left( x_{1,D}^* \right) \right] \right\}, \quad (C.8)$$

The value of the firm is then

$$v_1 (x, c_1) = \begin{cases} (A + E) x^\xi + (B + F) x^\nu + \Lambda x + \tau \frac{c_1}{r}, & x_{1,D}^* < x < x_{\delta,D}^*, \\ (C + G) x^\eta + \frac{1 - \tau + \delta \Delta \xi}{r + \delta}, & x \geq x_{\delta,D}^*, \end{cases}$$

where the initial coupon payment is defined by $\partial v_1 (x, c_1) / \partial c_1 = 0$.

### C.3. Values of corporate securities with equity financing

Assume that the capital expenditure is financed with equity. Using similar steps as above, it is immediate to show that the value of equity after investment satisfies

$$e_2 (x, c_1) = \pi \Lambda x - \frac{1 - \tau}{r} c_1 - \frac{(1 - \tau) c_1}{r (\nu - 1)} \left( \frac{x}{x_{2,D}^*} \right)^\nu, \quad (C.10)$$

where the default threshold after investment, denoted by $x_{2,D}^*$, satisfies equation (A.3) with $c$ replaced by $c_1$. The value of corporate debt after investment is in turn defined by

$$d_2 (x, c_1) = \frac{c_1}{r} \left[ 1 - \left( \frac{x}{x_{2,D}^*} \right)^\nu \right] + (1 - \alpha) \pi \Lambda x_{2,D}^* \left( \frac{x}{x_{2,D}^*} \right)^\nu, \quad (C.11)$$

and the boundary condition at the investment threshold is given by: $e_1 (x_E^*, c_1) = e_2 (x_{E}^*, c_1) - I$. To simplify the exposition of results, we denote by $\Sigma (x)$ the present value of $1$ to be received at the time of investment, contingent on investment occurring before default, and by $\Sigma (x)$ the present value of $1$ to be received at the time of default, contingent on default occurring before investment. Using these definitions, it is immediate to show that over the region $x_{1,D}^* \leq x \leq x_{E}^*$ the value of equity satisfies:

$$e_1 (x, c_1) = \Lambda x - \frac{1 - \tau}{r} c_1 - \Sigma (x) \left[ \Lambda x_{1,D}^* - \frac{1 - \tau}{r} c_1 \right] + \Sigma (x) \left[ (\pi - 1) \Lambda x_{E}^* - I + \frac{(1 - \tau) c_1}{r (\nu - 1)} \left( \frac{x_{E}^*}{x_{2,D}^*} \right)^\nu \right] \quad (C.12)$$
where [see e.g. Revuz and Yor (1999, pp72)]:

\[
\Sigma(x) = \frac{x^{\xi}(x_{1,D}^*)^\nu - x^\nu(x_{1,D}^*)^\xi}{(x_E^*)^\xi (x_{1,D}^*)^\nu - (x_E^*)^\nu (x_{1,D}^*)^\xi}, \quad \text{and} \quad \Sigma(x) = \frac{x^\nu (x_E^*)^\xi - x^\xi (x_E^*)^\nu}{(x_E^*)^\xi (x_{1,D}^*)^\nu - (x_E^*)^\nu (x_{1,D}^*)^\xi}.
\]

(C.13)

Similarly, the value of corporate debt is given by

\[
d_1(x, c_1) = \frac{c_1}{r} \left[ 1 - \Sigma(x) - \Sigma(x) \left( \frac{x_E^*}{x_{2,D}^*} \right)^\nu \right] + (1 - \alpha) \Lambda x_{1,D} \Sigma(x) + (1 - \alpha) \pi \Lambda x_{2,D} \Sigma(x) \left( \frac{x_E^*}{x_{2,D}^*} \right)^\nu,
\]

(C.14)

and the value of the firm satisfies

\[
v_1(x, c_1) = \Lambda x + \Sigma(x) \left[ (\pi - 1) \Lambda x_E - I \right] - \alpha \Lambda x_{1,D} \Sigma(x) + \frac{\tau c_1}{r} \left[ 1 - \Sigma(x) - \Sigma(x) \left( \frac{x_E^*}{x_{2,D}^*} \right)^\nu \right] - \alpha \pi \Lambda x_{2,D} \Sigma(x) \left( \frac{x_E^*}{x_{2,D}^*} \right)^\nu.
\]

(C.15)

The initial coupon payment is then obtained by plugging (C.18) in condition (C.10) while the value-maximizing investment and default triggers \(x_{1,D}^*\) and \(x_{2,D}^*\) solve the smooth-pasting conditions: \(e_1'(x_{1,D}^*) = 0\), and \(e_1'(x_{2,D}^*) = e_2'(x_{2,D}^*, c_1)\), where derivatives are taken with respect to \(x\). Assume that there exists a solution \(y > 1\) to the non-linear equation

\[
\frac{\xi}{\xi - \nu} [rI + (1 - \tau) c_1 y^\nu] = \frac{y^\nu + (\pi - 1) y \nu (\xi - 1) [rI + (1 - \tau) c_1 y^\xi]}{y^\xi + (\pi - 1) y^\nu} \frac{1}{(\nu - 1)(\xi - \nu)} + \frac{(1 - \tau)c_1}{(1 - \nu)} \left( \frac{y}{x_{2,D}^*} \right)^\nu \left( \frac{\nu rI + (1 - \tau) c_1 y^\xi}{\nu - 1 r\Lambda [y^\xi + (\pi - 1) y]} \right)^\nu.
\]

(C.16)

then the investment and default thresholds \(x_{1,D}^*\) and \(x_{2,D}^*\) respectively satisfy

\[
x_{1,D}^* = y x_{1,D}^* \quad \text{and} \quad x_{2,D}^* = \frac{\nu rI + (1 - \tau) c_1 y^\xi}{\nu - 1 r\Lambda [y^\xi + (\pi - 1) y]}.
\]

(C.17)

**D. Credit supply and credit lines**

Assume that the capital expenditure is financed with the credit line and equity. Using similar steps as above, it is possible to show that equity value after investment satisfies

\[
e_2(x, c_1) = \pi \Lambda x - \frac{1 - \tau}{r} (c_1 + r^f f I) - \frac{(1 - \tau)(c_1 + r^f f I)}{r(\nu - 1)} \left( \frac{x}{x_{2,f}^*} \right)^\nu,
\]

(D.1)

where the default threshold after investment, denoted by \(x_{2,f}^*\), satisfies equation (A.3) with \(c\) replaced by \(c_1 + r^f f I\). Before investment, the firm delivers a cash flow stream \((1 -
where the stochastic discount factors $\tau \cdot (x - c_1 - \phi f I)$. In addition to this cash flow stream, equity investors get capital gains $E[\delta e_{1,D}]$ over each interval $dt$. Using Itô’s lemma, it is then immediate to show that firm value solves the following ordinary differential equation before investment:

$$r e_1(x, c_1) = \mu x e_1'(x, c_1) + \frac{\sigma^2}{2} x^2 e_1''(x, c_1) + (1 - \tau)(x - c_1 - \phi f I), \quad x \geq x^*_1, D \quad (D.3)$$

The general solution to this equation is

$$e_1(x, c_1) = \Lambda x - \frac{1 - \tau}{r} (c_1 + \phi f I) + M x^\xi + N x^\nu, \quad (D.4)$$

where $M$ and $N$ are constant parameters and $\xi$ and $\nu$ are the positive and negative roots of the quadratic equation $\frac{1}{2} \sigma^2 y (y - 1) + \mu y - r = 0$. The value of equity solves the value-matching and smooth-pasting conditions $e_1(x^*_1, D, c_1) = 0$ and $e'_1(x^*_1, D, c_1) = 0$ at the default trigger. In addition, it solves the value-matching and smooth-pasting conditions $e_1(x^*_E, c_1) = e_2(x^*_E, c_1) - (1 - f) I$ and $e'_1(x^*_E, c_1) = e'_2(x^*_E, c_1)$ at the investment trigger. Using these equations and simple algebraic manipulations, we get that over the region $x^*_1, D \leq x \leq x^*_E$ the value of equity satisfies:

$$e_1(x, c_1) = \Lambda x - \frac{1 - \tau}{r} c_1 - \Sigma(x) \left[ \Lambda x_1^*, D - \frac{1 - \tau}{r} c_1 \right] - \left( \frac{1 - \tau}{r} \phi f I \right) \left[ 1 - \Sigma(x) - \Sigma(x) D \right]$$

$$+ \Sigma(x) \left\{ (\pi - 1) \Lambda x_E^* - (1 - f) I - (1 - \tau) \left[ \frac{r f I}{r} + \frac{c_1 + r f I}{r (\nu - 1)} \left( \frac{x_E^*}{x^*_2} \right)^\nu \right] \right\}$$

where the stochastic discount factors $\Sigma(x)$ and $\Sigma(x)$ are defined in equation (C.14). Similarly, the value of corporate debt is given by

$$d_1(x, c_1) = \frac{c_1}{r} \left[ 1 - \Sigma(x) - \Sigma(x) \left( \frac{x_E^*}{x^*_2} \right)^\nu \right] + (1 - \alpha) \pi \Lambda x^*_2, D \Sigma(x) \left( \frac{x_E^*}{x^*_2} \right)^\nu \quad (D.6)$$

$$+ (1 - \alpha) \Lambda x^*_1, D \Sigma(x) + \frac{\phi f I}{r} \left[ 1 - \Sigma(x) - \Sigma(x) \right] + \Sigma(x) \frac{r f I}{r} \left[ 1 - \left( \frac{x_E^*}{x^*_2} \right)^\nu \right],$$

and the value of the firm satisfies

$$v_1(x, c_1) = \Lambda x + \Sigma(x) \left[ (\pi - 1) \Lambda x^*_E - (1 - f) I \right] - \alpha \Lambda x^*_1, D \Sigma(x) \quad (D.7)$$

$$+ \frac{\tau c_1}{r} \left[ 1 - \Sigma(x) - \Sigma(x) \left( \frac{x_E^*}{x^*_2} \right)^\nu \right] + \frac{\tau \phi f I}{r} \left[ 1 - \Sigma(x) - \Sigma(x) \right]$$

$$+ \Sigma(x) \frac{r f I}{r} \left[ 1 - \left( \frac{x_E^*}{x^*_2} \right)^\nu \right] - \alpha \pi \Lambda x^*_2, D \Sigma(x) \left( \frac{x_E^*}{x^*_2} \right)^\nu$$

The initial coupon payment is then defined by $\partial v_1(x, c_1) / \partial c_1 = 0$. 

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E. Time-varying credit supply

This Appendix derives the value of equity $v_{i,D}(x)$ with time varying credit supply. In the following we slightly abuse notation and write $v_i(x)$ for $v_{i,D}(x)$. As long as it is in operation, the firm delivers a cash flow stream $(1-\tau)x$. In addition to this cash flow stream, investors also get capital gains $E[dv_i]$ over each interval $dt$. The required rate of return for investing in the firm’s assets is $r$. It is then immediate to show that the firm value solves:

- In the region $x \leq x_{L,D}^*$,
  
  \[
  rv_H(x) = \mu xv'_H(x) + \frac{\sigma^2}{2}x^2v''_H(x) + \lambda_H [v_L(x) - v_H(x)] + (1-\tau)x, \quad (E.1)
  \]
  
  \[
  rv_L(x) = \mu xv'_L(x) + \frac{\sigma^2}{2}x^2v''_L(x) + \lambda_L [v_H(x) - v_L(x)] + (1-\tau)x, \quad (E.2)
  \]

- In the region $x_{L,D}^* \leq x \leq x_{H,D}^*$,
  
  \[
  rv_H(x) = \mu xv'_H(x) + \frac{\sigma^2}{2}x^2v''_H(x) + \lambda_H [v_L(x) - v_H(x)] + (1-\tau)x, \quad (E.3)
  \]
  
  \[
  rv_L(x) = \mu xv'_L(x) + \frac{\sigma^2}{2}x^2v''_L(x) + \lambda_L [v_H(x) - v_L(x)] + (1-\tau)x + \delta_L \theta_H [\Phi x - I - v_L(x)]. \quad (E.4)
  \]

- In the region $x \geq x_{H,D}^*$,
  
  \[
  rv_H(x) = \mu xv'_H(x) + \frac{\sigma^2}{2}x^2v''_H(x) + \lambda_H [v_L(x) - v_H(x)] + (1-\tau)x + \delta_L \theta_H [\Phi x - I - v_H(x)], \quad (E.5)
  \]
  
  \[
  rv_L(x) = \mu xv'_L(x) + \frac{\sigma^2}{2}x^2v''_L(x) + \lambda_L [v_H(x) - v_L(x)] + (1-\tau)x + \delta_L \theta_L [\Phi x - I - v_L(x)]. \quad (E.6)
  \]

This system of ODEs is solved subject to the following boundary conditions:

\[
\begin{align*}
& v_i(0) = 0, \quad \text{for } i = L, H. \quad (E.7) \\
& v_i(x) \leq \psi x, \quad \text{for } i = L, H. \quad (E.8) \\
& \lim_{x \to x_i^+} v_i(x) = \lim_{x \to x_i^+} v_i(x), \quad \text{for } i = L, H, \quad (E.9) \\
& \lim_{x \to x_i^-} v_i'(x) = \lim_{x \to x_i^-} v_i'(x), \quad \text{for } i = L, H, \quad (E.10) \\
& \lim_{x \to x_i^+} v_i(x) = \lim_{x \to x_i^+} v_i(x), \quad \text{for } i = L, H, j = L, H, \text{ and } i \neq j, \quad (E.11) \\
& \lim_{x \to x_i^-} v_i'(x) = \lim_{x \to x_i^-} v_i'(x), \quad \text{for } i = L, H, j = L, H, \text{ and } i \neq j. \quad (E.12)
\end{align*}
\]

where derivatives are taken with respect to $x$. 

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In the analysis below, we denote the regions \( x \leq x_{L,D}^*, \ x_{L,D}^* \leq x \leq x_{H,D}^* \), and \( x \geq x_{H,D}^* \) as regions \( n = 1, 2, \) and \( 3 \) respectively. In addition, before presenting the solution to the firm's optimization problem, we introduce the following notations. For \( i = L, H, \) we define the polynomials \( P_{n}(\gamma) \) as

\[
P_{n}(\gamma) = r + \lambda_{i} - \gamma \mu - 0.5\sigma^{2}\gamma(\gamma - 1) + \delta_{i}\theta_{i}\{n=3]\cup\{n=2,i=L\}, \quad \text{where} \ n = 1, 2, 3, \quad (E.13)
\]

and we let

\[
h_{n}(\gamma) = P_{Hn}(\gamma)P_{Ln}(\gamma) - \lambda_{H}\lambda_{L}, \quad \text{for} \ n = 1, 2, 3. \quad (E.14)
\]

We then have the following intermediate result (see the Appendix):

**Lemma 6** Each of the quartic equations

\[
h_{n}(\gamma) = 0, \quad n = 1, 2, 3 \quad (E.15)
\]

has four real roots: two positive roots \( \gamma_{1n} \) and \( \gamma_{2n} \) and two negative roots \( \gamma_{3n} \) and \( \gamma_{4n} \). In addition, when \( r + \lambda_{i} + \delta_{i}\theta_{i} > \mu, \) the positive roots \( \gamma_{13} \) and \( \gamma_{23} \) are larger than 1.

**Proof.** Fix an \( n \). The quadratic equation \( P_{Hn}(\gamma) = 0 \) has one positive root \( \beta_{1} \) and one negative root \( \beta_{2} \) because \( \beta_{1}\beta_{2} = -2(r + \lambda_{H})/\sigma^{2} < 0 \). Therefore, the function \( h_{n} \) satisfies

\[
h_{n}(\infty) > 0, \ h_{n}(\beta_{2}) < 0, \ h_{n}(0) > 0, \ h_{n}(\beta_{1}) < 0, \ h_{n}(+\infty) > 0,
\]

and the first claim follows from the intermediate value theorem.

Assume without loss of generality that \( \gamma_{13} < \gamma_{23} \). Suppose on the contrary that one of the roots is smaller than one. Since \( h_{3}(\gamma) < 0 \) for all \( \gamma \in (\gamma_{13}, \gamma_{23}) \) and, by assumption, \( h_{3}(1) > 0 \), we must have \( \gamma_{23} < 1 \). Since the function \( h_{3}'(\gamma) \) has its largest zero between \( \gamma_{13} \) and \( \gamma_{23} \) (this is a local minimum of \( h_{n} \)), the function \( h_{3}(\gamma) \) is monotone increasing in \( \gamma \) for all \( \gamma > \gamma_{23} \). In particular, \( h_{3}(\gamma) \) is monotone increasing in a small neighborhood of \( \gamma = 1 \). Now, the functions \( P_{H3}(\gamma) \) and \( P_{L3}(\gamma) \) attain their maxima at \( \gamma = 0.5 - \mu \sigma^{-2} < 1 \). Therefore, both \( P_{H3}(\gamma) \) and \( P_{L3}(\gamma) \) are positive and monotone decreasing in a small neighborhood of \( \gamma = 1 \). Consequently, \( h_{3}(\gamma) = P_{H3}(\gamma)P_{L3}(\gamma) \) is also monotone decreasing in \( \gamma \) in a small neighborhood of \( \gamma = 1 \). This is a contradiction.

Using the set of ODEs (E.1)-(E.6) and the boundary conditions (E.7)-(E.12), it is possible to characterize value of equity in each of the three regions (no investment, investment in regime \( L \), and investment in both regimes). Let

\[
(\Delta H_{1}, \Delta L_{1}) = (0, 0), \ (\Delta H_{2}, \Delta L_{2}) = (0, \delta_{L}\theta_{L}), \ \text{and} \ (\Delta H_{3}, \Delta L_{3}) = (\delta_{H}\theta_{H}, \delta_{L}\theta_{L}).
\]

In addition, denote

\[
A_{L1} = A_{H1} = A_{H2} = 1 - \tau, \ A_{L2} = A_{L3} = \delta_{L}\theta_{L}\Phi + 1 - \tau, \ \text{and} \ A_{H3} = \delta_{H}\theta_{H}\Phi + 1 - \tau
\]

and

\[
B_{L1} = B_{H1} = B_{H2} = 0, \ B_{L2} = B_{L3} = -\delta_{L}\theta_{L}I, \ B_{H3} = -\delta_{H}\theta_{H}I.
\]

A direct calculation implies that the following is true:
Lemma 7 Fix a region \( n \). The general solution to the system
\[
(r + \lambda_H + \Delta_{Hn}) v_H(x) = \mu x v_H'(x) + \frac{\sigma^2}{2} x^2 v''_H(x) + \lambda_H v_L(x) + A_{Hn} x + B_{Hn},
\]
\[
(r + \lambda_L + \Delta_{Ln}) v_L(x) = \mu x v_L'(x) + \frac{\sigma^2}{2} x^2 v''_L(x) + \lambda_L v_H(x) + A_{Ln} x + B_{Ln},
\]
is given by
\[
v_H(x) = \sum_j K_{jn} x^{\gamma_{jn}} + x \frac{\lambda_H A_{Ln} + P_{Ln}(1) A_H}{h_n(1)} + \frac{\lambda_H B_{Ln} + P_{Ln}(0) B_{Hn}}{h_n(0)},
\]
\[
v_L(x) = \sum_j P_{Ln} \left( \frac{1}{\lambda_H} \right) K_{jn} x^{\gamma_{jn}} + x \frac{\lambda_L A_{Ln} + P_{Ln}(1) A_L}{h_n(1)} + \frac{\lambda_L B_{Ln} + P_{Ln}(0) B_{Ln}}{h_n(0)},
\]
where the \( K_{jn} \) are arbitrary constants.

Using conditions (E.7), we have \( K_{31} = K_{41} = 0 \). In addition, conditions (E.8) together with Lemma 8 imply \( K_{13} = K_{23} = 0 \). Define
\[
\Theta = \lambda_H A_{L} + \frac{P_{L}(1) A_{H}}{h_2(1)} - \frac{\lambda_H A_{L} + P_{L}(1) A_{H}}{h_1(1)},
\]
\[
\Xi = \lambda_H B_{L} + \frac{P_{L}(0) B_{H}}{h_2(0)} - \frac{\lambda_H B_{L} + P_{L}(0) B_{H}}{h_1(0)}.
\]
Using the set of smooth-pasting and value-matching conditions, it is immediate to show that \( K_{11} \) and \( K_{21} \) solve
\[
(\gamma_{21} - \gamma_{11}) K_{11} \left( x_{L,D}^{*} \right)^{\gamma_{11}} = \sum_{j=1}^{4} \left( \gamma_{21} - \gamma_{j2} \right) K_{j2} \left( x_{L,D}^{*} \right)^{\gamma_{j2}} + (\gamma_{21} - 1) x_{L,D}^{*} \Theta + \gamma_{21} \Xi
\]
and
\[
(\gamma_{11} - \gamma_{21}) K_{21} \left( x_{L,D}^{*} \right)^{\gamma_{21}} = \sum_{j=1}^{4} \left( \gamma_{11} - \gamma_{j2} \right) K_{j2} \left( x_{L,D}^{*} \right)^{\gamma_{j2}} + (\gamma_{11} - 1) x_{L,D}^{*} \Theta + \gamma_{11} \Xi.
\]
Similarly, define
\[
\Upsilon = \lambda_H A_{L} + \frac{P_{L}(1) A_{H}}{h_2(1)} - \frac{\lambda_H A_{L} + P_{L}(1) A_{H}}{h_3(1)},
\]
\[
\Psi = \lambda_H B_{L} + \frac{P_{L}(0) B_{H}}{h_2(0)} - \frac{\lambda_H B_{L} + P_{L}(0) B_{H}}{h_3(0)}.
\]
It is possible to show that the constant parameters \( K_{33} \) and \( K_{43} \) solve
\[
(\gamma_{43} - \gamma_{33}) K_{33} \left( x_{H,D}^{*} \right)^{\gamma_{33}} = \sum_{j=1}^{4} \left( \gamma_{43} - \gamma_{j2} \right) K_{j2} \left( x_{H,D}^{*} \right)^{\gamma_{j2}} + (\gamma_{43} - 1) x_{H,D}^{*} \Upsilon + \gamma_{43} \Psi.
\]
and

\[(\gamma_{33} - \gamma_{43}) K_{43} (x_{H,D}^*)^{\gamma_{43}} = \sum_{j=1}^{4} (\gamma_{33} - \gamma_{j2}) K_{j2} (x_{H,D}^*)^{\gamma_{j2}} + (\gamma_{33} - 1) x_{H,D}^* Y + \gamma_{33} \Psi.\]

To complete the proof of Proposition 6, we need to write down the system of equations for the parameters \(K_{j2}\) for \(j = 1, \ldots, 4\). To this end define

\[\kappa_{jH} = \frac{P_{H3}(\gamma_{33}) \gamma_{43} - \gamma_{j2}}{\lambda_H (\gamma_{43} - \gamma_{33})} - \frac{\gamma_{j2}}{\lambda_H (\gamma_{43} - \gamma_{33})} - \frac{P_{H2}(\gamma_{j2})}{\lambda_H} \]

and

\[\eta_{jH} = \frac{P_{H3}(\gamma_{33}) \gamma_{33}(\gamma_{43} - \gamma_{j2})}{\lambda_H (\gamma_{43} - \gamma_{33})} - \frac{P_{H3}(\gamma_{43}) \gamma_{43}(\gamma_{33} - \gamma_{j2})}{\lambda_H (\gamma_{43} - \gamma_{33})} - \frac{P_{H2}(\gamma_{j2})}{\lambda_H} \]

\[\kappa_{jL} = \frac{P_{H1}(\gamma_{11}) \gamma_{21} - \gamma_{j2}}{\lambda_H (\gamma_{21} - \gamma_{11})} - \frac{P_{H1}(\gamma_{21}) \gamma_{11} - \gamma_{j2}}{\lambda_H (\gamma_{21} - \gamma_{11})} - \frac{P_{H2}(\gamma_{j2})}{\lambda_H}, \]

\[\eta_{jL} = \frac{P_{H1}(\gamma_{11}) \gamma_{11}(\gamma_{21} - \gamma_{j2})}{\lambda_H (\gamma_{21} - \gamma_{11})} - \frac{P_{H1}(\gamma_{21}) \gamma_{11} - \gamma_{j2}}{\lambda_H (\gamma_{21} - \gamma_{11})} - \frac{P_{H2}(\gamma_{j2})}{\lambda_H} \]

Let also

\[\zeta_{H} = \Psi + \frac{P_{H3}(\gamma_{43}) \gamma_{33} - \gamma_{j2}}{\lambda_H (\gamma_{43} - \gamma_{33}) - \gamma_{j2}} \left( \frac{\lambda_H A_{L2} + P_{L2}(1) A_{H2}}{h_{2}(1)} - \frac{\lambda_H A_{L3} + P_{L3}(1) A_{H3}}{h_{3}(1)} \right) \]

and

\[\xi_{H} = \Psi + \frac{P_{H1}(\gamma_{43}) \gamma_{33} - \gamma_{j2}}{\lambda_H (\gamma_{43} - \gamma_{33}) - \gamma_{j2}} \left( \frac{\lambda_H B_{L2} + P_{L2}(0) B_{H2}}{h_{2}(0)} - \frac{\lambda_H B_{L3} + P_{L3}(0) B_{H3}}{h_{3}(0)} \right) \]

and

\[\mu_{H} = \frac{\lambda_L A_{H2} + P_{H2}(1) A_{L2}}{h_{2}(1)} - \frac{\lambda_L A_{H3} + P_{H3}(1) A_{L3}}{h_{3}(1)} \]

\[+ \frac{\lambda_{H} (\gamma_{43} - \gamma_{33}) - \gamma_{j2}}{\lambda_{H} (\gamma_{43} - \gamma_{33}) - \gamma_{j2}} \left( \frac{\lambda_{H} B_{L2} + P_{L2}(0) B_{H2}}{h_{2}(0)} - \frac{\lambda_{H} B_{L3} + P_{L3}(0) B_{H3}}{h_{3}(0)} \right) \]

and

\[\nu_{H} = \frac{P_{H1}(\gamma_{43}) \gamma_{43} \gamma_{33} - \gamma_{j2}}{\lambda_{H} (\gamma_{43} - \gamma_{33}) - \gamma_{j2}} \left( \frac{\lambda_{H} B_{L2} + P_{L2}(0) B_{H2}}{h_{2}(0)} - \frac{\lambda_{H} B_{L3} + P_{L3}(0) B_{H3}}{h_{3}(0)} \right) \]

and similarly,

\[\zeta_{L} = \frac{\lambda_{L} A_{H2} + P_{H2}(1) A_{L2}}{h_{2}(1)} - \frac{\lambda_{L} A_{H1} + P_{H1}(1) A_{L1}}{h_{1}(1)} + \frac{P_{H1}(\gamma_{21}) (\gamma_{11} - 1) - P_{H1}(\gamma_{11}) (\gamma_{21} - 1)}{\lambda_{H} (\gamma_{21} - \gamma_{11})} \]
and
\[\xi_L = \frac{\lambda_L B_{H2} + P_{H2}(0) B_{L2}}{h_2(0)} - \frac{\lambda_L B_{H1} + P_{H1}(0) B_{L1}}{h_1(0)} + \Xi P_{H1}(\gamma_{21}) \gamma_{11} - P_{H1}(\gamma_{11}) \gamma_{21}}{\lambda_H (\gamma_{21} - \gamma_{11})}\]
and
\[\mu_L = \frac{\lambda_L A_{H2} + P_{H2}(1) A_{L2}}{h_2(1)} - \frac{\lambda_L A_{H1} + P_{H1}(1) A_{L1}}{h_1(1)} + \Theta \frac{P_{H1}(\gamma_{21}) \gamma_{11} - P_{H1}(\gamma_{11}) \gamma_{21}}{\lambda_H (\gamma_{21} - \gamma_{11})}\]
and
\[\nu_L = \Xi P_{H1}(\gamma_{21}) \gamma_{11} - P_{H1}(\gamma_{11}) \gamma_{11} \gamma_{21}}{\lambda_H (\gamma_{21} - \gamma_{11})}\]

Now, let \( R = x_{L,D}^*/x_{H,D}^* \) and

\[M(R) = \begin{pmatrix}
\kappa_{1H} & \kappa_{2H} & \kappa_{3H} & \kappa_{4H} \\
\eta_{1H} & \eta_{2H} & \eta_{3H} & \eta_{4H} \\
\kappa_{1L} R_{\gamma_{12}} & \kappa_{2L} R_{\gamma_{22}} & \kappa_{3L} R_{\gamma_{32}} & \kappa_{4L} R_{\gamma_{42}} \\
\eta_{1L} R_{\gamma_{12}} & \eta_{2L} R_{\gamma_{22}} & \eta_{3L} R_{\gamma_{32}} & \eta_{4L} R_{\gamma_{42}}
\end{pmatrix}\]

Then, we can rewrite the system for the vector \( K = (K_{j2}) \) as

\[M(R) \text{diag}((x_{H,D}^*)^{\gamma_{j2}}) K = x_{H,D}^* \begin{pmatrix}
\xi_H \\
\mu_H \\
\xi_L R \\
\mu_L R
\end{pmatrix} + \begin{pmatrix}
\xi_H \\
\nu_H \\
\xi_L \\
\nu_L
\end{pmatrix}\]

Denote

\[\alpha(R) = (\alpha_j(R)) = (M(R))^{-1} \begin{pmatrix}
\xi_H \\
\mu_H \\
\xi_L R \\
\mu_L R
\end{pmatrix}, \quad \text{and} \quad \beta(R) = (\beta_j(R)) = (M(R))^{-1} \begin{pmatrix}
\xi_H \\
\nu_H \\
\xi_L \\
\nu_L
\end{pmatrix}\]

Then,

\[\text{diag}((x_{H,D}^*)^{\gamma_{j2}}) K = x_{H,D}^* \alpha(R) + \beta(R)\]

Combining these results, we can establish the following proposition, which gives the value of equity for any given investment policy.

**Proposition 8** Assume that assets in place that generate a continuous flow of operating income \( f(X_t) = (1 - \tau) X_t \) and that the firm has the opportunity to increase capacity and cash flows by paying a lump sum \( I \). Then, when the firm issues debt to finance the capital...
expenditure and the arrival rate of debt investors is given by $\delta_i > 0$, for $i = L, H$, the value of equity in regime $i = L, H$ is given by

$$v_L(x) = \begin{cases} 
\sum_{j=1}^{2} \frac{P_{H1}(\gamma_{j1})}{\lambda_H} K_{j1} x^{\gamma_{j1}} + \Lambda x, & x \leq x^*_L, \\
\frac{P_{H2}(\gamma_{j2})}{\lambda_H} K_{j2} x^{\gamma_{j2}} + x^{\lambda_H(1-\tau)+P_{H2}(1)(\delta_H H+1-\tau)} - \frac{(r+\lambda_H)\delta_{L1} I}{h_2(0)}, & x_L \leq x \leq x_{H,D}^*, \\
\frac{P_{H3}(\gamma_{j3})}{\lambda_H} K_{j3} x^{\gamma_{j3}} + x^{\lambda_H(\delta_H H+1-\tau)+P_{H3}(1)(\delta_H H+1-\tau)} - I \frac{\lambda_H \Phi + P_{H3}(0)\delta_H H}{h_3(0)}, & x \geq x_{H,D}^*, 
\end{cases}$$

and

$$v_H(x) = \begin{cases} 
\sum_{j=1}^{2} K_{j1} x^{\gamma_{j1}} + \Lambda x, & x \leq x^*_L, \\
\frac{4}{3} \sum_{j=1}^{4} K_{j2} x^{\gamma_{j2}} + x^{\frac{\lambda_H(\delta_H H+1-\tau)+P_{H2}(1)(1-\tau)}{h_2(0)}} - \frac{\lambda_H \delta_{L1} I}{h_2(0)}, & x_L \leq x \leq x_{H,D}^*, \\
\frac{4}{3} \sum_{j=3}^{4} K_{j3} x^{\gamma_{j3}} + x^{\frac{\lambda_H(\delta_H H+1-\tau)+P_{H3}(1)(\delta_H H+1-\tau)}{h_3(0)}} - I \frac{\lambda_H \Phi + P_{H3}(0)\delta_H H}{h_3(0)}, & x \geq x_{H,D}^*, 
\end{cases}$$

for $\delta_i = \delta_i \theta_i$ and where the endogenous default thresholds $x_{L,D}^*$ and $x_{H,D}^*$ are reported in Proposition 4, the parameters $K_{jn}$ are defined above, $\Lambda$ is defined in equation (2) and the exponents $\gamma_{jn}$ solve the quartic equations $h_n(\gamma) = 0$, for $n = 1, 2, 3$.

In Proposition 8, the investment thresholds $x_{L,D}^*$ and $x_{H,D}^*$ are determined endogenously by shareholders and depend on the arrival rate of investors. Specifically, the equity value-maximizing investment thresholds $x_{L,D}^*$ and $x_{H,D}^*$ uniquely satisfy the conditions:

$$v_L(x_{L,D}^*) = \Phi x_{L,D}^* - I, \quad \text{and} \quad v_H(x_{H,D}^*) = \Phi x_{H,D}^* - I.$$  

(38)

Using these value-matching conditions and Proposition 8, it is then possible to establish the following result:

**Proposition 9** If there exists a solution $R \in (0,1)$ to the nonlinear equation

$$I + \sum_{j=1}^{4} \frac{P_{H2}(\gamma_{j2})}{\lambda_H} R^{\gamma_{j2}} \beta_j(R) - \frac{(r+\lambda_H)\delta_{L1} I}{r\lambda_L + (r+\lambda_H)(r+\delta_{L1} I)} = 0$$

(39)

$$= \sum_{j=1}^{4} \frac{\lambda_H \delta_{L1} I}{r\lambda_L + (r+\lambda_H)(1-\tau)} + \frac{1}{\lambda_H(\delta_{H1} H+1-\tau)+P_{H2}(1)(\delta_H H+1-\tau)} - \Phi \sum_{j=1}^{4} \alpha_j(R) + \frac{\lambda_L(1-\tau) + (r+\lambda_H - \mu) (\delta_{L1} \Phi + 1-\tau)}{(r+\lambda_H - \mu)(r-\mu+\delta_{L1} \Phi+1-\tau)} - \Phi R$$

$$\times \left\{ \sum_{j=1}^{4} \frac{P_{H2}(\gamma_{j2})}{\lambda_H} R^{\gamma_{j2}} \alpha_j(R) + R^{\lambda_L(1-\tau) + (r+\lambda_H - \mu) (\delta_{L1} \Phi + 1-\tau)}{(r+\lambda_H - \mu)(r-\mu+\delta_{L1} \Phi+1-\tau)} - \Phi R \right\}$$
where the parameters \( \alpha_j(R) \) and \( \beta_j(R) \), for \( j = 1, \ldots, 4 \) are defined in Appendix E, then the investment policy that maximizes shareholder wealth when financing the capital expenditure with debt is characterized by the investment thresholds \( x_{L,D}^* \) and \( x_{H}^* \) satisfying

\[
x_{L,D}^* = R x_{H,D}^*,
\]

and

\[
x_{H,D}^* = \frac{I + \sum_{j=1}^{4} \beta_j(R) - \frac{\lambda_H \delta_L \theta_L I}{r \lambda_L + (r + \lambda_H)(r + \delta_L \theta_L)}}{\Phi - \sum_{j=1}^{4} \alpha_j(R) - \frac{\lambda_H (\delta_L \theta_L \Phi + 1 - \tau) + (r + \lambda_H - \delta_L \theta_L - \mu)(1 - \tau)}{(r + \lambda_H - \mu)(r + \delta_L \theta_L) + \lambda_L (r - \mu)}}.
\]

Figure 7 in Section 5 is based on the results of both Proposition 8 (right panel of the figure) and Proposition 9 (right panel of the figure).
References


Figure 1: Investment threshold with limited credit supply. Figure 1 plots the value-maximizing investment trigger $x_{\delta, D}^*$ as a function of the arrival rate of investors $\delta$, the share $\theta$ of the investment surplus captured by shareholders, the corporate tax rate $\tau$, and cash flow volatility $\sigma$. Input parameter values that roughly reflect a typical S&P 500 firm: the risk free rate $r = 5\%$; the growth rate and volatility of cash flows $\mu = 0.5\%$ and $\sigma = 20\%$, the corporate tax rate $\tau = 0.15$, liquidation costs $\alpha = 40\%$, and size of the firm’s growth option $\pi = 1.2$. 

![Graphs showing the investment threshold for different parameters](image-url)
Figure 2: Separating financing trigger. Figure 2 plots the separating financing trigger $x^*(\delta)$ as a function of the arrival rate of investors $\delta$, the share $\theta$ of the investment surplus captured by shareholders, the corporate tax rate $\tau$, and cash flow volatility $\sigma$. Input parameter values that roughly reflect a typical S&P 500 firm: the risk free rate $r = 5\%$; the growth rate and volatility of cash flows $\mu = 0.5\%$ and $\sigma = 20\%$, the corporate tax rate $\tau = 0.15$, liquidation costs $\alpha = 40\%$, and size of the firm’s growth option $\pi = 1.2$. 

![Graphs showing the separating financing trigger as a function of arrival rate, bargaining power, corporate tax rate, and bankruptcy costs.](image)
Figure 3: Probability of investment and credit supply. Figure 3 plots the probability of investment over a 3-year (left panel) and a 5-year (right panel) horizon as a function of the arrival rate of investors $\delta$, assuming that the capital expenditure is financed with debt. Input parameter values that roughly reflect a typical S&P 500 firm: the risk free rate $r = 5\%$; the growth rate and volatility of cash flows $\mu = 0.5\%$ and $\sigma = 20\%$, the corporate tax rate $\tau = 0.15$, liquidation costs $\alpha = 40\%$, and size of the firm’s growth option $\pi = 1.2$. 
Figure 4: Marginal financing decision. Figure 4 plots the separating financing trigger $x^*(\delta)$ as a function of the arrival rate of investors $\delta$, the share $\theta$ of the investment surplus captured by shareholders, the corporate tax rate $\tau$, and cash flow volatility $\sigma$ when the firm has debt outstanding before investment. Input parameter values that roughly reflect a typical S&P 500 firm: the risk free rate $r = 5\%$; the growth rate and volatility of cash flows $\mu = 0.5\%$ and $\sigma = 20\%$, the corporate tax rate $\tau = 0.15$, liquidation costs $\alpha = 40\%$, and size of the firm’s growth option $\pi = 1.2$. 
Figure 5: Initial financing decision. Figure 5 plots the value maximizing initial coupon payment $c_1^*$ as a function of the arrival rate of debtholders $\delta$ and the bargaining power of shareholders $\rho$. Input parameter values are set as in the base case environment.

Figure 6: Credit lines. Figure 6 plots shareholder wealth as a function of the arrival rate of debtholders $\delta$ for a high (left panel) and low (right panel) bargaining power of shareholders. The figure represents the value of the firm without the credit line (blue increasing curve) and with the credit line (red straight line). Input parameter values are set as in the base case environment.
Figure 7: Time-varying credit supply. Figure 7 plots the value-maximizing investment triggers (left panel) as well the financing separating trigger (right panel) as a function of the arrival rate of investors $\delta$ and the share $\theta$ of the investment surplus captured by shareholders. In this figure, we set the arrival rate of investors to $\delta_H = 3$ in the high state and let $\delta_L$ vary in the interval $[1,2.5]$. The blue (top) line in each panel represents the relevant threshold in the high state while the red (bottom) line represents the relevant threshold in the low state.