Intertemporal Trade-off and the Yield Curve

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This paper documents that, contrary to the standard economic intuition, estimates of the elasticity of intertemporal substitution are consistently negative and statistically significant around -0.5. The key for obtaining this estimate is to use a new measure of the real interest rate that avoids the use of realized inflation. In particular, the negative of the slope of the nominal Treasury yield curve closely tracks the ex-ante real interest rate. This new measure is then used to estimate the elasticity of intertemporal substitution from a standard Euler equation and aggregate consumption data. I rationalize the negative values of the aggregate elasticity of intertemporal substitution in a general equilibrium model with limited market participation. The negative value of the aggregate elasticity of intertemporal substitution helps explain why yield curve behavior is hard to reconcile with asset pricing models. The negative sign pertains to the aggregate output Euler equation and thus to models of monetary policy.

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I. Introduction

Intertemporal trade-off is one of the key concepts in asset pricing and macroeconomics. The trade-off is described by the elasticity of intertemporal substitution (EIS) defined as the change in expected consumption growth in response to a change in the real interest rate. The EIS does not only have a large impact on the behavior of asset prices but it also determines the magnitude of various policy effects. In consumption-based models, the willingness of agents to shift consumption intertemporally is a key determinant of the level and variation in yield on the risk-free asset.

Despite its importance, the literature has not yet settled on the value of the EIS. On one hand, estimates from aggregate data suggest that the EIS is close to zero (Hall, 1988; Campbell, 2003; Fuhrer and Rudebusch, 2004). On the other hand, evidence from disaggregated data indicates that the EIS is likely to be above unity for households with substantial asset holdings (Vissing-Jorgensen and Attanasio, 2003; Gruber, 2006). Micro evidence is often used to motivate the EIS above unity in calibrations of asset pricing models (Barro, 2009; Bansal, Kiku, and Yaron, 2012).

This paper provides two novel empirical results that help resolve the conflicting and inconclusive evidence on the magnitude of the EIS.

First, I propose a new concept for measuring the ex-ante real interest rate that relies solely on asset prices and avoids using realized inflation. Specifically, I show that the difference between the three-month and ten-year nominal Treasury yield, which is the negative of the yield curve slope, closely tracks the variation in the ex-ante short-term real rate. I label this new measure of the ex-ante real rate as $\tilde{r}_t = -\text{slope}_t$. This measure has two properties that are useful for the estimation of the EIS. It does not require any measure of expected inflation thus avoiding short samples with direct measures of inflation expectations or auxiliary assumptions on the time series dynamics of inflation such as linearity or rational expectations. As a result, my measure is available for a long sample and has favorable stationarity properties across different monetary policy regimes. Given that the nominal Treasury yield curve data are available for long periods, using $\tilde{r}_t$ allows me to estimate the EIS from the Euler equation using 135 years of data.

Second, EIS estimates using aggregate consumption data are consistently negative and statistically significant around -0.5. Similarly, replacing aggregate consumption by the
aggregate output data in the Euler equation yields negative estimates with magnitudes between -0.5 and -1.2. The results hold across sub-samples marked by different monetary policies and institutional arrangements. Hence, the stability of these EIS estimates requires a structural interpretation of the negative sign.

When is the yield curve slope a good measure of the real rate variation? Three restrictions on statistical properties of inflation expectations, risk premia and the real rate need to hold so that the slope of the nominal yield curve closely tracks the ex-ante real rate, all of which are met with theoretical and empirical support. First, the Fisher effect holds in the long run, i.e. the real interest rate is stationary and most of the variation in the long-term nominal yield is due to changing inflation expectations. Second, inflation expectations can be described by a single factor with a close-to-unit-root persistence. Third, the variation in risk premia is small and less persistent than the variation of the real rate.

I show that the negative sign of the EIS arises naturally in a standard limited market participation model. The key mechanism that changes the sign of the EIS operates through the interaction of two types of households. A fraction of households, rule-of-thumb consumers, do not participate in asset markets and consume their wage income every period (Campbell and Mankiw, 1989; Mankiw, 2000). The remaining households, savers, have positive EIS and own all the productive assets in the economy. Savers have two sources of income, wages and dividends. Changes in the real interest rate directly influence only the consumption and labor supply of savers. But the intertemporal choices of savers then have an impact on the equilibrium real wage of rule-of-thumb consumers and thereby alter their consumption. Changes in the real wage also lead to the variation in profit margins of firms, affecting the dividend income and the consumption of savers. The resulting effect of the real interest rate shock on aggregate consumption is negative if the fraction of rule-of-thumb households is sufficiently high and the labor supply is sufficiently inelastic. Standard values for the structural parameters of the model that are consistent with micro evidence imply the estimated value of the aggregate EIS.

The negative relationship between consumption growth and ex-ante real interest rates has a number of important implications for both asset pricing and monetary policy.

First, the negative value of the aggregate EIS helps explain why the yield curve behavior is hard to reconcile with asset pricing models (Duffee, 2012). Consumption-based models calibrated with a positive EIS imply a short-term real interest rate that is negatively correlated
with the real interest rate from the data (Canzoneri, Cumby, and Diba, 2007).\footnote{Canzoneri, Cumby, and Diba (2007) compare the model-implied real rate to the ex-ante and ex-post real rate obtaining similar results for both.} Intuitively, changing the sign of the EIS aligns the model-implied real rate with the ex-ante real rate from the data.

Second, persistent but mean-reverting variation in the real interest rate together with sizeable estimates of the EIS imply a time-varying expected consumption growth rate. Using this link, I extract the persistent component of aggregate consumption growth and study its asset pricing implications. In line with the intuition of long-run risk models, the extracted component has a high persistence, explains about 11% of the variation in consumption growth and is positively related to the price-dividend ratio. New and in contrast to the calibration favored by the long-run risk literature, the persistent component that is consistent with asset prices is obtained with the negative value of the EIS.

The implications for consumption-based asset pricing can be summarized as follows. To replicate the dynamics of asset prices one can either use the aggregate consumption data together with a negative value of the EIS in the representative agent Epstein-Zin framework, or one can use a positive value of the EIS close to unity and apply it only to the consumption of households participating in asset markets along the lines of Malloy, Moskowitz, and Vissing-Jorgensen (2009).

The negative EIS also matters for models of monetary policy because the central bank reacts to aggregate output. The aggregate Euler equation (IS curve) with a positive EIS is the core of dynamic general equilibrium models used in the monetary policy analysis, e.g. New-Keynesian models. The sign and magnitude of the aggregate EIS determine the real effect of monetary policy. Therefore, the negative sign of the EIS changes key predictions of these models. Most notably, it implies that an increase in the real interest rate is expansionary.

\textit{I.A. Related literature}

This paper is related to several areas in the finance and macro literature. First, numerous papers study the empirical properties of the real interest rate (Mishkin, 1981; Fama and Gibbons, 1982; Evans and Lewis, 1995; Ang, Bekaert, and Wei, 2008). The overall message of these papers is that statistical properties of the real rate change over time. Similarly, the literature studying inflation documents that both the persistence and the volatility of
inflation in the US vary over time (Barsky, 1987; Cogley, Primiceri, and Sargent, 2010). Despite this unambiguous evidence most of the recent studies constructing the ex-ante real interest rate assume linearity which leads to biased estimates. To avoid estimation issues, this paper proposes a measure of the ex-ante real rate that does not require estimating a statistical model for inflation.

Second, the empirical literature documents that the slope of the nominal yield curve predicts real output growth (Estrella and Hardouvelis, 1991; Plosser and Rouwenhorst, 1994; Ang, Piazzesi, and Wei, 2006) and consumption growth (Harvey, 1988). There is no consensus as to the economic mechanism behind this predictability. One way to explain the predictive power of the slope is monetary policy (Bernanke and Blinder, 1992), but other studies argue against this interpretation (Estrella and Hardouvelis, 1991; Plosser and Rouwenhorst, 1994). For a recent discussion see Benati and Goodhart (2008). In addition to the predictability of real output, Fama (1990) and Mishkin (1990) show that the slope predicts changes in inflation. I show that the slope tracks the variation in the ex-ante real rate and is unrelated to expected inflation, hence the relationship between the slope and aggregate consumption/output growth represents the intertemporal optimality condition that is consistent with a structural rather than monetary policy interpretation. Unlike most of the papers studying the predictive power of the slope, I study the sample 1875-2009 that includes a period in which the Fed was not operational. This helps distinguish monetary policy from structural interpretation.

Third, econometric studies estimating the EIS from aggregate data exploit the intertemporal optimality condition for consumption (Hansen and Singleton, 1982; Hall, 1988) or output (Fuhrer and Rudebusch, 2004; Bilbiie and Straub, 2012). Estimates of the EIS from aggregate data have sizeable standard errors and are usually close to zero. A variety of explanations for the low magnitude of the EIS estimates have been proposed: non-separability of non-durable and durable consumption (Ogaki and Reinhart, 1998), limited stock market participation (Guvenen, 2006), bounded rationality (Gabaix, 2012), or the structural break in the EIS (Bilbiie and Straub, 2012). These studies provide arguments for why the EIS estimates are biased toward zero but do not explain why the aggregate EIS is consistently negative. The lack of accurate EIS estimates from aggregates has motivated the analysis of household-level data. Vissing-Jorgensen (2002) and Attanasio, Banks, and Tanner (2002) show that the EIS is close to unity for stockholders. Similarly, Gruber (2006) estimates the EIS to be larger
than one in a sample of households paying substantial capital income tax. My estimates of the EIS from the aggregate data are negative which makes the discrepancy between the micro and macro estimates of the EIS larger than previously documented. I argue that the differences arise as a result of aggregation of heterogeneous households. The model presented in this paper is consistent with both the positive EIS for households participating in asset markets and the negative aggregate EIS.

Fourth, the evidence on violation of the permanent income hypothesis has led to models in which the consumption is generated by two types of consumers: forward-looking who consume their permanent income and rule-of-thumb who do not optimize intertemporally because they are excluded from asset markets (Campbell and Mankiw, 1989). Guvenen (2006) introduces heterogeneity in the EIS combined with limited stock market participation into a real business cycle model to replicate the empirical evidence on low (but positive) EIS at the aggregate level. I use the limited participation combined with nominal rigidities to study the implications of the negative aggregate EIS for dynamics of short-term interest rates.

Finally, asset pricing models usually represent the variation in the yield curve slope as a time-varying risk premium. Wachter (2006), Rudebusch and Swanson (2012), Bansal and Shaliastovich (2012) fall into this category. This paper provides an alternative interpretation of the yield curve slope. I first show that the slope closely tracks the variation in the real short-term interest rate. Then I use the estimated EIS to connect the variation in the short-term real rate to expected consumption growth.

II. The main result

This section states the main result and all the details are postponed to subsequent sections. I estimate the log-linearized version of the intertemporal optimality condition for aggregate consumption $C_t$ using annual data in the sample 1875-2009:

$$\Delta c_{t+1} = \bar{\mu}_t + \sigma \tilde{r}_t + \epsilon_{t+1},$$

(1)

where $\Delta c_{t+1} = \log C_{t+1} - \log C_t$, $\bar{\mu}_t$ collects constant terms and the time-varying second moments, $\sigma$ denotes the EIS, $\tilde{r}_t$ is the measure of the ex-ante real interest rate proposed in this paper and $\epsilon_{t+1}$ subsumes expectations errors. I argue that, to obtain unbiased EIS
estimates, the choice of instruments for the ex-post real rate $r_{t+1}$ is of great importance. I exclude realized inflation from the instrument set because, as I will argue below, its forecast errors are predictable and the dynamics are non-linear. Instead, I use $\tilde{r}_t$ as an instrument for $r_{t+1}$.

Table I reports the estimates of $\sigma$ across different sub-samples (estimation details are postponed to Section IV.A). The sub-samples are chosen so that they allow for an assessment of the stability of the estimates in different macroeconomic/policy environments and for comparison with the literature. Panel A reports the results for consumption and Panel B for the real output. The most important finding across sample periods and output measures is that the estimates are negative and statistically significantly different from zero.\(^2\) The full sample estimate of aggregate EIS is -0.51. The estimates provided in both panels of Table I counter the standard economic intuition which asserts that $\sigma > 0$, i.e. a positive shock to the real interest rate induces agents to postpone today’s consumption to the next period.

In the subsequent sections, I elaborate on this result. Section III provides arguments for why $\tilde{r}_t$ is an appropriate measure of the ex-ante real interest rate. Section IV discusses the estimation details and econometric issues. Section V shows that the negative EIS estimates can be replicated in a standard general equilibrium model with limited asset market participation for plausible values of structural parameters. Section VI discusses the key mechanism for obtaining the negative aggregate EIS and studies its implications for term structure modeling. Section VII relates this paper’s results to long-run risk models.

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\(^2\)Note that Hansen and Singleton (1984, 1996) in some cases obtain negative but insignificant estimates of the risk aversion parameter. Hall (1988) also estimates a negative value for $\sigma$ using annual data in the sample spanning 1924-1940 and 1950-1983. Similarly, Yogo (2004) reports negative albeit in most cases insignificant estimates of EIS for a panel of developed countries. Using the US data on the long sample period (1891-1997), Campbell (2003) estimates a negative though insignificant EIS parameter. These studies interpret negative estimates of EIS as implausible.
The table reports the results for the log-linearized Euler equation given by (1). The IV setup is implemented as a two-step GMM. Panel A reports the results for consumption and Panel B for the real output. Each column corresponds to a different sample period. The first column shows the full sample estimates, 1875-2009. The second column reports the results for 1875-1979, a period preceding the Great Moderation and Volcker chairmanship. The third column displays the results for the post-war period 1947-2009. The fourth column reports the estimates for 1960-2009, which includes the inflationary period and finally the fifth column reports the results for 1980-2009, a post-inflation/Great Moderation period. The last column in both panels reports the estimates using the survey-based ex-ante real rate. Inflation expectations are obtained from the Livingston panel for the period 1960-2009. The data are annual. The autocorrelation and heteroskedasticity-robust standard errors are reported in parentheses (bandwith=3, Bartlett kernel). The overidentification test refers to Hansen’s J statistic which is asymptotically \( \chi^2 \) distributed. The “Weak identification test” in each panel reports the F-statistic (Kleibergen-Paap). The critical values for the maximum relative bias are 16.85 for 5%, 10.27 for 10%, 6.71 for 20%, and 5.34 for 30%. The critical values for the maximal size are 24.58 for 10%, 10.26 for 20%, and 8.31 for 25%. The critical values are computed using the procedure proposed by Stock and Yogo (2002). The last row in each panel reports the Wald test p-values testing the equality of \( \sigma \) in the pre-Volcker (1875-1979) and post-Volcker periods (1980-2009). The following variables are used as instruments: two lags of \( \hat{r}_t \), lagged three-month nominal T-bill and twice lagged consumption/output growth.

### Table I: Estimates of Euler equation for total consumption and aggregate output, 1875-2009

<table>
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<td><strong>A. Consumption Euler equation</strong></td>
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<tr>
<td>( \hat{\sigma} )</td>
<td>-0.51</td>
<td>-0.63</td>
<td>-0.72</td>
<td>-0.45</td>
<td>-0.69</td>
<td>-0.38</td>
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<tr>
<td>Robust s.e.</td>
<td>(0.20)</td>
<td>(0.25)</td>
<td>(0.28)</td>
<td>(0.27)</td>
<td>(0.24)</td>
<td>(0.28)</td>
</tr>
<tr>
<td>Confidence interval (95%)</td>
<td>[-0.91, -0.12]</td>
<td>[-1.12, -0.14]</td>
<td>[-1.27, -0.18]</td>
<td>[-0.97, -0.07]</td>
<td>[-1.17, -0.21]</td>
<td>[-0.93, 0.18]</td>
</tr>
<tr>
<td>Overidentification test (p-val)</td>
<td>0.78</td>
<td>0.91</td>
<td>0.17</td>
<td>0.16</td>
<td>0.21</td>
<td>0.24</td>
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<tr>
<td>Weak identification test (F-stat)</td>
<td>20.78</td>
<td>22.10</td>
<td>15.26</td>
<td>19.43</td>
<td>14.94</td>
<td>4.73</td>
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<tr>
<td>Wald test (p-val): ( \sigma ) equal</td>
<td>–</td>
<td>–</td>
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<td>0.82</td>
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<td><strong>B. Output Euler equation</strong></td>
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<tr>
<td>( \hat{\sigma} )</td>
<td>-0.51</td>
<td>-0.85</td>
<td>-1.14</td>
<td>-1.16</td>
<td>-1.08</td>
<td>-0.86</td>
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<tr>
<td>Robust s.e.</td>
<td>(0.29)</td>
<td>(0.41)</td>
<td>(0.33)</td>
<td>(0.33)</td>
<td>(0.28)</td>
<td>(0.34)</td>
</tr>
<tr>
<td>Confidence interval (95%)</td>
<td>[-1.07, 0.04]</td>
<td>[-1.65, -0.05]</td>
<td>[-1.79, -0.49]</td>
<td>[-1.81, -0.50]</td>
<td>[-1.63, -0.53]</td>
<td>[-1.53, -0.20]</td>
</tr>
<tr>
<td>Overidentification test (p-val)</td>
<td>0.60</td>
<td>0.69</td>
<td>0.23</td>
<td>0.60</td>
<td>0.73</td>
<td>0.42</td>
</tr>
<tr>
<td>Weak identification test (F-stat)</td>
<td>19.51</td>
<td>19.82</td>
<td>12.69</td>
<td>15.92</td>
<td>13.05</td>
<td>5.02</td>
</tr>
<tr>
<td>Wald test (p-val): ( \sigma ) equal</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.41</td>
<td>–</td>
</tr>
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</table>
III. Measuring the real interest rate

The ex-ante short term real interest rate $r_t$ is defined as $r_t = i_t - E_t \pi_{t+1}$, where $i_t$ is the short term nominal yield and $E_t \pi_{t+1}$ represents the expected inflation one period ahead. While $i_t$ is observable, $r_t$ is not because the inflation expectations are not directly observable. There are two common ways to obtain inflation expectations.

The first and most often used method is to obtain $r_t$ from the ex-post real rate $i_t - \pi_{t+1}$ by projecting it on the information set available at time $t$. Two key assumptions underlie this approach: (i) the rational expectations (RE) which puts a structure on the inflation forecast errors and (ii) the stability of the data-generating process for inflation, henceforth linearity assumption. However, empirical evidence does not square with either of these assumptions. As a consequence, measures of expected inflation based on realized inflation and linear regressions lead to biased estimates. More details on these assumptions are provided in Section III.B.

Second, the real rate $r_t$ can be constructed using inflation survey data. The attractive feature of this approach is that a measure of $r_t$ is obtained without the use of assumptions (i) and (ii). One concern related to the survey data is their accuracy. The empirical evidence documents, however, that survey-based inflation forecasts outperform statistical methods out-of-sample, often by a wide margin (Ang, Bekaert, and Wei, 2007; Faust and Wright, 2012). Taken together, lack of accurate measures of the ex-ante real rate in long sample periods can explain imprecise estimates of the EIS from aggregate data.

This paper proposes an alternative measure of $r_t$ denoted by $\tilde{r}_t$ that uses only the nominal yield curve data. Specifically, I define $\tilde{r}_t = i_t - y_t^{(n)}$ with $n$ large. Appendix B provides details on data used to obtain $\tilde{r}_t$. Constructing $\tilde{r}_t$ by subtracting the long-term nominal yield $y_t^{(n)}$ from $i_t$ is based on the idea that most of the variation in long-term nominal yield is due to changing inflation expectations. In other words, I assume that the Fisher effect holds in the long-run. By this assumption, the real rate is less persistent than inflation expectations. This method has several advantages. First, $\tilde{r}_t$ relies neither on the RE or the linearity assumption, both of which are necessary when extracting inflation expectations from the realized inflation. That is to say, $\tilde{r}_t$ exploits the linear relationship between long-term yields and inflation expectations rather than modeling changing persistence and stochastic volatility of the inflation process. Second, given that the nominal Treasury yield curve data are available for much longer periods than inflation surveys, $\tilde{r}_t$ allows to study the real rate.
and its links to key macro variables over long periods. This is particularly important for the identification of the EIS.

To check its validity, Figure 1 compares \( \tilde{r}_t \) to the ex-ante real rate obtained from the inflation survey denoted by \( r_{t}^{\text{surv}} \). Inflation expectations are from the Livingston survey, the longest consistently available semi-annual inflation survey.\(^3\)

\[
\tilde{r}_t = r_{t}^{\text{surv}} = i_t - E_t^{0} \pi_{t+1} + 1
\]

The most important observation is that \( \tilde{r}_t \) closely tracks the variation in \( r_{t}^{\text{surv}} \) with the correlation \( \rho = 0.55 \). The only exception is a brief period at the beginning of 1980s, a period

\(^{3}\)The Livingston survey is conducted in June and December every year. Participants are asked to forecast the CPI level six and twelve months ahead. Survey results are usually collected several weeks before the publication date. For a detailed discussion of issues related to the publication lag see Carlson (1977).
marked by extremely high and volatile inflation and interest rates due to the monetary policy experiment (Volcker disinflation). Excluding the 1979-1984 period increases the correlation to 0.75. To formally show that $\tilde{r}_t$ is a valid instrument for the ex-ante real rate, I run the following regression in the sample 1875-2009 (un-smoothed annual data):

$$i_t - \pi_{t+1} = \alpha_0 + \alpha_1 \tilde{r}_t + \varepsilon_{t+1} \quad R^2 = 0.15,$$

Figure 2: Instrumental variables estimation of the ex-ante real interest rate, 1875-2009

The figure compares two estimates of the ex-ante real rate, each obtained with a different set of instruments. First, the ex-post real rate $i_t - \pi_{t+1}$ (dashed red line) is regressed on the nominal short-term interest rate $i_t$ represented by the three-month nominal T-bill and the past inflation $\pi_t$ (blue line). Second instrument set includes only $\tilde{r}_t$ (thick black line). The dashed red line represents the ex-post real rate. The data are annual and un-smoothed. The sample period is 1875-2009.

4Designed to contain surging inflation, on October 6, 1979, the Fed formally announced a change in the conduct of monetary policy with the focus on reserves targeting. As a consequence, this change induced a large increase in the level and volatility of short-term nominal interest rates. In October 1982, the Fed abandoned the formal M1 growth target.
where the t-statistics reported in parentheses are Newey-West-adjusted with three lags. The regression results show that $\tilde{r}_t$ is indeed a valid instrument. Moreover, the null that $\alpha_1 = 1$ cannot be rejected, indicating a one-to-one mapping between $\tilde{r}_t$ and the ex-post real rate. Figure 2 plots the ex-ante real rate from the regression given by (2) together with the ex-post real rate. The close match of $\tilde{r}_t$ with the survey-based real rate (Figure 1) together with the regression results in (2) motivate the use of $\tilde{r}_t$ as a direct measure of the ex-ante real interest rate variation. Appendix F provides additional results and discussion regarding the construction of $\tilde{r}_t$.

III.A. When is slope a good measure of the ex-ante real rate?

The idea of constructing a proxy for the real rate variation from the nominal yield curve builds on results in the earlier term structure literature (Fama, 1990; Mishkin, 1990). These papers document that the variation in real interest rates is concentrated at the short end of the yield curve and the slope is informative about the variation in the real rate. Cieslak and Povala (2011) show that the slope is highly correlated with the short-term nominal interest rate after extracting the long-term inflation expectations. Taken together, the empirical evidence locates most of the variation in slope at the short end of the real yield curve. Below, I discuss the conditions under which $\tilde{r}_t$ is an accurate measure of the variation in the short term real rate $r_t$. These assumptions impose weaker restrictions on the joint behavior of inflation and real rate than the implicit assumptions in parametric VAR models.$^5$

Assumption 1: The ex-ante real interest rate is mean-reverting. Therefore the variation in long-term nominal yields due to the fluctuating real interest rate is negligible. The theoretical support for this assumption comes from the fact that in consumption-based models, the real rate and expected consumption growth rates should have similar time series properties. Arguably, the consumption growth rate is not exposed to permanent shocks. The empirical support for this assumption is the fact that long-term yields do not exhibit large and persistent swings in periods characterized by a metallic monetary system such as Gold standard.$^6$

$^5$The idea is to combine the advantages of vector autoregressive models with time-varying parameters (TVP-VAR) and the parsimony of the constant parameter VAR. Therefore, the assumptions do not rule out shifts in parameters but also avoid the filtering of a large number of state variables as it is the case for TVP-VAR models.

$^6$Rose (1988) cannot reject the null of unit root in the ex-post real rate. However, Garcia and Perron (1996) identify several regime switches in the post-war period and argue that the failure to reject the
Assumption 2: The variation in risk compensation accruing to long-term nominal bond holders is small in magnitude and has lower persistence than the real rate variation. Assuming RE, the empirical literature has identified time-varying risk premia as the main source of the rejection of the expectations hypothesis. At the same time, there is evidence from survey data showing significant departures from the full information rational expectations assumption in the short rate expectations (Froot, 1989; Piazzesi and Schneider, 2011; Cieslak and Povala, 2013). Therefore, deviations from RE are likely to explain part of the failure of the expectations hypothesis. Indeed, Cogley (2005) evaluates the joint RE–expectations hypothesis and finds that departures from the prescriptions of rational expectations are important for describing the observed yield curve dynamics. The time-varying risk premia extracted from the long term nominal Treasuries, despite high predictability of returns, have a half-life of less than a year and contribute around 5% to the total yield variation (Cieslak and Povala, 2011). In most of the analysis, I use annual data, therefore the variation in risk premia is unlikely to have a sizable impact on my results.

Assumption 3: Inflation expectations are described by a single factor with unit root persistence and stochastic volatility, i.e. I assume an unobserved component model with stochastic volatility (UC-SV) for realized inflation. This assumption is motivated by the observation that such a model provides a good statistical representation of inflation in the US (Stock and Watson, 2007) and in the UK (Cogley, Sargent, and Surico, 2012a). In the UC-SV model, inflation expectations are modeled as a single-factor random walk process with stochastic volatility. The assumption has several empirical implications. First, the term structure of inflation expectations is flat. Second, the real and nominal slope of the yield curve are identical. Third, short- and long-term nominal yields are cointegrated. The empirical evidence is largely consistent with all of these implications. Kozicki and Tinsley (2006) use inflation survey data from multiple sources to estimate the term structure of expected inflation. The estimated term structure is virtually flat throughout their sample period starting in 1955. Additionally, the long-term survey data from the Blue Chip Economic Indicators (BCEI) panel for the 1984-2010 period, depicted in Panel a of Figure 3, confirm that the term structure of inflation expectations is indeed well described by a single level.

---

non-stationarity of the real interest rate is due to a small number of changes in mean rather than unit root. For measuring the real rate by $\tilde{r}_t$, it is important that the contribution of the real interest rate variation to the total variance of long term nominal yields is negligible. This continues to be the case even if the real rate undergoes a low number of regime switches.
factor. Finally, Panel b of Figure 3 plots the real slope superimposed with the nominal slope for the period 1979-2010. Their correlation is 0.84.

For simplicity, the illustration below restricts the aforementioned assumptions to the case with fixed parameters and constant volatility. The one-period nominal interest rate can be written as:

\[ i_t = r_t + E_t \pi_{t+1}. \]  

(3)

The \( n \)-period nominal Treasury yield can be decomposed as follows:

\[ y_t^{(n)} = \frac{1}{n} \sum_{i=0}^{n-1} E_t r_{t+i} + \frac{1}{n} \sum_{i=1}^{n} E_t \pi_{t+i} + r_{p_t^{(n)}}, \]  

(4)

where \( r_{p_t^{(n)}} \) represents the risk premium. Assume that the real rate follows an AR(1) process with zero mean: \( r_t = \phi r_{t-1} + \sigma r \varepsilon_t^r \), where \(|\phi_r| < 1\), further assume that \( \pi_t = \tau_t + \eta_t \) where inflation expectations \( \tau_t \) are described by the random walk: \( \tau_t = \tau_{t-1} + \sigma_{\tau} \varepsilon_t^\tau \) and \( \eta_t \) is a serially uncorrelated shock, then we have that \( \tilde{r}_t = i_t - y_t^{(n)} \):

\[ \tilde{r}_t = r_t - \frac{1}{n} \sum_{i=0}^{n-1} E_t r_{t+i} + E_t \pi_{t+1} - \frac{1}{n} \sum_{i=1}^{n} E_t \pi_{t+i} - r_{p_t^{(n)}} \]

\[ = \left( 1 - \frac{1}{n} \left( \frac{1 - \phi^n_r}{1 - \phi_r} \right) \right) r_t - r_{p_t^{(n)}}. \]

From Assumption 1 it follows that, for large \( n \), the variation in \( \frac{1}{n} \sum_{i=0}^{n-1} E_t r_{t+i} \) is negligible.\(^7\) Clearly, \( \tilde{r}_t \) is not informative about the unconditional mean of the real rate but this is inconsequential for the EIS estimation.\(^8\) Assumption 2 implies that the variation in \( r_{p_t^{(n)}} \) is of smaller magnitude than the variation in the real rate, i.e. \( \text{Var} (r_t) \gg \text{Var} \left( r_{p_t^{(n)}} \right) \). \( E_t \pi_{t+1} - \frac{1}{n} \sum_{i=1}^{n} E_t \pi_{t+i} = 0 \) which follows from Assumption 3. If Assumptions 1-3 hold, \( \tilde{r}_t \) or equivalently the yield curve slope provides an accurate measure of the real rate variation.

\(^7\)Empirically, \( \phi_r \approx 0.7 \) at semi-annual frequency, which is obtained using the Livingston survey-based proxy for \( r_t \) in the 1955-2010 sample. For \( y_t^{(n)} \), I use the ten-year yield, which implies \( \left( 1 - \frac{1}{n} \left( \frac{1 - \phi^n_r}{1 - \phi_r} \right) \right) \approx 0.83. \)

\(^8\)Indeed, one of the reasons for estimating the EIS from a linearized Euler equation is the fact that \( \tilde{r}_t \) is not informative about the unconditional mean of the real rate.
Panel a plots the survey-based inflation expectations obtained from the Blue Chip Economic Indicators survey. The data are semi-annual as the survey is conducted in March and October each year. The sample period is 1984 through 2010. $E_t^s \pi_{t+1Y}$ denotes the median survey response about the inflation one year ahead. $E_t^s \frac{1}{5} \sum_{i=1Y}^{5Y} \pi_{t+i}$ denotes the median survey response about the average inflation over the next five years and finally $E_t^s \frac{1}{5} \sum_{i=6Y}^{10Y} \pi_{t+i}$ represents the median response about the average inflation between five and ten years ahead. Panel b compares the real and nominal slope of the yield curve. The data are semi-annual. The sample period is 1979-2010. The start of the sample is dictated by the availability of the long term inflation survey data. The real slope is constructed as the difference of the ten-year real yield and real three-month Treasury bill. The real ten-year yield is obtained using the ten-year inflation expectations from Livingston and SPF surveys.
III.B. The RE and linearity assumptions

It is instructive to study the two key assumptions that underlie the standard method for obtaining the ex-ante real rate.

**RE assumption.** The RE assumption asserts that inflation forecast errors are not predictable. If the forecast errors are predictable, expected inflation obtained from standard time-series models will inherit the predictable part of these errors. The slope of the term structure of nominal yields varies because of changes in expected inflation and fluctuations in the real rate (Fama, 1990). Additionally, inflation and the real interest rate are negatively correlated (Mishkin, 1981; Fama and Gibbons, 1982). These stylized facts are based on the following regression:

\[
\pi_{t+m} - \pi_{t+1} = \delta_0 + \delta_1 \text{slope}_t + \varepsilon_{t+m},
\]

(5)

where \(\pi_{t+m}\) denotes the realized inflation between time \(t\) and \(t+m\), \(m > 1\) and \(\pi_{t+1}\) represents the realized inflation one period ahead.

Panel A of Table II reports the results for the regression (5) for \(m = 2, \ldots, 5\) years in the sample 1955-2010. The results confirm that slope is a significant predictor of realized inflation changes for all horizons considered with \(R^2\)’s up to 20% and the positive sign. Estimates of \(\delta_1\) are significantly below unity which indicates that the slope also predicts the real rate variation. To understand the source of the inflation predictability, I decompose the dependent variable in (5) as follows:

\[
\pi_{t+m} - \pi_{t+1} = \pi_{t+m} - E_t \pi_{t+1} - (\pi_{t+1} - E_t \pi_{t+1})
\]

where the Livingston inflation survey data are used to proxy for \(E_t \pi_{t+1}\). Panels B and C of Table II report the regression results for \((\pi_{t+m} - E_t \pi_{t+1})\) and \((\pi_{t+1} - E_t \pi_{t+1})\) as dependent variables, respectively. The regressions indicate that all of the predictability of realized inflation is due to the predictability of one-period inflation forecast errors. This result has several important implications. First, slope is not informative about the variation in expected inflation which supports the notion that slope predominantly captures the variation in the
Table II: Inflation changes and the slope, 1955-2010

Panel A reports the results for the regression (5) for different horizons $m = \{2, \ldots, 5\}$ years. Using inflation surveys, the dependent variable in (5) is decomposed as follows: $
_{t+m} - \n_{t+1} = \n_t - E_t \n_{t+1} - (\n_{t+1} - E_t \n_{t+1})$ where $\n_{t+1} - E_t \n_{t+1}$ is the one-year inflation forecast error. $\n_{t+m} = (\log P_{t+m} - \log P_t)/m$ where $P_t$ is the CPI index. Panels B and C report the regression results for the decomposed dependent variable. $E_t \n_{t+1}$ is obtained from the Livingston survey which is conducted semi-annually (June and December). Timing of the variables is as follows. For example, a June 1955 slope observation is constructed using end of May 1955 bond data. A June 1955 observation for the one-year inflation rate is computed from the May 1955 and May 1956 CPI. The sample covers the period 1955-2010. Both the sampling frequency and beginning of the sample period are determined by the availability of inflation survey data. Newey-West adjusted t-statistics are reported in parentheses, the number of lags is 6.

<table>
<thead>
<tr>
<th>m=2</th>
<th>m=3</th>
<th>m=4</th>
<th>m=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. $\n_{t+m} - \n_{t+1} = \delta_0 + \delta_1 \text{slope}<em>t + \varepsilon</em>{t+m}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.28</td>
<td>0.48</td>
<td>0.58</td>
</tr>
<tr>
<td>$R^2$</td>
<td>(2.97)</td>
<td>(3.04)</td>
<td>(2.80)</td>
</tr>
<tr>
<td>Panel B. $\n_{t+m} - E_t \n_{t+1} = \beta_0 + \beta_1 \text{slope}<em>t + \varepsilon</em>{t+1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-0.22</td>
<td>-0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$R^2$</td>
<td>(-1.32)</td>
<td>(-0.20)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>Panel C. $\n_{t+1} - E_t \n_{t+1} = \gamma_0 + \gamma_1 \text{slope}<em>t + \varepsilon</em>{t+1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>(-3.27)</td>
<td>0.14</td>
<td></td>
</tr>
</tbody>
</table>

real interest rate.\(^9\) Second, it indicates a violation of the RE assumption for inflation.\(^10\) As a consequence, the real rate obtained from the ex-post realized real rate contains the predictable part of inflation forecast errors which leads to more volatile and biased estimates of the ex-ante real rate. Third, the negative correlation between the real rate and the expected inflation reported in the literature disappears once the ex-ante real rate is obtained from the inflation survey data. In the 1955-2010 sample, the correlation between the ex-ante real rate and inflation expectations is 0.16.

Linearity assumption. A number of papers document that the statistical properties of inflation change over time (Barsky, 1987; Stock and Watson, 2007; Cogley, Sargent, and

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\(^9\)Equivalently, yield curve slope does not predict changes in survey-based inflation expectations (Livingston).

\(^10\)I interpret the deviations from the RE assumption broadly. For instance, these can include a peso problem, i.e. it can well be that investors anticipated a discrete shift in the inflation process which was then reflected in asset prices long before it actually occurred. Alternatively, the predictable forecast errors can arise in a setup with imperfect information. From the perspective of monetary policy, predictable inflation forecast errors are indicative of the inflation non-neutrality in the short-run. To explicitly account for the predictable inflation forecast errors documented above, it is straightforward to extend the UC-SV model by specifying an AR(1) dynamics for the transitory component.
Surico, 2012b). Barsky (1987) shows that the inflation evolved from a highly volatile white noise process in the period before World War I into a highly persistent one in the post-1960 period. Stock and Watson (2007) show that the postwar US inflation is well described by an UC-SV model.

To evaluate the bias introduced by assuming the linear dynamics for inflation in the presence of stochastic volatility, I simulate inflation from the UC-SV model and the real rate as a simple AR(1) process using parameter values consistent with the data. Details of the simulation exercise are provided in Appendix A. The median correlation between the true ex-ante real rate and the rate obtained by a linear projection is 0.34. In the simulation exercise, biased estimates of the real rate translate into a sizable bias in the EIS estimates. For the true value of EIS=1, the median estimate of the EIS from the linearized Euler equation is 0.30. The non-linearity can be handled by modeling the nominal short rate $i_t$ and inflation $\pi_t$ within a TVP-VAR with stochastic covariance matrix. Indeed, Cogley and Sargent (2005) show that models with time-varying parameters perform well under both random walk time variation and discrete structural breaks. However, a typical TVP-VAR with two lags and three observables (inflation, unemployment, nominal short rate) requires the filtering of more than 20 time-varying parameters, which necessarily introduces a significant degree of statistical uncertainty.

The degree to which the RE and linearity assumptions are violated can be seen by comparing the ex-ante real rate instrumented by $\tilde{r}_t$ and the ex-ante real rate obtained by regressing $i_t - \pi_{t+1}$ on $i_t$ and $\pi_t$. Figure 2 plots the ex-post real rate together with these two versions of ex-ante real rate. It is clear that including past inflation improves the statistical fit, in fact the $\bar{R}^2$ increases from 0.15 obtained in regression (2) to 0.51. However, the plot shows that most of the improvement in statistical fit comes from fitting extreme values of inflation during the two World Wars and the Great Depression. Despite the fact that inflation is clearly a valid instrument in a statistical sense, it is not a valid instrument economically because the RE and linearity assumptions are violated. Figure 2 also shows that in the post-1980 period, the ex-post real rate and both measures of ex-ante real rate increasingly move together. Indeed, post-1980, the correlation between the slope and inflation forecast errors has been declining.

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11Earlier work by Evans and Lewis (1995) argues that a two-component model of inflation where the volatility of shocks to each component follows a regime-switching Markov process closely replicates the inflation expectations from the Livingston survey.

12Note that all estimates of the EIS using household-level data (e.g. CEX survey) fall into the post-1980 period where the linear projection method for obtaining the ex-ante real rate works reasonably well,
Assuming an isoelastic period utility function and the absence of market frictions, the intertemporal optimality condition for consumption $C_t$ reads:

$$
E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + R_{rf,t+1}) \right] = 1, \tag{6}
$$

where $\gamma$ denotes the risk aversion parameter, $\beta$ represents the time discounting and $R_{rf,t+1}$ is the ex-post real interest rate. Further, assuming conditional log-normality leads to a linearized version of (6):

$$
\Delta c_{t+1} = \mu_t + \frac{1}{\gamma} r_t + u_{t+1}, \tag{7}
$$

where $r_t = E_t \log (1 + R_{rf,t+1})$, $u_{t+1}$ denotes the expectational error and:

$$
\mu_t = \frac{\log (\beta)}{\gamma} + \frac{\gamma}{2} \var_t (\Delta c_{t+1}) - \text{cov}_t (\Delta c_{t+1}, \log (1 + R_{rf,t+1})) + \frac{1}{2\gamma} \Var_t (\log (1 + R_{rf,t+1})).
$$

In the previous section, I argue that $r_t$ can be replaced by $\hat{r}_t$ after accounting for the unconditional mean of the real rate. Therefore, an estimable version of (7) reads:

$$
\Delta c_{t+1} = \bar{\mu}_t + \sigma \hat{r}_t + u_{t+1}, \tag{8}
$$

where $\sigma \equiv \frac{1}{\gamma}$ and $\bar{\mu}_t$ is $\mu_t$ plus the unconditional mean of $r_t$.\textsuperscript{13}

### IV.A. Estimation

I use the US real per capita annual data on consumption and output for the sample period 1875-2009. I estimate the consumption and output Euler equation on two data sets. First, I use the data set constructed by Barro and Ursua (1875-2009). Second, the consumption data from the Bureau of Economic Analysis (BEA) are used at annual frequency (1929-2009) and at quarterly frequency (1950-2009). More details regarding both data sets and their sources which explains the significant and plausible estimates of the EIS for households participating in asset markets.

\textsuperscript{13}Note that due to the log-linearization, the isoeelastic specification of the utility function is not crucial for relating the expected consumption growth to the real interest rate. Alternative utility specifications such as Epstein-Zin utility obtain the same relationship.
are provided in Appendix B. The output data allow me to evaluate the aggregate output Euler equation (IS equation) used in the New-Keynesian model below. The aggregate output Euler equation is the core of dynamic equilibrium models used in monetary policy analysis. Therefore, the EIS estimates from the output Euler equation are key for evaluating the real effects of monetary policy. Figure 4 plots the real consumption $\Delta c_{t+1}$ (Panel a) and output growth $\Delta y_{t+1}$ (Panel b) superimposed with $\hat{r}_t$ for the period 1875-2009. The unconditional correlation between $\Delta c_{t+1}$, $\Delta y_{t+1}$ and $\hat{r}_t$ is -0.34 and -0.30, respectively.

![Figure 4: Consumption and output growth and the real interest rate, 1875-2009](image)

Panel a plots real per capita consumption growth superimposed with the proxy for the short term real interest rate denoted by $\hat{r}_t$. Panel b plots the real output growth superimposed with $\hat{r}_t$. The data are annual and the sample period is 1875 through 2009.

If one is willing to assume homoscedasticity and a household’s decision interval of one year (in addition to Assumptions 1-3 from Section III.A), the log-linearized Euler equation given in (8) can be estimated via OLS. To account for possible correlation between $\hat{r}_t$ and
conditional variance terms collected in $\bar{\mu}_t$ and also to avoid potential issues induced by time-aggregation, I estimate (8) via instrumental variables. The linear IV estimation is standard and is implemented as a two-step linear GMM. The set of instruments for $\tilde{r}_t$ includes twice lagged consumption growth, two lags of $\tilde{r}_t$ and the lagged nominal three-month Treasury bill yield. The lags of instruments for $\tilde{r}_t$ are chosen such that there is no overlap with the consumption at time $t$.

**IV.B. Estimation results**

In addition to the main results reported in Table I based on Barro and Ursua’s data, this section discusses the estimates of EIS from the BEA consumption data. I evaluate if the estimated magnitude of the EIS from annual data on total consumption matches the estimates from non-durable consumption and quarterly data, respectively.

Barro and Ursua’s data set does not distinguish between non-durable and durable consumption expenditures due to limitations on data availability. Given that expenditures on durable goods are roughly 15% of the non-durable consumption and services, the potential bias in the EIS estimates is likely to be small. To quantify the impact of durable goods expenditures on the EIS estimates, I estimate the consumption Euler equation using the non-durable consumption and services in the sub-sample 1929-2009. The sample start is determined by the availability of the data on non-durable consumption and services from the BEA. Table III reports the results for three different sample periods. Panel A displays the estimates of EIS using expenditures on non-durable goods and services. Panel B shows the results obtained with durable goods expenditures and finally Panel C reports the EIS estimates using total consumption.

The estimate of EIS using non-durables in the 1929-2009 period is $-0.66$ and statistically significant. It compares to the estimate of $-0.78$ using total consumption in the same period. Similarly, the EIS estimates using non-durable and total consumption expenditures in the 1980-2009 period are $-0.45$ and $-0.64$, respectively. Panel B shows that the interest rate elasticity of durable expenditures is a multiple of the non-durable consumption sensitivity.

---

14The main motivation for excluding expenditures on durables is to minimize the measurement error originating from the discrepancy between expenditures and the consumption itself. However, with decreasing measurement frequency the error becomes smaller.
Table III: Euler equation estimates for non-durable consumption, 1929-2009

The table reports the estimation results for the log-linearized Euler equation given by (8). The IV setup is implemented as a two-step GMM. Panel A reports the results for non-durable consumption and services, Panel B for the durable consumption expenditures and Panel C for the total consumption. Each column corresponds to a different sample period. The first column shows the full sample estimates, 1929-2009. This is the longest sample period for which the split into durable and non-durable consumption expenditures is available. The second column reports the results for 1947-2009, a post-war period. The third column displays the results for the period 1980-2009. The data are annual. The autocorrelation- and heteroskedasticity-robust standard errors are reported in parentheses (bandwidth=3, Bartlett kernel). The overidentification test refers to Hansen’s J statistic which is asymptotically \( \chi^2 \) distributed. The “Weak identification test” in each panel reports the F-statistic (Kleibergen-Paap). The critical values for the maximum relative bias are 16.85 for 5%, 10.27 for 10%, 6.71 for 20% and 5.34 for 30%. The critical values for the maximal size are 24.58 for 10%, 10.26 for 20%, and 8.31 for 25%. The critical values are computed using the procedure proposed by Stock and Yogo (2002). The following variables are used as instruments: two lags of \( \tilde{r}_t \), the lagged three-month nominal T-bill and twice lagged corresponding consumption growth.

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
<td>A. Nondurable consumption &amp; services</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>-0.66</td>
<td>-0.47</td>
<td>-0.45</td>
</tr>
<tr>
<td>Robust s.e.</td>
<td>(0.30)</td>
<td>(0.21)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>Confidence interval (95%)</td>
<td>[-1.25, -0.07]</td>
<td>[-0.88, -0.05]</td>
<td>[-0.79, -0.12]</td>
</tr>
<tr>
<td>Overidentification test (p-val)</td>
<td>0.41</td>
<td>0.69</td>
<td>0.25</td>
</tr>
<tr>
<td>Weak identification test (F-stat)</td>
<td>21.77</td>
<td>14.47</td>
<td>15.67</td>
</tr>
<tr>
<td></td>
<td>B. Durable expenditures</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>-2.43</td>
<td>-2.59</td>
<td>-2.52</td>
</tr>
<tr>
<td>Robust s.e.</td>
<td>(0.88)</td>
<td>(0.74)</td>
<td>(0.79)</td>
</tr>
<tr>
<td>Confidence interval (95%)</td>
<td>[-4.14, -0.71]</td>
<td>[-4.05, -1.13]</td>
<td>[-4.07, -0.96]</td>
</tr>
<tr>
<td>Overidentification test (p-val)</td>
<td>0.90</td>
<td>0.29</td>
<td>0.19</td>
</tr>
<tr>
<td>Weak identification test (F-stat)</td>
<td>18.30</td>
<td>11.73</td>
<td>12.00</td>
</tr>
<tr>
<td></td>
<td>C. Total consumption</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>-0.78</td>
<td>-0.68</td>
<td>-0.64</td>
</tr>
<tr>
<td>Robust s.e.</td>
<td>(0.32)</td>
<td>(0.24)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>Confidence interval (95%)</td>
<td>[-1.40, -0.15]</td>
<td>[-1.14, -0.22]</td>
<td>[-1.05, -0.23]</td>
</tr>
<tr>
<td>Overidentification test (p-val)</td>
<td>0.43</td>
<td>0.58</td>
<td>0.20</td>
</tr>
<tr>
<td>Weak identification test (F-stat)</td>
<td>22.03</td>
<td>14.46</td>
<td>16.48</td>
</tr>
</tbody>
</table>

Higher sensitivity of durable goods expenditures slightly increases the magnitude of EIS estimates when total consumption is used. However, the difference is well within one standard deviation of the EIS estimate from the non-durable consumption. The estimates are stable across sub-samples. Particularly, in the most recent period 1980-2009, the estimates of aggregate EIS have relatively low standard errors despite the low number of observations. Overall, there is no evidence that the negative aggregate EIS is driven solely by the non-durable consumption expenditures.

The estimates of \( \sigma \) in Tables I and III are unaffected by the structural break in inflation and monetary policy which is usually located in the 1979-1982 period (Clarida, Gali, and...
Comparing the second and fifth columns in both panels of Table I shows neither a change of sign nor a significant difference in the magnitude of $\hat{\sigma}$. A formal test of equality of $\hat{\sigma}$ in the period 1875-1979 and 1980-2009 is reported in the last row of each panel. In both cases the null that the coefficients are equal is not rejected.\textsuperscript{15}

Throughout the paper, I use annual consumption and interest rate data. An important question is if the results extend to quarterly frequency. One concern is that if the household’s decision interval is shorter than one year, the aggregation introduces a spurious autocorrelation in $\Delta c_{t+1}$ and the estimates of the EIS are biased. Main arguments for measuring the consumption and output at annual rather than higher frequencies are the transaction costs and other frictions which might prevent households from instantaneous consumption smoothing (Gabaix and Laibson, 2001; Jagannathan and Wang, 2007). Additionally, the variation in real interest rates is persistent (half-life above one year), hence the consumption could react slowly to changes in the real interest rate because the utility loss induced by the slow adjustment is low. To evaluate whether my results are robust to time aggregation, I re-estimate the consumption Euler equation on quarterly data for the period 1950-2009. To obtain estimates of the EIS that are comparable to the annual data, quarterly consumption growth is annualized. Table IV reports the estimates of $\sigma$ for non-durables (Panel A), durable consumption expenditures (Panel B) and the total consumption (Panel C) across three post-war periods. Throughout the table, the magnitudes of EIS closely correspond to the estimates from annual data (Table III) which indicates that the estimated magnitudes are invariant to the decision interval of households.

It is possible that the negative EIS is specific to the US data. To evaluate this interpretation, Table V reports the EIS estimates for selected developed countries. The EIS estimates are consistently negative and have, in some cases, a larger magnitude than in the US. Arguably, each of these countries follow different monetary policies. Therefore, the evidence from a

\textsuperscript{15}Bilbiie and Straub (2012) argue that the main reason for the structural break and the change of sign in $\sigma$ is the financial market liberalization which stimulated an increase in asset market participation. However, they use realized inflation when constructing the ex-ante real rate which is responsible for the identified structural break.

\textsuperscript{16}In fact, in other countries that have not experienced a structural break in inflation in the recent past (e.g. Germany, the Netherlands or Canada), the standard method yields negative and significant estimates of the aggregate EIS of the similar magnitude compared to the US estimates using $\hat{r}_t$. 
Table IV: Euler equation estimates for quarterly data, 1950-2009

The table reports the estimation results for the log-linearized Euler equation given by (8). The IV setup is implemented as a two-step GMM. Panel A reports the results for non-durable consumption and services, Panel B for the durable consumption expenditures and Panel C for the total consumption. Quarterly consumption growth data are annualized to be comparable with \( \tilde{r}_t \) which is annual. Each column corresponds to a different sample period. The first column shows the full sample estimates for which the quarterly consumption data are available, 1950-2009. The second column reports the results for 1960-2009. The third column displays the results for the period 1980-2009. The data are quarterly. The autocorrelation- and heteroskedasticity-robust standard errors are reported in parentheses (bandwidth=8, Bartlett kernel). The overidentification test refers to Hansen’s J statistic which is asymptotically \( \chi^2 \) distributed. The “Weak identification test” in each panel reports the F-statistic (Kleibergen-Paap). The critical values for the maximum relative bias are 19.28 for 5%, 11.12 for 10%, 6.76 for 20%, and 5.15 for 30%. The critical values for the maximal size are 29.18 for 10%, 11.72 for 20%, and 9.38 for 25%. The critical values are computed using the procedure proposed by Stock and Yogo (2002). Following variables are used as instruments: lag one and lag four of \( \tilde{r}_t \), lags two through four of consumption growth, and lag one of the three-month nominal T-bill.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>( \hat{\sigma} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Nondurable consumption &amp; services</td>
<td>-0.24</td>
<td>-0.40</td>
<td>-0.50</td>
</tr>
<tr>
<td></td>
<td>Robust s.e.</td>
<td>(0.18)</td>
<td>(0.19)</td>
</tr>
<tr>
<td></td>
<td>Confidence interval (95%)</td>
<td>[-0.60, 0.12]</td>
<td>[-0.77, -0.03]</td>
</tr>
<tr>
<td></td>
<td>Overidentification test (p-val)</td>
<td>0.08</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Weak identification test (F-stat)</td>
<td>217.55</td>
<td>254.22</td>
</tr>
<tr>
<td>B. Durable expenditures</td>
<td>-2.80</td>
<td>-2.64</td>
<td>-2.67</td>
</tr>
<tr>
<td></td>
<td>Robust s.e.</td>
<td>(0.81)</td>
<td>(0.80)</td>
</tr>
<tr>
<td></td>
<td>Confidence interval (95%)</td>
<td>[-4.33, -1.28]</td>
<td>[-4.13, -1.14]</td>
</tr>
<tr>
<td></td>
<td>Overidentification test (p-val)</td>
<td>0.35</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>Weak identification test (F-stat)</td>
<td>200.82</td>
<td>173.01</td>
</tr>
<tr>
<td>C. Total consumption</td>
<td>-0.35</td>
<td>-0.50</td>
<td>-0.62</td>
</tr>
<tr>
<td></td>
<td>Robust s.e.</td>
<td>(0.20)</td>
<td>(0.22)</td>
</tr>
<tr>
<td></td>
<td>Confidence interval (95%)</td>
<td>[-0.74, 0.04]</td>
<td>[-0.93, -0.07]</td>
</tr>
<tr>
<td></td>
<td>Overidentification test (p-val)</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>Weak identification test (F-stat)</td>
<td>206.22</td>
<td>220.89</td>
</tr>
</tbody>
</table>

Panel of countries supports the structural interpretation of the negative EIS. Discussion of some additional econometric issues is provided in Appendix C.

IV.C. Econometric issues

Estimating the EIS parameter from the log-linearized Euler is subject to a number of potential biases. One obvious source is the stochastic volatility which has been mostly studied at the household level (Carroll, 2001; Attanasio and Low, 2004). Attanasio and Low (2004) show that, even in the presence of stochastic volatility, the log-linearized Euler
Table V: Euler equation estimates for selected developed countries

The table reports estimation results for the log-linearized Euler equation given by (8). The IV setup is implemented as a two-step GMM. Panel A reports the results for consumption and Panel B for the real output. Each column corresponds to a different country. The sample period for each country is 1950-2009, the only exception is Germany with the sample period 1956-2009 (due to the data availability). The data are annual and their sources are described in Appendix B.1. The autocorrelation and heteroskedasticity-robust standard errors are reported in parentheses (bandwidth=3, Bartlett kernel). The overidentification test refers to Hansen’s J statistic which is asymptotically \( \chi^2_3 \) distributed. The “Weak identification test” in each panel reports the F-statistic (Kleibergen-Paap). The critical values for the maximum relative bias are 16.85 for 5%, 10.27 for 10%, 6.71 for 20%, and 5.34 for 30%. The critical values for the maximal size are 24.58 for 10%, 10.26 for 20%, and 8.31 for 25%. The critical values are computed using the procedure proposed by Stock and Yogo (2002). The instruments are twice lagged consumption/output growth, two lags of \( \tilde{r}_t \), and the lagged nominal Treasury bill yield.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>Germany</th>
<th>Netherlands</th>
<th>Switzerland</th>
<th>United Kingdom</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Consumption Euler equation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>-0.93</td>
<td>-0.96</td>
<td>-0.91</td>
<td>-0.28</td>
<td>-0.19</td>
</tr>
<tr>
<td>Robust s.e.</td>
<td>0.21</td>
<td>0.34</td>
<td>0.28</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>Confidence interval (95%)</td>
<td>[-1.35, -0.51]</td>
<td>[-1.62, -0.30]</td>
<td>[-1.46, -0.37]</td>
<td>[-0.70, 0.15]</td>
<td>[-0.66, 0.27]</td>
</tr>
<tr>
<td>Overidentification test (p-val)</td>
<td>0.30</td>
<td>0.08</td>
<td>0.39</td>
<td>0.33</td>
<td>0.70</td>
</tr>
<tr>
<td>Weak identification test (F-stat)</td>
<td>11.00</td>
<td>22.79</td>
<td>38.06</td>
<td>12.69</td>
<td>27.05</td>
</tr>
<tr>
<td><strong>B. Output Euler equation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>-0.95</td>
<td>-1.02</td>
<td>-0.84</td>
<td>-1.15</td>
<td>-0.41</td>
</tr>
<tr>
<td>Robust s.e.</td>
<td>0.24</td>
<td>0.29</td>
<td>0.23</td>
<td>0.29</td>
<td>0.18</td>
</tr>
<tr>
<td>Confidence interval (95%)</td>
<td>[-1.42, -0.49]</td>
<td>[-1.58, -0.46]</td>
<td>[-1.30, -0.38]</td>
<td>[-1.72, -0.50]</td>
<td>[-0.76, -0.06]</td>
</tr>
<tr>
<td>Overidentification test (p-val)</td>
<td>0.55</td>
<td>0.66</td>
<td>0.34</td>
<td>0.53</td>
<td>0.30</td>
</tr>
<tr>
<td>Weak identification test (F-stat)</td>
<td>10.00</td>
<td>22.84</td>
<td>33.44</td>
<td>11.92</td>
<td>30.79</td>
</tr>
</tbody>
</table>

equation yields unbiased estimates of the EIS if the sample period is long enough, which is the case in this paper. Furthermore, the stochastic volatility is less of a concern in the aggregate data where the idiosyncratic volatility is averaged out. Beeler and Campbell (2012) show that the bias due to the stochastic volatility does not play an important role, at least for the volatility process calibrated to the US aggregate consumption data. Additionally, in Section VII below, I show that the proxy for stochastic volatility of consumption growth does not significantly alter the EIS estimates.

The observation that the linearized Euler equation (7) and its reversed form with the real rate as a dependent variable lead to very different estimates of the EIS is commonly interpreted as an evidence for weak instruments (Neely, Roy, and Whiteman, 2001; Yogo, 2004). This is not the case for \( \tilde{r}_t \). Using the identical instrument set, the estimate of \( \frac{1}{\sigma} \) from the reversed form of the consumption Euler equation (8) is -1.66 with the 95% confidence interval: \([-2.65, -0.66]\) in the 1875-2009 sample. The full sample estimate of \( \sigma \) from the original specification is -0.51 which is within the confidence interval given by the reversed form.
I construct moving averages of both quantities, demeaned $\Delta c_{t+1}$ and $\tilde{r}_t$, and run a univariate regression with filtered series. The goal of this exercise is to evaluate the relationship between consumption growth and the real rate across frequencies. The real rate variation is persistent and it should be reflected in the persistent variation of expected consumption growth if the EIS is non-zero. Conversely, if the significant estimates of the EIS reported above are driven by a few outliers, the filtered series shall not have a similar degree of co-movement. To compute moving averages, I use the filter proposed in Lucas (1980). For a covariance stationary series $x_t$, it is given by:

$$\bar{x}_t(\beta) = \alpha \sum_{k=-n}^{n} \beta^{|k|} x_{t+k}$$

(9)

$$\alpha = \frac{(1 - \beta)^2}{1 - \beta^2 - 2\beta^{n+1}(1 - \beta)},$$

(10)

where $\beta \in [0, 1)$. I study the following four degrees of smoothing $\beta \in \{0, 0.8, 0.95, 0.98\}$.

Panels $a - d$ of Figure 5 evaluate the comovement of consumption growth with the real rate for different frequencies as measured by values of $\beta$. As the figure shows, the negative relationship between $\tilde{r}_t$ and consumption growth is present at all frequencies. Moreover, the regression coefficient, which is given by the slope of the dash-dotted line, shows a remarkable stability for all degrees of smoothing. The estimated slope coefficients are close to the EIS estimates reported in Panel A of Table I. The close relationship between the consumption growth and the real interest rate indicates that expected consumption growth contains a time-varying and persistent component. Section VII studies this relationship in greater detail and links the results to the long-run risk literature.

IV.D. Alternative explanations

The relationship between the real interest rate and consumption growth can potentially be negative for other reasons than the negative aggregate EIS. For instance, the time discounting can be time-varying and strongly negatively correlated with the real rate. In such a case,

---

17 Note that $\beta = 0$ corresponds to a univariate regression of unfiltered series. For $\beta \rightarrow 1$, the regression approximately measures the relationship at frequency $\omega = 0$ in the frequency domain, for details see Whiteman (1984).

18 The discount factor shock is often introduced in the empirical implementations of dynamic stochastic general equilibrium models (Justiniano and Primiceri, 2008). Given the empirical failure of the consumption Euler equation, the time-varying discounting picks up the variation in consumption growth.
Panels a–d scatter-plot the filtered consumption growth and the real rate measure \( \tilde{r}_t \) for different filtering parameters \( \beta \). For \( \beta = 0 \) (Panel a), the filter corresponds to a linear regression of unfiltered series. The filter is given by equation (9). I set \( n = 50 \). Blue dash-dotted lines represent the regression coefficient obtained from a regression of the filtered consumption growth on filtered \( \tilde{r}_t \). The regression coefficients together with Newey-West adjusted t-statistics are reported in the lower left corner of each panel. The red dashed line represents the regression slope of minus one. The data are annual and the sample period is 1875 through 2009.

one obtains a negative estimate of \( \sigma \) even though the true EIS is positive. However, for this to happen, the discount factor shock would have to be more volatile than \( r_t \), which, together with the highly negative correlation seems implausible. Alternatively, it can be the case that the unobserved returns to human capital are negatively correlated with the real

The factor is usually interpreted as a shock to aggregate demand or demographics. See also Albuquerque, Eichenbaum, and Rebelo (2012).
short rate. However, Lustig, Van Nieuwerburgh, and Verdelhan (2012) show that returns on human wealth are closely positively related to real bond returns. Therefore, the negative relationship between consumption growth and the real interest rate cannot be replicated for positive values of $\sigma$ at the aggregate level unless one is willing to accept counterfactual assumptions.

V. Model

This section outlines a standard New Keynesian model with limited asset market participation. The model setup is closely related to Gali, Lopez-Salido, and Valles (2004) and Bilbiie (2008). While these papers focus on the equilibria determinacy in the presence of limited market participation, I evaluate the asset pricing implications of limited participation.\(^{19}\) The key ingredient of the model is limited asset market participation rather than a short-run inflation non-neutrality induced by sticky prices. To make the key mechanism transparent, I restrict the technology to be the only economic disturbance in the model. The main purpose of the model is to illustrate one potential mechanism that can replicate the negative sign and the magnitude of the aggregate EIS. As such, the model is too simple to be evaluated on quantitative predictions along other dimensions, but could be easily enriched.

The economy consists of two types of households, a representative final-good-producing firm, a continuum of intermediate-goods-producing firms and a central bank.

V.A. Households

There is a continuum of households on [0,1] where $1 - \lambda$ fraction of them are forward-looking, participate in asset markets to smooth consumption and own all assets in the economy—*savers* denoted by subscript $s$. The *rule-of-thumb* households on [0, $\lambda$] denoted by the subscript $r$ do not participate in financial markets to smooth their consumption and do not have any wealth. Both types of households have an identical period utility function $U$ which is standard:

\[
U(C_{i,t}, N_{i,t}) = \frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \frac{N_{i,t}^{1+\eta}}{1+\eta},
\]

\[\text{(11)}\]

\(^{19}\)The sample period considered in this paper includes a period before the Fed was operational. The outlined model is meant to describe the more recent period where the central bank has been actively managing the short term interest rate.
where \( i \in \{s, r\} \). The parameter \( \gamma > 0 \) represents the degree of risk aversion, \( \eta > 0 \) is the inverse of labor supply elasticity. Savers solve the usual optimization problem:

\[
\max E_t \sum_{j=0}^{\infty} \beta^j U (C_{s,t+j}, N_{s,t+j}), \quad \beta \in (0, 1)
\]  

(12)

by choosing the consumption \( C_{s,t} \) and worked hours \( N_{s,t} \) every period subject to the budget constraint expressed in nominal terms:

\[
B_{s,t+1} + \Theta_{s,t+1} V_t \leq B_s R_t + \Theta_s (V_t + P_t D_t) + W_t N_{s,t} - P_t C_{s,t}.
\]  

(13)

\( B_{s,t} \) is the quantity of one-period nominal bonds at the beginning of period \( t \), \( \Theta_{s,t} \) denotes the holdings of savers in firms, \( V_t \) is the market value of firms and \( D_t \) are real dividends. \( W_t \) denotes the nominal wage.

The no-arbitrage condition implies the existence of a stochastic discount factor \( M_{t,t+1} \) that prices all uncertain future cash flows:

\[
V_t = E_t [M_{t,t+1} (V_{t+1} + P_{t+1} D_{t+1})].
\]  

(14)

The stochastic discount factor together with the assumption of frictionless financial markets (for savers) allows to rewrite (13) in terms of present values:

\[
E_t \sum_{j=t}^{\infty} M_{t,t+j} P_j C_{s,j} \leq V_t + E_t \sum_{j=t}^{\infty} M_{t,t+j} W_j N_{s,j}.
\]  

(15)

The remaining first order conditions for savers read:

\[^{20}\text{Separable preferences are used for simplicity. The utility function given by (11) is not compatible with balanced growth path. To resolve this, one could introduce a non-separable utility specification. The following specification is common in the RBC literature: } U (C_t, N_t) = \frac{1}{1-\sigma} \left[ (C_t V(N_t))^{1-\sigma} - 1 \right]. \text{ The main result in this paper, namely } EIS < 0, \text{ is obtained also with this specification.}
\]

\[^{21}\text{ Nominal bonds are in the zero net supply because markets are complete and savers are homogenous. Hence, they can be replaced by a representative agent.}
\]

\[^{22}\text{In a model with the representative agents } \Theta = 1. \text{ In this model, it will depend on the fraction of savers on the whole population: } \Theta = \frac{1}{1-\lambda}. \]
\[
\beta U_C \left( C_{s,t+1} \right) / U_C \left( C_{s,t} \right) = M_{t,t+1} \frac{P_{t+1}}{P_t} \\
1 / R_t = \beta E_t \left[ \left( \frac{C_{s,t+1}}{C_{s,t}} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right] \\
\frac{N_{r,t}^\eta}{C_{r,t}^{-\gamma}} = \frac{W_t}{P_t}.
\]

The optimization problem of rule-of-thumb consumers involves only the intratemporal trade-off:

\[
\max_{C_{r,t}, N_{r,t}} \frac{C_{r,t}^{1-\gamma}}{1-\gamma} - \frac{N_{r,t}^{1+\eta}}{1+\eta},
\]

subject to the budget restriction \( C_{r,t} P_t = W_t N_{r,t} \). The first order condition for the rule-of-thumb household is:

\[
\frac{N_{r,t}^\eta}{C_{r,t}^{-\gamma}} = \frac{W_t}{P_t}.
\]

**V.B. Production**

The final good is produced with a CES production function with elasticity \( \varepsilon \), which aggregates the intermediate goods indexed by \( k \), i.e. \( Y_t = \left( \int_0^1 Y_t(k)(^{\varepsilon-1}) dk \right)^{\varepsilon/(\varepsilon-1)} \). Firms producing intermediate goods are characterized by linear production technology without capital:

\[
Y_t(k) = A_t N_t(k) - F \text{ if } N_t(k) > F \text{ and 0 otherwise,}
\]

where \( F \) denotes the fixed costs and \( A_t \) represents the technology, where \( a_t = \log A_t \) is assumed to evolve as:

\[
a_t = \rho_a a_{t-1} + \varepsilon_t^a,
\]

with \( 0 \leq \rho_a \leq 1 \) and \( \varepsilon_t^a \sim N \left( 0, \sigma_a^2 \right) \). Firms producing intermediate goods face a downward-sloping demand curve: \( C_t(k) = \left( \frac{P_t(k)}{P_t} \right)^{-\varepsilon} C_t \). They take wages as given and the cost minimization implies the following nominal marginal cost: \( MC_t = W_t/A_t \). The total profit \( D_t \) is given by \( D_t = (1 - (MC_t/P_t) \Delta_t) Y_t \), where \( \Delta_t = \int_0^1 (P_t(k)/P_t)^{-\varepsilon} dk \) is defined as

\[23\]The technology shock can have a unit-root, in which case the output, consumption, and real wages would need to be stochastically detrended to obtain stationary dynamics.
relative price dispersion. Savers, who hold shares in firms, maximize their total value by choosing price $P_t(k)$. The optimal price reads:

$$P_{opt}^t(k) = E_t \sum_{j=0}^{\infty} \frac{\omega^j M_{t,t+j} P_{t+j}^{\varepsilon-1} Y_{t+j}}{E_t \sum_{m=0}^{\infty} \omega^m M_{t,t+m} P_{t+m}^{\varepsilon-1} Y_{t+m}} MC_{t+j}.$$  \hspace{1cm} (23)

The aggregate price index evolves as:

$$P_t^{1-\varepsilon} = (1 - \omega) \left( P_{opt}^t \right)^{1-\varepsilon} + \omega P_{t-1}^{1-\varepsilon},$$ \hspace{1cm} (24)

where $\omega$ represents the fraction of firms being unable to adjust their prices, i.e. Calvo staggered pricing. The Philips curve follows from linearized versions of (23) and (24), for details see standard references, e.g. Woodford (2003).

\textbf{V.C. Equilibrium}

The equilibrium requires that all markets clear. Labor markets clearing implies $N_t = \lambda N_{r,t} + (1 - \lambda) N_{s,t}$, goods market clearing delivers $C_t = Y_t$ and $C_t = \lambda C_{r,t} + (1 - \lambda) C_{s,t}$, and finally asset market clearing implies that $\Theta_{s,t} = \frac{1}{1-\lambda}$ for all $t$. Bonds are in zero net supply.

\textbf{V.D. Key model equations}

The aggregate dynamics are described by the following three equations (derivations and the discussion of steady states are in Appendix E). Lower case letters below denote the log-deviation of a given variable from its steady state value.

\textit{Aggregate Euler equation}

The Euler equation of savers has the usual form:

$$E_t c_{s,t+1} - c_{s,t} = \frac{1}{\gamma} \left( i_t - E_t \pi_{t+1} \right).$$ \hspace{1cm} (25)

The output Euler equation is obtained by aggregating the consumption of savers and rule-of-thumb agents:

$$E_t y_{t+1} - y_t = \frac{1}{\pi^*} \left( i_t - E_t \pi_{t+1} \right) + \chi \left[ E_t a_{t+1} - a_t \right].$$ \hspace{1cm} (26)
where \( y_t \) represents the aggregate output, \( \pi_t = \log P_t/P_{t-1} \) is the inflation and \( \chi = -\frac{1}{\lambda \eta \eta (1+\kappa)} \frac{1+\mu}{1+\mu} \). Estimates of \( \frac{1}{\chi \eta} \) from the aggregate intertemporal optimality condition in Table I are consistently negative. Given that savers are risk averse (\( \gamma > 0 \)), the negative sign can arise through \( \delta < 0 \). The magnitude of the aggregate EIS is largely determined by \( \delta \), which is a function of structural parameters:

\[
\delta = \frac{(\kappa \lambda + 1 - \lambda)(1 + \mu) - \lambda \eta (1 + \kappa)}{(1 - \lambda)(1 + \mu) [\kappa \lambda \eta + \gamma \lambda (1 + \kappa) + 1 - \lambda]},
\]

where \( \kappa \equiv \frac{1-\gamma}{\eta+\gamma} \). In case of full market participation (\( \lambda = 0 \)) \( \delta = 1 \), and the standard IS curve is recovered. Importantly, \( \delta \) can have either sign depending on the combination of structural parameters in the numerator.

**Phillips curve**

Nominal rigidities introduce a trade-off between the output gap \( x_t = y_t - y_t^{*} \) and inflation:

\[
\pi_t = \beta E_t \pi_{t+1} + \psi \vartheta (y_t - y_t^{*}).
\]  

(27)

Natural level of output \( y_t^{*} \), which depends only on the exogenous technology shock \( a_t \), is achieved when prices are flexible, i.e. \( \omega = 0 \) (see Appendix E.3 for the derivation). Unlike most of the recent New-Keynesian model specifications, both the aggregate output equation and the Phillips curve are purely forward-looking. In my setup, the Phillips curve is not influenced by limited participation.

**Monetary policy**

The central bank sets the policy rate according to the following interest rate rule:

\[
i_t = \phi_i i_{t-1} + (1 - \phi_i) [\phi_{\pi} \pi_t + \phi_x (y_t - y_t^{*})].
\]  

(28)

Consistent with the empirical evidence, I assume that the central bank moves the policy rate gradually, which introduces a substantial degree of smoothing captured by \( \phi_i \).

---

24The backward indexation of prices is usually added to the Phillips curve to make inflation more persistent and fit the data, but it has little support in micro data, e.g. (Chari, Kehoe, and McGrattan, 2009).
V.E. Model calibration

Where possible, the choice of parameter values is guided by microeconomic evidence.

Elasticity of labor supply. Chetty, Guren, Manoli, and Weber (2011), argue that the Frisch elasticity of labor supply should be set to 0.75 which implies $\eta = 1.333$. However, the magnitude of labor supply elasticity is not uncontroversial. For this reason, I report the results for a range of values reported in the literature, $\eta \in \{1, 1.333, 2, 4\}$.

Asset market participation. I set $\lambda = 0.7$ which is motivated by the evidence from Vissing-Jorgensen (2002) who classifies 21.75 percent of households as stockholders and 31.40 percent as bondholders, based on the Consumer Expenditure Survey (CEX) for the period 1980-1996. Similarly, Guvenen (2006) argues that $\lambda = 0.7$ represents a lower bound for the fraction of non-participating households. One concern with choosing high values for $\lambda$ is the fact that stock market participation of households has increased significantly in the last decades. I provide a detailed discussion of this issue in Appendix D. To assess the impact of $\lambda$ on my results, I vary its values between 0 and 1.

Rigidities. The elasticity of substitution among intermediate goods is set to $\varepsilon = 6$ which implies the steady-state markup value $\mu = 0.2$, (Eichenbaum and Fisher, 2007). This parameter has a minute effect on $\delta$ for any plausible markup value. The Calvo parameter of $\omega = 0.75$ implies that firms can reset prices once a year.

Risk aversion. I study parameter values ranging from 0.5 through 10. Values of $\gamma$ that are substantially higher than unity are commonly used in the asset pricing literature. In contrast, Chetty (2006) obtains an upper bound for $\gamma$ equal to two by exploiting the labor supply behavior.

Monetary policy. The smoothing parameter $\phi_t$ is set to 0.85, $\phi_x = 0.4$ and $\phi_x = 1.2$. In the standard New-Keynesian model without limited participation, coefficients above one for $\phi_x$ are necessary to ensure the determinacy (Taylor rule). However, in the case of limited participation, it is possible that $0 < \phi_x < 1$ leads to determinate equilibria (Gali, Lopez-Salido, and Valles, 2004; Bilbiie, 2008). In this model, $\pi_t$ is a deviation of inflation from its steady state. Post-war US inflation has a stochastic trend, and Taylor rule estimates that explicitly account for it, indicate that the central bank reacted by more than one-for-one to
the inflation trend, but the coefficient on the deviations from the stochastic trend is indeed less than unity. Here, I abstract from the stochastic inflation target.\footnote{See Cogley and Sbordone (2008) and Coibion and Gorodnichenko (2011) for the analysis of the New Keynesian models with inflation trend.}

Technology shock. The single exogenous shock of the model is the technology shock. I set $\rho^a = 0.3$ and $\sigma_a = 0.01$.

### Table VI: Model parameters

This table collects the parameter values used to calibrate the model. All parameters refer to quarterly frequency.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td><strong>Preference parameters</strong></td>
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<td>Discount factor</td>
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</tr>
<tr>
<td>Risk aversion</td>
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<tr>
<td>Labor elasticity parameter</td>
<td>$\eta$ 1.33</td>
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<tr>
<td><strong>Price rigidities</strong></td>
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<tr>
<td>Steady state markup</td>
<td>$\mu$ 0.2</td>
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<tr>
<td>Calvo parameter</td>
<td>$\omega$ 0.75</td>
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<tr>
<td><strong>Limited participation</strong></td>
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<tr>
<td>Degree of limited participation</td>
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<td>Interest rate smoothing</td>
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<td>Inflation sensitivity</td>
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<td>Persistence of technology shock</td>
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</tr>
<tr>
<td>Volatility of technology shock</td>
<td>$\sigma_a$ 0.01</td>
</tr>
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</table>

### V.F. Model solution

Equations (26)–(28) together with the exogenous technology shock determine the law of motion of the endogenous variables in the model, which is given by:

$$z_t = \Gamma z_{t-1} + \Xi a_t,$$

where $z_t = [\pi_t, y_t, i_t, y^*_t]'$. More details about the model solution are provided in Appendix E.4. Table VI collects all the parameters. Numerically solving the model for these parameter values gives the following dynamics:
\[
\Gamma = \begin{bmatrix}
0 & 0 & 0.9570 & 0 \\
0 & 0 & 0.2224 & 0 \\
0 & 0 & 0.9475 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad \Xi = \begin{bmatrix}
0.3335 \\
1.2423 \\
0.1087 \\
0.7496 \\
\end{bmatrix}.
\]

The single most important observation about (30) is that the dynamics of endogenous variables are persistent. This is surprising given that the Phillips curve and the aggregate Euler equation are purely forward-looking. A full participation model does not generate such a persistence. Thus, limited market participation, could replace habits and backward indexation, both of which are typically relied upon but lack strong support in the data.

VI. Model discussion

This section inspects the key mechanism in the model that can switch the sign of the aggregate EIS. Subsequently, it discusses the link between the aggregate consumption and the consumption of savers. Finally, it shows how the negative sign of the EIS helps reconcile the consumption-based models with the data.

VI.A. Justifying \( \tilde{r}_t \) as a proxy for the real rate

I simulate the model outlined above and construct a term structure of yields implied by this model. The details of the New-Keynesian term structure model and of the simulation exercise are provided in Appendix E.5. The simulation shows that \( \tilde{r}_t \), which is the negative of the slope of the simulated term structure, closely tracks the real rate constructed as \( i_t - E_t \pi_{t+1} \), where an AR(1) model for realized inflation is used to compute \( E_t \pi_{t+1} \). Median correlation is around 0.7, see Figure A.2 in Appendix E.5. As can be seen from the law of motion of endogenous variables given by (30), inflation is more persistent than the output. Therefore, the slope captures the variation in the real rate.

VI.B. What makes \( \delta \) negative?

Figure 6 plots the values of the aggregate EIS given by \( \frac{1}{\pi_\gamma} \) for a range of structural parameters \( \gamma, \lambda, \) and \( \eta \) together with its estimate from the total consumption data in the period 1875-2009 (Panel A of Table I). Parameter values for the degree of limited asset market
Figure 6: Parameterizations of the output Euler equation.

Panels a – d plot the parameter values for $\frac{1}{\delta \gamma}$ in the aggregate output Euler equation given by (26). Three structural parameters are varied. First, the coefficient of relative risk aversion $\gamma$ takes values between 0.5 and 10. Each panel represents different values of $\gamma$. Second, the Frisch elasticity of labor supply is varied between 0.25 and 1 which implies parameter values for $\eta$ between 1 and 4. Third, the share of rule-of-thumb consumers $\lambda$ varies between 0 and 1 (x-axis). The red dashed line represents the estimate of $\frac{1}{\delta \gamma}$ based on the annual data for total consumption and interest rates in the period 1875-2009, see the first column of Panel A in Table 1.

participation $\lambda$, the risk aversion parameter $\gamma$ and the labor supply elasticity $\eta$, motivated above are consistent with the estimate of $\frac{1}{\delta \gamma}$.

Conditional on a moderate degree of risk aversion $\gamma = 2$, Figure 7 displays two key ingredients that drive the sign of $\delta$: (i) inelastic labor supply as measured by $\eta$ and, (ii) limited market participation. $\delta$ changes its sign at the threshold value of market participation defined as $\hat{\lambda}$.
The figure shows the dependence of $\delta$ on the elasticity of labor supply parameter $\eta$ and the asset market participation parameter $\lambda$. Expression for $\delta$ is given by (27). Note that $\eta$ is the inverse of the Frisch elasticity of labor supply. Hence, the elasticity of labor supply is varied between 0.25 and 1. The risk aversion parameter is $\gamma = 2$ and the steady-state markup is $\mu = 0.2$.

Appendix E.2 derives $\hat{\lambda}$ in terms of structural parameters. For the whole range of labor supply elasticities considered, the sign of $\delta$ switches for non-participation levels between 0.25 and 0.55.

The intuition for the negative aggregate EIS is as follows. A positive exogenous technology shock increases the natural level of output $y_t^*$ and thus changes the output gap $x_t$. Depending on the configuration of the parameter values, in particular those in the monetary policy reaction function, the increase in $y_t^*$ translates through the monetary policy reaction into a positive or negative shock to the real interest rate. In the full participation case, a positive shock to the real interest rate reduces the demand today as it induces the agents to seize

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26 The aggregate EIS has a discontinuity point at $\hat{\lambda}$. For this reason, the negative sign of the EIS cannot be achieved by introducing agents with different non-zero EIS (Guvenen, 2006). To obtain the negative sign, a fraction of households needs to be excluded from all asset markets.
better investment opportunities and substitute today’s consumption into the future. In the limited participation model, only savers react to the real rate shock and substitute consumption intertemporally. Intertemporal choices of savers shift the labor supply curve (derived in Appendix E.2):

$$w_t = \frac{\eta}{\eta\lambda\kappa + (1 - \lambda)} n_t + \frac{(1 - \lambda)\gamma}{\eta\lambda\kappa + (1 - \lambda)} c_{s,t}. \quad (31)$$

For the parameter values given in Table VI, \(\frac{(1 - \lambda)\gamma}{\eta\lambda\kappa + (1 - \lambda)} > 0\), i.e. a positive shock to the real interest rate reduces \(c_{s,t}\) and thus the real wage \(w_t\). The lower real wage further reduces the output because rule-of-thumb agents consume their wages. So far, this is the strengthened standard logic by which a positive shock to the real interest rate induces a fall in output. To invert this logic, one needs to consider the dividend income of savers. As can be seen directly from the budget constraint, the consumption of savers has two components, wages and dividends:

$$c_{s,t} = w_t + n_{s,t} + \frac{1}{1 - \lambda} d_t, \quad (32)$$

where real profits \(d_t\), accruing only to savers, are given by:

$$d_t = \frac{\mu}{1 + \mu} y_t - w_t + a_t. \quad (33)$$

If the fall in the aggregate output \(y_t\) brought about by the interest rate increase is paired with a correspondingly larger decrease in wages, it leads to a positive income effect for savers (equation (33)). The positive income effect generates additional labor demand and the real wage increases. In equilibrium, both the aggregate output and real wages increase. Hence, a higher real interest rate coincides with an increase in aggregate output induced by the positive income effect for savers. This mechanism implies that firm markups are counter-cyclical which is consistent with the empirical evidence (Rotemberg and Woodford, 1999).

VI.C. Consumption of savers and rule-of-thumb consumers

The negative \(\delta\) implies that the consumption growth of rule-of-thumb consumers is negatively correlated with the variation in the real interest rate. This link arises as a result
of intertemporal choices of savers and does not imply that rule-of-thumb consumers have
a negative EIS. Consistent with this prediction, using UK household-level data, Attanasio,
Banks, and Tanner (2002) report a negative relationship for non-shareholders (Panel C of
Table II in their paper).

The model implies that the consumption of savers loads negatively on the aggregate con-
sumption and positively on technology shocks, which can be seen in equation (34) below:

\[ c_{s,t} = \delta y_t + \zeta a_t, \tag{34} \]

where \( \zeta > 0 \) for \( \lambda > 0 \) and is increasing in \( \lambda \):

\[ \zeta = \frac{\lambda \eta(1 + \kappa)}{(1 - \lambda)[\kappa \lambda \eta - \lambda + \gamma \lambda(1 + \kappa) + 1]}, \]

(derivation of (34) and \( \zeta \) is in Appendix E.2). Depending on the magnitude of \( \delta \) and \( \zeta \),
the consumption growth of savers is more volatile than the aggregate consumption growth.
For \( \delta < 0 \) and moderate risk aversion parameter values, i.e. \( \gamma \ll 10 \), we have that \( \zeta > 1 \)
as illustrated in Panels a–d of Figure 8. With larger \( \lambda \), a smaller fraction of savers holds
all stocks in the economy and is thus more exposed to technology shocks.\(^{27}\) Equation (34)
does not imply that the consumption of savers is negatively correlated with the aggregate
consumption in the data. The technology shock \( a_t \) is positively correlated with \( y_t \) and can
induce a positive correlation in the data despite negative \( \delta \). Equation (34) also helps explain
why the aggregate consumption appears too smooth when compared to substantial volatility
of returns on risky assets. The model suggests that it is not driven by efficient risk sharing
but rather by intertemporal choices of savers which induce the negative correlation of the
consumption of savers with the total consumption.

VI.D. Why does the standard consumption Euler equation fail?

The Euler equation for aggregate consumption fails to replicate the variation and the level of
interest rates in the data (Weil, 1989). Recently, Canzoneri, Cumby, and Diba (2007) show
that a broad range of preference specifications in consumption-based models imply a real

\(^{27}\)Setting \( \lambda = 0 \) implies \( \delta = 1 \) and \( \zeta = 0 \), which restores the representative agent setup.
Figure 8: Exposure of consumption of savers to the technology shocks.

Panels \(a - d\) plot the parameter values for \(\zeta\) in the equation linking the consumption of savers to the total output given by (34). Three structural parameters are varied. First, the coefficient of relative risk aversion \(\gamma\) takes values between 0.5 and 10. Each panel represents different values of \(\gamma\). Second, the Frisch elasticity of labor supply is varied between 0.25 and 1 which implies parameter values for \(\eta\) between 1 and 4. Third, the share of rule-of-thumb consumers \(\lambda\) varies between 0 and 1 (x-axis). All plotted values of \(\zeta\) are obtained for \(\delta < 0\).

short rate that is either negatively correlated with the observed ex-post real interest rate or it is extremely volatile and uncorrelated with the observed one.

The model outlined in this paper offers one way to reconcile the mismatch between the consumption-based models and observed short-term yield dynamics. The log of nominal pricing kernel implied by the model reads:

\[
mt,t+1 = \log(\beta) - \gamma \Delta c_{s,t+1} - \pi_{t+1},
\]  

(35)
where $\Delta c_{s,t+1} = \log C_{s,t+1} - \log C_{s,t}$ denotes the consumption growth of savers. Using the differenced version of (34) and the market clearing condition $y_t = c_t$, rewrite (35) as:

$$m_{t,t+1} = \log (\beta) - \gamma \delta \Delta c_{t+1} - \gamma \zeta \Delta a_{t+1} - \pi_{t+1}. \quad (36)$$

The comparison of (35) with (36) provides the intuition for the mismatch between the observed short term interest rates and the pricing kernel. For $\delta < 0$, which is empirically the case, the aggregate consumption enters the pricing kernel with the opposite sign compared to the consumption of savers who are marginal bond investors. The presence of rule-of-thumb consumers introduces a wedge between the aggregate Euler equation and the Euler equation of bond market participants (savers). Given that the consumption of savers is not directly observable, when equating the aggregate consumption Euler equation with observed interest rates, one needs to use $\frac{1}{\delta \gamma}$.

The literature has proposed solutions to the empirical failure of the consumption Euler equation that are largely risk-based and/or depend on the specifics of monetary policy (Gallmeyer, Hollifield, Palomino, and Zin, 2007; Atkeson and Kehoe, 2008; Reynard and Schabert, 2012). However, the negative relationship between real interest rates and consumption growth appears to be structural, hence exists irrespective of monetary policy. Indeed, the results in Table I indicate that the relationship exists across periods marked by very different monetary policy regimes.

### VII. Is the negative EIS consistent with long-run risk models?

Empirical results in previous sections imply that the expected consumption growth varies in a persistent way. This section shows that the persistent component of expected consumption growth rate extracted from the Euler equation with the negative EIS has asset pricing implications that are consistent with models featuring long-run cash flow risk (Bansal and Yaron, 2004; Hansen, Heaton, and Li, 2008).

In the standard long-run risk model (Bansal and Yaron, 2004), both the consumption and dividend growth, $g_t$ and $g_{d,t}$ contain the persistent component $x_t$ and their innovations have stochastic volatility $\sigma_t^2$. Further consider Epstein-Zin preferences with $\gamma$ and $\sigma$ denoting the

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28 Models with time-varying risk premia add the stochastic second-order terms into the Euler equation while keeping the first-order relationship unchanged. This is done either through time-varying risk aversion (e.g. habits) or stochastic volatility (e.g. long-run risk models).
risk aversion and the EIS, respectively. The essence of the model are two equations given by (37)–(38). Both, the short term real rate \( r_t \) and the log price-dividend ratio \( z_{m,t} \) are affine functions of \( x_t \) and \( \sigma^2_t \):

\[
\begin{align*}
    r_t &= A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma^2_t, \\
    z_{m,t} &= A_{0,m} + A_{1,m}x_t + A_{2,m}\sigma^2_t,
\end{align*}
\]

(37)–(38)

where \( A_{1,f} = \frac{1}{\gamma} \).\(^{29}\) Hence, the expected consumption growth \( x_t \) has a first-order effect on the real rate \( r_t \). \( \sigma^2_t \) represents the fluctuations in risk and is the key determinant of risky asset valuations represented by \( z_{m,t} \). The long-run risk literature restricts its attention to positive values of preference parameters. Therefore, both the EIS and the risk aversion parameter have to be greater than unity to get plausible asset pricing implications, i.e. price-dividend ratio increasing in \( x_t \) and decreasing in \( \sigma^2_t \). However, it can readily be seen from \( \theta = \frac{1-\gamma}{1-\gamma} \) that the negative value of EIS also implies \( \theta < 0 \) and thus preserves the desirable asset pricing properties of long-run risk models.

To empirically evaluate the asset pricing properties of \( x_t \), I first extract the persistent component of consumption growth. From (37), we have:

\[
x_t = -\sigma A_{0,f} + \sigma r_t - \sigma A_{2,f}\sigma^2_t.
\]

(39)

The affine relationship given in (39) is implemented via OLS:

\[
\Delta c_{t+1} = \delta_0 + \frac{\delta_1}{-0.81 [-5.23]} \hat{r}_t + \frac{\delta_2}{0.005 [1.32]} \hat{\sigma}^2_t + \varepsilon_{t+1} \hat{R}^2 = 0.11,
\]

(40)

where \( \hat{\sigma}^2_t \) is an empirical proxy for the consumption volatility: \( \hat{\sigma}^2_t = \log \sum_{i=1}^5 |\eta_{t-i}| \) with \( \eta_t \) being the innovation in consumption growth obtained from an AR(1) estimated on \( \Delta c_t \) recursively using the window size of 30 years. The regression results show that the volatility of consumption growth is unrelated to future consumption growth. Contrary to the argument that stochastic volatility induces a downward bias in the EIS estimates from the linearized Euler equation, the estimated \( \delta_1 \) in (40) shows that this is not the case. The extracted persistent component denoted by \( \hat{x}_t \), is the fitted value from a restricted version of (40) with

\(^{29}\)The solution coefficients \( A_1, A_2, A_{0,m}, A_{1,m}, A_{2,m}, A_{0,f}, A_{1,f}, A_{2,f} \) are defined in Bansal and Yaron (2004) and are not repeated here for brevity.
\[ \delta_2 = 0. \] This is justified given the insignificant estimate of \( \delta_2 \) in the unrestricted version. Figure 9 plots \( \hat{x}_t \) together with the realized consumption growth. Consistent with the basic

**Figure 9: Time-varying expected consumption growth, 1875-2009**

The figure plots the annual consumption growth together with the estimated persistent component \( \hat{x}_t \), which is obtained as a fitted value from the regression of \( \Delta \hat{c}_{t+1} \) on \( \hat{r}_t \). Grey areas represent the recessions. The data are annual and the sample period is 1875 through 2009.

intuition, the expected consumption growth explains a fairly small portion (\( \bar{R}^2 = 0.11 \)) of the overall variation in consumption growth.

One of the key implications of the long-run risk model is that \( x_t \) can be predicted by the price-dividend ratio which is based on the relationship given by (38). However, establishing this link empirically has been a challenge. I use \( \hat{x}_t \) to estimate (38):

\[
\begin{align*}
  z_{m,t} &= \gamma_0 + \gamma_1 \hat{x}_t + \gamma_2 \hat{x}_t^2 + \epsilon_t \quad R^2 = 0.57.
\end{align*}
\]
In both regressions, (40) and (41), Newey-West-adjusted $t$-statistics with three lags are reported in parentheses. Due to data limitations, regression (41) is estimated on the sample 1926-2009 using annual data. Importantly, both $\hat{x}_t$ and $\hat{\sigma}_t^2$ are statistically significant and have the expected signs. Higher expected consumption growth rate increases the price-dividend ratio while an increase in consumption volatility depresses it. This result provides supportive evidence for the validity of $\hat{x}_t$ as a persistent component of consumption growth.

The key takeaway from this exercise is that additional moments in the form of equity valuation ratios are consistent with the negative value of the EIS. However, this is not to say that working with the negative EIS is appropriate. Optimally, one should use the EIS close to unity but apply it to the consumption data aggregated only over households actively participating in asset markets as has been done in Malloy, Moskowitz, and Vissing-Jorgensen (2009). In other words, the discrepancy between the aggregate EIS estimates and the micro evidence is largely a result of aggregation.

**VIII. Conclusions**

This paper shows that the aggregate elasticity of intertemporal substitution is negative around -0.5. This empirical finding might seem counterintuitive at first but it is sensible if one considers that a substantial fraction of aggregate consumption is attributed to agents whose financial wealth is negligible. These agents have little incentive to participate in asset markets and largely consume their wage income. This single friction is responsible for the negative sign of the aggregate EIS and helps explain several empirical puzzles in the asset pricing and monetary policy literature.

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30 In constructing the price-dividend ratio $z_{m,t}$, I follow the literature and infer the dividend payments from monthly returns on the value-weighted portfolio of all NYSE, Amex, and NASDAQ stocks. Monthly dividend distributions are reinvested in the three-month T-bill. These data are available starting in 1926 which determines the start of the sample for (41). Correspondingly, $\hat{x}_t$ used in (41) is constructed using the data from the 1926-2009 sample.

31 Note that $\hat{x}_t$ can be extracted in real-time as $\tilde{r}_t$ is constructed from asset prices. Estimating (40) recursively gives a stable and significant estimate of $\delta_1$. 

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References


A. A two-component model for inflation

The US inflation can be well described by a two-component model with stochastic volatility (UC-SV), (Stock and Watson, 2007):

\[
\pi_t = \tau_t + \eta_t \quad \text{with} \quad \eta_t = \sigma_\eta \zeta_{\eta,t} \\
\tau_t = \tau_{t-1} + \varepsilon_t \quad \text{with} \quad \varepsilon_t = \sigma_\varepsilon \zeta_{\varepsilon,t} \\
\ln \sigma^2_{\eta,t} = \ln \sigma^2_{\eta,t-1} + \nu_{\eta,t} \\
\ln \sigma^2_{\varepsilon,t} = \ln \sigma^2_{\varepsilon,t-1} + \nu_{\varepsilon,t},
\]

where \( \zeta_t = (\zeta_{\eta,t}, \zeta_{\varepsilon,t}) \) is i.i.d. \( N(0, I_2) \), \( \nu_{\eta,t} \sim N(0, \gamma_1) \) and \( \nu_{\varepsilon,t} \sim N(0, \gamma_2) \). All four innovations are mutually independent.

The simulation exercise replicates the linearized Euler equation given by (8), falsely assuming constant volatility. In the first step, I construct the nominal interest rate by simulating the real rate and inflation. I simulate realized inflation from the UC-SV model above and set \( \gamma_1 = 0.2 \) and \( \gamma_2 = 0.5 \) to reflect the higher volatility fluctuations in the transitory component of inflation, observed in the early part of the 1875-2009 sample. I set \( T=135 \) to be consistent with the sample size used in this paper. I simulate the real rate as an AR(1) process, assuming the following: \( \phi_r = 0.75, \sigma_r = 1 \) and \( \bar{r} = 1.9 \), where the data are annual percentages. For consumption growth, I assume \( \sigma_c = 3.2 \) and EIS=1. The ex-ante real rate is obtained by linearly projecting the ex-post real rate observed at \( t+1 \) on past inflation and the nominal interest rate. I simulate the model 1000 times and report median values.

B. Data

This section details the data series used to estimate the consumption and output Euler equations.

Yield data. I study US yields in the period January 1875 through December 2009 unless otherwise stated. The raw yield data are monthly and are obtained from the Global Financial Data (GFD) database. I consider the ten-year constant maturity yield (series IGUSA10D) as long term yield. To construct the short term yield for the whole sample period, I combine the three-month AA non-financial commercial paper yield (series IPUSAC3D) (1875-1933) with the three-month Treasury bill (1934-2009). The reason for using the three-month commercial paper in the earlier part of the sample is the data availability. The ten-year constant maturity yield is interpolated from available long term non-inflation-indexed Treasury securities. GFD itself combines various data sources such as the National Monetary Statistics from the Federal Reserve to obtain the ten-year CMT yield. \( \tilde{r}_t \) is an average of monthly observations of \(-\text{slope}_t\) within each year.

Consumption and output data (1875-2009). The consumption and output data are constructed by Robert Barro and Jose Ursua and downloaded from their website.\(^{32}\) These data contain the total real per capita consumption/output index without further split into durable and non-durable consumption. Barro and Ursua compile the consumption data from various sources: For the period 1869-1899, the data are from Rhode (2002). For the 1900-1928 period, consumption data are taken from Lebergott (1996) and the period 1929-2009 is covered by data from the Bureau of Economic Analysis. The output data are sourced from Balke and Gordon (1989) for the period 1869-1928 and from BEA for the 1928-2009 period.

\(^{32}\)The dataset is available at [http://rbarro.com/data-sets/](http://rbarro.com/data-sets/).
Consumption data (1929-2009). The consumption data for this period are obtained from the Bureau of Economic Analysis (US Dept. of Commerce), Tables 2.3.4 and 2.3.5. Non-durable goods and services as well as durables are deflated by their corresponding price indexes (2005=100). Population data are obtained from the Bureau of Labor Statistics for the period 1948-2009. In the period 1929-1947, the population over 16 years is obtained from the Population Estimates Program of the U.S. Census Bureau\footnote{http://www.census.gov/popest/data/national/totals/pre-1980/tables/popclockest.txt.} by dividing the whole population by 1.5. This ratio of population over 16 years to the total population has been stable in the 1950s.

Output and non-Treasury interest rate data (1869-1983). The real GNP data for the period 1869-1983 are from Table I, Appendix B in Balke and Gordon (1986). The data in Balke and Gordon (1986) are assembled from various sources, see Notes on Section 1 of their paper for more details. The proxy for the real rate \( \tilde{r}_t \) is computed as a difference between the commercial paper rate and yield on corporate bonds. The commercial paper has a maturity of six-months in most of the sample. Yields on corporate bonds are Baa-rated (Moodys) corporate bond yields (1919-1983) and railroad bond yields (1869-1918).

B.1. International data

Consumption and output data (1950-2009). Annual consumption and output data for Canada, Germany, Netherlands, Switzerland and the United Kingdom are obtained from the Barro and Ursua’s data set.

Yield data. All yield data are from the GFD database, see their data descriptions for the original sources. For all countries, short-term nominal yield is represented by a three-month Treasury bill. The only exception is Switzerland in the period 1950-1979 where the commercial paper rate is used instead. The long-term yield is represented by a ten-year Treasury bond yield for Canada, Germany and Netherlands. For the UK, I use the 20-year government bond and the confederate long-term bond for Switzerland. \( \tilde{r}_t \) is a difference between the three-month Treasury bill yield and the long-term bond yield averaged over monthly observations in a given year.

C. EIS estimation: non-linear GMM and capital taxes

There are at least two reasons for estimating the log-linearized version of the Euler equation as opposed to estimating equation (6) via non-linear GMM (Hansen and Singleton, 1982). First, it is likely that consumption is measured with error which, if present, causes a more severe bias in the non-linear estimation. This is especially relevant for the consumption data in the period before 1947. Romer (1986) shows that macro data before World War II are less accurate and are constructed using a different methodology than the post-World War II data. Second, \( \tilde{r}_t \) is not informative about the unconditional mean of the real rate. Given that I want to avoid the use of the ex-post real interest rate in the estimation, the non-linear estimation is not feasible.

I estimate the aggregate EIS from the pre-tax data. Applying the capital income tax rate to \( \tilde{r}_t \) directly is not appropriate because this measure does not represent the real interest income but rather its variation. Additionally, Mishkin (1981) argues that the effective tax rate varies substantially across households which makes it difficult to know the appropriate tax rate at macro
level. Therefore, I do not adjust \( \tilde{r}_t \) for capital income taxes which might bias the EIS estimates toward zero. Hence, my estimates of the EIS represent a lower bound on its magnitude.\(^{34}\)

D. Empirical evidence on asset market participation

Wolff (2004) reports that the fraction of households with direct holdings of stocks increased from 13.10 percent in 1989 to 21.30 percent in 2001. Moreover, the share of stock-owning households including indirect holdings through mutual funds or pension accounts rose even more from 31.70 to 51.90 percent in the same period.

However, these numbers might misrepresent the degree of asset market participation for the purpose of this model. In the model, \( \lambda \) represents the fraction of households that actively participate in asset markets to smooth consumption which requires substantial liquid wealth. According to Wolff (2004), only 35 percent of households owned USD 10 000 (in 1995 dollars) or more of stocks in 2001. Using the Survey of Consumer Finances (SCF), Poterba and Samwick (1995) report that 24.60 and 29.30 percent of households had direct and indirect stock holdings of more than USD 2 000 (in 1992 dollars) in 1983 and 1992, respectively.

I use the SCF data to show that the financial wealth of low income households has not risen in real terms in the last two decades. Panel a of Figure A.1 reports the fraction of stockholders by income quintiles for the period 1989-2007, and Panel b reports the corresponding median values of stock holdings adjusted for inflation. The important observation is that while the degree of participation has increased, the value of stock holdings for the lowest income quintile has remained virtually unchanged in the last two decades. In line with this evidence, Carroll (2000) reports that according to the SCF, the lowest 66 percentiles of the US population by wealth have liquid assets of only 10 percent of their annual wage income. Overall, these numbers indicate that \( \lambda = 0.7 \) is justified. The likely reason for the limited asset market participation are various forms of costs attached to it. Section 5 in Vissing-Jorgensen (2004) provides a thorough discussion of participation costs and their ability to explain the non-participation of a substantial portion of households.\(^{35}\)

E. Model

This section provides details on some derivations of the model outlined in Section V.

E.1. Steady state

The steady state risk-less interest rate is \( R = \frac{1}{\beta} \), the net mark-up is \( \mu = \frac{1}{\varepsilon - 1} \) and define \( F_Y = F/Y \). Fixed costs are introduced for convenience. Following Bilbiie (2008), I set the fixed costs share \( F_Y = \mu \) which implies that profits are zero in steady-state. The share of the real wage on the total output is: \( WN/PY = (1 + F_Y)/(1 + \mu) \), share of profit is \( D/Y = (\mu - F)/(1 + \mu) \). Assume that both savers and rule-of-thumb consumers have the same hours worked in the steady state: \( N_s = N_r = N \). It follows:

\(^{34}\)For illustration, the average marginal tax rate for capital income was slightly below 5% before World-War II, see Table 2 in Barro and Sahasakul (1983). Since the mid-1940s, the rate has been increasing to a level around 30%.

Figure A.1: Stock market participation and holdings by income, 1989-2007

Panel a plots the percentiles of households with direct or indirect stock holdings split by their income. Two lower quintiles and two upper deciles are displayed for the period 1989-2007. Panel b plots the median value of stock holdings of households by income. The values are in 2007 dollars and thus directly comparable across time. The data are obtained from the Survey of Consumer Finance.

\[ \frac{C_s}{Y} = \frac{1 + F_Y}{1 + \mu} + \Theta \frac{\mu - F_Y}{1 + \mu} \]

using the assumption \( F_Y = \mu \), we have:

\[ \frac{C_s}{Y} = 1 \]
\[ \frac{C_r}{Y} = \frac{1 + F_Y}{1 + \mu} = 1, \]

Hence, both types of agents have the same consumption in the steady state: \( C_{s,t} = C_{r,t} = \left[ \frac{N_sP}{W} \right]^{-1/\gamma} \).

E.2. Aggregate Euler equation

The aggregate Euler equation is derived by manipulating the the equilibrium conditions and budget constraints of both types of agents. First, I provide the derivation of the threshold level of
market participation $\hat{\lambda}$ which is the discontinuity point at where the sign of the output Euler equation switches. Combining the budget constraint of rule-of-thumb consumers and their first order condition we get:

$$N_{r,t}^{\eta} = \left( \frac{W_t}{F_t} N_{r,t}^{\eta} \right)^{-\gamma} \frac{W_t}{F_t}$$

$$N_{r,t}^{\eta+\gamma} = \left( \frac{W_t}{F_t} \right)^{1-\gamma}.$$

Taking logs and rearranging yields:

$$n_{r,t} = 1 - \gamma \frac{\eta}{\eta + \gamma} w_t. \quad (A.1)$$

To simplify the notation, define $\kappa \equiv 1 - \gamma \frac{\eta}{\eta + \gamma}$. From the budget constraint of rule-of-thumb consumers we have: $c_{r,t} = (1 + \kappa)w_t$. Total consumption $c_t$ and hours worked $n_t$ are defined as:

$$c_t = \lambda c_{r,t} + (1 - \lambda) c_{s,t} \quad (A.2)$$

$$n_t = \lambda n_{r,t} + (1 - \lambda) n_{s,t}. \quad (A.3)$$

Combining (A.1) and (A.3) with the labor supply of savers, we obtain the expression for the wage:

$$w_t = \frac{(1 - \lambda) \gamma}{\eta \lambda \kappa + (1 - \lambda)} c_{s,t} + \frac{\eta}{\eta \lambda \kappa + (1 - \lambda)} n_t. \quad (A.4)$$

Plugging the consumption of rule-of-thumb consumers into (A.2) and using (A.4) yields:

$$c_t = \frac{[(1 - \lambda) [\gamma \lambda (1 + \kappa) + (\eta \lambda \kappa + (1 - \lambda))]]}{\eta \lambda \kappa + (1 - \lambda)} c_{s,t} + \frac{\lambda \eta (1 + \kappa)}{\eta \lambda \kappa + (1 - \lambda)} n_t. \quad (A.5)$$

Expressing (A.5) in terms of output gives:

$$c_t = \frac{[(1 - \lambda) [\gamma \lambda (1 + \kappa) + (\eta \lambda \kappa + (1 - \lambda))]}{\eta \lambda \kappa + (1 - \lambda)} c_{s,t} + \frac{\lambda \eta (1 + \kappa)}{\eta \lambda \kappa + (1 - \lambda)} n_t - \frac{\lambda \eta (1 + \kappa)}{\eta \lambda \kappa + (1 - \lambda)} a_t. \quad (A.6)$$

The differenced version of equation (A.6) is similar to the regression used in Campbell and Mankiw (1989) to estimate the fraction of rule-of-thumb households. The co-movement of consumption and output arises endogenously in the model. However, $\frac{\lambda \eta (1 + \kappa)}{\eta \lambda \kappa + (1 - \lambda)} \frac{1}{1 + \mu}$ is not linear in $\lambda$, therefore linear regressions yield biased estimates of the fraction of rule-of-thumb households. The output Euler equation changes its sign when $\frac{\partial c_t}{\partial y_t} > 1$, therefore the threshold reads:

$$1 = \frac{\lambda \eta (1 + \kappa)}{\eta \lambda \kappa + (1 - \lambda)} \frac{1}{1 + \mu}$$

$$\lambda \eta (1 + \kappa) = (1 + \mu) \eta \lambda \kappa + (1 - \lambda) (1 + \mu)$$

$$\hat{\lambda} = \frac{1 + \mu}{1 + \mu - (1 + \mu) \eta \kappa + \eta (1 + \kappa)}$$

$$= \frac{1}{1 + \frac{\eta (1 - \eta \mu)}{1 + \mu}}.$$

The risk aversion $\gamma$ influences the threshold value $\hat{\lambda}$ in a non-linear way through $\kappa$. Rearrange (A.6) and impose the clearing condition $c_t = y_t$ to obtain:
\[ c_{s,t} = \frac{(\eta \lambda \kappa + 1 - \lambda) (1 + \mu) - \lambda \eta (1 + \kappa)}{(1 - \lambda) (1 + \mu) [\kappa \lambda \eta + \gamma \lambda (1 + \kappa) + 1 - \lambda]} y_t + \frac{\lambda \eta (1 + \kappa)}{(1 - \lambda) [\kappa \lambda \eta - \lambda + \gamma \lambda (1 + \kappa) + 1]} a_t. \quad (A.7) \]

The Euler equation of savers reads:

\[ E_t c_{s,t+1} - c_{s,t} = \frac{1}{\gamma} (i_t - E_t \pi_{t+1}), \quad (A.8) \]

substituting (A.7) into the Euler equation of savers obtains the aggregate Euler equation:

\[ E_t y_{t+1} - y_t = \frac{1}{\delta \gamma} (i_t - E_t \pi_{t+1}) - \frac{1}{\lambda \eta (1 + \kappa)} [E_t a_{t+1} - a_t] \quad (A.9) \]

\[ E_t y_{t+1} - y_t = \frac{1}{\delta \gamma} (i_t - E_t \pi_{t+1}) + \chi [E_t a_{t+1} - a_t], \quad (A.10) \]

where \( \chi = -\frac{1}{\lambda \eta (1 + \kappa)} - \frac{1}{\mu} \).

E.3. Natural real rate

I start by deriving the relationship between the real wage and total output. Combining (A.7) with the production function given by \( y_t = (1 + \mu) a_t + (1 + \mu) n_t \) and the labor supply of savers given by \( \eta n_{s,t} = w_t - \gamma c_{s,t} \) and rearranging obtains:

\[ w_t = \eta + (1 + \mu) (1 - \lambda) \gamma \delta \frac{y_t}{(1 + \mu) (1 - \lambda + \lambda \eta \kappa)} + \frac{(1 - \lambda) \gamma \zeta - \eta}{1 - \lambda + \lambda \eta \kappa} a_t. \quad (A.11) \]

The New Keynesian Phillips curve reads:

\[ \pi_t = \beta E_t \pi_{t+1} + \psi mc_t, \quad (A.12) \]

with \( \psi = \frac{(1 - \omega \beta)(1 - \omega)}{\omega} \). Real marginal costs can be expressed as \( mc_t = w_t - a_t \). Using (A.11), \( mc_t \) can be expressed as:

\[ mc_t = \vartheta y_t + (\nu - 1) a_t, \quad (A.13) \]

therefore:

\[ \pi_t = \beta E_t \pi_{t+1} + \psi [\vartheta y_t + (\nu - 1) a_t]. \quad (A.14) \]

The natural level of output \( y^*_t \) is obtained from (A.14) by setting the inflation to zero:

\[ y^*_t = \frac{1}{\vartheta} (1 - \nu) a_t, \quad (A.15) \]

the Phillips curve can be rewritten in terms of output gap \( x_t = y_t - y^*_t \):

\[ \pi_t = \beta E_t \pi_{t+1} + \psi \vartheta x_t. \quad (A.16) \]

From the IS curve given by (A.10) evaluated at zero inflation one obtains the natural level of the real interest rate:
\[ r^*_t = \delta \gamma \left( \frac{1}{\theta}(1 - \nu) - \chi \right) [E_t a_{t+1} - a_t]. \] (A.17)

The IS curve in terms of output gap reads:

\[ E_t x_{t+1} - x_t = \frac{1}{\delta \gamma} (i_t - E_t \pi_{t+1} - r^*_t) \] (A.18)

### E.4. Model solution

Collect variables into a vector: \( z_t = [\pi_t, y_t, i_t, y^*_t]' \). The equilibrium conditions are collected in matrix form:

\[
\begin{bmatrix}
1 & -\psi \vartheta & 0 & \psi \vartheta \\
0 & 1 & \delta \gamma & 0 \\
-(1 - \phi_i) \phi_x & -(1 - \phi_i) \phi_x & 1 & (1 - \phi_i) \phi_x \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
z_t \end{bmatrix} = \begin{bmatrix}
\beta & 0 & 0 & 0 \\
\frac{1}{\delta \gamma} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
E_t z_{t+1} + 0 & 0 & 0 & 0 \\
0 & 0 & \phi_i & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
z_{t-1} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
-\chi(\rho_a - 1) \\
\frac{1}{\theta}(1 - \nu) \\
\end{bmatrix}
a_t.
\] (A.19)

In a compact form:

\[ B_1 z_t = AE_t z_{t+1} + B_2 z_{t-1} + C a_t, \] (A.21)

where matrices \( A, B_1, B_2, C \) are implicitly defined by comparing (A.20) with (A.21). The rational expectations equilibrium follows:

\[ z_t = \Gamma z_{t-1} + \Xi a_t. \] (A.22)

The model is solved by the forward method proposed in Cho and Moreno (2011). For convenience, I define \( X_t = (z'_t, a_t)' \) and rewrite (A.22) in a compact form:

\[ X_t = \varpi X_{t-1} + \Psi \epsilon^s_t, \] (A.23)

where \( \varpi = \begin{bmatrix}
\Gamma & \Xi \\
0_{1 \times 4} & \rho_a \\
\end{bmatrix} \) and \( \Psi = \begin{bmatrix}
\Xi \\
1 \\
\end{bmatrix} \).

### E.5. Affine term structure model

In setting up the affine term structure model, I follow Bekaert, Cho, and Moreno (2010) and keep the risk premia constant. Given that the paper focuses on intertemporal trade-off and expectations, time-varying risk premia are of secondary importance. The log of the nominal pricing kernel implied by the model outlined in Section V, reads:

\[ m_{t, t+1} = \log(\beta) - \gamma \Delta c_{s, t+1} - \pi_{t+1}, \] (A.24)

where \( \Delta c_{s, t+1} = \log C_{s, t+1} - \log C_{s, t} \) denotes the consumption growth of savers. Assuming log-normality, the pricing equation for short-term bonds implies:
\[ E_t m_{t,t+1} + \frac{1}{2} \text{Var}_t m_{t,t+1} = -i_t. \]  
(A.25)

Rewrite (A.24) using the relationship between the consumption of savers and the total output given in (A.7):

\[ m_{t,t+1} = \log (\beta) - \gamma \delta \Delta y_{t+1} - \gamma \zeta \Delta a_{t+1} - \pi_{t+1}. \]  
(A.26)

Define:

\[ \Lambda' = \begin{bmatrix} 1 & \gamma \delta & 0 & 0 & \gamma \zeta \end{bmatrix} \Psi, \]

and \( \text{Var} \varepsilon_t^0 = D \), then \( \text{Var}_t m_{t,t+1} = \Lambda'D\Lambda \). Given the assumptions, \( m_{t,t+1} \) can be written as:

\[ m_{t,t+1} = -i_t - \frac{1}{2} \Lambda'D\Lambda + \Lambda'\varepsilon_{t+1}, \]  
(A.27)

and the \( n \)-period zero coupon yield reads:

\[
\begin{align*}
\bar{y}_t^{(n)} &= -A_n \frac{1}{n} B_n X_t \\
A_n &= A_{n-1} + \frac{1}{2} B_{n-1}' \Psi D\Psi' B_{n-1} - \Lambda' D\Psi' B_{n-1} \\
B_n &= B_{n-1}' \phi \\
B_1 &= -\varepsilon_3'.
\end{align*}
\]

To test if the model-implied term structure recovers the short term real rate, I simulate the model at the parameters given in Table VI for 220 quarters and construct the yield curve. Then, I construct the measure of the real rate \( \bar{r}_t = i_t - y_t^{(10)} \) as in the previous sections and compare it to \( i_t - E_t \pi_{t+1} \) where \( E_t \pi_{t+1} \) is obtained from an AR(1) model. Figure A.2 shows the histogram of correlations between \( \bar{r}_t \) and \( i_t - E_t \pi_{t+1} \). The median correlation is around 0.7 which supports the validity of \( \bar{r}_t \) as a measure of the short term real rate.

**F. Robustness and additional results**

**F.1. Ex-ante real rate: robustness checks**

This appendix compares \( \bar{r}_t \) to alternative measures of the real rate variation. First, I compare \( \bar{r}_t \) constructed using the nominal UK yield curve to the real interest rate obtained from the inflation-indexed government bonds. As shown in Figure A.3, in the period 1985-2011, \( \bar{r}_t \) closely follows the real interest rate from UK inflation-indexed bonds with the correlation \( \rho = 0.6 \). The shortest continuously available maturity for the real interest rate is 3.5 years. Intuitively, the one-year real rate shall exhibit more variation than the one plotted in Figure A.3 and therefore it is more suitable for studying the intertemporal trade-off. Second, I compare \( \bar{r}_t \) in the US to the real rate constructed using the inflation expectations from the SPF panel in the period 1970-2011. Figure A.4 superimposes \( \bar{r}_t \) with the survey-based real interest rate \( i_t - E_t^s \pi_{t+1} \) where \( E_t^s \pi_{t+1} \) represents the three-month Treasury bill and \( E_t^s \pi_{t+1} \) denotes the survey-based measure of inflation expectations over the next year. Similarly to the real rate obtained from the Livingston survey, both \( \bar{r}_t \) and \( i_t - E_t^s \pi_{t+1} \) closely track each other (\( \rho = 0.53 \)).
Correlation of $\tilde{r}_t$ and real rate, simulated model

Figure A.2: Correlation of $\tilde{r}_t$ and real rate in a simulated model.

The figure reports the distribution of the correlation between $\tilde{r}_t$ and the ex-ante real interest rate from a simulated model. $\tilde{r}_t$ is obtained as a difference between the nominal short rate and the ten-year yield. Ex-ante real rate is computed as a difference between short term nominal yield and expected inflation. To obtain the expected inflation, I estimate an AR(1) model on realized inflation and forecast two quarters ahead. The model is simulated for 220 quarters with 1000 replications.

F.2. Accuracy of the pre-1929 data

The interest rate and consumption data from the period before 1929 are not well-researched. Therefore, there is a potential issue that at least part of the results are driven by the selection of the dataset. To address these concerns, I evaluate the main result, namely the estimate of the EIS using alternative data sources.

First, I re-estimate the consumption Euler equation using the annual data by Robert Shiller. The sample period is 1888-2009. $\tilde{r}_t$ is defined as the difference between the one-year interest rate and the long term government bond yield, which is a ten-year Treasury note post 1953. The results are quantitatively similar, i.e. the estimate of $\sigma$ is close to -0.5 for the full sample and it is statistically significant.

Second, between the Civil War and 1920, yields of government bonds are potentially downward-biased due to the fact that government bonds were held as reserves by banks and these could issue

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The figure plots the proxy for the ex-ante real rate $\tilde{r}_t$ which is computed as the difference between the one- and ten-year nominal Gilt. Superimposed is the real yield with maturity 3.5 years denoted by $r^Gilt_t$. Shorter maturities are not continuously available for real yields in the UK. The data are annual. The sample period is 1985 through 2011.

bank notes against them. To evaluate the potential bias, I use the non-Treasury yields to construct $\tilde{r}_t$ and use it to estimate the Euler equation. The results are quantitatively similar to those using Treasury yields.
Figure A.4: Ex-ante real interest rate and $\tilde{r}_t$, 1970-2011

The figure plots the proxy for the ex-ante real rate $\tilde{r}_t$ which is computed as the difference between the three-month Treasury bill yield and ten-year Treasury yield together with the survey-based real interest rate $i_t - E_t^\pi_t \pi_{t+1}$, where $i_t$ represents the three-month Treasury bill and $E_t^\pi_t \pi_{t+1}$ denotes the survey-based measure of inflation expectations over the next year. The survey data are obtained from the SPF panel and the median response is a proxy for the expectations. The data are quarterly. The sample period is Q2:1970 through Q3:2011.