Overbidding in Fixed-Rate Tenders: The Role of Exposure Risk

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Overbidding in fixed rate tenders: the role of exposure risk*

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Abstract. The fixed rate tender is one of the main operational formats used by central banks in the implementation of their monetary policies. While academic research has largely dismissed the procedure for its tendency to encourage overbidding, central banks such as the ECB and the Bank of England have continued using it. We elaborate on this apparent conflict by considering an auction-theoretic setting with privately known declining marginal valuations. Since overbidding entails exposure risk, an equilibrium may exist even if bids are costless and the intended amount is pre-announced. In fact, rationing may occur with certainty. The equilibrium is also robust under adaptive expectations. However, the resulting allocation is typically inefficient. Empirical proxies of exposure risk are significant in both Euro and Sterling operations. Our findings have implications, in particular, for the potential re-introduction of rationing in the main refinancing operations of the Eurosystem.

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1. Introduction

The fixed rate tender is one of the main operational formats used by central banks in the implementation of their monetary policies. In a routine application of the procedure, a central bank would first announce the provision of an intended amount of interbank liquidity against payment of the interest rate and delivery of eligible collateral. Market participants would then submit “bids” that specify an amount of liquidity. Finally, the central bank would determine the allotment to each of the bidders. If all bids can be allotted in full, allotments correspond in size to individual bid amounts. In the more typical case, however, there is excess demand, and bids need to be prorated.

It is probably fair to say that academic research has largely dismissed the fixed rate tender because of its tendency to encourage manipulation and strategic overbidding. Indeed, the simple rationing scheme cannot work in a complete-information setting when there is excess demand. For example, consider \( n = 10 \) bidders with a demand of \( \theta_i = 0.2 \) each when the intended amount is 1 unit. If bidding was sincere, each bidder would obtain \( q_i = 0.1 \) after rationing. Therefore, bidders have an incentive to overstate their demand. But if bids are expected to be inflated, then there is an incentive to overbid even more. And so on. Thus, myopic best replies diverge, and there is no equilibrium. Also in reality, the fixed rate tender has not always
performed to the satisfaction of all market participants. Specifically, during a well-documented episode at the start of Stage III of the European Monetary Union, escalating levels of aggregate bids lost any relation to intended amounts. As a consequence, the procedure has been abandoned in June 2000.¹

Notwithstanding this experience, central banks such as the European Central Bank (ECB) and the Bank of England (BoE) have continued using the fixed rate tender. More specifically, in the Euro area, the fixed rate tender with rationing has been employed to absorb liquidity in fine-tuning operations, to distribute funds provided through the USD term auction facility, and to allot €/CHF foreign exchange swaps. In the UK, the procedure has been used in the weekly open market operations and in fine-tuning operations. Interestingly, the mechanism has tended to work quite smoothly in all these cases. Indeed, as Table I shows, the overbidding episode in the Euro area, with a mean log bid-to-cover ratio of 2.91, is a rather exceptional observation.²

To elaborate on the apparent conflict between existing theory and practical uses, and to better understand the determinants of operational per-

²The bid-to-cover ratio is the ratio of total bids over the intended amount. All logs are with respect to the natural base.
formance, we study the possibility of an equilibrium in a richer theoretical framework. Specifically, we assume that bidders have declining marginal valuations that are not observed by competing bidders. In this setting, strategic overbidding entails exposure risk, i.e., the risk of ending up with an overly large allotment. As will be shown, the presence of exposure risk has important consequences for bidding behavior.\footnote{A related exposure problem arises in the theory of multi-unit auctions. There, a bidder with increasing marginal valuations may end up making losses when competition is unexpectedly strong. See, e.g., Krishna and Rosenthal (1996).}

For example, in a liquidity-providing operation, a bidder that strongly overstates demand may end up with a very large allotment if competing bids realize significantly below expectations. That bidder not only receives excess liquidity but is also obliged to forward the corresponding amount of collateral. However, holding excess liquidity is costly, e.g., in terms of interest rate spreads. Moreover, the obligation to deliver collateral may cause high fees for securities lending when there is a shortage of eligible collateral after the tender. Thus, in this example, bidders dislike an oversized allotment, i.e., there is scope for exposure risk.\footnote{Nyborg and Strebulaev (2001) stress the possibility that bidders fear the scarcity of liquidity in the after-market. In the present example, exposure risk is essentially the mirror image of that effect, capturing a scarcity of collateral in the after-market.}

Because overbidding causes exposure, we can show that an equilibrium is sustainable even if bids are costless and the intended amount is pre-
announced. Intuitively, with exposure risk in place, incentives to overbid remain limited provided that competitors are expected to likewise overbid to a moderate extent only. Based on this result, existence can be verified using a standard existence theorem for Bayesian games. The identified equilibrium turns out to be robust. For example, it exists even when rationing is expected to occur with certainty. Moreover, in a dynamic setting, myopic best responses lead to a steady decline of overbidding factors down to the equilibrium level. Still, exposure risk is undesirable, i.e., the equilibrium allocation is typically inefficient.

To test the empirical relevance of exposure risk for bidding behavior in fixed rate tenders, we ran regressions of bid amounts in the weekly Euro and Sterling operations. Proxies of exposure risk turned out to be significant for both currency areas, confirming our main hypothesis. Moreover, exposure risks were less pronounced in the Euro area than in the UK. Our interpretation of these findings will be that, apparently, there was a mismatch between the collateral framework of the Eurosystem and its tender procedures, while there was no such mismatch in the case of the Bank of England. We also comment on the potential re-introduction of fixed rate tenders in the main refinancing operations of the Eurosystem.

The rest of the paper is structured as follows. In Section 2, we survey
some recent uses of the fixed rate tender. Section 3 introduces the theoretical framework. The existence theorem is stated and proved in Section 4. Section 5 contains an example which illustrates the possibility of rationing with probability one. A dynamic extension is considered in Section 6. Section 7 deals with the efficiency of the equilibrium allocation. An empirical test is conducted in Section 8. Section 9 reviews some related literature. Section 10 concludes. The Appendix contains a technical proof.

2. Recent uses of the fixed rate tender

This section briefly surveys recent uses of the fixed rate tender in the euro area and in the UK. As elsewhere in the paper, we focus on the fixed rate tender with proportional rationing, in which potential bidders are informed upfront about the intended amount (or have at least a good clue about it.) Not consistent with this definition are operations with pre-announced full allotment as used, e.g., by the Eurosystem following October 2008 in its main and longer-term refinancing operations. Also not consistent with the definition are operations as conducted, e.g., by the Swiss National Bank, where the central bank remains silent about the intended allotment prior to the operation.\footnote{See, e.g., Jordan and Kugler (2004).}

Altogether, we found six recent uses of the fixed rate tender with rationing
(cf. also Table I, where the Sterling samples have been split).

(i) Euro fine-tuning. Between May 2003 and October 2008, the ECB conducted 32 of its fine-tuning operations as fixed rate tenders with rationing. These operations were liquidity-absorbing, and bidders could anticipate the intended allotment relatively well. Figure 1 shows the development of the allotment quota along the settlement date. An occurrence of excessive over-bidding would appear in the figure as a very short bar. As can be seen, the performance of the rationing scheme was quite satisfactory in these operations.

(ii) Term auction facility. Between December 2007 and October 2008, the Eurosystem provided US dollar liquidity through 21 fixed rate tenders. These operations were conducted within the framework of the term auction facility. Two features were remarkable. First, the tender rate was determined as the marginal rate of the simultaneous Federal Reserve auction. Second, there was a maximum bid, equal to 10 percent of the pre-announced intended amount. Table II lists some key data for these operations. We remark that there is a clearly discernible downward trend in the allotment quota.

(iii) FX swaps. Between October 2008 and January 2010, the Eurosystem conducted 70 liquidity-absorbing €/CHF foreign exchange swap operations.

6The allotment quota is the ratio of the allotted amount over total bids.
The announcement included the price (i.e., swap points) as well as intended allotments. Figure 2 compares the intended amounts for the 1-week swaps to total bids. Intended amounts were €20 bn before February 2009, and €25 bn thereafter. The bid-to-cover ratio initially rose, with a tendency to destabilize, but recovered later without changes to the intended amount.

(iv) The overbidding experience. From January 1999 to June 2000, the Eurosystem conducted 76 main refinancing operations as fixed rate tenders. The weekly tenders offered credit for two weeks. The ECB followed a neutral allotment policy, i.e., provided the banking system with an amount of liquidity that allowed banks to smoothly satisfy their reserve targets. Moreover, from the information provided to them before the auction, bidders were able to derive the intended amounts quite precisely.\footnote{Cf. ECB (2002).} As already mentioned in the Introduction, the performance of these operations was unsatisfactory due to an escalating bid-to-cover ratio.

(v) Sterling fine-tuning operations. Between June 2006 and March 2009, the Bank of England conducted 45 fine-tuning operations as fixed-rate tenders. Somewhat less than half of those operations were liquidity-providing, a mild majority was liquidity-absorbing. These operations performed well.

(vi) Sterling weekly open market operations. During the period May 2006
through February 2009, the Bank of England conducted altogether 149 regular open market operations as fixed rate tenders with a maturity of one week. The operations were liquidity-providing until early October 2008, after that time they were liquidity-absorbing. The objective of the Bank of England was to allow reserve account holders a smooth fulfillment of reserve bands. There was significant overbidding, yet the bid-to-cover ratio always remained within reasonable bounds.

The examples show that an escalation of bids in fixed rate tenders with pre-announced volumes is by no means a necessity. In fact, excessive overbidding occurred in only one of the nine series. Before continuing the discussion of the evidence, we move to the theoretical modeling of the fixed rate tender.

3. The model

The central bank intends to distribute one unit of a perfectly divisible good at price $p_0$. There are $n$ bidders. Each bidder $i = 1, ..., n$ is asked to submit a bid $b_i \geq 0$. The allotment to bidder $i$ is then

$$
\hat{q}_i(b_i, b_{-i}) = \begin{cases} 
    b_i & \text{if } b_i + b_{-i} \leq 1 \\
    \frac{b_i}{b_i + b_{-i}} & \text{if } b_i + b_{-i} > 1,
\end{cases}
$$

(1)

where $b_{-i} = \sum_{j \neq i} b_j$ denotes the aggregate bid of bidders $j \neq i$.

Bidder $i$’s marginal valuation $v_i(q_i, \theta_i)$ at quantity $q_i$ depends on a type parameter $\theta_i$, drawn from some interval $\Theta_i = [\underline{\theta}_i; \overline{\theta}_i]$, where $0 \leq \underline{\theta}_i < \overline{\theta}_i < 1$. 


Only bidder $i$ observes $\theta_i$. We will assume that $v_i(q_i, \theta_i)$ is continuously differentiable with $\partial v_i / \partial q_i < 0$. Type $\theta_i$’s demand at $p_0$ is defined as the maximum quantity $q_i$ such that $v_i(q_i, \theta_i) \geq p_0$. In fact, for expositional reasons, we will assume that type $\theta_i$ has a demand of just $\theta_i$, as illustrated in Figure 3.

Given a strategy profile $\{\beta_j(\theta_j)\}_{j \neq i}$ for bidders $j \neq i$, expected payoffs for a bidder $i$ of type $\theta_i$ resulting from a bid $b_i$ read

$$\Pi_i(b_i, \theta_i) = \int_0^\infty \left\{ \int_0^{\tilde{q}_i(b_i, b_{-i})} v_i(q_i, \theta_i) dq_i - p_0 \tilde{q}_i(b_i, b_{-i}) \right\} dG_{\theta_i}(b_{-i}),$$

(2)

where $G_{\theta_i}(\cdot)$ denotes the distribution of the random variable $b_{-i} = \sum_{j \neq i} \beta_j(\theta_j)$ conditional on $\theta_i$.

In our set-up, the fixed rate tender is a game of incomplete information (cf. Harsanyi, 1967-68). Following the auction-theoretic literature initiated by Vickrey (1961), we will be searching for a Bayesian Nash equilibrium of this game. In equilibrium, each bidder is envisaged to correctly anticipate the bid functions chosen by the other bidders. Via the bid functions, the uncertainty about types translates into an uncertainty about the size of total competing bids. A bid thereby determines a probability distribution for the allotment. Each bidder then chooses her bid so as to maximize expected payoffs from this uncertain outcome.

The trade-off for bidder $i$ is as follows. When bidding $b_i$, type $\theta_i$ ends up
with too much (too little) of the good if \( b_{-i} < b_{-i}^0 \equiv b_i \frac{1 - \theta_i}{\theta_i} \) (if \( b_{-i} > b_{-i}^0 \)).

Correspondingly, the first-order condition may be written as

\[
\int_0^{b_{-i}} \{ p_0 - v_i(\tilde{q}_i(b_i, b_{-i}), \theta_i) \} \frac{\partial \tilde{q}_i}{\partial \theta_i}(b_i, b_{-i})dG_{\theta_i}(b_{-i}) = \int_{b_{-i}^0}^\infty \{ v_i(\tilde{q}_i(b_i, b_{-i}), \theta_i) - p_0 \} \frac{\partial \tilde{q}_i}{\partial \theta_i}(b_i, b_{-i})dG_{\theta_i}(b_{-i}). \quad (3)
\]

Figure 4 illustrates the various factors entering the first-order condition, viz. marginal valuations, marginal allotments, and the density \( g_{\theta_i}(b_{-i}) \) of total competing bids. This will be of relevance in the next section.

4. Equilibrium bidding

This section deals with equilibrium existence in the fixed rate tender with pre-announced allotment volume. The crucial issue for existence is to exclude the possibility of mutually reinforcing overbidding. For this, bidders should find it in their own interest to overbid only moderately when they expect competitors to do the same. Thus, excess demand should be small compared to the risk and potential detriment from overbidding.

Excess demand will be measured by \( \varepsilon(i, \theta_i) = \int \max\{0; \theta_i + \theta_{-i} - 1\}dF_{\theta_i}(\theta_{-i}) \), where \( F_{\theta_i}(\theta_{-i}) \) denotes the distribution of competing demand \( \theta_{-i} = \sum_{j \neq i} \theta_j \), conditional on \( \theta_i \). Taking the supremum across all bidders and types, we obtain the proxy \( \varepsilon = \sup_{(i, \theta_i)} \varepsilon(i, \theta_i) \).

To construct proxies for risk and potential detriment from overbidding,
we focus on allotments exceeding $\theta_i/M$, where $M$ is a factor such that $\max\{\bar{\theta}_1, ..., \bar{\theta}_n\} < M < 1$. The allotment for bidder $i$ is $\theta_i/M$ when aggregate competing bids amount to $b^M_i = b_i(M-\theta_i)/\theta_i$. Figure 4 illustrates the relative position of $b^M_i$. To capture the risk of a low demand realization, we consider the size of competing demand when aggregate demand falls short of $M$.\footnote{It turns out that this condition suffices to drive bidder $i$’s allotment above $\theta_i/M$.}

Formally, write $R(i, \theta_i) = \int_{\theta_{-i}+\theta_i \leq M} \theta_{-i}dF_{\theta_i}(\theta_{-i})$, and $R = \inf_{(i, \theta_i)} R(i, \theta_i)$.

Finally, we define a proxy for the relative detriment of an overly large allotment compared to that of an allotment that is too small. As illustrated in Figure 3, define a maximum slope $S = \max\{|\partial v_i/\partial q_i| : i = 1, ..., n \text{ and } q_i \leq \theta_i\}$ for allotments below demand, and a minimum slope $s = \min\{|\partial v_i/\partial q_i| : i = 1, ..., n \text{ and } \theta_i \leq q_i \leq \theta_i/M\}$ for allotments exceeding demand by a factor less than $1/M$. We will use the proxy $D = s/S$.

The following lemma is the key result driving the existence theorem.

**Lemma 1.** Let $\alpha$ satisfy $1/M < \alpha < \alpha^* \equiv RD(1-M)/M^3 \varepsilon$. Assume that each bidder $j \neq i$ follows some strategy $\beta_j(\cdot)$ such that $\beta_j(\theta_j) \leq \alpha \theta_j$ for all $\theta_j$. Then bidder $i$’s best response $\beta_i(\cdot)$ satisfies $\beta_i(\theta_i) \leq \alpha \theta_i$ for all $\theta_i$.

The proof is in the Appendix. Consistent with intuition, it is easier to put a non-escalating bound on the best response to strategic overbidding when excess demand $\varepsilon$ is small, and risks $R$ as well as potential detriments $D$ are
The following theorem is the main theoretical result of this paper.

**Theorem 1.** Assume \( \varepsilon < RD(1 - M)/M^2 \). Then the fixed rate tender with pre-announced allotment volume allows an equilibrium in mixed bid strategies. In this equilibrium, bidders exaggerate their demand by a factor \( \alpha < \alpha^* \equiv 1/M \), and the allotment quota never drops below \( M/(\sum_{i=1}^{n} \bar{\theta}_i) \).

**Proof.** Consider the game in which each bidder \( i \) of type \( \theta_i \in \Theta_i \) chooses a multiplier \( \alpha_i(\theta_i) \in [1; \alpha] \) corresponding to a bid \( \beta_i(\theta_i) = \alpha_i(\theta_i)\theta_i \). Existence of a mixed strategy equilibrium \( \{\alpha^*_i(\theta_i, \xi_i)\}_{i=1,...,n} \), where \( \xi_i \in [0, 1] \) is bidder \( i \)'s random signal, follows from an existence result for Bayesian games with compact strategy sets and continuous utility functions (see Milgrom and Weber, 1985, Theorem 1 and Proposition 3). But this equilibrium is an equilibrium also in the game with unconstrained strategy sets. Indeed, by the proof of Lemma 1, if \( \alpha_i(\theta_i) > \alpha \), then \( i \)'s marginal expected profit would be negative for each \( \{\xi_j\}_{j \neq i} \). Hence, marginal expected profit would be negative, a contradiction. \( \square \)

5. **Rationing with probability one**

The following example illustrates the Bayesian equilibrium. It also shows that rationing may occur with probability one.

**Example 1.** There are two bidders, with independent types. Bidder \( i \)'s type
\( \theta_i \) is drawn from the interval \( \left[ \frac{1}{\lambda+1}, \frac{\lambda}{\lambda+1} \right] \) according to the density

\[
g(\theta_i) = \frac{2(\lambda + 1)}{\lambda - 1} (1 - \theta_i),
\]

(4)

where \( \lambda > 1 \). When of type \( \theta_i \), bidder \( i \)'s marginal valuation for an allotment \( q_i \) is given by

\[
v_i(q_i, \theta_i) = p_0 + \frac{\theta_i - q_i}{q_i^2 (1 - q_i)}.
\]

(5)

There is a pure strategy Nash equilibrium in which type \( \theta_i \) bids \( \beta_i(\theta_i) = \lambda \theta_i / (1 - \theta_i) \). Moreover, all bids are rationed.

To verify these claims, assume that \( \beta_j(\theta_j) = \lambda \theta_j / (1 - \theta_j) \) for all \( \theta_j \), where \( j \neq i \). Then, \( j \)'s bids are distributed on the interval \( [1, \lambda^2] \), so that rationing occurs with probability one. Hence, when bidding \( b_i \) against type \( \theta_j \), bidder \( i \) obtains the allotment \( \hat{q}_i = b_i / (b_i + \beta_j(\theta_j)) \). Bidder \( i \)'s problem is strictly concave, with first-order condition

\[
\int_{1/(1+\lambda)}^{\lambda/(1+\lambda)} \frac{\theta_i - \hat{q}_i}{\hat{q}_i} g(\theta_j) d\theta_j = 0.
\]

(6)

Replacing \( \hat{q}_i \) by the explicit expression derived above yields the assertion.

Thus, in this specification, marginal valuations fall quickly, and the probability of high types, that overbid more excessively, is comparably low.

The reader will note that an equilibrium in which rationing occurs with probability one differs structurally from an equilibrium in which with positive
probability all bids are fulfilled. In the former, there is a coordination problem with respect to the extent of overbidding. That is, equilibrium strategies of all bidders may be scaled up by an arbitrary factor without affecting the resulting allocation. In an equilibrium in which with positive probability all bids are fulfilled, however, re-scaling of all bids does not in general lead to a new equilibrium. This is because the allocation would change for those bids for which the bidder is uncertain as regards to whether they lead to rationing or not.

The potential indeterminacy of the equilibrium has implications in a repeated setting where expectations about bidding may depend on the outcome of earlier operations. The prevalent view in the literature sees the development of myopic and adaptive expectations as a consequence of the perceived non-existence of the equilibrium. A potential problem with that interpretation, however, is that participants in a bidding race would be required to uphold the hypothesis of, say, an unchanged allotment quota despite the recurrent empirical rejection of that hypothesis. In an alternative interpretation, suggested by Example 1, expectations in an overbidding episode are in fact rational, only the coordination along the linear trend is adaptive.

6. Dynamics

This section studies the dynamic robustness of the equilibrium identified
in Section 4. Time $t = t_0, t_0 + 1, \ldots$ is discrete and tenders are organized sequentially. We envisage a setting in which expectations are not necessarily aligned, but there is initially some common understanding as to the extent of overbidding.

**Theorem 2.** Under the assumption of Theorem 1, assume that all bidders have exaggerated their true demand in some period $t$ by a factor $\alpha_t$ in the range $\alpha_* < \alpha_t < \alpha^*$. Then, with adaptive expectations, it is optimal for all bidders to exaggerate true demand in period $t+1$ by some factor $\alpha_{t+1} < \alpha_t$. In fact, the sequence $\{\alpha_t\}$ declines exponentially until it undercuts the threshold $\alpha_*$. 

**Proof.** By assumption, $\varepsilon < RD(1 - M)/M^2$. For $\delta > 0$ small, define $\varepsilon(i, \theta_i, \delta) = \int \max\{0; \theta_i + \theta_{-i} - 1 + \delta\} dF_{\theta_i}(\theta_{-i})$, and $\varepsilon(\delta) = \sup_{(i, \theta_i)} \varepsilon(i, \theta_i, \delta)$. By continuity, there is a $\delta > 0$ such that $\varepsilon(\delta) < RD(1 - M)/M^2$. Let then $\alpha_{t+1} = (1 - \delta)\alpha_t$. Assume that $b_{t-1}^i \leq \alpha_t \theta_{t-1}^i$. Adaptive expectations predict $b_{t-1}^{t+1} \leq \alpha_t \theta_{t-1}^{t+1}$. We claim that then $b_t^{t+1} \leq \alpha_{t+1} \theta_t^{t+1}$. To provoke a
contradiction, assume \( b_{i}^{t+1} > \alpha_{t+1} \theta_{i}^{t+1} \). Then,

\[
\begin{align*}
  b_{i}^{t+1} - b_{0}^{0} &\leq \alpha_{t} \theta_{i}^{t+1} - \frac{1 - \theta_{i}^{t+1}}{\theta_{i}^{t+1}} b_{i}^{t+1} \\
  &\leq \alpha_{t} \theta_{i}^{t+1} - (1 - \theta_{i}^{t+1}) \alpha_{t+1} \tag{8} \\
  &\leq \alpha_{t} (\theta_{i}^{t+1}(1 - \delta) + \theta_{i}^{t+1} - 1 + \delta) \tag{9} \\
  &\leq \alpha_{t} (\theta_{i}^{t+1} + \theta_{i}^{t+1} - 1 + \delta). \tag{10}
\end{align*}
\]

Following now the lines of the proof of Lemma 1, we obtain a contradiction.

Hence, \( b_{i}^{t+1} \leq \alpha_{t+1} \theta_{i}^{t+1} \). The second assertion follows from \( \alpha_{t+1} = (1 - \delta) \alpha_{t} \).

\( \square \)

Thus, myopic best responses may lead to a steady decline of overbidding factors to the level predicted for the equilibrium. The range of overbidding factors over which this type of convergence is obtainable clearly depends on excess demand, risk, and potential detriment of overbidding, as discussed before. Still, Theorem 2 can be seen as an “optimistic” counterpart to Nautz and Oechssler’s (2003) divergence prediction.

7. Welfare

It is a common experience that the fixed rate tender renders an inefficient allocation whenever bidders find it difficult to coordinate. As we are going to show in this section, however, the situation is not much better when bidders have equilibrium expectations. Intuitively, in any equilibrium with
excess demand, the individual bidder has to be uncertain about the resulting allocation. Therefore, with strictly positive probability, the allotment will be larger (or smaller) than optimal, causing the inefficiency.

Formally, denote by
\[
W(\hat{q}, \theta) = \sum_{i=1}^{n} \left\{ \int_{0}^{\hat{q}_i} v_i(q, \theta_i) dq_i - p_0 \hat{q}_i \right\}
\] (11)
the welfare associated with an ex-post allocation \( \hat{q} = (\hat{q}_1, \ldots, \hat{q}_n) \) in a state \( \theta = (\theta_1, \ldots, \theta_n) \). No positive welfare is associated with any fraction of the good left with the central bank. An ex-post allocation \( q \) is called feasible if \( q_i \geq 0 \) for all \( i \) and \( \sum_{i=1}^{n} q_i \leq 1 \). A feasible ex-post allocation \( q \) is efficient if it maximizes \( W \) under the feasibility constraint.

**Theorem 3.** Assume that \( \sum_{i=1}^{n} \theta_i > 1 \) with strictly positive probability. Then any equilibrium allocation of the fixed-price tender is ex-post inefficient.

**Proof.** Consider a pure-strategy Nash equilibrium \( \{\beta_i^*(.)\}_{i=1, \ldots, n} \). Ignoring the zero set on which the rationing rule is only continuous, but not differentiable, the necessary first-order condition for bidder \( i \) of type \( \theta_i \) reads
\[
\int \left\{ v_i(\hat{q}_i(\beta_i^*(\theta_i), b_{-i}), \theta_i) - p_0 \right\} \frac{\partial \hat{q}_i}{\partial \beta_i}(\beta_i^*(\theta_i), b_{-i}) dG_{\theta_i}(b_{-i}) = 0. \quad (12)
\]
Integrating over \( \theta_i \) yields
\[
\int \int \left\{ v_i(\hat{q}_i(\beta_i^*(\theta_i), b_{-i}), \theta_i) - p_0 \right\} \frac{\partial \hat{q}_i}{\partial \beta_i}(\beta_i^*(\theta_i), b_{-i}) dG_{\theta_i}(b_{-i}) dF_i(\theta_i) = 0, \quad (13)
\]
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where $F_i(.)$ denotes the marginal distribution of $\theta_i$. An ex-post allocation $q^* = (q_1^*, ..., q_n^*)$ that is efficient in state $\theta = (\theta_1, ..., \theta_n)$ satisfies $v_i(q_i^*, \theta_i) \geq p_0$. Moreover, efficiency implies $v_i(q_i^*, \theta_i) > p_0$ for all $i$ whenever $\sum_{i=1}^{n} \theta_i > 1$. As $\frac{\partial q_i}{\partial \theta_i} > 0$, this contradicts (13). The extension to mixed equilibria is immediate. □

Thus, when there is a positive probability that true demand exceeds intended supply, then any equilibrium will be inefficient as a consequence of exposure risk.

8. Empirical test

To assess the practical importance of exposure risk for bidding behavior in fixed rate tenders, we use data on liquidity-providing operations conducted by the ECB in the period January 1999 through June 2000, and by the Bank of England in the period May 2006 through October 2008.

The test is based on the following reduced-form model of bidding behavior. The endogenous variable is the log of total bids $b_t$ submitted in tender $t$, where $t = 1, 2, 3, \ldots$ counts the weekly tenders in chronological order.

$$\ln b_t = \gamma_0 + \gamma_1 t + \gamma_2 \ln a_t + \gamma_3 \ln a_{t-1} + \gamma_4 \ln b_{t-1} + \gamma_5 (\rho - r)_t + \gamma_6 (\rho - \sigma)_t + \varepsilon_t$$

(14)

In this specification, $a_t$ is the intended amount. We include one lagged vari-
able of intended amount and bids, respectively. The tender rate is denoted by \( r \). Arbitrage possibilities are proxied by the spread between the unsecured rate over the tender rate, \((\rho - r)_t\), where the unsecured rate \( \rho \) corresponds to the two-week EONIA swap for the Euro area and to the one-week LIBOR for the UK, respectively. The hypothesis of exposure risk is operationalized by the spread between unsecured and secured rates, \((\rho - \sigma)_t\), where the secured rate \( \sigma \) is the ECB market repo rate for the Euro area and the one-week general collateral rate for the UK, respectively.

The time series for the Bank of England was split into operations that settled on or before August 2, 2007, and operations that settled on or after August 9, 2007. We refer to the former as the pre-crisis sample, and to the latter as the crisis sample. The model has been estimated by ordinary least squares for the three samples, i.e., ECB, BoE pre-crisis, and BoE crisis. Table III contains the results of the regressions.

The left column shows the regression for the Eurosystem. Consistent with earlier findings, arbitrage possibilities had a strong influence on bids in the Euro area. Indeed, the coefficient of the spread \((\rho - r)_t\) is highly significant and positive. Also the time trend is highly significant and positive.\(^{11}\) The

\(^{9}\)Higher lags are not significant.
\(^{10}\)For the Euro area, we actually used the one-week repo spread, for which the longest time series was available (settlement not before April 21, 1999). However, the results are almost identical for the two-week repo.
\(^{11}\)The usual interpretation of the time trend is that it captures the adaptive response
new element here is the regressor for exposure risk. Its coefficient is highly significant and negative, as predicted by the theoretical analysis.

The regression results for the Bank of England during the pre-crisis period are shown in the mid column. While the coefficient for arbitrage is positive, it is not significant. Moreover, the trend is weakly significant, but negative. In sum, this suggests that neither the arbitrage hypothesis nor the rationing hypothesis had a strong role in the pre-crisis Sterling auctions. There is, however, a significant and strongly negative effect of exposure risk.

Finally, in the right column, the table shows the coefficients for the Bank of England during the crisis. Intended volume is significant, potentially because bidders became more responsive to information after August 2007. More importantly for our present analysis, the coefficient for exposure risk is again highly significant and negative. The somewhat smaller coefficient for exposure risk, compared to the pre-crisis period, might reflect the fact that relative scarcity of cash and high-quality collateral became more balanced during the crisis. That interpretation would also be consistent with the significant arbitrage coefficient. In sum, we find that exposure risk has been a significant determinant of bidding behavior in all three samples.

The results of the regressions are consistent with anecdotal evidence on
central bank collateral in Euro and Sterling markets. In the Euro area, eligibility criteria for collateral have traditionally been broad. In fact, the first mentioning of a scarcity of collateral was when banks in some member countries felt at a disadvantage during the most excessive phase of the overbidding episode.\footnote{Cf. Bundesbank (2000).} The situation was very different in Sterling markets. Indeed, the largest fraction of collateral used in Sterling operations was gilts (i.e., UK government bonds), and these were, typically, not owned outright by the bidders, i.e., by banks and building societies, but borrowed through securities lending transactions from the ultimate holders of gilt securities, viz. pension funds and insurance companies. Therefore, when the bid-to-cover ratio happened to be unexpectedly low, bidders with insufficient provisions were indeed in search of collateral.\footnote{A squeeze in the gilt market occurred, e.g., on July 31, 2006. Cf. Bank of England (2007).}

In the remainder of this section, we discuss informally the performance of the rationing scheme in the other six cases (cf. Table I). Consider first the three samples in which operations merely withdrew liquidity from the market, i.e., Euro fine-tuning, Sterling weekly absorbing, and Sterling fine-tuning absorbing. Exposure risk in these liquidity-absorbing operations differed from the exposure risk in liquidity-providing operations discussed above. An exces-
sive allotment in a liquidity-absorbing operation means that the bidder has an unexpected outflow of liquidity. To compensate, the bidder must then either borrow in the market or have recourse to the central bank’s lending facility. Typically, both options will be costly in terms of the interest rate spread. More severely, credit limits may be exhausted in the market, and there may also be a reputational damage from seeking additional credit. Finally, it has been suggested that not only in the US but also in Europe, there may be a stigma attached to the use of the central bank facility. Therefore, even if protected by anonymity, the bidder might still be exposed to rumors and suggestions. We conclude that an oversized allotment in a liquidity-absorbing operation is potentially even more harmful than an oversized allotment in a liquidity-providing operation. In particular, this would explain the quite satisfactory performance of the fixed rate tender in this class of examples, all of which have very low bid-to-cover ratios.

Next, we turn to the liquidity-providing Sterling fine-tuning operations. On the one hand, operations in that sample tend to exhibit a somewhat higher bid-to-cover ratio than their liquidity-absorbing counterparts, which is consistent with our discussion above. On the other hand, the better performance of the liquidity-providing fine-tuning operations of the BoE compared to the weekly liquidity-providing tenders might have to do with the fact that
reserve account holders in the UK have some flexibility with regard to their reserve fulfillment, which should lower demand in the fine-tuning operations.

Regarding foreign exchange swap operations, Figure 2 shows that in contrast to the other examples, the liquidity policy for these operations was biased towards excess supply, which explains the very low bid-to-cover statistics of -0.73. During the short period with excess demand, however, the rationing scheme generated a rather special exposure risk profile. An oversized allotment meant here a large Swiss Franc liquidity inflow as well as a large Euro liquidity outflow. The exposure risk associated with the latter, i.e., with the liquidity outflow, might have contributed to the relatively swift decline of total bids after the peak.

Finally, the USD tender was similar to an ordinary liquidity-providing Eurosystem operation in terms of exposure risks. That is to say, exposure risk was probably not very pronounced due to the relatively broad definition of eligible collateral. This view is consistent with the downward trend in the allotment quota. Indeed, our results suggest that even lower quotas might have been possible if these operations had been continued in an unchanged market environment.

9. Related literature

Our main theoretical result is a sufficient condition for equilibrium existence
when the intended allotment volume is pre-announced and there are no costs of bidding. To the best of our knowledge, this result is new in the literature. Ayuso and Repullo (2003) explain the overbidding observed in the Eurosystem over the period January 1999 through June 2000 as a consequence of an asymmetric objective function for the central bank. In contrast to the present paper, however, Ayuso and Repullo have a cost function that depends on the size of the bid. Nyborg and Strebulaev (2001) consider equilibria in fixed rate tenders with subsequent short-squeezes, assuming that bids are constrained by collateral endowments. Nautz and Oechssler (2003) document the overbidding phenomenon. They show that the rationing game does not allow an equilibrium under complete information, and explain the development of aggregate bids during the overbidding period as driven by a myopic best reply dynamics. Bindseil (2005) provides a survey of the experience with fixed rate tenders by modern central banks, stressing in particular the case of the Eurosystem. He also analyzes the macro behavior of a banking system facing a cost of bidding that depends on the aggregate bid. Ehrhart (2002) extends and refines the non-existence result in several directions, analyzing in particular the case of repeated interaction. That paper also contains a numerical example of an equilibrium with uncertainty about supply. Välimäki

\footnote{See also Bindseil (2004).}
(2003) assumes a two-part penalty consisting of a rate on missing collateral and a fixed amount for non-compliance. He then studies the decision of an individual bank to bid optimally against a given probability distribution of aggregate bids submitted by the other banks. Catalão-Lopes (2010) compares the fixed rate tender with the uniform-price tender, stressing the non-existence result and collateral constraints on bids. Thus, it appears that the case considered in this paper has indeed not been covered by the existing literature.\footnote{This statement includes related work on market disequilibrium (e.g., Bénassy, 1977) and supply-chain management (e.g., Lee et al., 1997).}

10. Conclusion

Central banks are fond of the fixed rate tender because it conveys the monetary signal to the market usually with very little noise. Proportional rationing is applied in these tenders when the total of incoming bids exceeds the amount intended by the central bank. In the present paper, we have argued that exposure risk is a critical determinant of bidding behavior in fixed rate tenders. Due to exposure risk, an equilibrium exists even when bids are costless and the intended amount is pre-announced. In this equilibrium, the extent of overbidding is limited, and there is a bound below which the allotment quota never falls. We also found conditions under which temporarily
elevated overbidding factors will return, with adaptive expectations, to the levels predicted for the equilibrium. In particular, this suggests a rationale for the continued use of the fixed rate tender by central banks such as the ECB and the Bank of England.\footnote{Having an equilibrium is also a prerequisite for further theoretical analysis such as comparing the efficiency properties of the fixed rate tender with that of the variable rate tender (cf. Ewerhart et al., 2010). This last point might be an interesting area for future research.}

By clarifying the role of exposure risk for bidding behavior in fixed rate tenders, our analysis allows to put the overbidding phenomenon in the Eurosystem operations into a somewhat broader context. The comparison to the Sterling market strongly suggests that an absence of exposure risk, due to the market-friendly eligibility criteria for central bank collateral, was critical for the escalation of bids in the Euro area. In particular, our findings suggest that a potential re-introduction of rationing in the main refinancing operations of the Eurosystem would presuppose a substantial review of the existing collateral framework.

**Appendix: Proof of Lemma 1**

We claim that under the assumptions of the lemma, for $b_i > \alpha \theta_i$, the left-hand side (LHS) of the first-order condition (3) exceeds the right-hand side (RHS). Note that $\beta_i(0) = 0$, and $\beta_i(0) \geq \theta_i$ for all $\theta_i$. We may therefore assume in the sequel that $\theta_i > 0$ and $b_i > 0$.\footnote{Having an equilibrium is also a prerequisite for further theoretical analysis such as comparing the efficiency properties of the fixed rate tender with that of the variable rate tender (cf. Ewerhart et al., 2010). This last point might be an interesting area for future research.}
**RHS.** By the construction of $S$,

$$v_i(\tilde{q}_i(b_i, b_{-i}), \theta_i) - p_0 \leq S(\theta_i - \tilde{q}_i(b_i, b_{-i}))$$  \(15\)

for all $b_{-i} \geq b_{-i}^0$. Since the right-hand side of (15) is concave in $b_{-i}$,

$$\theta_i - \tilde{q}_i(b_i, b_{-i}) \leq (b_{-i} - b_{-i}^0) \left. \frac{\partial}{\partial b_{-i}} \right|_{b_{-i}=b_{-i}^0} \{ \theta_i - \tilde{q}_i(b_i, b_{-i}) \}$$  \(16\)

$$= (b_{-i} - b_{-i}^0) \frac{\theta_i^2}{b_i}. \quad (17)$$

Using $b_{-i} \leq \alpha \theta_{-i}$ and $b_i > \alpha \theta_i$, one finds $b_{-i} - b_{-i}^0 \leq \alpha (\theta_i + \theta_{-i} - 1)$. Moreover, for $b_{-i} \geq b_{-i}^0$,

$$\frac{\partial \tilde{q}_i}{\partial b_i}(b_i, b_{-i}) = \frac{b_{-i}}{(b_i + b_{-i})^2} \leq \frac{1}{b_i + b_{-i}} \leq \frac{1}{b_i + b_{-i}^0} = \frac{\theta_i}{b_i}. \quad (18)$$

Thus,

$$\text{RHS} \leq \frac{S \theta_i^3}{b_i^2} \int_{b_{-i}^0}^{\infty} (b_{-i} - b_{-i}^0) dG_{\theta_i}(b_{-i})$$  \(19\)

$$\leq \frac{\alpha S \theta_i^3}{b_i^2} \int \max\{0, \theta_i + \theta_{-i} - 1\} dF_{\theta_i}(\theta_{-i}). \quad (20)$$

**LHS.** Note that $b_{-i}^M > 1 - b_i$. As marginal valuations are declining, and because $\tilde{q}_i(b_i, b_{-i})$ is weakly decreasing in $b_{-i}$, one obtains for $b_{-i} \leq b_{-i}^M$ that

$$p_0 - v_i(\tilde{q}_i(b_i, b_{-i}), \theta_i) \geq p_0 - v_i(\tilde{q}_i(b_i, b_{-i}^M), \theta_i)$$  \(21\)

$$= v_i(\theta_i, \theta_i) - v_i(\frac{\theta_i}{M}, \theta_i)$$  \(22\)

$$\geq \frac{1 - M}{M} \theta_i s. \quad (23)$$
Moreover, for $1 - b_i < b_{-i} \leq b^M_i$, 

$$\frac{\partial \hat{q}_i}{\partial b_i}(b_i, b_{-i}) = \frac{b_{-i}}{(b_i + b_{-i})^2} = \left(\frac{b_i}{b_i + b_{-i}}\right)^2 \frac{b_{-i}}{b_i^2} \geq \left(\frac{\theta_i}{b_i M}\right)^2 b_{-i}. \quad (24)$$

Also for $b_{-i} \leq 1 - b_i$, 

$$\frac{\partial \hat{q}_i}{\partial \theta_i}(b_i, b_{-i}) = 1 \geq \frac{b_{-i}}{(\alpha M)^2} \geq \left(\frac{\theta_i}{b_i M}\right)^2 b_{-i}, \quad (25)$$

because $\alpha > 1/M$. Hence, 

$$\text{LHS} \geq \frac{(1 - M)\theta_i^3 s}{M^3 b_i^2} \int_{b_{-i}}^{b_i} b_{-i} dG_{\theta_i}(b_{-i}). \quad (26)$$

It is straightforward to check that if $\theta_i + \theta_{-i} \leq M$ then $b_{-i} \leq b^M_i$. Therefore, 

$$\text{LHS} \geq \frac{(1 - M)\theta_i^3 s}{M^3 b_i^2} \int_{\theta_{-i} + \theta_i \leq M} \theta_{-i} dF_{\theta_i}(\theta_{-i}). \quad (27)$$

This implies that LHS>RHS for all $b_i > \alpha \theta_i$. Thus, a bidder $i$ of type $\theta_i$ will bid at most $\alpha \theta_i$. □
References


Ehrhart, K.-M., 2002, A well-known rationing game, *mimeo*, University of Karlsruhe.


Figure 1: Allotment quotas in ECB fine-tuning operations
Figure 2: Intended amount and total bids in the weekly EUR/CHF foreign exchange swap operations
Figure 3: Marginal valuations

\[ v_i \]

\[ p_0 \]

\[ \theta_i \]

\[ \theta_i / M \]

\[ q_i \]
Figure 4: First-order condition
<table>
<thead>
<tr>
<th>Operation series</th>
<th>Period</th>
<th>Mean log bid-to-cover</th>
<th>No. of operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro main refinancing</td>
<td>Jan 1999 – Jun 2000</td>
<td>2.91</td>
<td>76</td>
</tr>
<tr>
<td>USD term auction facility</td>
<td>Dec 2007 – Oct 2008</td>
<td>1.01</td>
<td>21</td>
</tr>
<tr>
<td>Sterling weekly, providing, pre-crisis</td>
<td>May 2006 – Aug 2007</td>
<td>0.94</td>
<td>64</td>
</tr>
<tr>
<td>Sterling fine-tuning, providing</td>
<td>Aug 2006 – Feb 2009</td>
<td>0.37</td>
<td>22</td>
</tr>
<tr>
<td>Euro fine-tuning</td>
<td>May 2003 – Oct 2008</td>
<td>-0.03</td>
<td>32</td>
</tr>
<tr>
<td>Sterling weekly, absorbing</td>
<td>Oct 2008 – Feb 2009</td>
<td>-0.14</td>
<td>23</td>
</tr>
<tr>
<td>Sterling fine-tuning, absorbing</td>
<td>Jun 2006 – Mar 2009</td>
<td>-0.37</td>
<td>17</td>
</tr>
<tr>
<td>CHF foreign-exchange swaps</td>
<td>Oct 2008 – Jan 2010</td>
<td>-0.73</td>
<td>70</td>
</tr>
</tbody>
</table>

Notes. The table lists recent uses of the fixed rate tender in the Euro area and the UK. Shown are the period of usage, the mean of the log bid-to-cover ratio, and the number of operations conducted. The term pre-crisis (crisis) refers to operations with settlement on or before August 2, 2007 (on or after August 9, 2007). The nine samples are ordered along the bid-to-cover statistics, starting with the highest value.
<table>
<thead>
<tr>
<th>Settlement date</th>
<th>Duration (days)</th>
<th>Intended amount (€ bn)</th>
<th>Maximum bid (€ bn)</th>
<th>Number of bidders</th>
<th>Total bids (€ bn)</th>
<th>Allotment quota</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-Dec-2007</td>
<td>28</td>
<td>10'000</td>
<td>1'000</td>
<td>39</td>
<td>22'080</td>
<td>45%</td>
</tr>
<tr>
<td>27-Dec-2007</td>
<td>35</td>
<td>10'000</td>
<td>1'000</td>
<td>27</td>
<td>14'115</td>
<td>71%</td>
</tr>
<tr>
<td>17-Jan-2008</td>
<td>28</td>
<td>10'000</td>
<td>1'000</td>
<td>22</td>
<td>14'790</td>
<td>68%</td>
</tr>
<tr>
<td>31-Jan-2008</td>
<td>28</td>
<td>10'000</td>
<td>1'000</td>
<td>19</td>
<td>12'400</td>
<td>81%</td>
</tr>
<tr>
<td>27-Mar-2008</td>
<td>28</td>
<td>15'000</td>
<td>1'500</td>
<td>34</td>
<td>31'237</td>
<td>48%</td>
</tr>
<tr>
<td>10-Apr-2008</td>
<td>28</td>
<td>15'000</td>
<td>1'500</td>
<td>32</td>
<td>30'760</td>
<td>49%</td>
</tr>
<tr>
<td>24-Apr-2008</td>
<td>28</td>
<td>15'000</td>
<td>1'500</td>
<td>33</td>
<td>30'128</td>
<td>50%</td>
</tr>
<tr>
<td>8-May-2008</td>
<td>28</td>
<td>25'000</td>
<td>2'500</td>
<td>57</td>
<td>84'830</td>
<td>29%</td>
</tr>
<tr>
<td>22-May-2008</td>
<td>28</td>
<td>25'000</td>
<td>2'500</td>
<td>59</td>
<td>90'075</td>
<td>28%</td>
</tr>
<tr>
<td>5-Jun-2008</td>
<td>28</td>
<td>25'000</td>
<td>2'500</td>
<td>56</td>
<td>78'460</td>
<td>32%</td>
</tr>
<tr>
<td>19-Jun-2008</td>
<td>28</td>
<td>25'000</td>
<td>2'500</td>
<td>57</td>
<td>84'830</td>
<td>29%</td>
</tr>
<tr>
<td>3-Jul-2008</td>
<td>28</td>
<td>25'000</td>
<td>2'500</td>
<td>59</td>
<td>90'075</td>
<td>28%</td>
</tr>
<tr>
<td>17-Jul-2008</td>
<td>28</td>
<td>25'000</td>
<td>2'500</td>
<td>56</td>
<td>78'460</td>
<td>32%</td>
</tr>
<tr>
<td>31-Jul-2008</td>
<td>28</td>
<td>25'000</td>
<td>2'500</td>
<td>63</td>
<td>101'683</td>
<td>25%</td>
</tr>
<tr>
<td>14-Aug-2008</td>
<td>84</td>
<td>10'000</td>
<td>1'000</td>
<td>57</td>
<td>38'522</td>
<td>26%</td>
</tr>
<tr>
<td>14-Aug-2008</td>
<td>28</td>
<td>20'000</td>
<td>2'000</td>
<td>66</td>
<td>91'100</td>
<td>22%</td>
</tr>
<tr>
<td>28-Aug-2008</td>
<td>28</td>
<td>20'000</td>
<td>2'000</td>
<td>69</td>
<td>89'249</td>
<td>22%</td>
</tr>
<tr>
<td>11-Sep-2008</td>
<td>84</td>
<td>10'000</td>
<td>1'000</td>
<td>53</td>
<td>43'340</td>
<td>23%</td>
</tr>
<tr>
<td>11-Sep-2008</td>
<td>28</td>
<td>10'000</td>
<td>1'000</td>
<td>53</td>
<td>43'340</td>
<td>23%</td>
</tr>
<tr>
<td>25-Sep-2008</td>
<td>28</td>
<td>25'000</td>
<td>2'500</td>
<td>71</td>
<td>110'100</td>
<td>23%</td>
</tr>
<tr>
<td>9-Oct-2008</td>
<td>85</td>
<td>20'000</td>
<td>2'000</td>
<td>70</td>
<td>88'650</td>
<td>23%</td>
</tr>
</tbody>
</table>

Notes. The table lists data related to the USD operations conducted by the Eurosystem within the framework of the term auction facility. Shown are the settlement date, the duration, the intended amount, the maximum bid, the number of bidders, total bids, and the allotment quota.
Table III
Bid functions for fixed rate tenders

<table>
<thead>
<tr>
<th></th>
<th>ECB</th>
<th>Bank of England</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Apr 1999 – Jun 2000</td>
<td></td>
<td>Pre-crisis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bank of England</td>
<td></td>
</tr>
<tr>
<td>trend $\times 10^2$</td>
<td>0.825*** (0.271)</td>
<td></td>
<td>- 0.441** (0.216)</td>
</tr>
<tr>
<td></td>
<td>0.097 (0.182)</td>
<td></td>
<td>1.147 (1.303)</td>
</tr>
<tr>
<td></td>
<td>- 0.153 (0.188)</td>
<td></td>
<td>- 0.141 (1.236)</td>
</tr>
<tr>
<td>$b_{t-1}$</td>
<td>0.342*** (0.065)</td>
<td></td>
<td>0.649*** (0.080)</td>
</tr>
<tr>
<td></td>
<td>4.017*** (0.371)</td>
<td></td>
<td>0.733 (0.800)</td>
</tr>
<tr>
<td></td>
<td>- 0.933*** (0.242)</td>
<td></td>
<td>- 5.218*** (1.457)</td>
</tr>
<tr>
<td>$\rho-r$</td>
<td>9.272** (3.657)</td>
<td></td>
<td>- 5.972** (7.953)</td>
</tr>
<tr>
<td>$\rho-\sigma$</td>
<td>0.902</td>
<td></td>
<td>0.688</td>
</tr>
<tr>
<td>Observations</td>
<td>61</td>
<td></td>
<td>63</td>
</tr>
</tbody>
</table>

Notes. The table shows the estimated bid ($b_t$) functions for the fixed rate tenders of the ECB (left column) and the Bank of England (mid and right columns), compare Eq. (11). Confidence levels are *** for p<0.01, ** for p<0.05, and * for p<0.1. Standard errors are noted in parentheses.