Wealth Inequality and the Optimal Level of Government Debt

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Abstract

In this paper, we compute the welfare-maximizing level of government debt in an incomplete markets economy with production in which households are subject to uninsurable income shocks. We calibrated the model to match the wealth and the earnings distribution of the U.S. economy. In our model, the optimal level of government debt is significantly negative, implying that the government should hold assets, not debt. The finding is in sharp contrast to the previous literature, which concludes that a benevolent government should issue debt when markets are incomplete. According to the results, the high debt levels that are currently observable in most developed countries will decrease welfare in the long-run. However, we find that raising income taxes in order to reduce government debt generates large welfare losses over the transition and is thus not politically feasible. The skewed wealth distribution of our calibration is key for explaining both the long-run and the short-run effects of government debt.

Key words: Government Debt, Borrowing Limits, Incomplete Markets, Crowding Out

JEL classification: E2, H6, D52

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1 Introduction

Many countries - including the United States - have experienced a dramatic surge in their public debt levels in the aftermath of the 2007-08 financial crisis. In this paper, we analyze the welfare consequences of government debt, both in the long-run and in the short-run.

We find that in the long-run, government debt has large and negative welfare consequences. The debt/GDP ratio which maximizes aggregate welfare in stationary equilibrium is -50 percent. That is, a benevolent government should hold assets worth 50 percent of GDP in the long-run, and not debt.

Given this result, it is perhaps surprising that government debt is on rise. This apparent contradiction can be explained by considering the combination of short-run and long-run welfare effects. The sum of the two turns out to be positive, which means that long-run and short-run welfare effects have the opposite sign. Reducing government debt is associated with welfare losses, which more than offset the welfare gains that can be reaped in the long-run. This shows that both short-run and long-run welfare effects are important in order to understand the total impact of government debt, an aspect which has been ignored by much of the previous literature.

Our findings suggest that austerity plans which are meant to reduce government debt will lack broad support among the population, even though they are beneficial in the long-run. The fact that debt reduction plans are difficult to implement is consistent with the experience of many countries in which debt consolidation plans have been discussed. Interestingly, our paper shows that reducing government debt redistributes resources from the poor to the rich, which explains why austerity plans are so unpopular.

We conduct our analysis with the help of an incomplete markets framework in the tradition of Aiyagari (1994). Households are subject to idiosyncratic productivity shocks. These shocks are uninsurable because insurance markets are absent. Households can self-insure against adverse shocks by accumulating precautionary savings or by borrowing. In our framework, borrowing is limited. This restricts the ability of households to self-insure.

This framework is ideal for our purposes for two reasons. First, issuing government debt can play a welfare enhancing role if markets are incomplete or if borrowing is restricted. This was shown in the seminal papers of Woodford (1990) and Aiyagari and McGrattan (1998). In Woodford (1990), government debt relaxes binding borrowing constraints.

Aiyagari and McGrattan (1998) argue that government debt helps to 'complete' markets because it facilitates precautionary saving. Issuing government debt might thus be an effective way to improve risk-sharing and aggregate welfare (Flodén, 2001, Shin, 2006, Albanesi, 2008).

In contrast, theories that assume the existence of a representative infinitely-lived household cannot explain why benevolent governments - with full commitment - accumulate debt. If lump-sum taxes are permitted, there is no role for government debt and its welfare effects are zero. The optimal level of government debt is indeterminate. If taxes are distorting, government debt is used to smooth distortions over time. The long-run level of debt depends only on initial conditions, as in Lucas and Stokey (1983). If government debt is non-contingent, as in Aiyagari et al. (2002), a benevolent government accumulates assets.

Second, the fact that markets are incomplete and risk-sharing is limited generates a non-trivial distribution of assets and consumption. We show that the degree of inequality implied by the model
is crucial for assessing the welfare effects of government debt. In order to generate a distribution of earnings and assets that resembles the skewed distributions of earnings and wealth in the U.S. economy, we follow Castañeda, Díaz-Giménez, and Ríos-Rull (2003) in our calibration of the stochastic process that governs the evolution of idiosyncratic earning shocks. Moreover, the fact that our model generates the skewed wealth distribution observable in the U.S. allows us to adequately pin down the impact of different debt reduction strategies on several subgroups of the population. It also allows us to precisely compute the political support that various austerity policies could achieve.

Our results are as follows. If we compute the optimal level of government debt in the long-run, i.e. by comparing the aggregate welfare of different stationary equilibria, we find that it is significantly negative. The optimal debt/GDP ratio depends on whether government debt is financed by adjusting labor income taxation or capital income taxation. If capital income taxation is used, a debt/GDP ratio of -50 percent is optimal. If we adjust the labor income tax, the optimal debt/GDP ratio is even lower. The welfare gains from being at the optimal debt/GDP ratio, relative to the long-term average debt/GDP ratio of the U.S. economy (67 percent), are substantial. They correspond to a permanent increase in consumption of 0.4 percent. The result that a negative debt/GDP ratio maximizes welfare is perhaps surprising, given that there is a welfare enhancing role of government debt in our incomplete markets framework.

In order to understand our finding, it is important to consider the impact of borrowing constraints in greater detail. If borrowing constraints are binding, raising government debt crowds out private capital. This is because households that face binding borrowing constraints will not increase their savings one-to-one in response to an increase in debt, and the Ricardian Equivalence breaks down. This implies that the demand for private bonds will not meet the supply of private bonds issued by the firm. We say that public debt crowds out the capital stock, and therefore also production and output. As a result, the equilibrium interest rate that clears the private bond market will increase, and the marginal product of labor will decrease. Moreover, if taxation is distortionary instead of lump-sum, the negative effect of government debt on capital and output is even stronger, due to an inefficiently low supply of labor and capital.

In a world in which markets are incomplete and in which there is heterogeneity among households, the changes in the interest rate and the wage rate resulting from crowding out and distortionary taxation affect aggregate welfare via two additional channels. First, a higher interest rate facilitates self-insurance of private households, since saving yields a higher return (Aiyagari and McGrattan, 1998). As a result, the price of the riskless production factor (capital) increases, while the price of the risky factor (labor) decreases. We call this the insurance channel of government debt. The fact that changes in market clearing prices affect welfare when markets are incomplete is also key in Gottardi, Kajii, and Nakajima (2010) and Davila et al. (2011), who analyze whether the laissez-faire outcome is constrained efficient if markets are incomplete. \(^1\)

Second, government debt also affects the distribution of consumption via the composition of income, because households that receive capital income benefit and households that mainly rely on labor income lose. The insurance and the income composition channel might have counteracting effects on total

\[^{1}\text{Gottardi, Kajii, and Nakajima (2010) and Davila et al. (2011) find that, depending on the structure of uncertainty, the constrained efficiency requires a higher level of capital compared to the competitive equilibrium outcome when markets are incomplete.}\]
welfare, since the households that profit the most from additional insurance are the consumption-poor, which also suffer the most from a decline in wages.

We show that our finding that the government should hold assets instead of debt hinges on the fact that wealth and income in the United States are very unequally distributed across the population. Because households that are consumption-poor also hold no wealth or are even in debt, the positive insurance effect of government debt has only a small impact on aggregate welfare. As a result, in our calibration, the negative welfare effects more than offset the insurance effect. Instead, Aiyagari and McGrattan (1998) and Flodén (2001) conclude that the opposing effects almost cancel out, leading to only weak overall welfare effects of government debt.

An important contribution of our project with respect to the previous literature is that we explicitly target the high wealth and earnings inequality observed in the U.S. in our calibration procedure, following Castañeda, Díaz-Giménez, and Ríos-Rull (2003).\(^2\) We show that taking into account the high degree of inequality in the U.S. leads to very different conclusions about the optimal level of government debt.

The second important contribution of our paper with respect to the previous literature is that we also compute the transitional welfare effects of government debt. In particular, we analyze the welfare effects of changes of debt/GDP ratio, starting from an initial debt/GDP ratio which is approximately equal to the long-run average of the U.S. economy (67 percent). We focus on Ramsey policies, where changes in debt/GDP ratios are financed by either adjustments in the (linear) tax rate on labor income or by changes in the (linear) tax rate on capital income. The welfare costs of a public debt reduction using these policies are high. In fact, in most of the experiments that we consider, the costs of reducing public debt easily outweigh the welfare gains associated with the new stationary equilibrium, in which the debt/GDP ratio is lower.

Consequently, if the benevolent government takes both short-run and long-run welfare gains into account, it should increase debt in order to maximize aggregate welfare. We find that in this case, welfare gains corresponding to a permanent increase in consumption of up to 4 percent of consumption are possible.\(^3\)

In order to understand the determinants of the large welfare losses that arise if government debt is reduced, it is again important to consider the role of inequality. Reducing government debt means that taxes need to be increased. This primarily affects the wealth-poor, independently of whether labor income taxes or capital income taxes are raised. In order to see this, consider first an increase in labor income taxes. Taxing labor income has adverse effects on the wealth-poor, which rely heavily on labor income to finance their consumption expenditures. If instead capital income is taxed, the saving decisions are distorted, which reduces the aggregate capital stock and therefore also equilibrium output. As a result, the equilibrium wage rate is lower as well, which again hurts those who need to rely on labor income.\(^4\)

\(^2\)The technique by Castañeda, Díaz-Giménez, and Ríos-Rull (2003) is widely applied in the literature, see e.g. Davila et al. (2011).

\(^3\)The optimal debt/GDP ratio depends on the number of periods during which the debt increase takes place. More periods allow for larger debt increases. Our simulations suggest that no combination of periods/debt resulted in a welfare gain above 4 percent.

\(^4\)If the implementation of the capital income tax is unanticipated, there are no saving distortions in the initial period. This somehow mitigates the welfare costs of debt reductions, but does not avoid them. This is due to the fact that the capital stock may still shrink because the capital income tax is larger than 100 percent. In this case, wages still fall
Interestingly, because of inequality, short-run and long-run welfare effects of public debt have the opposite sign. Hence, policy makers face an important trade-off. If government debt is increased, the poor will benefit, but only in the short-run. In the long-run, however, the increase in debt will reduce the welfare of the poor. This mechanism also underlines the importance of studying both long-run and short-run effects of government debt.

Given the high welfare costs of debt reductions, it is thus not surprising that only few policies would enjoy enough support to be implemented by a majority vote. Our findings thus provide an explanation for the fact that fiscal consolidation leads to social unrest, as documented by Ponticelli and Voth (2011). In earlier work, Alesina and Perotti (1996) have established a link between inequality and social unrest. In a related recent contribution, D’Erasmo and Mendoza (2011) study the link between domestic inequality and the likelihood to default on domestic debt in an incomplete markets model similar to ours. They find that inequality increases the probability to default.

An additional advantage of our set-up is that it allows us to compare the welfare costs of certain debt reduction policies. We compare linear and non-linear reduction policies. Non-linear policies reduce debt at an increasing or a decreasing rate. We find that policies in which debt is reduced at an increasing rate are preferred, all other things equal, as long as labor income is taxed. If debt is reduced at an increasing rate, the tax burden associated with this policy is back-loaded. This is beneficial for those households who are at the borrowing constraint. Since a debt reduction results in an increasing income profile over time, stemming from the fact that lower debt today leads to lower taxes in the future, forward-looking households would like to borrow in order to smooth out their consumption profile across periods. If debt is reduced such that the tax burden is front-loaded, this entails a huge utility loss for all those who are constrained in their borrowing already. If the reduction in debt is financed by taxing capital income, it is better to decrease debt as fast as possible, in order to minimize the distortions that are associated with this policy.

So far, the related literature that studies models in which households face uninsurable earnings shocks has largely ignored transitional effects. Notable exceptions are Desbonnet and Weitzenblum (2011), Gottardi, Kajii, and Nakajima (2011), Domeij and Heathcote (2004) and Heathcote (2005). Desbonnet and Weitzenblum (2011) also compute the optimal level of government debt but do not account for the high earnings and wealth inequality observed in the U.S. economy. They thus find results that are very different from ours. In Desbonnet and Weitzenblum (2011), short-run and long-run welfare effects of government debt reinforce each other, while our calibration implies that they point in opposite directions. Gottardi, Kajii, and Nakajima (2011) instead study a different source of uncertainty. They assume that household can invest in government bonds, physical capital and human capital. Investment in human capital is subject to idiosyncratic risk. Most importantly, they abstract from the presence of borrowing constrained households which are crucial for the crowding out effect of public debt on the capital stock emphasized by our paper. Domeij and Heathcote (2004) and Heathcote (2005) also take account of transitional effects, but their focus is on tax policies, not on the welfare effects of government debt. In sum, we are the first to emphasize in combination the importance of taking into account a high degree of inequality, the presence of borrowing constrained households and transitional effects.

initially, and because there are borrowing constraints, those without assets cannot respond by increasing their borrowing in order to smooth out their consumption.
Our findings are complementary to the recent literature analyzing the macroeconomic consequences of the recent financial crisis through the lens of incomplete markets models. Gomes, Michaelides, and Polkovnichenko (2010), for example, quantify the distortionary costs resulting from the increase in government expenditures which were necessary to ‘bailout’ parts of the private sector during the crisis.

In an important contribution, Oh and Reis (2010) analyze the effects of an increase in government expenditure, which we keep constant throughout our analysis. In particular, they focus on targeted transfers, such as retirement benefits, disability payments and medical care. The model which Oh and Reis (2010) use for this purpose also features incomplete markets which manifest themselves in uninsurable, idiosyncratic shocks. Oh and Reis (2010) conclude that the multiplier on employment and output associated with increases in transfers is small.

Another recent interesting contribution by Azzimonti, de Francisco, and Quadrini (2012) links the increase in public debt that has been observed over the last decades to the liberalization of international financial markets and the increase in income inequality. As in our model, Azzimonti, de Francisco, and Quadrini (2012) assume that insurance markets are absent. Different from our set-up, they abstract from capital accumulation and crowding-out. Government debt is thus the only consumption-smoothing device.

The remainder of the paper is structured as follows. We present the baseline model in the next section. In section 3 we discuss the calibration of the model. Section 4 shows the quantitative results. Section 5 concludes.

2 The Baseline Model

The economy we consider is a neoclassical growth model with incomplete markets where households face uninsurable income shocks, as in Aiyagari (1994). The economy consists of three sectors: households, firms and a government. In the following, we describe the three sectors in greater detail. We start by describing the bonds that households in our economy use to accumulate savings.

2.1 Supply and Demand for Bonds

Households self-insure against income fluctuations by saving in one-period risk-free bonds. Bonds are issued by firms and the government, as in Aiyagari and McGrattan (1998) and Flodén (2001). Bonds issued by firms are claims to physical capital. We abstract from aggregate risk, which implies that claims to physical capital and government bonds are perfect substitutes and thus yield the same return, \( r_t \).\(^5\) Different from Aiyagari and McGrattan (1998) and Flodén (2001), we also allow households to borrow up to a certain limit. We view this as an important modification, given that the fraction of households that actually borrow in the data is substantial.\(^6\)

\(^5\)In Gomes, Michaelides, and Polkovnichenko (2008, 2010), government bonds and private capital are imperfect substitutes due to aggregate uncertainty.

\(^6\)Borrowing by households can be interpreted as bonds that are issued to other households (‘IOUs’) or to the government.
2.2 Household Sector

The economy is populated by a continuum of ex-ante identical, infinitely lived households with total mass of one. Households maximize their expected utility by making a series of consumption, leisure and savings choices subject to a budget constraint and a borrowing limit on assets. In period $t = 0$, before any uncertainty has realized, their expected utility is given by

$$U(\{c_t, l_t\}_{t=1,2,\ldots}) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

where $\beta$ is the subjective discount factor. The per-period utility function, $u(\cdot)$, is assumed to be strictly increasing, strictly concave and continuously differentiable. Additionally the first derivative is assumed to satisfy the following limiting (Inada) conditions:

$$\lim_{c \to 0} u_c(c, l) = \infty, \quad \lim_{c \to \infty} u_c(c, l) = 0$$
$$\lim_{l \to 0} u_l(c, l) = \infty$$

Household productivity is subject to a shock, $\epsilon_t$, that follows a Markov process with transition matrix $\pi(\epsilon'|\epsilon)$.

A household faces the following per-period budget constraint:

$$c_t + a_{t+1} = y_t + a_t$$

where $a_{t+1}$ denotes the bond holdings of a household. $y_t$ is the household’s (after-tax) income. Notice that $a_{t+1}$ may also be negative, in which case the household borrows. Borrowing is restricted:

$$a_{t+1} \geq 0$$

The fact that we impose an exogenous borrowing constraint is important. In the Appendix we show that with a natural borrowing constraint and lump sum taxes government debt is neutral (Ricardian equivalence holds).

The government can tax labor income at some proportional tax rate, $\tau_{l,t}$, as well as financial income at some proportional tax rate, $\tau_{a,t}$, and can redistribute income via lump sum transfers, $tr_t$. In our benchmark model we assume that the transfers stay constant over time, i.e. $tr_t = tr$, but as a robustness check we will later also analyze the case, where the government varies transfers and debt at the same time. We assume that only non-negative financial income is taxed or in other words there are no proportional subsidies in the face of financial losses. More precisely, we define the tax on financial income $\tau_{a,t}$, as follows:

$$\tau_{a,t}(a_t) = \begin{cases} 
\tau_{a,t} & \text{if } a_t \geq 0 \\
0 & \text{if } a_t < 0
\end{cases}$$

The after-tax interest rate is therefore given by $\bar{r}_t = (1 - \tau_{a,t}(a_t))r_t$. The after-tax wage rate is given by $\bar{w}_t = (1 - \tau_{l,t})w_t$ where $w_t$ is the price of labor in the economy. After-tax income is thus given by:

$$y_t = \bar{w}_t \epsilon_t (1 - l_t) + \bar{r}_t a_t + tr_t$$
The optimization problem of a household in recursive formulation looks as follows:

\[
W_t(a, \epsilon) = \max_{c,l,a'} \left\{ u(c, l) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) W_{t+1}(a', \epsilon') \right\} \\
\text{s.t. } c + a' = \bar{w}_t \epsilon(1-l) + (1 + r_t) a + \tau r_t a' \\
a' \geq a
\]

The household’s problem is time-dependent because we do not only study steady-states but also transitions between steady-states.\(^7\)

### 2.3 Firm sector

We assume that the aggregate production technology which is operated by a representative firm to produce output, \(Y_t\), using aggregate capital, \(K_t\), and aggregate labor, \(L_t\), as inputs is given as follows:

\[
Y_t = F(K_t, X_t L_t)
\]

where \(X_t\) denotes exogenous labor-augmenting technological progress. This technology is assumed to grow exogenously at a constant rate \(X_{t+1} = (1 + g) X_t\). For simplicity we normalize initial technology to \(X_0 = 1\), such that:

\[
X_t = (1 + g)^t
\]

The presence of technological progress implies that households, on average, become richer over time. Technological progress thus increases the propensity of households to borrow.\(^8\)

The aggregate production function, \(F\), is assumed to have the standard properties, in particular constant returns to scale. This ensures that in competitive equilibrium, the number of firms is indeterminate and we can assume the existence of a representative firm, without loss of generality.

### 2.4 Government sector

The government has to finance a time-invariant amount of government spending, \(G\), and the total transfers to households, \(TR\), by issuing new government bonds, \(B_{t+1}\), and levying taxes on positive asset and labor income. Furthermore, the government services its debt, \(B_t\), and makes interest payments, \(r_t B_t\). The government budget constraint is thus given by:

\[
G + r_t B_t + TR = B_{t+1} - B_t + \tau w_t L_t + \tau_a r_t A_t
\]

where \(\hat{A}_t \geq A_t\) is the tax base for the asset income tax. As explained above taxes are only levied on positive financial income (no proportional transfers from the government for indebted people) and thus the tax base is defined as:

\[
\hat{A}_t = \int_{a_t \geq 0} a_t d\theta(\epsilon_t, a_t)
\]

\(^7\)Notice that as long we focus on transition between steady-states, the dynamic programming problem of the household falls into the class of stationary dynamic programming problems for which the principle of optimality is satisfied. We numerically show that our economy converges to a new steady-state.

\(^8\)Technically, in the detrended version of the household problem which is presented in the appendix, technological progress reduces the discount factor. This reduction also decreases the propensity to save and thus increases borrowing, all other things equal.
where \( \theta(\epsilon_t, a_t) \) denotes the distribution of households over income and asset states. Aggregate transfers have to equal the sum of all individual transfers:

\[
\int \text{trd} \theta(\epsilon_t, a_t) = TR
\]

### 2.5 Recursive competitive equilibrium

Using the characterization of the three sectors we can now define the recursive competitive equilibrium.

**Definition 1. Recursive Competitive Equilibrium:** Given a transition matrix \( \pi \), a government policy \( \{B_t, \tau_{a,t}(a_t), \tau_{l,t}, G\}_{t=0}^\infty \), and an initial distribution of the idiosyncratic productivity shocks and of the asset holdings \( \theta_0(\epsilon_0, a_0) \) a recursive competitive equilibrium is defined by a law of motion \( \Gamma \), factor prices \( (r_t, w_t) = (r(K_t), w(K_t)) \), the value function \( W = W(\theta, a, \epsilon) \) and policy functions \( (c, a') = (\gamma(\theta, a, \epsilon), \zeta(\theta, a, \epsilon)) \) such that

1. Households’ utility maximization problem is defined in equation (1).

2. Competitive firms maximize profits, such that factor prices are given by

\[
\begin{align*}
  w_t &= F_L(K_t, X_t L_t) \\
  r_t &= F_K(K_t, X_t L_t) - \delta
\end{align*}
\]

3. The government budget constraint as defined in equation (2) holds.

4. Factor and goods markets have to clear:

   - **Labor market clearing:**
     \[
     N_t = \int \epsilon_t (1 - l_t) d\theta_t(\epsilon_t, a_t) = L_t
     \]

   - **Asset market clearing:**
     \[
     A_{t+1} = \int a_{t+1} d\theta_t(\epsilon_t, a_t) = K_{t+1} + B_{t+1}
     \]

   - **Goods market clearing:**
     \[
     \int c_t d\theta_t(\epsilon_t, a_t) + G + I_t = F(K_t, X_t L_t)
     \]

   where investment, \( I_t \) is the sum of private investment and public investment:

   \[
   I_t \equiv K_{t+1} - (1 - \delta)K_t + B_{t+1} - (1 + r_t)B_t
   \]

5. Rational expectations of households about the law of motion of the distribution of shocks and asset holdings, \( \Gamma \) reflect the true law of motion, as given by

\[
\theta_{t+1}(a_{t+1}, \epsilon_{t+1}) = \Gamma[\theta_t(a_t, \epsilon_t)]
\]

where \( \theta(a, \epsilon) \) denotes the joint distribution of asset holdings and productivity shocks.
2.6 Welfare measure

In order to be able to compare the welfare effects of different government policies, we have to define a welfare criterion. Following the previous literature as for example, Aiyagari and McGrattan (1998) and Flodén (2001), we compute the aggregate value function:

\[ \Omega = \int W(a, c; \theta) d\theta(a, c) \]

This criterion can either be interpreted as (1) a utilitarian social welfare function where every individual has the same weight for the planner, (2) a steady-state ex ante welfare of an average consumer before realizing income shocks and initial asset holdings or (3) the probability limit of the utility of an infinitely lived dynasty where households utilities are altruistically linked to each other (for more details see Aiyagari and McGrattan, 1998).

In order to facilitate the interpretation, we compute the average welfare change in consumption equivalent units, i.e. the consumption that needs to be given to each household in order to make households indifferent on average between two specific policies. More details are provided in the Appendix.

3 Calibration

We calibrate our model such that it is consistent with long run features of the U.S. economy. The resulting allocation serves as a benchmark for our welfare calculations.

The parameter values that result from our calibration procedure are shown in Table 1. Parameter values that are adopted from the existing literature are given in Table 2. In the following, we discuss the rationale behind our parameter choices in greater detail.

3.1 Utility Function and Production Technology

We assume that preferences can be represented by a constant relative risk aversion utility function:

\[ u(c) = \frac{(c^{\eta}(1-\eta))^{1-\mu}}{1-\mu} \]

\( \mu \) is the coefficient of relative risk aversion, which we set to 2. This is well in the range (between 1 and 3) commonly chosen in the literature. \( \eta \) denotes the share of consumption in the utility function. We calibrate \( \eta \) such that the average share of time worked is 0.3. This results in \( \eta = 0.31 \).

We assume that the aggregate technology is given by a Cobb-Douglas production function:

\[ F(K, XL) = K^{\alpha}(XL)^{1-\alpha} \]

Initial technology is normalized to \( X_0 = 1 \), such that \( X_t = (1 + g)^t \). We set \( g = 0.02 \), which implies that our economy growth at a rate of 2 percent per year. The parameter \( \alpha \), which denotes the share of capital in total production, is set to 0.3. This implies a labor share of 0.7. The discount factor \( \beta \)

\[ \beta \text{More precisely, this measure provides the percentage increase in benchmark consumption at every date and state (with leisure at every date and state held fixed at benchmark values) that leads to the same welfare (under the benchmark policy) as under the new policy.} \]
Table 1: Calibrated Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target Data Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor, $\beta$</td>
<td>0.96</td>
<td>Capital to output ratio 3.1</td>
</tr>
<tr>
<td>Labor supply elasticity, $\eta$</td>
<td>0.31</td>
<td>Average labor supply 0.3</td>
</tr>
<tr>
<td>Borrowing constraint $\alpha$</td>
<td>-0.3</td>
<td>% of HH with no assets or debt 0.24</td>
</tr>
<tr>
<td>Gov. spending, $G$</td>
<td>0.15</td>
<td>gov. budget constraint clearing -</td>
</tr>
</tbody>
</table>

Table 2: Parameters Set Exogenously

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital’s share, $\alpha$</td>
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<tr>
<td>Growth rate, $g$</td>
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<tr>
<td>Debt to GDP ratio, $b$</td>
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<td>Labor tax, $\tau^l$</td>
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<tr>
<td>Capital tax, $\tau^k$</td>
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<tr>
<td>Transfers, $tr$</td>
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<tr>
<td>Risk Aversion $\mu$</td>
<td>2</td>
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</tbody>
</table>

is chosen such that the model reproduces a wealth-output ratio of 3.1 (cf. Cooley and Prescott (1995) or Ábrahám and Cárcules-Poveda (2010)). Since we do not model housing, wealth is defined as net financial assets excluding housing and other real estate. The resulting $\beta$ is equal to 0.96. The annual depreciation rate $\delta$ is set to 7 percent, which is a common value in the literature (see e.g. Trabandt and Uhlig (2009)).

3.2 Taxes and Government Debt

Following Trabandt and Uhlig (2009), we set the labor income $\tau_l$ and capital income tax rate $\tau_k$ to 0.28 and 0.36, respectively. Similar values are also reported by Mendoza, Razin, and Tesar (1994). Lump-sum transfers $tr$ are set to 0.083, in accordance with Trabandt and Uhlig (2009). Following Aiyagari and McGrattan (1998), we use a debt to GDP ratio of 0.67. Government spending $G$ is set such that the government’s budget constraint clears, given all other parameters.

3.3 Income process

We calibrate the vector of income states, $s$, and the transition matrix, $\Pi$, such that the distribution of earnings and net worth generated by the model are consistent with the data. Disciplining the model such that it is consistent with the skewed distribution of earnings and wealth observable in the U.S. economy is key for assessing the optimal level of government debt. In particular, we are interested in the share of (consumption-) poor households in the economy, who receive more weight in our utilitarian welfare criterion. In addition, matching inequality is also important in order to determine the factor
income composition of the poor, which in turn determines whether poor households gain or lose from changes in government debt.

We compute the distribution of earnings and net worth from the 2007 Survey of Consumer Finances (SCF) (see Table 3 and 4). Since we do not model housing, we define net worth as net financial assets excluding housing and other real assets (see also Abrahám and Cárcedes-Poveda (2010)). Earnings are defined as labor earnings (wages and salaries) plus a fraction of business income before taxes, excluding government transfers. This definition is close to the concept of earnings that is implied by our model as well.

Table 3 and 4 show that both earnings and net financial assets are very unequally distributed in the data. The richest 20 percent of the population hold more than 90 percent of all financial assets, net of debt. The distribution of earnings is less skewed. Households in the top quintile earn around 60 percent of the total earnings.

It is well known that for a standard parameterization of the earnings process, incomplete markets models in the tradition of Aiyagari (1994) generate too little inequality compared to the data (see e.g. Quadrini and Ríos-Rull (1997)). This can also be seen from Tables 3 and 4, where we report the earnings and wealth distribution that is implied by our model if we parameterize the earnings process as well as all other parameters as in Aiyagari and McGrattan (1998) (see row "Model fitted to AR(1)").

Aiyagari and McGrattan (1998) assume that the earnings process is given by an AR(1) process with persistence \( \rho = 0.6 \) and variance of innovations of \( \sigma = 0.3 \).

We follow Castañeda, Díaz-Giménez, and Ríos-Rull (2003) and calibrate the vector of income states \( s \) and the transition matrix \( \mathbf{I} \) to match the Lorenz curves of U.S. earnings and wealth as found in our analysis of the 2007 SCF.

\[ \text{Unfortunately, it is not declared exactly in the SCF how much of the business income is actually labor and how much is capital income. We take business income from sole proprietorship or a farm to be labor earnings, whereas we define business income from other businesses or investments, net rent, trusts, or royalties as capital income.} \]

\[ \text{Our calibration of the income process is one main difference with respect to Aiyagari and McGrattan (1998). Another difference is that we consider distinct tax rates on capital and labor income, whereas Aiyagari and McGrattan (1998) assume one tax rate on total income. However, in the Appendix, we show that implementing separate tax rates on capital and labor income in Aiyagari and McGrattan (1998) changes the welfare function only slightly. Furthermore the level of the interest rate is higher in our benchmark calibration, because of a higher depreciation rate (0.075 instead of 0.07). The wage rate is slightly lower in our calibration because of a different target for aggregate hours (0.3 instead of 0.28). Those deviations are not substantial enough to account for important differences in results. Robustness checks are not reported, but available from the authors upon request.} \]

\[ \text{We also compute an intermediate case where we calibrate the model to match the targets outlined above, but keep the income process as in Aiyagari and McGrattan (1998). The results of this experiment are presented in the Appendix. Table 4 in the Appendix shows that earnings and wealth inequality are very similar to the distributions that are implied by the calibration in Aiyagari and McGrattan (1998).} \]

\[ \text{In the model, wealth inequality (and consumption inequality) are the result of households’ optimal decisions with respect to consumption and saving. Household use assets in order to smooth consumption over time and in order to save against uninsurable income shocks. Households’ decision making in turn depends on preference parameters and the specification of the earnings process. In our calibration, we modify the uninsurable income shocks such that the equilibrium asset allocation resembles the observable wealth inequality in the U.S. An alternative approach, which is due to Krusell and Smith (1998), would be to introduce preference heterogeneity. Krusell and Smith (1998) assume that some households are more patient than others, and thus also accumulate more savings. For the purpose of our model, applying the method of Castañeda, Díaz-Giménez, and Ríos-Rull (2003) has two advantages. First, it allows us to also match the observable degree of earnings inequality, which is important given that we need to know the factor income composition of the poor. Second, it is commonly argued in the literature that government debt improves self-insurance (see Aiyagari and} \]
Table 3: Distributional Properties at Benchmark Stationary Economy

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net financial assets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>−1.60%</td>
<td>0.10%</td>
<td>1.64%</td>
<td>8.29%</td>
<td>91.57%</td>
<td>0.90</td>
</tr>
<tr>
<td>Benchmark Calibration</td>
<td>−1.57%</td>
<td>0.88%</td>
<td>3.92%</td>
<td>7.23%</td>
<td>89.54%</td>
<td>0.83</td>
</tr>
<tr>
<td>Model fitted to AR(1)</td>
<td>3.24%</td>
<td>10.07%</td>
<td>16.96%</td>
<td>25.71%</td>
<td>44.03%</td>
<td>0.41</td>
</tr>
<tr>
<td><strong>Earnings</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>−0.40%</td>
<td>3.19%</td>
<td>12.49%</td>
<td>23.33%</td>
<td>61.39%</td>
<td>0.62</td>
</tr>
<tr>
<td>Benchmark Calibration</td>
<td>0.00%</td>
<td>2.38%</td>
<td>12.58%</td>
<td>22.73%</td>
<td>62.31%</td>
<td>0.65</td>
</tr>
<tr>
<td>Model fitted to AR(1)</td>
<td>1.21%</td>
<td>9.70%</td>
<td>16.18%</td>
<td>26.85%</td>
<td>46.07%</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Remarks: Quintiles (Q1-Q5) denote net financial assets (resp. earnings) of a group in percent of total net financial assets (resp. earnings). The entries in 'data' are computed from the 2007 SCF. See main text for precise definitions. Notice that earnings can be negative due to the fact that labor earnings also contain part of the gains (or losses) of small enterprises.

Table 4: Upper Percentiles of Wealth Distribution at Benchmark

<table>
<thead>
<tr>
<th></th>
<th>upper 10%</th>
<th>upper 5%</th>
<th>upper 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net financial assets</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>79.64%</td>
<td>66.83%</td>
<td>39.09%</td>
</tr>
<tr>
<td>Benchmark Calibration</td>
<td>70.58%</td>
<td>47.03%</td>
<td>13.53%</td>
</tr>
<tr>
<td>Model fitted to AR(1)</td>
<td>26.16%</td>
<td>15.00%</td>
<td>3.81%</td>
</tr>
</tbody>
</table>

Remarks: The table shows the percent of net financial assets held by the wealthiest 10% (upper 10%), 5% (upper 5%) and 1% (upper 1%).

Tables 3 and 4 show that our model tracks the observable wealth and earnings distribution very closely, with the exception of the top decile of the wealth distribution. Here, our earnings process generates less inequality than the data, but much more than a standard AR(1) process. We find the following vector of income states:

\[ s = \{0.055, 0.551, 1.195, 7.351\} \]

It should be noted that the highest income state is more than 130 times as high as the lowest income state.

McGrattan (1998)). Given that the method by Castañeda, Díaz-Giménez, and Ríos-Rull (2003) emphasizes savings for self-insurance as the main driving force behind wealth inequality, we would thus expect that the welfare enhancing role of government debt, which stems from the fact that government debt facilitates self-insurance, is even more prominent in our context, compared to e.g. Aiyagari and McGrattan (1998).
Furthermore, we get the following transition matrix for the income states:

\[
\Pi = \begin{bmatrix}
0.940 & 0.040 & 0.020 & 0.000 \\
0.034 & 0.816 & 0.150 & 0.000 \\
0.001 & 0.080 & 0.908 & 0.012 \\
0.100 & 0.015 & 0.060 & 0.825
\end{bmatrix}
\]

As can be seen from the transition matrix, there is a 10 percent probability of moving from the highest income state today to the lowest income state tomorrow. This generates a strong saving motive for income-rich households, leading to the high degree of wealth inequality that we also observe in the data. The same mechanism is also present in the transition matrix found by Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

### 3.4 Borrowing limit

We calibrate the ad-hoc borrowing limit to match the percentage of households with negative or zero financial assets in the 2007 SCF (24 percent). We find a borrowing limit of \( a = -0.3 \).

### 4 Results

We are now ready to compute the optimal amount of government debt with the help of our quantitative model. We proceed as follows. First, we analyze the welfare consequences of government debt in the long run. This is done by comparing stationary equilibria that are characterized by different debt/GDP ratios, keeping all other parameters constant. In order to keep the budget of the government balanced, we also adjust either the labor tax rate or the capital tax rate. We show that a benevolent government would choose a stationary equilibrium in which debt is negative, i.e. in which the government holds assets. In a second step, we also incorporate the welfare effects that occur over the transition between a high-debt stationary equilibrium to a low-debt equilibrium. We find that the costs of debt reductions occurring over the transition can be substantial, depending on the policy. The short-run costs of debt reductions can easily outweigh the long-run gains.

#### 4.1 Effects of Public Debt in the Stationary Equilibrium

In Figure 1, we plot the aggregate welfare changes for stationary equilibria that differ with respect to the public debt/GDP ratios, relative to the benchmark in which debt amounts to 2/3 of GDP. Figure 1 conveys a clear message: stationary equilibria with lower debt/GDP ratios offer more aggregate welfare, compared to the benchmark economy. The increase in welfare is however not monotone. There is a peak at around \(-50\) percent of GDP or \(-110\) percent of GDP, depending on whether we adjust the capital income tax (blue crosses) or the labor income tax (red squares) in order to clear the government’s budget constraint. In sum, Figure 1 points to the fact that the government should hold assets, not debt.

It is interesting to compare the welfare function implied by our calibration with the result from the seminal work by Aiyagari and McGrattan (1998), who were the first to analyze the optimal level of government debt in a model with incomplete markets. We reproduce their welfare function in Figure 1 (black circles). The most striking difference is that the welfare function in Aiyagari and McGrattan
Figure 1: **Comparing Welfare Between Different Stationary Equilibria.** In this exercise we plot the welfare change in consumption equivalent units implied by our model (on the ordinate) for different stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa), relative to the benchmark in which public debt amounts to \( \frac{2}{3} \) of GDP (green diamond and vertical line). Two cases: (1) the capital income tax is adjusted (blue crosses); (2) the labor income tax is adjusted (red squares). Black circles show for comparison results from the seminal paper by Aiyagari and McGrattan (1998).

(1998) has its peak at about 66 percent, which is also the (long run) average ratio of government debt to GDP in the U.S. economy. In contrast, the welfare function that results from our calibration indicates that the optimal level of government debt is negative and thus far away from the long run level in the U.S. economy. As a consequence, in our benchmark economy, large welfare gains are possible by reducing government debt.

In the following, we explain the driving forces behind these results in more detail. In particular, we stress the role that inequality plays in explaining the difference between the findings of Aiyagari and McGrattan (1998) and our results. There are three channels through which changes in aggregate production and aggregate prices affect welfare.\(^{14}\) We describe each in greater detail in the following.

**Level effect:** As we showed in Section 3, government debt crowds out the capital stock if borrowing constraints are binding. For our benchmark economy, this effect can be seen in Figure 2, where we depict assets (i.e. aggregate private savings) and the capital stock (private savings minus public debt), relative to GDP.

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\(^{14}\)We adopt the labels of Flodén (2001), who distinguishes a "level" effect, an "uncertainty" effect and an "inequality" effect. By re-labeling the inequality effect into "income composition effect", we would like to stress the origin of inequality in the model, namely the fact the composition of income between households differs.
Figure 2: Capital, Assets, Income Before Taxes, Income After Taxes. This figure shows the changes in selected aggregate economic variables (on the ordinate) for different stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa). In the benchmark public debt amounts to 2/3 of GDP (green diamond and vertical line). Two cases: (1) capital income tax is adjusted (blue crosses); (2) labor income tax is adjusted (red squares). All variables relative to GDP in the benchmark.

Higher levels of public debt decrease capital. This means that the increase in assets supplied by households cannot compensate for the increase in asset demand exerted by the government. The reverse also holds. If government debt is reduced, the capital stock is crowded in. Households do not reduce their asset holdings as much as the government reduces its debt. As a result, the capital stock increases.$^{15}$

If the capital stock in the benchmark economy is below its efficient level, crowding out reduces aggregate output (net of depreciation) and thus lowers welfare, all other things equal. This is indeed the case, as Figure 2 shows. In the second row, we plot income before and after taxes for different public debt/GDP ratios. The higher the debt/GDP ratio, the lower is income (first panel in the second row of Figure 2). The same is true even if we look at income after taxes (second panel in the second row of

$^{15}$One might wonder to what extent the crowding out (or crowding in) that is observed is indeed due to the interaction between binding borrowing constraints and public debt, or rather due to the fact that taxes are distortionary. We address this point with the help of the following experiment, which we present in the Appendix. We fix the labor and the capital tax rate at their benchmark levels. When we adjust the debt/GDP ratio, we modify only the lump-sum transfer in order to keep the government’s budget constraint balanced. Distortions are thus constant, independent of the level of public debt. This experiment shows that the development of the aggregate capital stock depends mainly on public debt/GDP ratio. The impact of tax distortions is limited, unless the debt/GDP ratio is far below the optimal level.
Insurance effect: Figure 3 shows the reaction of the interest rate $r$ and the wage rate $w$ if we change the ratio of public debt/GDP. An increase in debt, relative to GDP, raises the interest rate and reduces the wage rate. This is consistent with our previous result that an increase in public debt crowds out private assets.

All other things equal, an increase in the interest rate and a fall in the wage rate reduces uncertainty in total income. This is because the weight of the uncertain income component, namely labor income, is reduced relative to capital income, which is certain in our economy. As a result, uncertainty about consumption is reduced as well, and households experience an ex-ante welfare gain.

Figure 3: Interest Rate and Wage Rate. This figure shows the changes in equilibrium prices for capital and labor (on the ordinate) for different stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa). In the benchmark public debt amounts to $2/3$ of GDP (green diamond and vertical line). Two cases: (1) capital income tax is adjusted (blue crosses); (2) labor income tax is adjusted (red squares). Also shown: results from Aiyagari and McGrattan (1998) (black circles).

Interestingly, Figure 3 indicates that the elasticity of the wage rate and the interest rate with respect to public debt is much higher in our benchmark calibration, compared to Aiyagari and McGrattan (1998). In the Appendix, we show that this result that crowding out is more pronounced for our calibration compared to the calibration of Aiyagari and McGrattan (1998) is independent of the distortiveness of taxation by presenting the same graph for a lump sum tax experiment. Instead, we argue that the force that drives crowding out is the fact that government debt is non-neutral in the presence of borrowing constrained households.17

16However as one can see from the second panel in the second row of Figure 2 at very high reductions of government debt in the case of capital taxation income after taxes goes down again. The reason is that taxation of capital must be very large to be able to decrease debt by so much and then the effect of taxes outweighs the crowding in effect.

17The exact mechanism is as follows: If government debt increases because of the presence of borrowing constrained agents the demand for assets does not rise as much as the supply of assets (or bonds) which pushes the interest rate up. Since our calibration matches more precisely the wealth distribution the number of borrowing constrained agents should also correspond more closely to what it actually is in the data. We thus argue that the elasticity of aggregate prices with
**Income composition effect:** As we just argued, higher public debt to GDP ratios are associated with higher interest rates and lower wage rates. We now ask whether this implies that also total asset income $r_A$ is higher and total labor income, $w_L$, is lower if debt/GDP ratios are higher. The answer is not obvious, given that government debt crowds out the capital stock. Therefore, $r_A$ may actually be lower for higher levels of public debt.

Figure 4: Asset Income, Asset Income After Taxes, Labor Income, Labor Income After Taxes. This figure shows the changes in aggregate household income components (on the ordinate) for different stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa). In the benchmark public debt amounts to 2/3 of GDP (green diamond and vertical line). Two cases: (1) capital income tax is adjusted (blue crosses); (2) labor income tax is adjusted (red squares). "Asset Income" and "Asset Income After Taxes" ("Labor Income" and "Labor Income After Taxes") is relative to asset income (labor income) at benchmark.

Figure 4 indicates that this is not the case. Aggregate asset income $r_A$ is indeed higher for higher levels of public debt (first panel of first row), while aggregate labor income $w_L$ is lower (first panel of second row). These patterns still pertain, even if we consider disposable asset income $(1 - \tau_k)r_A$ and disposable labor income $(1 - \tau_l)w_L$ (second column in the Figure).

These results show that asset owners experience, on average, a gain in their income if there is more government debt, while those households who primarily depend on labor income experience a loss. This means that in an economy where wealth and thus income from assets are very unequally distributed, only few households actually benefit from the redistribution of resources associated with an increase in government debt. Since in the U.S. a large fraction of the population is indebted or holds almost no respect to public debt is better captured with our calibration.
assets, the income composition effect predicts that more public debt reduces aggregate welfare. Last but not least, it is important to notice that the income composition effect would also exist if markets were complete and if idiosyncratic shocks were fully insurable. This distinguishes the income composition effect from the uncertainty channel that was previously outlined.

In sum, government debt affects aggregate welfare via a level effect, an uncertainty channel and through the composition of individual income. The level effect and the income composition effect imply that higher levels of public debt reduce aggregate welfare, whereas higher levels of public debt increase welfare according to the uncertainty effect. The relative strength of each channel depends on the degree of wealth and income inequality in the economy. Therefore, the two negative effects outweigh the positive effect in our calibration, while the calibration of Aiyagari and McGrattan (1998) implies that all channels approximately cancel out. This highlights the importance of matching the observable degree of wealth and income inequality for computing the optimal level of government debt.

Figure 5: Capital Income Taxes and Labor Income Taxes. This figure shows the changes in capital and labor income taxes (on the ordinate) for different stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa). In the benchmark public debt amounts to 2/3 of GDP (green diamond and vertical line). Two cases: (1) capital income tax is adjusted (blue crosses) and (2) labor income tax is adjusted (red squares).

In the following, we shed more light on the origins of the two different cases for taxes. In order to do so, we first analyze the behavior of the two taxes for different debt/GDP ratios. Figure 5 shows that the capital income tax (left panel) changes more drastically, compared to the labor income tax (right panel). In order to understand why, compare two fictitious stationary equilibria, one with a high the other one with a low debt/GDP ratio. We would expect that in the stationary equilibrium with less public debt, taxes are lower, since the government needs fewer resources to service its debt. However, we know from our previous analysis that lower public debt levels are associated with less asset income $rA$ and more labor income $wL$. This implies that, all other things equal, tax revenues from capital taxation are falling in the debt/GDP ratio. This means that taxes have to increase at some point, in order to compensate for decreasing tax revenue from capital taxation. According to Figure 5, taxes are

---

18 Formally, since we have indebted agents who are not subject to taxation aggregate assets are not exactly equal to
hump shaped with minimum around a debt/GDP ratio of 0 increasing for higher as well as for lower debt/GDP ratios. The increase on the left hand side is substantially more pronounced if the government uses capital taxation to balance its budget, because an increase in the capital tax makes saving even less attractive and thus reduces the tax base further. A vicious circle starts.

Figure 5 also shows that the government needs to increase the labor income tax for debt/GDP ratios below 0. This is because the revenues from capital income taxation are falling. In order to compensate for this, the government needs to raise the tax rate on labor income to clear its budget. However, there is no vicious circle in this case, since aggregate labor income \( wL \) is increasing the lower the debt/GDP ratio. Therefore, the labor tax adjustments that are necessary to keep the government’s budget balanced are quite small.

It is not surprising that the substantial differences in the behavior of capital and labor income taxes for different debt/GDP ratios are also reflected in the welfare functions shown in Figure 1. We would like to highlight three issues. First, if the government adjusts the labor income tax, it can achieve a higher aggregate welfare. This result is a direct consequence of the fact that a raise in labor taxation does not cause the beginning of a vicious circle, as it is the case for capital taxation.

Second, the optimal amount of government debt for the case in which the labor income tax rate adjusts is twice as low, compared to the case in which the capital income tax adjusts. Put differently, if the government adjusts the labor tax to clear its budget, it should accumulate assets worth 110 percent of GDP, while it should optimally accumulate only assets worth 50 percent of GDP if the government adjusts the capital tax rate. Again, the reason for this discrepancy is the high capital tax rate which is necessary to achieve a balanced budget for low debt/GDP ratios. This affects the after-tax income such that aggregate welfare is decreased. This can be seen by again considering Figure 4 shown above, where we depict aggregate capital income \( rK \) and aggregate labor income \( wL \) as well as after-tax capital and labor income, \((1 - \tau_k) rK\) and \((1 - \tau_l) wL\). While both \( rK \) and \( wL \) are monotonically increasing if the government accumulates assets, independently of which of the two taxes adjusts, we see that \((1 - \tau_k) rK\) falls sharply for debt/GDP ratios which are lower than \(-0.5\) if the capital income adjusts. There is no such effect if the labor income tax adjusts. After-tax labor income \((1 - \tau_l) wL\) instead declines slightly for debt/GDP ratios below \(-1.1\) if the labor income tax adjusts. Again, there is no change in \((1 - \tau_l) wL\) if the capital tax adjusts. It is important to notice that the fact that tax rates are higher for low debt/GDP levels implies that the welfare function is not monotonically increasing, but concave.

In the next subsection, we decompose the aggregate steady-state welfare effects into group-specific welfare effects. This helps us to analyze the welfare changes of various debt-reducing policies in the section thereafter.

**4.2 Welfare of different wealth-groups: poor, middle class, rich**

In this subsection, we show that the aggregate welfare effect that we documented before is highly unequally distributed among wealth groups. We analyze three groups of households that are defined as follows:

1. Poor: Households with zero or negative assets.

The tax base. Quantitatively, it turns out that the difference is not that large. Therefore, we do not show the tax base separately.
2. Rich: Group of households who together own 70 percent of total assets.

3. Middle class: Households who do not belong to either of the previous categories.

The logic behind these definitions is as follows. According to our definition, the poor do not receive asset income and are thus only affected by changes in the after-tax wage rate. The rich instead care primarily about the after-tax interest rate. Finally, the middle class is affected by both changes in the after-tax wage rate and in the after-tax interest rate.\footnote{We keep the definition constant in all of the following experiments.}

Figure 6: \textbf{Group Size and Welfare Change of Wealth Groups.} This figure shows the changes in group size and welfare of a group (on the ordinate) for different stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa). In the benchmark public debt amounts to $2/3$ of GDP (green diamond and vertical line). Three groups: (1) the poor have no assets or are in debt (black circles), (2) the rich own 70\% of assets as a group (red squares), (3) the middle class is the rest of the households that are neither rich nor poor (blue crosses). Two cases: (1) capital income tax is adjusted (left panel) and (2) labor income tax is adjusted (right panel).

In Figure 6, we plot the share of population belonging to the respective groups. In the benchmark steady-state with debt/GDP of 66 percent, around 20\% of all households are poor according to our definition. Around 2\% are rich and the rest belongs to the middle class.

The relative group-sizes are not invariant to changes in the debt/GDP ratio, as the first row of Figure 6 makes apparent. The lower debt/GDP, the bigger the fraction of households that are poor.
Intuitively, lower interest rates that are associated with lower debt/GDP ratios discourage saving.\textsuperscript{20}

The group-specific welfare functions are shown in the second row of Figure 6. The between-group differences are enormous. If one compares the benchmark economy to the economy with the optimal debt/GDP ratio, poor households gain on average 13 percent of their yearly consumption if the capital tax is changed and 20 percent if the labor tax is changed. The difference is mainly due to the fact that the optimal level of debt is much lower if the labor tax adjusts (−1.1 compared to −0.5).

The welfare function of the rich is almost exactly the mirror image of the welfare function of the poor. Compared to the benchmark, the rich lose 7 percent of their yearly consumption if they live in an economy where the optimal debt/GDP ratio with adjustments in capital taxation is implemented. The loss is 11 percent if labor taxation is adjusted. Interestingly, the welfare function of the rich appears to be slightly concave, at least if adjustments in the capital taxation are considered. The reason is that the capital tax adjustments between different debt/GDP ratios are non-monotonic. For high debt/GDP ratios, the rich gain from a higher interest rate. At the same time, they lose because taxes are higher, the more public debt needs to be serviced. For low debt/GDP ratios, the rich lose because of a lower interest rate. If debt/GDP falls below zero, they on top of that lose because tax rates are higher compared to the benchmark stationary equilibrium.

This section has shown that there are substantial differences in the welfare effects between different subgroups of the population. This again highlights the importance of inequality for assessing the optimal level of government debt. In the following subsection, we move beyond the mere comparison of stationary equilibrium debt/GDP ratios by incorporating the transition into our results.

4.3 Welfare over the transition path

We now incorporate the transition between two stationary equilibria into our analysis. In this subsection, we aim at answering two questions. First, we suppose that there is a government which wants to reduce its government debt. How should it proceed, given that it can only adjust either the capital income tax or the labor income tax.\textsuperscript{21} This question is motivated by our long-run analysis above, where we found that the government should optimally hold assets. Given this, it is natural to ask how the government should reduce debt in order to achieve the debt/GDP level which we identified as the long-run optimum. Moreover, the question of how to reduce government debt is also a topical one. Many countries in the world face very high debt/GDP levels, which are becoming less and less sustainable. Provided that the calibration is adapted, our analysis could also have implications for other countries facing high debt/GDP ratios.

The second question we ask is about the optimal level of government debt, if both long-run and short-run welfare effects are incorporated. A government that wants to reduce debt needs to raise taxes. This leads to a welfare reduction in the short run. It is therefore interesting to see whether the long-run

\textsuperscript{20}Reducing government debt thus increases wealth inequality (but reduces consumption inequality) in the long run, a result that was already emphasized by Flodén (2001).

\textsuperscript{21}In the tradition of the literature on Ramsey taxation, we consider only linear taxation and assume that government expenditures are exogenous and fixed. In particular, we do not change transfers. Since transfers benefit mainly the poor, a reduction in transfers which would be necessary to finance a reduction in public debt would mean a huge welfare loss for the poor. For simplicity, we analyze changes in either labor income taxation or capital income taxation. Our results also have implications for the case in which both taxes can be adjusted simultaneously or sequentially.
gains outweigh the short-run losses.

**Preview of our results.** For most of the policies that we consider, the total welfare effect of a debt reduction is negative. However, some policies do better than others. And some would even find the approval of the majority of the population. Under the conditions that capital income taxes are used to finance the debt reduction, that the tax burden is stretched over a sufficiently long time horizon, and that the tax burden is front-loaded, meaning that taxes are high in the beginning and phase out slowly over time, a reduction in debt would win a majority vote.\footnote{We also study the extreme case of front-loading, which is an unanticipated one period increase in capital income taxation. This policy would not only win a majority vote, but is on top of that also welfare improving.} We also show that front-loading is not a good strategy if labor income is taxed. In general, welfare losses are substantially higher if the reduction in debt is financed with the help of labor income taxation. This result is perfectly in line with our previous findings.

If we compute the optimal level of government debt, taking into account both long-run and short-run welfare effects, we find that it is indeterminate. This is because the optimal debt level depends on the time horizon within which debt is increased. Longer time horizons lead to larger welfare gains, but only if the rise in debt is high. By suitably choosing time horizon and a new debt level, the government can achieve large welfare gains (equivalent to an increase of almost 4 percent in consumption). Interestingly, we show that wealth inequality and borrowing constraints are important determinants of our results.

**Our experiments.** We reduce government debt by either raising the (linear) tax on labor income or on capital income. We assume that the government adjusts only one of the two tax rates along the transition towards the new stationary equilibrium. The tax increment depends on the target debt/GDP ratio as well as on the time span during which government debt is reduced. There are three scenarios: ‘front-loaded’, where debt/GDP is reduced faster in the beginning and slower towards the end; ‘linear’, where debt/GDP is reduced linearly over time; ‘back-loaded’, where debt/GDP is reduced slower in the beginning and faster towards the end. In order to highlight the role of the time horizon, we compare the result of two different experiments (15 and 25 years). For capital income taxation, we also discuss the case of a one-time tax increase in the initial period. This policy does not generate distortions and is thus treated as a special case.

Using a small debt reduction as an example, Figure 7 shows the time path of public debt for the three scenarios (front-loading, linear, back-loading) if the tax on capital income is adjusted. The second column of Figure 7 depicts the three scenarios for labor income taxation.

In the second row of Figure 7, we plot the tax paths that generate the debt paths. The tax path mirror the debt path. If debt declines quickly, then taxes must be high in the beginning. The opposite is true if debt is declining only slowly in the beginning, but faster towards the end.

Independently of our assumed scenario, we compute the transition as follows. After public debt has reached its target value, we keep it constant and the economy starts converging towards its new stationary equilibrium, in which tax rates take the values that we presented in the previous subsection. Figure 8 shows the resulting total welfare change from debt reduction experiments where different debt/GDP ratios are targeted (for time horizon 15 and 25 and the three scenarios discussed above).

We now discuss several important features of our results in greater detail.
Figure 7: Example of Debt and Tax Path. This figure shows the shape of the debt/GDP (first row) and tax path (second row) over the transition where the government reduces the public debt/GDP ratio from an initial stationary equilibrium value of $2/3$ to a new stationary equilibrium value of 0.6. As an example, we focus on the case where the debt/GDP ratio is reduced in 25 periods. Two cases: capital income tax adjusted (first column); labor income tax adjusted (second column). Three scenarios: front-loaded (blue line with circles), linear (red line with squares) and back-loaded (green line with crosses).

Labor income vs. capital income taxation Reducing government debt leads to a welfare loss with respect to the benchmark stationary equilibrium. This result is independent of whether the reduction in debt is financed by increasing capital income taxation or by raising labor income taxation.

Interestingly, welfare losses are smaller if debt is reduced by raising capital income taxation. This is because capital income taxes affects only those households who own assets. Since wealth is very concentrated in our calibration, a considerable share of the population in our benchmark calibration owns no assets or is even indebted. Because these households receive only income from labor, they are not affected by higher capital taxes. Moreover, since wealth-poor households are also consumption-poor, they have a high weight in the utilitarian welfare criterion. Consequently, the welfare losses from increasing capital income taxes are smaller than the respective welfare losses that result from a similar increase in labor income taxes.

Slow vs. fast reduction of government debt. A key element of any debt reduction plan is the timing. Increasing the number of periods lowers the additional tax burden per period, all other things equal. This is particularly relevant if debt is reduced with the help of labor income taxation. In this
Figure 8: Welfare Including the Transition. In this exercise we plot the welfare change in consumption equivalent units due to a specific debt-reduction experiment (on the ordinate), where public debt/GDP ratio is reduced until a certain target level is reached (on the abscissa), relative to the initial benchmark. Four possible kinds of experiments: adjustment in either capital (first row) or labor (second row) tax rate and duration of policy is either 15 periods (first column) or 25 periods (second column). Three scenarios: (1) front-loaded (blue line with circles), (2) linear (red line with squares), (3) back-loaded (green line with crosses). Also shown: welfare change in new stationary equilibrium alone with transition path excluded (black line).

In this case, the welfare costs associated with a debt reduction are lower, the longer the time span during which the debt reduction takes place (see Figure 8). This is intuitive. A longer time horizon implies a lower tax burden and thus also less distortions in each period.

If we consider a debt reduction that is financed by an increase in capital income taxation, the effect of extending the number of periods is ambiguous. If taxes are front-loaded, debt should be decreased as fast as possible. Instead, if taxes are back-loaded, it is better to stretch out the debt reduction over a longer time horizon. The reason is that the combination of front-loading and capital income taxation already minimizes distortions, a finding which is also important if we separate the effects of front-loading and back-loading. This is done in the next paragraph.

Front-loading vs. back-loading. Our experiments indicate that if the government aims at reducing its debt quickly with the help of an increase in capital income taxation, it is better to front-load the tax burden (see again Figure 8). If instead labor income taxation is used, it is better to choose...
back-loading.

The intuition for this finding is as follows. Front-loading the tax burden means that the tax path is high in the beginning and declining over time. If the debt reduction is financed by an increase in capital income taxation, front-loading leads to lower transitional welfare costs because households cannot adjust their assets in the first period. It is important to notice that the government can only profit from front-loading the tax burden if debt is reduced quickly, otherwise it is better to back-load the tax burden.

In contrast, if labor income taxation is used as a means to finance the debt reduction, back-loading is always the preferred option. An increase in labor income taxation distorts mainly the supply of labor. Households can adjust their labor supply freely in any period. Therefore, front-loading does not reduce welfare costs. On the contrary, it hurts all those households who are borrowing constrained. This is because the additional tax burden further reduces the consumption of the constrained, which is already suboptimally low. Constrained households prefer a tax path which is increasing over time, such that the negative effects of taxes and currently binding borrowing constraints do not amplify each other. Therefore, because of binding borrowing constraints, we find that it is optimal to deviate from the principle of tax smoothing, as established by Lucas and Stokey (1983) for an environment with complete markets.

It is also interesting to notice that the differences between front-loading and back-loading are sizable if the labor income taxation adjusts. However, the impact of the timing is negligible in the case of an adjustment in capital taxation. The reason is that front-loading and back-loading are each associated with offsetting forces on welfare. For example, back-loading is preferred by those who are borrowing constrained. On the other hand, it turns out that back-loading results in a sharp increase in the interest rate. This, in turn, is bad for those who are highly indebted, i.e. those who are close to the borrowing constraint. As a result, the welfare effects of back-loading and front-loading are approximately equal to each other.

The behavior of the interest rate deserves a more detailed explanation. If capital income taxation is adjusted and the tax burden is back-loaded, households foresee an increasing path of capital income taxation. This decreases the incentives to save, leading to higher interest rates.

**Special case of one-period policy.**

Before we proceed, we want to shed further light on a special case, which we call one-period policy. Here, we assume that the government needs to implement the new debt level in just one period. One-period policies might be appealing in some cases, as the following results show. Moreover, this special case contains a lot of useful information about the general mechanisms at work.\(^{23}\)

This case is the one exception to the rule that reducing debt decreases overall welfare once the transition is taken into account. If the debt reduction is financed by a one-period increase in capital income taxation, total welfare rises compared to the benchmark stationary equilibrium, albeit only slightly. This is because households’ assets are fixed for one period. Only after one period, assets can

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\(^{23}\)Despite of these advantages, we nevertheless decided to treat one-period debt changes as a special case, because the implicit assumption behind all of our policy experiments is that households are surprised by the policy change, which is unlikely to be the case in practice. If the policy is implemented in only one period, households have very little time (or no time at all, if capital income taxation is concerned).
be transformed back into consumption. Therefore, a one-period change in the capital income tax does not distort households’ behavior.\textsuperscript{24}

Figure 9: \textbf{Welfare Change for the 1-Period Policy.} In this exercise we plot the welfare change in consumption equivalent units (on the ordinate) when economies transit to new stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa), relative to the initial benchmark equilibrium in which public debt amounts to \(2/3\) of GDP. While reducing government debt to equalize the government budget a tax rate has to be adapted. We focus on the case where the capital tax is raised for only one period. Furthermore, we also show the welfare change with respect to the new stationary equilibrium, when the transition path is excluded (black line).

Interestingly, the welfare gain associated with this policy is very small. For large debt reductions, it is even negative. This may come as a surprise, given that the tax increase used to finance the debt reduction does not distort the optimal decisions of households. The reason is that the tax increase which is needed to reduce debt can be very large. For example, if government debt is reduced from 66 percent of GDP to \(-40\) percent, a capital income tax of 689 percent is needed. A capital tax which is larger than 100 percent means that capital owners are expropriated because they need to reduce their assets in order to pay the tax. As a result, the aggregate capital stock shrinks, leading to lower wages and to a reduction of labor income as well. This effect is exacerbated by the fact that the reduction of capital income constitutes a negative wealth shock for asset owners. In response, they increase their labor income, which puts even more pressure on wages. This severely affects the income of the poor.

Therefore, the income of the wealth-poor falls in the first period of the transition. From the second period onwards, the poor experience an income gain with respect to the benchmark equilibrium. This gain stems from the fact that the capital stock is converging to its new stationary equilibrium value, which is higher than in the initial stationary equilibrium. Therefore, wages are converging to a

\textsuperscript{24}For this reason, the literature that studies optimal taxation typically restricts \(\tau_k^0\) to be smaller than one. See Greulich and Marcet (2008). Otherwise, the optimal tax problem becomes meaningless. We do not impose this restrictions. \(\tau_k > 1\) can be interpreted as a wealth tax. The recent past has seen many incidences where a tax on wealth was implemented on surprise in order to reduce government debt, e.g. the 'patrimoniale' in Italy at the beginning of the 1990s.
higher value as well. Consequently, the welfare of households over the transition is higher than in the initial equilibrium, but lower than in the new stationary equilibrium, which is characterized by a lower debt/GDP ratio. Recall from our long-run analysis above that within the range of debt reductions that we consider here, labor income and thus the welfare of the wealth-poor is unambiguously higher for lower debt/GDP ratios.

This welfare loss occurring over the transition is exacerbated by the presence of borrowing constraints. Without borrowing constraints, poor households could borrow against their future income to smooth out their consumption over time. Thus the loss the poor face over the transition is larger the higher the debt reduction. That is, the more the capital tax rate is increased in the first period, the larger the negative impact on the capital stock and on wage rate in the first period. Since borrowing is not permitted, this depresses the welfare of the poor over the transition even further.

Figure 11 shows this case. In this Figure, we plot the welfare of different wealth groups for various debt/GDP levels. We analyze three groups: (1) the poor have no assets or are in debt (first subfigure), (2) the rich own 70% of assets as a group (second subfigure), (3) the middle class is the rest of the households that are neither rich nor poor (third subfigure). The lower the debt/GDP level, the larger the discrepancy between the welfare of the wealth-poor over the transition and the welfare in the new stationary equilibrium. Obviously, households who actually own assets face even larger welfare losses.
over the transition.

Figure 11: Welfare Change for Different Wealth Groups under the 1-Period Policy. This figure shows the changes in welfare of a group (on the ordinate) for different public debt/GDP ratios (on the abscissa). In the benchmark public debt amounts to 2/3 of GDP (green diamond and vertical line). Three groups: (1) wealth poor (first subfigure), (2) wealth middle class (second subfigure), (3) wealth rich (third subfigure). Both are shown: Welfare change excluding t (blue line with circles) and including (red line with squares) the transition path.

In sum, reducing government debt with the help of a one-period capital tax increase may lead to overall welfare gains if the amount by which debt is reduced is fairly small. Otherwise, a very high tax is needed which expropriates asset owners, reduces the total capital stock and thus also total output and the income of the wealth-poor. Because borrowing is not permitted, this means that welfare of the poor increases less relative to the long run and thus also aggregate welfare increases less.

Majority voting vs. Utilitarian welfare criterion. We now analyze which of the debt reduction policies are politically feasible. A debt reduction policy is politically feasible if it is supported by a majority of households. Analyzing the political support of debt reduction policies is a topical question, given that many developing countries will have to implement debt reduction plans in the near future.

We find that many policies lowering public debt would be favored by a majority of households. This is perhaps surprising, given that our previous analysis has shown that almost all policies reduce aggregate welfare, measured by the Utilitarian welfare criterion.

In Figure 12, we plot the political support for various debt reduction policies. As before, we assume that the economy is in the benchmark stationary equilibrium where debt/GDP amounts to 66 percent. We distinguish debt reduction policies with respect to the targeted debt/GDP ratio and whether the tax burden is front-loaded, back-loaded or linear. We only present results for policies that involve an adjustment of capital income tax.

25The fraction of households that is in favor of a specific policy reform is computed as \( \int I(W_0|\text{reform}(a, \epsilon) - W_0|\text{benchmark}(a, \epsilon) > 0) \theta_0(a, \epsilon) \), where \( I(.) \) is an indicator function which is 1 whenever the statement in brackets is true and zero otherwise, \( W_0|\text{reform}(a, \epsilon) \) is the value function of an agent with assets \( a \) and productivity state \( \epsilon \) in period 0 given the reform will be implemented, \( W_0|\text{benchmark}(a, \epsilon) \) is the value function of an agent with assets \( a \) and productivity state \( \epsilon \) in period 0 given nothing will happen (the benchmark), \( \theta_0(a, \epsilon) \) denotes the measure of agents for each asset and productivity state in period 0.

26Debt reductions which are financed by an increase in labor income taxation are never politically feasible. The results can be obtained from the authors upon request.
Figure 12: Welfare According to Majority Voting Criterion. In this exercise we plot the percentage of the population that would profit from a reform implied by our model (on the ordinate) where a reform means transiting to a different stationary equilibrium with respect to the public debt/GDP ratio (on the abscissa). Four different time horizons: 1-period policy (first panel in first row), 2-period policy (second panel in first row), 15-period policy (first panel in second row), 25-period policy (second panel in second row). Three different scenarios: (1) front-loaded (blue line with circles), (2) linear (red line with squares), (3) back-loaded (green line with crosses). Also shown: number of pro-reform voters with respect to the new stationary equilibrium, when the transition path is excluded (black line).

A vast majority of households would support a policy that involves only an one-period increase in capital income taxation. Support is decreasing, the larger the debt reduction. However, support is still sufficient to implement a direct jump to the long-run optimum of −0.4.

This result hinges on the fact that the tax increase is unexpected. This assumption is certainly not very realistic. However, it is not necessary to surprise asset owners to ensure political support. For example, if the tax increase is distributed over two periods, it is still possible to sustain the long-run optimal debt level. Even if the tax increase is distributed over 15 periods, one can get very close to the long-run optimum if the tax burden is front-loaded.

Interestingly, households favor front-loading over back-loading, even if the tax increase is spread over many periods. This is surprising, given that back-loading is beneficial for borrowing constrained households. Recall that aggregate welfare is slightly higher for back-loading policies, if we consider a policy horizon of 25 periods. The reason is that the situation for poor households worsens if the tax burden is back-loaded. This is because interest rates are higher and wage rates are lower if the time path...
of capital income taxes is increasing over time. This in turn stems from the fact that households save less if they foresee higher capital income taxes in the future. High interest rates and low wage rates, in turn, hurt those households who are highly indebted and draw low income shocks. Since a substantial fraction of households belongs to this group, back-loading receives little support in our experiments.\textsuperscript{27}

In sum, our results show that taxing income from capital is key in order to maximize support for a policy that aims at lowering public debt/GDP. Our experiments suggest that government debt should be reduced as fast as possible. The tax burden should be front-loaded. Thus, the policies that are designed to maximize support can be quite different from those that aim at maximizing social welfare.

**Searching for a 'global' optimum of government debt.**

We now answer the question how a benevolent government should set the public debt/GDP ratio optimally, if both short-run and welfare changes are taken into account. We have already shown that reductions of public debt creates massive welfare losses, which outweigh the long-run gains associated with lower debt levels. Therefore, from our previous exercise, we conclude that a benevolent government will not reduce, but increase, debt.

Our previous experiments have also shown that welfare reacts more sensitively to changes in labor taxation. Hence, in the following, we assume that government debt affects only the tax rate on labor income. Transfers, government expenditure and capital income taxation remain at their benchmark level.

In Figure 13, we depict the welfare changes associated with different debt/GDP ratios, again expressed in consumption equivalents. We study different time horizons during which debt/GDP ratios are increased (up to 25 years). Within these time spans, debt increases at a linear rate.\textsuperscript{28} In all our experiments, we require that the tax rate is non-negative.

We find that the overall welfare gains of increasing debt are large. For example, if we raise the debt/GDP ratio from 0.67 to 1 within 1 period, we get a welfare gain of around 1 percent, measured in terms of consumption. This is about twice the welfare gain of the debt/GDP ratio which was optimal in our long-run analysis above. This again underlines that the short-run welfare effects of government debt are large. Moreover, it also establishes that long-run and short-run welfare effects point in opposite directions.

The following two observations are worth commenting on. First, if we study different debt/GDP ratios but keep the time span during which government debt is increased constant, we observe that the welfare function is concave. Second, these welfare functions are becoming flatter, the longer the time span and thus the smaller the per-period increase in government debt.

We now turn to the first observation, the concavity of the welfare function. Given a time span, total welfare is initially rising if debt is increased, and later falling. The fact that total welfare is rising for higher debt levels stems from the fact that increasing debt helps borrowing constrained households. Raising public debt allows the government to lower the labor income tax. Households who do not have access to private credit react by increasing their consumption, despite the fact that increasing debt today implies higher taxes in the future, when the additional amount of debt needs to be serviced. Public debt

\textsuperscript{27}Notice that not all households who are close to the borrowing constraint are actually constrained in their borrowing. Those who are close to the constraint and receive a good income shock are more likely to be constrained, given that income shocks are very persistent, implying that these households expect to see their income rising during the near future.

\textsuperscript{28}Given our previous results, we expect that front-loading the tax adjustments would give even higher welfare gains.
thus acts as a substitute for private debt. Moreover, those who are at the borrowing constraint in the current period face a positive probability of receiving a sequence of good income shocks in the future. This means that there is certain probability that they will turn from net borrowers into net savers. As a consequence, their income composition will also change, as the share of capital income will increase and the share of labor income will fall. This makes the future increase in labor income taxation less severe for the current poor. This makes public debt as a substitute for private debt even more attractive.

However, this welfare-increasing effect of higher public debt levels disappears as soon as the labor income tax rate cannot be decreased further because the non-negativity constraint becomes binding. After this point, raising more government debt means that the government is wasting resources, which is welfare-decreasing. As a consequence of this, the welfare function becomes concave.

If we instead extend the time span within which public debt is increased, the non-negativity constraint is less likely to be of an issue. Because longer time spans also imply that the tax reduction are smaller, given an increase in government debt, the slope of the welfare function is less steep. This is why the welfare functions become flatter if we extend the number of periods within which debt is raised.
In sum, our results suggest that the welfare gains that can be achieved by increasing public debt are large, but bounded. According to our experiments, the upper bound is a 4 percent increase in welfare, measured in terms of consumption. There are several combinations of debt/GDP ratios and time horizons which give welfare gains that are close to the upper bound. In this sense, the optimal policy is not ‘unique’.

5 Conclusion

In this paper, we analyze the welfare effects of government debt. An important contribution of our project is to compute both the long-run and the short-run welfare consequences of public debt. A key result of our study is that the welfare effects of government debt depend crucially on the degree of wealth and income inequality.

The high degree of wealth and income inequality which can be observed in the US implies that the long-run welfare effects of government debt are large and negative. A benevolent government should hold assets worth at least 50 percent of GDP, instead of debt. By holding assets, the government raises the amount of capital available for production, which in turn increases the equilibrium wage rate and thus benefits those households who depend heavily on labor income.

The short-run welfare costs of reducing government debt are large. If both short-run and long-run welfare costs are taken into account, there overall welfare consequences of government debt turn positive. According to our simulation, welfare changes that correspond to a permanent consumption increase of up to 4 percent are possible if government debt is raised. Therefore, a benevolent government indeed has an incentive to increase government debt. However, as a negative side effect, this will - in the long-run - adversely affect the welfare of the poor.

Borrowing constraints play an important role in our analysis. They govern to what extent households are indebted, a crucial determinant of the overall welfare effects. Moreover, the response of households to changes in distorting taxes also depends on their ability to borrow. We assumed that borrowing limits are exogenous, and do not change if public policy changes. In an extension to this project, we relax this assumption and endogenize borrowing constraints by assuming limited commitment (Röhrs and Winter, 2012). Preliminary results suggest that endogenous borrowing constraints reinforce the negative effects of government debt.

Other possible extensions are the introduction of aggregate shock as an additional motive to smooth taxes, along the lines of Barro (1979), Lucas and Stokey (1983), Aiyagari et al. (2002) or Heathcote (2005). Our framework could be readily used to analyze the welfare effects of government debt in other countries, in which wealth and income follow a different distribution. An interesting application is the Euro zone, where currently many proposals to curb government debt are discussed. One of these proposals is to outsource all debt of the individual member countries which exceeds the threshold of so-called Maastricht criteria to a special fund. From this fund, a certain fraction will have to be repaid during a fixed number of predetermined years, by a joint effort of all Euro zone countries.29 According

29This proposal, also labeled “Schuldentilgungsfond”, is due to, inter alia, the German Council of Economic Advisors. A time horizon of 20 to 25 years was put forward. See Frankfurter Allgemeine Zeitung, http://www.faz.net/aktuell/wirtschaft/eu-schuldenfonds-harsche-kritik-am-vorschlag-der-wirtschaftsweisen-11525389.html, retrieved May 12, 2012.
to our results, which were obtained for the U.S. economy, the debt repayment scheme should be non-linear. In this paper, we also implicitly assumed that within families, parents are perfectly altruistic towards their offspring. This does not necessarily need to be the case. We expect that allowing for imperfect altruism would alter both the trade-off between short- and long run welfare effects as well as the relationship between private capital and public debt due to crowding out. If there is no altruism at all, our model would then collapse to a model with overlapping generations, as in e.g. Conesa, Kitao, and Krueger (2009). Another interesting extension could be to allow for limited commitment. By the choice of our welfare function, we implicitly assumed that the government can commit itself to policies that maximize Utilitarian welfare over an infinite number of periods. If the government cannot commit, a different optimization problem arises as analyzed by Acemoglu, Golosov, and Tsyvinski (2011) in the context of a neoclassical model with Ramsey taxation. Given the tension between short-run and long-run debt policies that our paper reveals, studying the no-commitment case could be potentially fruitful for our understanding of why governments accumulate so much debt. We leave these interesting extensions for future research.
References


Appendix

A: Neutrality of Government Debt - Natural vs. Ad-hoc Borrowing Constraints

We now shed more light on the interaction between private borrowing limits and public debt. Understanding the interplay between the two is crucial for the interpretation of our results. In order to separate the effects of borrowing constraints from the distortions that arise from proportional taxes, we assume in the following that the government uses lump-sum taxes to finance debt. Also see Heathcote (2005).³⁰

It is well known that in a world in which income is deterministic, in which the government uses lump-sum taxation and in which there are no binding borrowing constraints, government debt is neutral: households foresee future tax changes and adapt their savings behaviour accordingly (see e.g. Ljungvist and Sargent, 2004, Ch. 11). This is the famous "Ricardian Equivalence" result. Woodford (1990) has shown that Ricardian Equivalence breaks down in the presence of binding borrowing constraints.

In the following, we extend the textbook model by allowing for income uncertainty and endogenous labor supply, features that also characterize our quantitative model. We show that the classical Ricardian Equivalence result is unaffected by these additions. Moreover, within our framework, we also analyze the short-run and long-run consequences of public debt on the budget constraint of private households. Our framework thus allows us to examine under which conditions public debt and private debt are isomorphic. If public debt and private debt were isomorphic, an increase in public debt would relax borrowing constraints that are binding at the individual level. That is, government debt enhances private liquidity, see e.g. Aiyagari and McGrattan (1998) or Guerrieri and Lorenzoni (2011).

The households’ problem that we study in this period is similar to the one we presented in the previous section, with the only exception that we introduce lump-sum taxes instead of proportional taxes:

\[
W_t(a, \epsilon) = \max_{l, a', c} \left\{ u(c, l) + \beta \sum_{c'} \pi(c'|\epsilon)W_{t+1}(a', c'; \theta') \right\}
\]

s.t. \(w_t(1 - l) + (1 + r_t)a - T_t = a' + c\)

\(c \geq 0\)

\(a' > \bar{a}\)

where \(T\) denotes a lump sum tax and all other variables are defined as above.

In the following, we consider the two different specifications for the borrowing limit \(\bar{a}\). First, we set \(\bar{a} = \bar{a}_{\text{natural}} = \frac{w \epsilon_{\text{min}} - T}{1 - \beta}\), where \(\epsilon_{\text{min}}\) is the smallest possible realization of our earnings process. \(\bar{a}_{\text{natural}}\) then describes the net present value of worst possible sequence of realization of the earnings process, in which \(\epsilon_{\text{min}}\) are drawn in every period. Put differently, \(\bar{a}_{\text{natural}}\) is the amount of borrowing that is feasible for a household to repay, even in the worst case. Aiyagari (1994) therefore calls this the natural borrowing limit. However, sustaining \(\bar{a}_{\text{natural}}\) when \(\epsilon = \epsilon_{\text{min}}\) in all periods would require to

³⁰Heathcote (2005) analyzes the quantitative short run effects of changes in the timing of proportional income taxes in heterogeneous agent economies with incomplete markets. He also distinguishes between effects via tax distortions and effects arising due to the presence of borrowing constraints.
set \( c = 0 \) forever. Since there is a positive probability that this case occurs, given that we impose the Inada conditions on the utility function, this implies that a household in our economy will never want to borrow up to \( \mathbf{a}_{\text{natural}} \). The second case we consider is \( \mathbf{a} = \mathbf{a}_{\text{natural}} \) where \( \mathbf{a}_{\text{natural}} < \mathbf{a} \leq 0 \), where \( \mathbf{a} \) is assumed to be fixed at some constant level. We call this an ‘ad-hoc’ borrowing limit.

Moreover, it is interesting to note that if borrowing limits are given by \( \mathbf{a}_b = \mathbf{a}_{\text{natural}} \), a change in government debt translates into an equal change in the borrowing capacity.

This can be directly seen from the definition of the natural borrowing limit:

\[
\mathbf{a}_{\text{natural}} = \frac{w \epsilon_{\text{min}} - T}{r} = \frac{w \epsilon_{\text{min}}}{r} - B
\]

We made use of the fact that the government constraint is \( T = rB \) for all periods from \( t = 2 \) onwards.\(^{31} \)

It follows that \( \Delta \mathbf{a}_{\text{natural}} = -\Delta B \), i.e. the natural borrowing limit becomes tighter if \( B \) is increased.

Now suppose that \( \mathbf{a} = \mathbf{a}_{\text{natural}} \) and that in period 1 the government issues debt \( \Delta b \) and redistributes the proceeds as a supplementary transfer, such that \( T_{\text{new},1} = T_{\text{old},1} - \Delta b \). Furthermore suppose that the taxes needed to service the additional amount of government debt in future periods will also be lump sum: \( T_{\text{new},t} = T_{\text{old},t} + r_t \Delta b \) for \( t > 1 \). Clearly, this policy does not affect the permanent income of an agent: the present value of the cost (higher taxes in the future) and benefits (transfer today) of this policy are equal. A rational, forward-looking household increases its savings such that \( a_{\text{new},2}^* = a_{\text{old},2}^* + \Delta b \), where \( a_{\text{old},2}^* \) denotes the optimal savings that would prevail in the situation without government debt.

As a result, the household’s budget in the period of the change (period 1) is unaffected and the additional debt cancels out:

\[
c_1 + a_{\text{new},2}^* = w_1 \epsilon_1 (1 - l_1) + (1 + r_1) a_1 - T_{\text{new},1} \\
\iff c_1 + a_{\text{old},2}^* + \Delta b = w_1 \epsilon_1 (1 - l_1) + (1 + r_1) a_1 - T_{\text{old},1} + \Delta b
\]

The neutrality holds also in all future periods (including in the stationary equilibrium), in which the income from savings can be used to pay the additional lump sum tax, i.e. in all periods \( t > 1 \), \( T_{\text{new},t} = T_{\text{old},t} + r_t \Delta b \) and

\[
c_t + a_{\text{new},t+1}^* = w_t \epsilon_t (1 - l_t) + (1 + r_t) a_{\text{new},t}^* - T_{\text{new},t} \\
\iff c_t + (a_{\text{old},t+1}^* + \Delta b) = w_t \epsilon_t (1 - l_t) + (1 + r_t)(a_{\text{old},t}^* + \Delta b) - T_{\text{old},t}
\]

This means that because households exactly offset the government’s policy, households’ budget constraints are unchanged. As a consequence, the optimal paths of consumption and leisure are unaffected. Moreover, the demand for assets is increased exactly by the amount of government debt such that firms continue to face the same demand for their assets as before. As a consequence the interest rate and wage rate in the economy will remain the same. Government debt is neutral.

Suppose we have instead an ad hoc borrowing limit which is binding for some households. In this case, for some agents savings are determined by \( a^* = a \) and their Euler equation is not satisfied:

\[
\frac{\beta \sum_{t=1}^\infty \pi(\epsilon_{t+1} | \epsilon_t) u_c(c_{t+1}, l_{t+1})}{u_c(c_t, l_t)} < \frac{1}{1 + r_{t+1}} \text{ for agents with } a^* = a
\]

\(^{31}\)The government’s budget in period \( t = 1 \) is irrelevant for the calculation of the natural borrowing limit. By definition, the natural borrowing limit is computed by taking only those periods into account, in which (private) debt is serviced.
Given this, we can show by contradiction that Ricardian equivalence does not hold. Suppose Ricardian equivalence would hold. This would imply that any borrowing constrained household would adjust its saving to $a_{new}^{*} = a + \Delta b$, keeping the path for consumption and leisure constant. But this cannot be optimal, as it violates the Euler equation as shown by equation (6) above. But if the Euler equation doesn’t hold for agents with $a_{new}^{*} = a + \Delta b$ it means they would like to go more into debt and they can do so as $a_{new}^{*} = a + \Delta b > a$. Consequently they will not save $a_{new}^{*} = a + \Delta b$. This contradicts our initial statement. We have thus shown that Ricardian equivalence cannot hold in this case.

In this case, households need more funds today and a transfer from the government thus relaxes their borrowing constraint. More formally, an increase in government debt in period $t = 1$ reduces the tax rate such that $T_{new, 1} = T_{old, 1} - \Delta b$. For a household for which $a$ is binding in period $t = 1$, implying that $a_{new, 2}^{*} = a_{old, 2}^{*} = a$, this is equivalent to relaxing the borrowing limit $a$ to $a_{b} = a - \Delta b$.

\[
c_1 + a = w_1 \epsilon_1 (1 - l_1) + (1 + r_1) a_1 - T_{new, 1}
\]

\[

\equiv \\

\[
c_1 + a - \frac{\Delta b}{2} = w_1 \epsilon_1 (1 - l_1) + (1 + r_1) a_1 - T_{old, 1}
\]

In the case of an ad-hoc borrowing constraint, an increase in government debt only relaxes households’ borrowing conditions in the period in which the increase occurs. In all later periods, the budget constraint reads as follows:

\[
c_t + a_{old, t+1}^{*} = w_t \epsilon_t (1 - l_t) + (1 + r_t) (a_{old, t}^{*} - \Delta b) - T_{old, t}
\]

Hence, the increase in debt in period 1 actually shrinks the amount of resources that borrowing constrained households have at their disposal in all following periods, if borrowing limits are ad-hoc.

Our previous results are also important to understand the welfare effects of government debt in stationary equilibrium. If there are ad-hoc borrowing limits and the government finances itself through lump-sum taxation, the part of the population that is initially borrowing constrained will prefer a stationary equilibrium which is associated with a lower debt/GDP ratio. The rest of the population will be indifferent, because they can offset higher taxes by higher savings.

It is important to realize that this statement would only be correct if interest rates and wages were constant across equilibria. This is, of course, not the case, because debt is not neutral if borrowing limits are ad-hoc.

The non-neutrality results from the fact that aggregate demand for assets does not increase as much as new government debt is added to the economy, due to fact that borrowing constraints are binding. Hence, public debt crowds out the capital stock, leading to an increase in the interest rate and a decline in the wage rate.

The increase in the interest rate makes it less likely that households are borrowing constrained. This is our interpretation of what Aiyagari and McGrattan (1998) call the "liquidity enhancing effect of government debt" (p. 448). Moreover, the rise in the interest rate associated with higher debt levels

\[32\text{Some authors say that government debt is isomorphic to private debt in the presence of ad-hoc borrowing constraints, see e.g. Guerrieri and Lorenzoni (2011).} \]
benefits households who accumulate precautionary savings. Whether the positive or the negative effects of government debt outweigh depends on the calibration, which we discuss in the next section.

B: Detrended Formulation of the Households’ Maximization Problem

In our model, there is a balanced growth path along which variables will be growing at the rate of technology growth. To find the stationary equilibrium of the model or to compute the transition from one stationary equilibrium to another it is useful to first detrend variables with respect to this exogenous productivity growth component to obtain a formulation where variables are constant in the balanced growth equilibrium. (This procedure was also used in the earlier literature, for example by Aiyagari and McGrattan, 1998 and Flodén, 2001). Denote a detrended variable by “tilde”: \( \tilde{x} = \xi \). The present value of lifetime utility (for a Cobb-Douglas can then be denoted as follows:

\[
U(\{\tilde{c}_t\}_{t=1,2,...}, \{l_t\}_{t=1,2,...}) = E_0 \sum_{t=0}^{\infty} \beta^t Y_t^{\eta(1-\mu)} u(\tilde{c}_t, l_t)
\]

Now using the fact that \( Y_t = Y_0(1+g)^t \), where \( Y_0 \) is output in period 0, we can write:

\[
U(\{\tilde{c}_t\}_{t=1,2,...}, \{l_t\}_{t=1,2,...}) = Y_0^{\eta(1-\mu)} E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t (1+g)^{t\eta(1-\mu)} u(\tilde{c}_t, l_t)
\]

where \( \tilde{\beta} = \beta \cdot (1+g)^{\eta(1-\mu)} \).

Similarly, we can find a detrended version of the household budget constraint by dividing it by \( Y_t \):

\[
\frac{c_t}{Y_t} + \frac{a_{t+1}}{Y_t} = \frac{\tilde{\omega}_t}{Y_t} c_t (1-l_t) + (1+\tilde{\gamma}_t) \frac{a_t}{Y_t} + \tilde{r}_t
\]

\[
\tilde{c}_t + (1+g) \tilde{a}_{t+1} = \tilde{\omega}_t \epsilon (1-l_t) + (1+\tilde{\gamma}_t) \tilde{a}_t + \tilde{r}_t
\]

Also the borrowing constraint can be detrended:

\[
\tilde{a}_{t+1} \geq \tilde{\omega}_t
\]

The resulting recursive formulation in detrended variables is given by:

\[
W(\tilde{a}, \epsilon; \theta) = \max_{\tilde{a}', \epsilon'} Y_0^{\eta(1-\mu)} u(\tilde{c}, l) + \tilde{\beta} \sum_{\epsilon'} \pi(\epsilon' | \epsilon) W(\tilde{a}', \epsilon'; \theta')
\]

s.t. \( \tilde{c} + (1+g) \tilde{a}' = \tilde{\omega}_t \epsilon (1-l) + (1+\tilde{\gamma}) \tilde{a} + \tilde{r} \)

\[
\tilde{a}' \geq \tilde{\omega}_t
\]

\[
\theta' = \Gamma[\theta]
\]

C: Definition of the consumption equivalent welfare change

A New Stationary Equilibrium versus the Benchmark. The consumption equivalent welfare change for the average household is defined as the percentage change in consumption that the household
must incur in the old situation in order to be indifferent between staying in the old situation and being in a new stationary equilibrium with different policies for debt and taxes. Let the old stationary equilibrium be denoted by the subscript old and be characterized by a (detrended) debt level \( \hat{b}_{old} = \frac{B_{old}}{Y_{old}} \) and a resulting density \( \theta_{old} \). In our computations this point of comparison will always be the benchmark equilibrium with \( \hat{b}_{old} = \frac{\hat{b}_0}{2} \). Let the new situation be denoted by the subscript new and characterized by the debt level \( \hat{b}_{new} \neq \hat{b}_{old} \) and a resulting density \( \theta_{new} \). Using this notation, the consumption equivalent change for the average household, \( x_{old \rightarrow new} \), is defined as follows:

\[
\int W_{old}(\bar{a}, \epsilon; x_{old \rightarrow new})d\theta_{old}(\bar{a}, \epsilon) = \int W_{new}(\bar{a}, \epsilon)d\theta_{new}(\bar{a}, \epsilon)
\]

\[
\int E_0 \sum_{t=0}^{\infty} \beta^t \left( (c_{old}(\bar{a}, \epsilon)(1 + x_{old \rightarrow new}))^{\eta} l_{old}(\bar{a}, \epsilon)^{1-\eta} \right)^{1-\mu} d\theta_{old}(\bar{a}, \epsilon) = \\
\int E_0 \sum_{t=0}^{\infty} \beta^t \left( c_{new}(\bar{a}, \epsilon)^{\eta} l_{new}(\bar{a}, \epsilon)^{1-\eta} \right)^{1-\mu} d\theta_{new}(\bar{a}, \epsilon)
\]

Solving this equation for \( x_{old \rightarrow new} \) we obtain:

\[
x_{old \rightarrow new} = \left( \frac{\int E_0 \sum_{t=0}^{\infty} \beta^t \left( c_{new}(\bar{a}, \epsilon)^{\eta} l_{new}(\bar{a}, \epsilon)^{1-\eta} \right)^{1-\mu} d\theta_{new}(\bar{a}, \epsilon)}{\int E_0 \sum_{t=0}^{\infty} \beta^t \left( c_{old}(\bar{a}, \epsilon)^{\eta} l_{old}(\bar{a}, \epsilon)^{1-\eta} \right)^{1-\mu} d\theta_{old}(\bar{a}, \epsilon)} \right)^{\frac{1}{\eta(1-\eta)}} - 1
\]

If \( x_{old} \rightarrow x_{new} \) is positive, the average household would prefer changing to the new equilibrium, even without being compensated. If instead \( x_{old} \rightarrow x_{new} \) is negative, compensation in terms of \( x_{old} \rightarrow x_{new} \) consumption units is required in order to make the household indifferent.

**Comparing the Transitional Path to a New Stationary Equilibrium and the Benchmark.**

When we include the transitional path into our considerations, we compare welfare of the average household in period 0 when a change in policy is announced and when staying at the benchmark. Let the detrended debt level in period 0 be denoted by \( \hat{b}_0 \) and the density by \( \theta_0 \). We assume that households are surprised by the new policy. Similar to above we will denote the old situation with the subscript old and the new situation with the subscript new. The only difference is that we now use subscripts \( t \) (since we are not always in the stationary equilibrium) and the initial debt level \( b_0 \) and density \( \theta_0 \) are the same in both situations. The consumption equivalent welfare change for the average household \( x_{old \rightarrow new} \) is again defined as the percentage change in consumption in situation old that makes the household indifferent between staying in old and going to new:

\[
\int W_{old,t=0}(\bar{a}, \epsilon; x_{old \rightarrow new})d\theta_0(\bar{a}, \epsilon) = \int W_{new,t=0}(\bar{a}, \epsilon)d\theta_0(\bar{a}, \epsilon)
\]

\[
\int E_0 \sum_{t=0}^{\infty} \beta^t \left( (c_{old,t}(\bar{a}, \epsilon)(1 + x_{old \rightarrow new}))^{\eta} l_{old,t}(\bar{a}, \epsilon)^{1-\eta} \right)^{1-\mu} d\theta_0(\bar{a}, \epsilon) = \\
\int E_0 \sum_{t=0}^{\infty} \beta^t \left( c_{new,t}(\bar{a}, \epsilon)^{\eta} l_{new,t}(\bar{a}, \epsilon)^{1-\eta} \right)^{1-\mu} d\theta_0(\bar{a}, \epsilon)
\]
Solving this equation for $x_{\text{old} \to \text{new}}$ we obtain:

$$
x_{\text{old} \to \text{new}} = \left( \frac{\int W_{\text{new},t=0}(\tilde{a}, \epsilon) d\theta_0(\tilde{a}, \epsilon)}{\int W_{\text{old},t=0}(\tilde{a}, \epsilon) d\theta_0(\tilde{a}, \epsilon)} \right)^{1/(1-\rho)} - 1
$$

D: The Aiyagari and McGrattan (1998) economy with two tax rates

In the result section, we argue that differences in steady-state welfare effects between our calibration and the one presented in Aiyagari and McGrattan (1998) are due to the different parameterization of the earnings process. Aiyagari and McGrattan (1998) approximate an AR(1) income process estimated from the data by a Markov chain (using the procedure proposed by Tauchen (1986)), we propose to calibrate the income process to fit moments of the U.S. wealth and earnings distribution.

Moreover, Aiyagari and McGrattan (1998) also assume a single tax rate of 37.6% at the benchmark on the sum of capital and labor income, whereas we allow for two distinct tax rates on capital and labor income. More specifically, following Trabandt and Uhlig (2009), we set - at the benchmark - the labor income tax to 28% and the capital income tax to 36%.

In this section, we show that the differences between our results and Aiyagari and McGrattan (1998) are indeed due to the parameterization of the earnings process and not due to the differences in the tax rates. In order to make this point, we calibrate the parameters as in Aiyagari and McGrattan (1998), with the sole differences that we now allow for two distinct tax rates for capital and labor income, as in our benchmark calibration. More specifically, for the income process, we use Tauchen’s procedure with the parameters $\rho = 0.6$ and $\sigma = 0.3$. We recalibrate the discount factor, $\beta$, such that the asset to output ratio is 3.1 and the labor supply elasticity, $\eta$, to match the labor supply of 0.3 (see Table 5). All other parameters are set equal to the ones from Aiyagari and McGrattan (1998) and do not differ very much from the ones in our benchmark calibration (see Table 6). With respect to the results, notice that changing the calibration to include two tax rates alters the wealth and earnings distribution generated by the model only modestly (see Table 7). More importantly, to see how welfare changes in reaction to a change in the debt/GDP ratio compared to our benchmark exercise, we again vary the debt/GDP ratio and calculate the welfare change in consumption equivalents under the differently calibrated parameter values. Again we have to decide which tax rate to adjust to equalize the government budget. Figure 14 shows that introducing two distinct tax rates changes the results of Aiyagari and McGrattan (1998) only by very little. The optimal level of government debt is still very close to the benchmark level and the welfare changes of varying government debt around that level are very small. Thus we conclude that it is the calibration of the income process which matters most for explaining the different welfare effects of government debt that we found.

| Parameter                        | Value  | Target                | Data | Model |
Table 6: Parameters Set Equal to Values Used by Aiyagari and McGrattan (1998)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital’s share, $\alpha$</td>
<td>0.3</td>
</tr>
<tr>
<td>Depreciation rate, $\delta$</td>
<td>0.075</td>
</tr>
<tr>
<td>Risk aversion, $\mu$</td>
<td>1.5</td>
</tr>
<tr>
<td>Borrowing limit, $\bar{a}$</td>
<td>0</td>
</tr>
<tr>
<td>Growth rate, $g$</td>
<td>0.0185</td>
</tr>
<tr>
<td>Debt to GDP ratio, $b$</td>
<td>0.670</td>
</tr>
<tr>
<td>Transfers, $tr$</td>
<td>0.082</td>
</tr>
</tbody>
</table>

Figure 14: An Intermediate Case. In this exercise we plot the welfare change in consumption equivalent units (on the ordinate) for different stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa), relative to the benchmark in which public debt amounts to 2/3 of GDP (green diamond and vertical line). In the stationary equilibrium to equalize the government budget with a different tax base and different debt/GDP ratio a tax rate has to be adapted. The left hand side of the figure shows the case where the capital income tax adjusts and the right hand side where the labor income tax adjusts. We present the welfare results implied by our model (red squares) and compare it to two alternative specifications. More precisely, we show also the results from the seminal paper by Aiyagari and McGrattan (1998) where the difference lies in the way the wealth inequality is accounted for and the specification of one common tax rate for both capital and labor income (black circles). Furthermore, we present an intermediate case where the model is calibrated in the same way as Aiyagari and McGrattan (1998), but allowing only the tax on capital income to adjust (blue crosses). More details on the computational procedure and on the way the welfare change is calculated are provided other parts of the Appendix.

E: The Impact of Distortive Taxation
In this section we want to address the concern that our results about crowding out and welfare for stationary equilibria with different debt/GDP ratios are solely due to the distortiveness of taxation
Table 7: Distributional properties at benchmark stationary economy

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net financial assets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>−1.60%</td>
<td>0.10%</td>
<td>1.64%</td>
<td>8.29%</td>
<td>91.57%</td>
</tr>
<tr>
<td>Benchmark Calibration</td>
<td>−1.57%</td>
<td>0.88%</td>
<td>3.92%</td>
<td>7.23%</td>
<td>89.54%</td>
</tr>
<tr>
<td>Model fitted to AR(1), one tax rate</td>
<td>3.24%</td>
<td>10.07%</td>
<td>16.96%</td>
<td>25.71%</td>
<td>44.03%</td>
</tr>
<tr>
<td>Model fitted to AR(1), two tax rates</td>
<td>2.17%</td>
<td>9.40%</td>
<td>16.72%</td>
<td>26.04%</td>
<td>45.67%</td>
</tr>
<tr>
<td><strong>Earnings</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>−0.40%</td>
<td>3.19%</td>
<td>12.49%</td>
<td>23.33%</td>
<td>61.39%</td>
</tr>
<tr>
<td>Benchmark Calibration</td>
<td>0.00%</td>
<td>2.38%</td>
<td>12.58%</td>
<td>22.73%</td>
<td>62.31%</td>
</tr>
<tr>
<td>Model fitted to AR(1), one tax rate</td>
<td>1.21%</td>
<td>9.70%</td>
<td>16.18%</td>
<td>26.85%</td>
<td>46.07%</td>
</tr>
<tr>
<td>Model fitted to AR(1), two tax rates</td>
<td>1.82%</td>
<td>10.31%</td>
<td>16.51%</td>
<td>26.59%</td>
<td>44.77%</td>
</tr>
</tbody>
</table>

Remarks: Quintiles (Q1-Q5) denote net financial assets (resp. earnings) of a group in percent of total net financial assets (resp. earnings). The last column denotes percent of population with no or negative assets. The entries in ‘data’ are computed from the 2007 SCF. See main text for precise definitions. Notice that earnings can be negative due to the fact that labor earnings also contain part of the gains (or losses) of small enterprises.

instead of the effect of government debt itself. To do this we analyze the following additional experiment: We fix the labor and the capital tax rate at their benchmark levels. When we adjust the debt/GDP ratio, we modify only the lump sum transfer in order to keep the government’s budget balanced.

Figure 15 compares the three possible tax adjustment policies in terms of their implied welfare effects and crowding out effect. Clearly the welfare effect is even more pronounced for the lump sum tax policy. The reason is that firstly a lump sum tax involves no distortion in savings or labor supply and secondly the lump sum component/transfer also has an insurance role if there is income uncertainty. In terms of crowding out however Figure 15 shows that the difference between tax policies is not very pronounced. We thus conclude that crowding out is mainly due to the change in the level of government debt and thus only marginally change for different tax policies.
Figure 15: Welfare Analysis and Crowding Out with Lump Sum Tax. In this exercise we plot two selected model implied measures (on the ordinate) for different stationary equilibria that differ with respect to the public debt/GDP ratio (on the abscissa), relative to the benchmark in which public debt amounts to 2/3 of GDP (green diamond and vertical line). In the stationary equilibrium to equalize the government budget with a different tax base and different debt/GDP ratio either the capital income tax (blue crosses), the labor income tax (red squares) or the transfer/lump sum component (black circles) has to be adapted. The left hand side of the figure shows the consumption equivalent welfare change in percent. The right hand side shows the capital Stock relative to benchmark GDP as a measure for crowding out. More details on the computational procedure and on the way the welfare change is calculated are provided other parts of the Appendix.


We can see that for our calibration welfare effects and crowding out are much bigger. This suggests the following conclusion: even under lump sum taxation the way the wealth inequality is calibrated matters.
Figure 16: Aiyagari and McGrattan (1998) and Lump Sum Taxes. Aiyagari and McGrattan with lump sum taxes (black circles), Our Calibration (blue crosses)