When Uncertainty Blows in the Orchard: Comovement and the Equilibrium Volatility Risk Premia

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Abstract

In a Lucas orchard with heterogeneous beliefs, we study the link between market-wide uncertainty, difference of opinions and co-movement of stock returns. We show that this link plays an important role in explaining the dynamics of equilibrium volatility and correlation risk premia. In our economy, uncertainty is linked to both firm-specific and market-wide signals. These two uncertainty channels drive the co-movement of stock returns even when dividends are weakly dependent: Greater subjective uncertainty or higher disagreement on the market-wide signal imply a larger correlation of beliefs, a stronger co-movement of stock returns, and a substantial correlation risk premium generated by the endogenous optimal risk-sharing among investors. These features are different and ancillary to the pure market-clearing effects distinct to any multiple-trees economy. We test empirically the implications of the model-implied co-movement channels and derive testable predictions for the dynamics and the cross-sectional features of index and individual stock options. Our empirical tests provide five novel results to the literature: (a) Volatility risk premia are linked to a counter-cyclical common disagreement component about future earning opportunities by financial analysts; (b) This common component helps to explain the cross-sectional differences in the volatility risk premia of index and individual stocks. (c) At the same time, firm-specific earnings disagreement is highly significant in explaining the cross-section of individual volatility risk premia; (d) The common disagreement component also helps to explain the time-series of the correlation risk premium extracted from option prices; (e) The excess returns of dispersion portfolios reflect a large time-varying correlation risk premium compensating investors for risk exposure to the common belief disagreement component. These findings are robust to several standard control variables and to transaction costs. They are also not implied by competing theories of volatility risk premia.

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Prices of index and individual stock options differ in a number of striking empirical dimensions. These differences and their time-series and cross-sectional properties present an important challenge to modern asset pricing modeling. First, the absolute size of the typical volatility risk premium of index options is larger than the volatility risk premium of single stock options. For instance, the average implied volatility of index options on the S&P 100 is about 19.2% per year, but the realized volatility is only about 16.7%: the volatility risk premium is -2.51%. This translates in expected absolute returns that dwarf the equity risk premium puzzle. At the same time, the average implied volatility of single-name options on all S&P 100 constituents is about 32.7% and the average realized volatility is 31.8%: the volatility risk premium is much smaller in absolute values. Such large differences in ex-ante compensation for volatility risk of index and individual option markets result in large average excess returns for index volatility sellers and for dispersion investors being short index volatility and long individual stock volatility, thus locking-in the correlation risk premium. Second, as noted in Bakshi, Kapadia, and Madan (2003), the index option implied volatility smile is on average steeper and more negatively skewed than the smile of single stock options. Third, the size of the volatility risk premium is highly time-varying, with the first and third quartile of its distribution being -4.75% and -1.31 for the S&P 100 index option (-3.56% and -0.41% for its constituents) and state dependent. Fourth, in the case of individual stock options the cross-sectional difference in the volatility risk premium is as large as their time-series variation. 

While these stylized facts are well documented, very little is known about the possible structural explanations for such distinctive features in volatility patterns. We address these questions by investigating a Lucas’ orchard economy with partial information and propose a structural explanation based on the role of market wide uncertainty and heterogeneity in beliefs. In this economy, rational investors update their beliefs according to Bayes’ Law but they have different perceptions of the degree of economic uncertainty affecting economic fundamentals. In equilibrium, as the heterogeneity in beliefs varies over time, both the average level of uncertainty and the dispersion in the subjective perception of uncertainty affect the volatility of asset returns and their co-movement. Stock volatilities and their correlations are stochastic and co-move with the equilibrium stochastic discount factor. Volatility is priced and the size of the volatility risk premium is linked to the perceived level of uncertainty. In this context, the volatility risk premium on index options is higher than on single stock options because of the presence of an endogenous correlation risk premium. The model generates important testable implications both on the cross-section and time-series of volatility and correlation risk premia. In the time-series, the model implies that the correlation risk premium increases when market-wide uncertainty is large or when the endogenous common component in the disagreement processes for the dividends of different stocks is larger. In the cross-section, option prices of stocks with a larger exposure to economic uncertainty have higher correlation risk premia. We study these model implications both theoretically and empirically and test the extent to which this explanation can shed some light on the different pricing patterns of index and individual options.
The recent global financial crisis has brought asset volatilities and correlations even more into the limelight of the financial press, due to the unusually high levels of volatility and correlation. The VIX, labeled by the financial press as the fear gauge index, reached an intraday all time high of almost 90% in October 2008. It is often suggested that implied volatility measures from derivative prices provide useful information on investors’ perceptions of future uncertainty. However, how this link may develop in equilibrium is not fully understood. To motivate the potential importance of the empirical link between volatility risk premia, economic uncertainty and belief disagreement among investors, Figure 1 (right panel) plots the volatility risk premium on the S&P 500, together with what we propose as a proxy of economy-wide uncertainty. This proxy is derived by estimating a dynamic single-factor model using a large panel of differences of analysts’ future earning forecasts for a cross-section of firms in the S&P 500 (full details are provided in the empirical section).

Figure 1 (right panel) highlights a strong co-movement of the volatility risk premium and this uncertainty proxy. For instance, both after the LTCM collapse in late summer 1998 and the terrorists’ attacks in September 2001 (yellow bars), they both increased very substantially, at the same time. The volatility risk premium increased tenfold three months before October 2001 and dropped by a factor of 11 three months after the attacks. In the same period, the economy-wide uncertainty increased threefold and then dropped by a factor of two. A simple regression of the volatility risk premium on the uncertainty proxy confirms this evidence with a statistically highly significant standardized coefficient of 0.48 and an adjusted $R^2$ of 23%. A strong positive relationship between volatility risk premia and uncertainty proxies linked to disagreement among investors prevails also in the cross-section. The left panel of Figure 1 plots on the abscissa (ordinate) the average uncertainty proxy of 14 different sectors (the average volatility risk premium of these sectors) in three different time periods. After conditioning by sectors, the previous results are even stronger and the statistical relationship is strengthened.

It may not come as a surprise, given the previous stylized facts, that variance and correlation have emerged as new asset classes and trading strategies have been standardized to create a new generation of financial products. Usually, the products that allow trading in the variance risk premium are based on combinations of straddles (via listed products) and/or variance swaps (over-the-counter). Correlation exposure, on the other hand, is normally obtained by mean of dispersion portfolios (via listed products) and correlation swaps (over-the-counter). In both cases, a

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1 We compute the index volatility risk premium as the difference between the VIX and the 30 day (annualized) realized volatility on the S&P 500. The volatility risk premia on the individual sectors are calculated from implied volatilities of 30 day at-the-money options on individual stocks. We then average across time and across all firms within each sector. The realized volatility is calculated from daily log returns over one month.

2 A dispersion strategy is designed to lock-in the correlation risk premium generated by the difference between the volatility risk premia of index and single-stock options. It involves a short position in the index volatility and a long position in the constituents volatility, which can be based on either straddles or strangles. Since at-the-money straddles have a delta exposure close to zero, a dispersion portfolio short an index straddle and long the constituents straddles is well hedged against market movements. Thus, they are quite popular among practitioners. See Appendix C for details.
large ex-ante (negative) risk premium can be locked-in by selling implied variance or correlation. Trading volumes in these products have increased dramatically in the last five years and this could be indicative either of simple speculative interest or of their economic importance in the broader process of economic risk-sharing.

[Insert Table 1 approximately here.] The nature of the time-variation in the excess return of these strategies is insightful. Even a quick glimpse at Table reveals that the largest forfeitures of the dispersion portfolio occur during crisis periods. For instance, the largest loss occurred in September 2001 at the time of the terrorists’ attacks. Similarly, when LTCM defaulted in August 1998, the dispersion portfolio lost -122% in one month. Intuitively, being long the constituents volatility and short the index volatility implies a net short position in the average correlation. The extent of the change in realized correlation during these crisis periods and therefore in losses for dispersion trades is impressive. For instance, in August 1998 the realized correlation increased by more than 100% on one month. What is more interesting, however, from a financial economics perspective is that such large correlation increases almost always occurred in correspondence with large surges in a proxy of economy-wide uncertainty obtained from differences in analysts’ earnings forecasts. For instance, in the month of August 1998 this proxy of market-wide uncertainty increased by more than 30%.

Overall, the preliminary evidence in Figure and Table suggests the possibility of an important economic relation between (a) index and single stock volatility risk premia, (b) returns of factor mimicking portfolios exposed to unexpected changes in equity correlations, (c) economic uncertainty, and (d) the degree of difference of opinions among investors. In this paper, we investigate in more details the equilibrium link between these four elements in the context of a structural two-trees Lucas (1978) exchange economy with disagreeing investors. We make two key assumptions. First, the growth rate of the two firms’ dividend stream are unknown to all agents in the economy. Second, agents have different levels of perceived economic uncertainty (volatility of the dividend growth rate). Because of the first assumption, to implement optimal portfolio and consumption choices agents needs to estimate expected growth rates using available information. However, because of the heterogeneity in perceived uncertainty agents process information differently and eventually disagree. In this context, disagreement is a stochastic process with a non-degenerate asymptotic distribution. In this economy, the optimal consumption share of the two agents depend on the level of disagreement: The equilibrium consumption share of the pessimist (optimist) is larger in low (high) dividend states. Since agents have decreasing marginal utility, the price of all states is lower in economies with

3 The aim of a dispersion strategy is to maximize the so-called dispersion, which is defined as the square root of the difference between the weighted average variance of constituents stocks and the index variance, thus generating exposure to changes in correlation. The dispersion is defined by:

\[ D = \sqrt{\sum_{i=1}^{n} \omega_i \sigma_i^2 - \sigma_{\text{Index}}^2}, \]

where \( \omega_i \) is stock \( i \)'s weight in the index, \( \sigma_i \) is the volatility of stock \( i \) and \( \sigma_{\text{Index}} \) is the index volatility.

4 Variance trading is said to have increased by 100% in 2008; see, for instance, http://www.euromoney.com/Article/2059815. Over the past few years, volatility products have emerged as an asset class on their own right as more and more hedge funds engaged in so called volatility arbitrage strategies. E.g., LTCM was labeled the “Central Bank of Volatility”; see Lowenstein, 2000.

5 In our model, such disagreement is not axiomatic, in the sense that it is generated by priors on parameters that cannot be learned even if high frequency data were available to investors.
larger disagreement. Disagreement among investors, in addition to aggregate endowment, becomes a key priced state variable that have important asset pricing implications. In this paper, we want to study four of these implications which are directly related to the dynamics of volatility and the correlation risk premium.

First, realized equity volatility can be stochastic even if dividend are homoscedastic. This is due to the time variation in the discount factor, which is induced by the time-variation in the difference in beliefs. Second, difference in beliefs create a wedge between risk-neutral and realized volatilities. The larger the difference in beliefs, the larger this spread (e.g. volatility risk premium) as the ex-ante compensation required by the optimist to hedge the pessimist in bad states is larger. Third, the model provides an economic rationale for the existence of an endogenous correlation risk premium, which drives the spread between the index and the single-stock volatility risk premia, due to the (endogenous) stochastic stock return correlation generated by the optimal risk sharing between agents. When marginal utilities are convex, the state price of a bad aggregate dividend state is lower than the average state price of a bad dividend in either of the two firms. Index returns have a larger volatility risk premium than the returns of each individual stock. In the context of our economy, greater subjective economic uncertainty or higher disagreement among investors on the market-wide signal implies a higher correlation of beliefs, which then implies a higher correlation risk premium. Fourth, the correlation risk premium depends on the degree to which market-wide information is used by agents to form their beliefs on dividends. During periods of market stress, it has been argued that agents are affected by limited attention, thus increasing their reliance on market-wide signal to conduct their inference and limiting attention to firm specific information. We model this element by introducing a market-wide signal which is correlated to the aggregate dividend process. When subjective economic uncertainty is greater and agents rely more on the common signal, the correlation in their beliefs is higher, which implies greater correlation of asset prices and eventually a larger correlation risk premium. Last, a higher disagreement on the future dividends of one firm induces a higher negative skewness of the stock returns of that particular firm. This effect is larger when agents disagree on the market-wide signal and market-wide uncertainty is high.

To quantify the magnitude of these implications, we design an experiment that is based on factor mimicking long-short portfolio that is exposed to correlation risk: a “dispersion” portfolio. Then, we simulate our economy and calculate the model-implied returns of this portfolio. This allows to document the extent to which differences in beliefs can reproduce realistic levels of correlation risk premia. The answer is positive: Not only the the model can generate realistic average risk premia, with respect to the data, but it can also replicate several characteristics that describe the differences between “dispersion” trades and pure short positions in put index options.

We study the empirical implications of the model and use data on S&P 100 index options and on single-stock options for all the index constituents in the period January 1996 to June 2007. We merge this dataset with analysts earning forecasts from the Institutional Brokers Estimate System (I/B/E/S) and stock return data from CRSP. We first

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These findings also give theoretical support to the empirical evidence that correlations between asset returns vary over time and are state-dependent; see Bollerslev, Engle, and Wooldridge (1988), and Moskowitz (2003), among many others. In a standard Lucas economy with homogeneous agents, the state price density varies only due to dividend fluctuations. Stock returns are correlated due to the correlation of the individual stock with the aggregate endowment (“diversification effect”), a feature that is documented in Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2009). In our economy, the state price density depends additionally on the cross-sectional wealth that is shifted across agents (“optimal-risk sharing effect”). This additional component dominates the two trees effect and is independent of the number of firms and their size in the aggregate market.
compute a belief disagreement proxy for each individual firm and then apply dynamic factor analysis to construct a common factor that proxies for the overall belief disagreement across firms. We obtain a number of novel results.

First, as predicted by our theoretical model, we find that belief disagreement increases (i) the volatility risk premium of both index and individual stock options and (ii) the correlation risk premium, in a way that is remarkably robust with respect to the inclusion of several other control variables. E.g., in a regression for the market volatility risk premium, the $R^2$ is about 12% higher when disagreement is accounted for. The impact of the firm-specific disagreement on the individual volatility risk premia is also economically significant: A one standard deviation change in firm specific disagreement implies an increase of almost 1% in individual risk premia. The impact of the common disagreement proxy for the correlation risk premium is equally large: A one percent increase in the common DiB implies an increase of almost 50% of the average correlation risk premium. To the best of our knowledge, there are only a few academic studies on the determinants of volatility risk premia and no work exists that studies determinants of the correlation risk premium. Carr and Wu (2009) find that classical risk factors, such as the market excess return, the Fama and French (1993) factors, a momentum factor, or two-bond market factors, cannot explain the variance risk premia of a limited set of individual options. Bollerslev, Gibson, and Zhou (2009) argue that the volatility risk premium in index options comes from time-varying risk aversion. In particular, they find that macro-finance variables have a statistically significant effect on the index volatility risk premium.

Second, we study a simple option-based trading strategy aimed at exploiting the correlation risk premium in option prices. More precisely, we study an at-the-money straddle dispersion portfolio. In a standard (naive) dispersion trade, each month the investor shorts the index straddles and buys individual straddles of the individual stocks that compose the index. Since we investigate the effect of the dispersion in beliefs, we consider portfolios of individual straddles sorted by the difference in beliefs. This approach has two advantages. First, it reduces transaction costs. Second, it motivates a model-consistent way of constructing factor-mimicking portfolios. In the context of our model, higher disagreement is linked to a higher (negative) volatility risk premium. Thus, if the model is correct, the quintile of firms with the lowest belief disagreement are associated with individual stock options implying higher expected returns. The results are compared with the two alternative most popular measures of cheapness in the cross-section of stock options: Liquidity risk and equity market beta. We find that a dispersion portfolio, in which the long leg is based on a portfolio of straddles sorted by the difference in beliefs, generates attractive expected excess returns, which are much higher than when sorting for equity market beta or liquidity factors. For example, the straddle dispersion portfolio yields an annualized Sharpe ratio of 1.5, which is 20% higher than the Sharpe ratio derived from investing all wealth in a short index put. Goyal and Saretto (2009) find similarly high returns and Sharpe ratios for straddle strategies, when sorting their portfolios of single-stock options according to the difference in implied and realized volatility, and interpret this finding as evidence of some form of volatility mispricing. While these features are indeed inconsistent with a single factor option-pricing model, such as the Black and Scholes (1973) and Merton (1973) model, in our economy they are fully compatible with the existence of a priced common disagreement factor.

Third, we test whether the dispersion strategy returns can be explained by other systematic risk factors, which are typically used to explain differences in the cross-section of stock returns. We sort the dispersion portfolio by size,
book-to-market, and momentum. We find that all sorted portfolio are exposed to the common difference in belief factor. On the other hand, when we regress the excess return of dispersion portfolio on the Fama and French (1993) and momentum factors we do not find statistically significant result. In order to understand in more depth the tail risk characteristics of uncertainty on the distribution of these portfolio returns, we use quantile regressions techniques and study the asymmetric effect of uncertainty. Indeed, we find that the effect of uncertainty risk is asymmetric: large losses are concentrated during few periods characterized by unusually large disagreement risk. This is important since it is further empirical support of the existence of a risk factor, rather than misspricing.

Finally, we check the robustness of the results with respect to alternative potential explanation. We consider transaction costs and margins on the profitability of option strategies. We find that transaction costs indeed lower the profitability of dispersion strategies, but the Sharpe ratio of the disagreement sorted dispersion portfolio still exceeds the Sharpe ratio of a standard short index put trading strategy, see for instance Bondarenko, (2003). We also find that even after considering transaction costs the ex post returns of the dispersion portfolio load significantly on the common disagreement proxy. The results are robust with respect to other potential explanations of volatility risk premia, as for instance those related to the effects of earning announcements.

Related Literature. This paper draws from the literature that studies equilibrium models with multiple assets, such as Menzly, Santos and Veronesi (2004), Santos and Veronesi (2006), Pavlova and Rigobon (2007), Cochrane, Longstaff, and Santa-Clara (2008), and Martin (2009). Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006) study a multi-asset economy with external habit where the dividend shares of assets are mean-reverting processes. Pavlova and Rigobon (2007) study a two country two good economy with demand shocks and log-linear preferences. Martin (2009) studies an economy with a collection of Lucas trees, so called Lucas orchards, and its impact on asset prices, risk premia, and the term structure. The model is able to replicate many salient features of asset returns such as momentum, mean-reversion, contagion, fight-to-quality, the value-growth effect, and excess volatility. Cochrane, Longstaff, and Santa-Clara (2008) study a Lucas (1978) economy with two assets and its implications for stock returns, correlations, and the equity risk premium. Ehling and Heyerdahl-Larsen (2008) study stock return correlation in a multi-asset Lucas economy when heterogeneous agents have different risk aversion coefficients and dogmatic beliefs. Stock return correlation is counter-cyclical due to the time-varying risk aversion. Our contribution is different from the one of the above papers along several dimensions. First, we model heterogeneous agents with the same risk aversion but different beliefs, which evolve stochastically over time. Second, we highlight the co-movement channels arising in presence of market wide uncertainty and disagreement. Third, we derive the link between uncertainty, disagreement, and volatility risk premia of index and individual options, eliciting the role of the correlation risk premium. We empirically show that disagreement impacts volatility risk premia not only in the time-series but also in the cross-section.

Our work is also related to the literature that investigates the characteristics of the volatility risk premium. The early literature on volatility risk premia is large and deals mostly with index options. Fleming, Ostdiek, and Whaley (1995), Jackwerth and Rubinstein (1996), and Christensen and Prabhala (1998) observe that average realized index
volatilities tend to be substantially lower than implied volatilities of index options. These papers mainly focus on the forecasting power of implied volatility for realized volatility, and study different measures of volatility. The more recent literature on single-name options has found a more mixed evidence of the existence of a nonzero volatility risk premium. Bakshi and Kapadia (2003b) show that individual equity option prices embed a negative market volatility risk premium, although much smaller than for index options, but they focus on a sample of 25 stocks only. Bollen and Whaley (2004) report that the average deviation between the Black and Scholes implied volatility and the realized volatility is approximately zero for all 20 individual stocks they study. Carr and Wu (2009) find evidence of a volatility risk premium using a subset of 35 stocks. Driessen, Maenhout, and Vilkov (2009) find insignificant differences between implied and realized volatilities studying average model-free volatility measures. They do find a significant difference in index options and interpret this finding as evidence of a correlation risk premium. A trading strategy that exploits this correlation risk premium yields high Sharpe ratios, which however tend to vanish after considering the impact of transaction costs. Duarte and Jones (2007) study the impact of systematic risk on volatility risk premia and find evidence of a volatility risk premium that varies with the overall level of market volatility. Our empirical findings provide a new perspective into this literature, by showing that common disagreement is a potentially key determining factor of volatility and correlation risk premia.

The literature aiming at giving a structural explanation for the emergence of volatility risk premia is sparse. Motivated by the empirical results in Bollen and Whaley (2004), who show that changes in implied volatility are correlated with signed option volume, Gürkaynak, Pedersen, and Poteshman (2009) study the relationship between the end user option demand and the overall shape of implied volatility curves. They document that end users tend to have a net long index option position and a short equity-option position, thus helping to explain the relative expensiveness of index options. They also show that there is a strong downward skew in the net demand of index but not equity options, which helps to explain the difference in the shapes of their overall implied volatility curves. Their framework is effective in explaining the steeper slope of index options, due to excess demand of out-of-the-money puts, but less so in differentiating the pricing of individual options in the cross-section. Eraker (2008) studies an equilibrium with long-run risk coupled with a highly persistent volatility process. Drechsler and Yaron (2008) consider infrequent, but potentially large, spikes in the level of volatility, together with infrequent jumps in the small, persistent components of consumption and dividend growth. The volatility shocks from a standard long-run risk model have a market price of risk which is sufficiently large to generate a variance risk premium. However, higher moments of the variance risk premium and short-horizon stock return predictability by the variance risk premium can only be generated in a setting based on non-Gaussian shocks. Zhou (2009) attributes the time-variation of the market volatility risk premium to the stochastic volatility of volatility of consumption growth in an economy where agents have recursive preferences. In contrast to these papers, we do not focus on the long-run component of consumption growth. Also,

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8These findings are extended by recent work in Lakonishok, Lee, Pearson, and Poteshman (2007) who document that end users are more short than long for both individual equity calls and puts.
we study both the time-series and the cross-sectional features of volatility risk premia of both index and individual options, and derive their strong link to the degree of heterogeneity in beliefs among agents.

The paper also draws from the literature that studies the risk-reward features of option strategies. Driessen, Maenhout, and Vilkov (2009) argue that priced correlation risk is the main driving factor of index volatility risk premia. They find that a simple option-based dispersion strategy that locks in the correlation risk premium earns large Sharpe ratios. They also study trading frictions, such as margins and transaction costs, and show that these can have a substantial effect on the attractiveness of their dispersion strategy. Other important papers studying the characteristics of volatility risk premia include Coval and Shumway (2001), Eraker (2007), Driessen and Maenhout (2008), and Broadie, Chernov, and Johannes (2007). Most of these papers focus to a large extent on the apparent over-pricing of out-of-the-money index put options. We depart from these papers in the following directions. First, we investigate the link between economic co-movement and correlation risk premia in a structural setting with heterogenous beliefs. We study the dynamics of volatility risk premia of index and individual options by considering the role of uncertainty and belief disagreement. Second, we show that volatility and correlation risk premia are compensations for priced disagreement risk. Third, we empirically show that belief disagreement is indeed priced both in the time series and in the cross-section of option returns, even after accounting for the impact of transaction costs.

Our results are related and contribute to the literature related to the role of earning announcements. Empirical evidence has shown that volatility risk premia tend to be high prior to an earning announcement. For example, on August 10th, 2004, Cisco announced their earnings and managed to beat analysts’ forecast with a 41% leap in net income. Prior to the announcement, the volatility risk premium increased by 43% from 0.10 to 0.14. Beber and Brandt (2006) study state-price densities of bond prices before and after macro announcements. They document a strong decrease in implied volatility and changes in skewness and kurtosis of the state-price density of bond option returns after a macro announcement. They attribute such changes in the higher moments of the state-price density to a time-varying risk aversion. Dubinsky and Johannes (2006) find similar effects for stock options and earning announcements: Implied volatilities of single-stock options increase prior to and decrease subsequent to an earning announcement. They argue that anticipated uncertainty surrounding the fundamental information about the firm causes the implied volatility to increase. Once the uncertainty is revealed, the implied volatility drops. This interplay of uncertainty, news revelation, and volatility risk premia is interesting also in relation to our model. For instance, we find that in the month previous to the earning announcement of Cisco, belief disagreement rose by more than 30% from 0.68 to 0.95. This evidence suggests a non-trivial interdependence of earning announcements, belief disagreement, and the volatility risk premium of individual options. We test this hypothesis and find that earning announcements have a significant impact on the volatility risk premium. However, an interaction term of belief disagreement and earnings announcements is not significant.

Finally, we are not the first to study the impact of belief disagreement on the implied volatility of options. Buraschi and Jiltsov (2006) demonstrate in a single-tree Lucas (1976) economy with heterogeneous beliefs that belief disagreement increases the implied volatility smile of index options. In this paper, we go several steps beyond and provide a structural explanation for the difference in volatility risk premia of index and single-stock options, both
Theoretical and empirically. We consider a Lucas orchard economy and study the impact of belief disagreement on both volatility and correlation risk premia embedded in index and individual options. We show how uncertainty and market-wide versus firm-specific disagreement impact on the cross-section of options. In addition, we empirically identify a powerful proxy for the common disagreement as the first dynamic component out of the cross-section of individual differences in first specific earning forecasts.

The rest of the paper is organized as follows. Section I provides the model setup. Section II derives the main theoretical model predictions. Section III describes our panel data set and Section IV presents the results of our empirical study. Section V concludes.

I. The Economy with Uncertainty and Heterogeneous Beliefs

A. The Model

We extend the standard single-asset Lucas-tree pure-exchange framework to the case with two assets and two investors. The economy has infinite horizon $[0, \infty)$ with uncertainty represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ on which is defined a standard Brownian motion $W = (W_{D_1}, W_{D_2}, W_z, W_{\mu_{D_1}}, W_{\mu_{D_2}}, W_{\mu_z})'$. All stochastic processes are assumed adapted to $\{\mathcal{F}_t; t \in [0, \infty)\}$, the augmented filtration generated by the Brownian motion $W$. The two firms produce their perishable good and dividends of firm $i = 1, 2$ have the following dynamics:

$$d\log D_i(t) = \mu_{D_i}(t)dt + \sigma_{D_i} dW_{D_i}(t),$$

where $\sigma_{D_i} \in \mathbb{R}^+$ is the dividend volatility. Dividends are observable, but their expected growth rate $\mu_{D_i}(t)$ is not and has to be estimated given available information. The dynamics of $\mu_{D_i}(t)$ is given by:

$$d\mu_{D_i}(t) = (a_{0D_i} + a_{1D_i}(\mu_{D_i}(t))) dt + \sigma_{\mu_{D_i}} dW_{\mu_{D_i}}(t).$$

Parameter $\sigma_{\mu_{D_i}} \in \mathbb{R}^+$ measures the "economic uncertainty" about the individual growth rate of firm $i$ dividends. In order to estimate the dividend growth rates, investors can also make use of the information from a market-wide indicator $z(t)$, whose rate of growth is related to the aggregate growth rate in the economy. This market-wide indicator has the following dynamics:

$$dz(t) = (\alpha_{D_1}\mu_{D_1}(t) + \alpha_{D_2}\mu_{D_2}(t) + \beta\mu_z(t)) dt + \sigma_z dW_z(t),$$

$$d\mu_z(t) = (a_{0z} + a_{1z}\mu_z(t)) dt + \sigma_{\mu_z} dW_{\mu_z}(t),$$

where $\sigma_z \in \mathbb{R}^+$ is a signal precision parameter. The drift of $z(t)$ provides an unbiased signal for $\alpha_{D_1}\mu_{D_1}(t) + \alpha_{D_2}\mu_{D_2}(t) + \beta\mu_z(t)$ and it is therefore linked to the growth rate of both firms in the economy. When $\beta = 0$, it provides an unbiased signal for a weighted average of firms' growth rates. If, in addition, $\alpha_{D_1} = \alpha_{D_2}$, the drift is an unbiased indicator of the actual aggregate growth rate in the economy.
LIMITED ATTENTION. When $\beta \neq 0$, the signal $z(t)$ is affected by another unobservable variable $\mu_z(t)$, which is independent of dividends. The relative importance of parameters $\alpha_{D_1}$, $\alpha_{D_2}$, and $\beta$ determines the extent to which the information provided by $z(t)$ is useful to make more precise inference on the expected dividend growth. When agents assume $\beta$ to be small with respect to $\alpha_{D_1}$ and $\alpha_{D_2}$, the relative importance of the aggregate indicator to conduct inference on the firm-specific growth opportunity $\mu_{D_1}(t)$ and $\mu_{D_2}(t)$ increases as agents reliance on firm-specific information is reduced. The relative size of $\beta$ is a parsimonious way to capture “limited attention” (or rational inattention). A vast literature in psychology studies rational inattention, namely the fact that agents can only process a limited amount of information during a given time period. At times, investors are assailed by a clang of market bellwethers and have to be selective in information processing. This issue is particularly severe during financial crises, as the daily amount of information and rumors increases. Peng and Xiong (2006) and Peng, Xiong, and Bollerslev (2007) show that after a macroeconomic shock investors mainly focus their attention on information about aggregate market factors. Only subsequently, as the crisis gets less boisterous, they switch their attention again to firm-specific information.\footnote{Gilbert, Kogan, Lochstoer, and Ozyildirim (2008) find that the market response of investors focusing on summary statistics, instead of a wide array of fundamental information, can have an impact on stock prices, volatility and trading volume.}

The parameter $\sigma_{\mu_z} \in \mathbb{R}^+$ is the volatility of the component $\mu_z(t)$, which is orthogonal to $\alpha_{D_1}$ and $\alpha_{D_2}$. A large value of $\sigma_{\mu_z}$ tends to imply larger revision of agents’ estimates of the drift of $z(t)$ and the market-wide indicator will have a larger impact on agents’ posterior beliefs. In this context, a larger co-movement in disagreement across firms can arise. Moreover, if agents have heterogeneous priors, a larger value of $\sigma_{\mu_z}$ is associated with a larger degree of disagreement across investors. If $\sigma_{\mu_z} = 0$, any change in the expected growth rate of $z(t)$ derives from a change in dividend growth rates. Finally, $-a_{0z}/a_{1z}$ is the long-term growth rate of the expected change in $\mu_z(t)$ and $a_{1z} < 0$ its mean-reversion parameter.

B. Modeling Uncertainty and Disagreement

We consider a simple specification for the uncertainty and disagreement in our economy. Investors are rational in the sense that they update their beliefs using all available information using Bayes’ rule. Differences in their posteriors can arise either from a difference in agents’ priors or a difference in some of the subjective parameters governing the dynamics of dividends and the market-wide indicator. We consider the case in which the subjective uncertainty parameters $\sigma_{\mu}$ is agent dependent, i.e. $\sigma_{\mu}^A \neq \sigma_{\mu}^B$. This assumption allows for a non-degenerate steady-state distribution of the disagreement process.

Let $m^n(t) := (m^n_{D_1}(t), m^n_{D_2}(t), m^n_z(t))' := E^n ((\mu_{D_1}(t), \mu_{D_2}(t), \mu_z(t))' | F_t^Y)$ where $F_t^Y := F_t^{D_1,D_2,z}$ is the information generated by dividends and the market-wide signal up to time $t$, and $E^n(\cdot)$ denotes expectation with respect to the subjective probability of investor $n = A, B$. It is convenient to write the state dynamics in vector form. Let us
define $b^n = \text{diag}(\sigma^\mu_{D1}, \sigma^\mu_{D2}, \sigma^\mu_{D3})$, and let the state vector be $Y(t) = (\log D_1(t), \log D_2(t), z(t))$, given the (common) parameters $a_0 = (a_{0D_1}, a_{0D_2}, a_{0z})', a_1 = \text{diag}(a_{1D_1}, a_{1D_2}, a_{1z})$, $B = \text{diag}(\sigma_{D1}, \sigma_{D2}, \sigma_{D3})$, and

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \alpha_{D1} & \alpha_{D2} & \beta \end{pmatrix}.$$ 

Since the model implied state dynamics is conditionally Gaussian, the posterior beliefs of agent $A$ can be obtained using a Kalman-Bucy filter:

$$dm^A(t) = (a_0 + a_1 m^A(t))dt + \gamma^A(t) A'B^{-1}dW^A(t),$$  

$$d\gamma^A(t)/dt = a_1 \gamma^A(t) + \gamma^A(t)a_1' + b^A b'^A \gamma^A(t)' A'(BB')^{-1} A \gamma^A(t),$$

with initial conditions $m^A(0) = m_0^A$ and $\gamma^A(0) = \gamma_0^A$, where $dW^A(t) := B^{-1} ((dY(t) - Am^A(t)) dt)$ is the innovation process induced by investor $A$’s belief and filtration. The matrix $\gamma^n(t) := E^n ((\mu(t) - m^n(t)) (\mu(t) - m^n(t))' | F^Y_t)$ is the posterior variance-covariance matrix of agent $n$.

The agent specific parameter $b^A$ affects the distribution of $m^A(t)$ indirectly, by influencing the Riccati differential equation for $\gamma^A(t)$. If the subjective perception of uncertainty is homogeneous across investors, i.e. $b^A = b^B$, the economy reduces to one in which rational Bayesian investors, even though they initially disagree due to different priors, eventually agree asymptotically. When $b^A \neq b^B$, however, heterogeneity in beliefs does not vanish asymptotically and it follows a stochastic dynamics. Note that since dividend growth rates $\mu_{D_k}$ and the signal growth rate $\mu_z$ are unobservable, the true parameter value for $b$ cannot be recovered from the quadratic variations of observable variables, even if one could sample at asymptotically high frequency. This implies that, at least in this setting, disagreement does not necessarily entail irrational agents.

To complete the specification of the disagreement structure in our economy, let us define the three-dimensional scaled difference in belief process $\Psi(t) = B^{-1} [m^A(t) - m^B(t)]$. This process is the key state variable driving all equilibrium quantities. The first two components of $\Psi(t)$ measure the disagreement about the expected growth rates of future dividends. The third component captures the disagreement about the market-wide indicator $z(t)$. Since the market-wide uncertainty parameter $\sigma_{\mu_z}$ influences the subjective dynamics of each individual belief, it also has implications for the stochastic properties of the disagreement process itself. The dynamics for $\Psi(t)$ follows, after a standard application of Itô’s Lemma:

$$d\Psi(t) = B^{-1} (A(B + \gamma^B(t) A'B^{-1}) \Psi(t) dt + B^{-1} (\gamma^A(t) - \gamma^B(t)) A'B^{-1}dW^A(t),$$

$^{10}$A formal proof of this result can be found in Liptser and Shiryaev (2000).

$^{11}$With $\Psi(t) := [\Psi_{D1}(t), \Psi_{D2}(t), \Psi_z(t)]$. 


with initial conditions $\Psi(0) = \Psi_0$ and $\gamma^B(0) = \gamma^B_0$. The average level and the heterogeneity of the subjective uncertainty parameters across agents are linked to the steady state distribution of the joint disagreement dynamics. This feature implies an interesting link between the latent market-wide uncertainty, which is embedded in the parameter matrix $b$, and the properties of the heterogeneity in beliefs.

**Remark.** A sufficient condition for agent disagreement not to converge asymptotically and to support a non-degenerate asymptotic distribution is $b^A \neq b^B$. Under this assumption, the disagreement process $d\Psi(t)$ is stochastic even in steady state. Moreover, the conditional covariance of the beliefs on individual firm growth opportunities, i.e. $\text{Cov}(\Psi_{D_1}(t), \Psi_{D_2}(t))$, depends on agents reliance on aggregate information and the overall structure of the learning process. This is important since correlation in differences in beliefs across firms can be obtained even when the dividend processes are weakly dependent. The correlation between $\Psi_{D_1}(t)$ and $\Psi_{D_2}(t)$ can naturally motivate a large degree of commonality in asset prices and larger correlation risk premia. The next section studies in greater detail the first part of this link, while the asset pricing implications are discussed after we solve for the general equilibrium dynamics of the stochastic discount factor.

### B.1. Limited Attention and the Link Between Uncertainty and Co-movement in Belief Disagreement

Given the assumed independence of Brownian motions $W_{D_1}, W_{D_2},$ and $W_z$, the disagreement dynamics imply that $\Psi_{D_1}$ and $\Psi_{D_2}$ are conditionally independent when the market-wide signal $z(t)$ does not contain information about expected dividend growth, i.e., when $\alpha_{D_1} = \alpha_{D_2} = 0$. When the market-wide signal is perceived to be related to dividend growth, agents use this common information to obtain more precise estimates $m^b_{D_1}(t)$ of expected dividend growth.

This feature finds supporting evidence in the data. Figure 1 suggests the existence of a large common component extracted from the cross-section of individual proxies of belief heterogeneity. This common belief component is counter-cyclical and shared across several market sectors. Moreover, we find that it is larger during crisis periods than during normal times (see also Buraschi, Trojani, and Vedolin, 2008, for more detailed discussion). The importance of this common component in firm-specific disagreement can be highlighted in a simple regression framework. On average, the standardized slope coefficient of regressions of the proxy of firm disagreement on their common component is about 0.641, indicating a sensitivity to the common disagreement in the cross section of firms, with a $R^2$ of 0.50. The existence of a large common component in belief heterogeneity is particularly evident during periods of heightened uncertainty. For instance, following the terrorists’ attacks in September 2001 – an unexpected and sector-independent exogenous event – the conditional correlations of the difference in beliefs across all sectors

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12 Using data from the whole I/B/E/S universe, Yu (2009) finds that the common component of firm-specific disagreement has substantial explanatory power for the time series variation of both the equity and the value premium.

13 A growing body of empirical literature has documented so-called spillover effects from one market to another, or from one sector to another, even when fundamentals are very weakly linked. The literature distinguishes between two different channels driving these effects: Real and financial linkages. The macro finance literature has mostly studied real linkages, such as demand or supply shocks; see Pavlova and Rigobon (2007), among others. However, the empirical evidence for these links is rather weak; see, e.g., Kaminsky and Reinhart, (2000). The second channel has adopted either asymmetric information or correlated default and correlated liquidity as possible vehicles of contagion.
increased and moved almost in lock-step. Uncertainty affected all sectors simultaneously (see Figure 2). The 2008 credit crisis highlights a richer evolution in uncertainty as this was affected by government interventions. First, uncertainty affected differences in beliefs in the financial sector; then, it spread to the real sector as the full implications of the financial crisis unfolded and the government intensified and broadened the scope of its policy interventions. Accordingly, the conditional correlations between the financial and real sectors first decreased and then increased.

[Insert Figure 2 approximately here.]

In the following, we study co-movement in disagreement in dependence of the corresponding model parameters. To this end, we summarize in the next technical Lemma the expressions for the steady-state variances and covariances of $\Psi_{D_1}(t)$, $\Psi_{D_2}(t)$, and $\Psi_z(t)$.

**Lemma 1.** Let $\gamma^n_{ij}$, be the steady-state conditional covariance between $\Psi_i(t)$ and $\Psi_j(t)$ implied by the Riccati equation \ref{eq:riccati} for the belief dynamics of agent $n = A, B$, where $i, j = D_1, D_2, z$. It then follows:

$$
\text{Cov}_i (d\Psi_i, d\Psi_j) = \left[ \frac{\gamma_A^i - \gamma_B^i}{\sigma^i} \right] \left[ \frac{\gamma_A^j - \gamma_B^j}{\sigma^j} \right] + \left[ \frac{\gamma_{D_1 D_2}^A - \gamma_{D_1 D_2}^B}{\sigma_{D_1} \sigma_{D_2}} \right] \left[ \frac{\gamma_{D_2 j}^A - \gamma_{D_2 j}^B}{\sigma_{D_2} \sigma_j} \right] \alpha_{D_1} \left( \gamma_{D_1 j}^A - \gamma_{D_1 j}^B \right) + \alpha_{D_2} \left( \gamma_{D_2 j}^A - \gamma_{D_2 j}^B \right) + \beta \left( \gamma_{D_2 j}^A - \gamma_{D_2 j}^B \right)
$$

where for brevity we have set $\gamma^i_n := \gamma^i_n$ for $i = D_1, D_2, z$.

We use Lemma 1 to study more rigorously, via comparative statics, the co-movement in belief disagreement with respect to parameters $\alpha_{D_1}$, $\alpha_{D_2}$, and $\beta$, and uncertainty. To investigate the link between market-wide uncertainty and co-movement in beliefs, we first focus on two aspects of uncertainty: (a) the average perceived uncertainty, measured by $\bar{\sigma}_{\mu z} \equiv 0.5 \left( \sigma_{\mu z}^A + \sigma_{\mu z}^B \right)$, and (b) the heterogeneity in perceived uncertainty, measured by $\Delta \sigma_{\mu z} \equiv \sigma_{\mu z}^A - \sigma_{\mu z}^B$.

Figure 3 plots the instantaneous correlations between disagreement about dividends ($\rho(d\Psi_{D_1}, d\Psi_{D_2})$, left column) and between the disagreement about dividends and market-wide signal ($\rho(d\Psi_{D_1}, d\Psi_z)$, right column), as a function of $\alpha_{D_1}$ and $\alpha_{D_2}$ such that $\alpha_{D_1} + \alpha_{D_2} + \beta = 1$. For simplicity of interpretation, we consider a symmetric economy in which all firm-specific parameters are identical. As either $\alpha_{D_1}$ or $\alpha_{D_2}$ increase, relative to $\beta$, the correlation $\rho(d\Psi_{D_1}(t), d\Psi_{D_2}(t))$ increases. The intuition is simple: in this case, the market-wide signal becomes more informative about individual dividends. Similarly, we find that the correlation $\rho(d\Psi_{D_1}(t), d\Psi_{D_2}(t))$ is largest when $\alpha_{D_1} = \alpha_{D_2}$, because in this case the market-wide signal carries information about aggregate consumption. Thus, the market-wide signal generates a large common component in the differences in beliefs across firms, consistent with what observed in the data. In the other extreme case, when the market-wide signal is uninformative for dividends ($\beta = 1$), the co-movement in the degree of disagreement about firm dividends vanishes. Therefore, the degree of excess co-movement in disagreement implied by the presence of a common market-wide indicator can be large. For instance,
for $\alpha_{D_1} = \alpha_{D_2} = 0.4$, it can be as high as 30% depending on the choice of the subjective uncertainty parameters $\sigma^2_{\mu_1}$ across agents. In a related way, we find that the correlation $\rho(d\Psi_{D_1}, d\Psi_z)$ is largest when the market-wide signal contains mostly information about $D_1$, i.e., $\alpha_{D_1}$ is large but $\alpha_{D_2}$ is small.

Let us now focus on the effect of the heterogeneity in beliefs, i.e. $\Delta \sigma_{\mu_z} \equiv \sigma^A_{\mu_z} - \sigma^B_{\mu_z}$. The two top (bottom) panels of Figure 3 show that an increase in either the heterogeneity of subjective uncertainty $\Delta \sigma_{\mu_z}$ or the average uncertainty $\bar{\sigma}_{\mu_z}$ strengthens the correlation between disagreement processes. This feature arises because a higher value of $\Delta \sigma_{\mu_z}$ tends to increase the sensitivity of $\Psi(t)$ to shocks in the market-wide indicator, implying a larger effect of the common factor $\mu_z$ on disagreement. Similarly, a higher average uncertainty $\bar{\sigma}_{\mu_z}$ increases the sensitivity of $m^A(t)$ and $m^B(t)$ to shocks in the market-wide indicator, which leads to a larger impact of the common factor $z(t)$ on the individual beliefs about future firm dividends. In times of high market-wide uncertainty, i.e. high $\bar{\sigma}_{\mu_z}$, the relative importance of market-wide information is larger and the correlation between market-wide and firm-specific beliefs tends to be higher. During these periods, firm-specific information adds less to the information provided by the market-wide signal. If agents face limited attention constraints, then it is more efficient to focus primarily on market-wide signals, instead of processing a whole cross-section of firm-specific information together with the market-wide information.

C. Investors’ Preferences and Equilibrium

The two investors in our economy have different subjective beliefs but are identical in all other aspects, such as preferences, endowments, and risk aversion. They maximize the life-time expected power utility subject to the relevant budget constraint:

$$V^n = \sup_{c^n_{D_1}, c^n_{D_2}} \mathbb{E}^n \left( \int_0^\infty e^{-\delta t} \left( \frac{c^n_{D_1}(t)^{1-\gamma}}{1-\gamma} + \frac{c^n_{D_2}(t)^{1-\gamma}}{1-\gamma} \right) dt \right),$$

where $c^n_{D_i}(t)$ is the consumption of agent $n$ of good $i$, $\gamma > 0$ is the relative risk aversion coefficient, and $\delta \geq 0$ is the time preference parameter. We assume time-separable utility functions. This simplifies not only the computation of the equilibrium, but also the interpretation, since we can sum over individual beliefs without making any further assumptions on aggregation.

Remark: Traditional consumption-based asset pricing models usually assume that there is only one single consumption good. Models that feature multiple trees (see e.g. Menzly, Santos, and Veronesi, 2004, Cochrane, Longstaff, and Santa-Clara, 2008, and Martin, 2009) maintain the one fruit assumption and thereby implicitly assume that all the goods in the economy are perfect substitutes. In our economy, we assume that each tree produces its own fruit. Note however, that return correlation in the setting of Cochrane, Longstaff, and Santa-Clara (2008) or Martin (2009) arises automatically due to a market-clearing effect which is not present in our model.

Agents can trade in the risk-free bond, shares in the two firms, and in options written on the stocks and an index (of the two firms). We denote by $r(t)$ the risk-free rate of the zero-coupon bond, assumed in zero net supply, by
Thus, differences in beliefs affect real allocation of resources and equilibrium asset prices. Let belief heterogeneity:

\[ \text{belief heterogeneity} \]

result to highlight is that in equilibrium the two agents engage in risk sharing which depends on their difference in

\[ \text{risk sharing} \]

simple. The stochastic discount factor of each agent is equal to the product of two terms, the first is the standard

\[ \text{standard setting} \]

extension to the case with heterogeneous beliefs is due, among others, to Cuoco and He (1994), Karatzas and Shreve (1998), and Basak and Cuoco (1998). In this extension, the utility function of the representative agent is a weighted average of the utility functions of the individual agents: \( U_t(c^A, c^B) = u^A_t(c^A) + \lambda(t)u^B_t(c^B) \). Different than in a standard setting, however, the relative weight \( \lambda(t) \) is stochastic and a function of the heterogeneity in beliefs across agents. Thus, differences in beliefs affect real allocation of resources and equilibrium asset prices. Let \( \xi^n(t) \) be the stochastic discount factor of agent \( n \).

In our economy, from the market clearing assumption and the optimality conditions \( \text{[4]} \), closed form expressions for \( \xi^A(t) \) and \( \xi^B(t) \) in terms of exogenous variables follow using standard martingale solution methods. The important result to highlight is that in equilibrium the two agents engage in risk sharing which depends on their difference in beliefs. Thus, the (agent specific) stochastic discount factor depends on an additional term which is a function of belief heterogeneity:

\[ \xi^A(t) = \frac{e^{-\delta t}}{y_A}D_1(t)^{-\gamma} \left( 1 + \lambda(t) \right)^{1/\gamma}, \quad \xi^B(t) = \frac{e^{-\delta t}}{y_B}D_1(t)^{-\gamma} \left( 1 + \lambda(t) \right)^{1/\gamma} \lambda(t)^{-1}, \]  

(5)

where \( y_A \) and \( y_B \) are the Lagrange multipliers in the (static) budget constraint of agent \( A \) and \( B \), respectively. The separability assumption in preferences make the representation of the individual stochastic discount factors \( \xi^n(t) \) simple. The stochastic discount factor of each agent is equal to the product of two terms, the first is the standard homogeneous economy term which is proportional to the equilibrium marginal utility of consumption, the second is a function of the weighting process \( \lambda(t) \). In equilibrium, the weighting process \( \lambda(t) \) and the stochastic discount factors are related by \( \lambda(t) = y_A \xi^A(t)/(y_B \xi^B(t)) \). The dynamics of the weighting process follows

\[ \frac{d\lambda(t)}{\lambda(t)} = - \left( \sum_{i=1}^{2} \Psi_{D_i}(t)dW^A_{D_i}(t) + \left( \sum_{i=1}^{2} \sigma_{D_i}(t)\frac{\sigma_{D_i}}{\sigma^2} + \beta \Psi_z(t) \right) dW^A_z(t) \right). \]  

(6)

Given \( \xi^n(t) \), the price of any contingent claim in equilibrium can be computed from the expectation of the contingent claim payoffs weighted by \( \xi^n(t) \). The equilibrium relative price of good 1 and good 2 is \( p(t) = (D_2(t)/D_1(t))^{-\gamma} \).

The dynamics of the weighting factor \( \lambda(t) \) depends on perceived shocks to dividends (\( W^A_{D_1}(t) \) and \( W^A_{D_2}(t) \), respectively) and the market-wide indicator (\( W_z(t) \)). The market-wide shocks impact on \( \lambda(t) \) proportionally to the
disagreement about dividends and the market-wide signal \((\Psi_{D_1}(t), \Psi_{D_2}(t), \Psi_z(t))\), the relative precision of market-wide and firm-specific shocks \((\sigma_{D_1}/\sigma_z, i = 1, 2)\), and the informativeness of the market-wide signal for estimating dividend growth \((\text{coefficients } \alpha_{D_1} \text{ and } \alpha_{D_2})\). The state price volatility is stochastic and increasing in both \(\Psi_{D_i}(t), i = 1, 2\), and \(\Psi_z(t)\). The difference in the agent-specific \(\xi^a(t)\) reflects the different consumption plans of the two agents in the economy which are necessary to induce market clearing ex-ante. Assume, for illustration purposes, that investor A is optimistic about future dividends of both firms. Then, in equilibrium, investor B will select a relatively higher consumption in states of low dividends of either firm 1 or 2. Therefore, the relative consumption share in this economy is stochastic and its cyclical behavior is reflected by the stochastic weight \(\lambda(t)\).

To finance her consumption plan, the pessimistic investor needs to buy financial protection, e.g., put options, from the more optimistic agent. This excess demand increases the price of securities with negative exposure to dividend shocks. In this way, part of the risk embedded in bad dividend states is transferred from the pessimist to the optimist. Ex post, if a negative state occurs, the more optimistic agent is hit twice: First, because the aggregate endowment is lower, second, as a consequence of the protection agreement which makes her consumption share lower in those states. Ex ante, the more optimistic agent is compensated by a premium for having entered the insurance contract with the pessimist and this premium is increasing in the degree of disagreement among agents about the probability of future bad dividend states. Individual put options on a single stock offer financial protection only against low dividend states of one firm. Index options give protection also against a low dividend of both firms. When dividends are positively correlated under agents’ subjective beliefs, i.e., because of the presence of relevant market-wide information, investor can perceive a higher probability of suffering ex-post a low consumption state due to low dividends received from both firms. This feature increases the price of index put options relative to individual put options. Moreover, investors’ decreasing marginal utility implies a larger state price for a low dividend of both firms, which further increases the price of financial protection against those states, i.e., index out-of-the money put, relative to financial instruments providing protection only against a low dividend of one firm. These features can help to explain the larger volatility premia observed for index options relative to those of individual options.

### D. Security Prices

Given the expressions for the individual state price densities \(\xi^A(t)\) and \(\xi^B(t)\), we can price any contingent claim in the economy by computing expectations of its contingent claim payoffs weighted by state price densities. For convenience, we give the relevant expressions for the prices of financial assets from the perspective of agent A and using the first good as a numéraire. The equilibrium stock index is the weighted sum of the individual stock prices, 

\[ ID(t) = \omega_1 S_1(t) + \omega_2 S_2(t), \]

where \(\omega_1\) and \(\omega_2\) are the market capitalizations of stock 1 and 2, respectively. The price of a European call option on stock \(i\) is computed as:

\[ O_i(t, T) = E^A_i \left( \frac{\xi^A(T)}{\xi^A(t)} (S_i(T) - K_i)^+ \right) , \tag{7} \]
Lemma 2. Under the steady state distribution, the joint Laplace transform of \( D_1(t), D_2(t) \) and \( \lambda(t) \) with respect to the belief of agent \( A \) is given by:

\[
E^A \left( (D_1(T) \overset{D_1}{\sim} D_1(t)) (\frac{D_2(T)}{D_1(t)}) (\frac{\lambda(T)}{\lambda(t)}) \right) = F_{m,A}(m^A, t; \epsilon_{D_1}, \epsilon_{D_2}) \times F_{\Psi}(\Psi, t, T; \epsilon_{D_1}, \epsilon_{D_2}, \chi),
\]

where

\[
F_{m,A}(m^A, t; \epsilon_{D_1}, \epsilon_{D_2}) = \exp \left( A_{m,A}(\tau) + B_{m,A}(\tau)m^A \right),
\]

with \( \tau = T - t \) and

\[
F_{\Psi}(\Psi, t, \epsilon_{D_1}, \epsilon_{D_2}, \chi, u) = \exp \left( A_{\Psi}(\tau) + B_{\Psi}(\tau)\Psi + \Psi' C_{\Psi}(\tau)\Psi \right).
\]

for functions \( A_{m,A}, B_{m,A}, A_{\Psi}, B_{\Psi} \) and \( C_{\Psi} \) detailed in the proof in the Appendix.

The relevant pricing expressions for the contingent claims in our economy are summarized in the next technical Lemma.

Lemma 3. Let

\[
G(t, T; x_{D_1}, x_{D_2}; \Psi) \equiv \int_0^\infty \left( \frac{1 + \lambda(t)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^{x_1} \left[ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{\lambda(t)}{\lambda(T)} \right)^{x_2} d\chi \right] F_{\Psi}(\Psi, t, T; x, i\chi) dx_1 \frac{d\lambda(T)}{\lambda(T)}
\]
1. The equilibrium price of stock 1 is:

\[
S_1(t) := S_1 \left( D_1, m^A, \Psi \right),
\]

\[
= D_1(t) \int_{t}^{\infty} e^{-\delta(u-t)} F_{m^A} \left( m^A, t, u; 1 - \gamma, 0 \right) G \left( t, u, 1 - \gamma, 0; \Psi \right) du.
\]

2. The equilibrium price of stock 2 is:

\[
S_2(t) := S_2 \left( D_1, D_2, m^A, \Psi \right),
\]

\[
= D_2(t) \int_{t}^{\infty} e^{-\delta(u-t)} F_{m^A} \left( m^A, t, u; -2\gamma, 1 + \gamma \right) G \left( t, u, -2\gamma, 1 + \gamma; \Psi \right) du.
\]

3. The equilibrium price of the index is:

\[
ID(t) := ID \left( D_1, D_2, m^A, \Psi \right) = \omega_1 S_1(t) + \omega_2 S_2(t).
\]

4. The equilibrium price of the European option on stock 1 is:

\[
O_1(t) := O_1 \left( D_1, m^A, \Psi \right),
\]

\[
= E_t^A \left( e^{-\delta(T-t)} \left( \frac{D_1(t)}{D_1(T)} \right)^{1+\gamma} \left( S_1(T) - K_1 \right)^+ \right) E_t^A \left( e^{-\delta(T-t)} \left( \frac{D_1(t)}{D_1(T)} \right)^{1+\gamma} \left( ID(T) - K_{ID} \right)^+ \right).
\]

The formula for the option on stock 2 is identical with the corresponding replacements, and with \( S_2(T) \) and \( K_2 \) replacing \( S_1(T) \) and \( K_1 \), respectively.

5. The equilibrium price of the European option on the index is:

\[
I(t) := I \left( D_1, D_2, m^A, \Psi \right),
\]

\[
= E_t^A \left( e^{-\delta(T-t)} \left( \frac{D_1(t)}{D_1(T)} \right)^{1+\gamma} \left( ID(T) - K_{ID} \right)^+ \right).
\]

Stock and risk-less zero bond prices are in semi-closed form, up to a numerical integration, and do not require a Monte Carlo simulation step for their computation. This feature already largely reduces the computational costs of the equilibrium in our economy. Index and individual options require one Monte Carlo simulation. The formulas above link in a semi-explicit form the price of stock and option prices with the degree of disagreement.

II. Model Predictions

We use the solutions of the model to develop empirical predictions for the joint behavior of index and individual option markets, as a function of the difference in beliefs. The analysis is organized in three steps: First, we document
the extent to which uncertainty and co-movement in the DiB affects co-movement in asset prices (under the physical measure). Second, we investigate how uncertainty and heterogeneity in beliefs affects implied volatilities (under the risk-adjusted measure). In particular, we document the different behavior of index and individual stock implied volatilities via risk-neutral skewness. Last, we document the impact on the equilibrium volatility and correlation risk premia. We document these three implications for different levels of the average market-wide uncertainty \( \bar{\sigma}_{\mu_z} \) and the heterogeneity in subjective uncertainty \( \Delta \sigma_{\mu_z} \). Equilibrium quantities are computed with respect to the steady state distribution of beliefs, which is non-degenerate when agents disagree about \( \sigma_{\mu_z}^i \).

We calibrate the model to the dividend dynamics of the S&P 500 and assume for simplicity of exposition a symmetric economy. Calibrated parameters are summarized in Table 2. We assume a level of risk aversion equal to 2 and a dividend volatility of both firms equal to 4\%. The median difference in beliefs of firms’ future earnings in our I/B/E/S forecast data is 0.22. Consequently, in our comparative statics we consider disagreement \( \Psi(t) \) between zero and 0.3.

A. Endogenous Stock Return Correlation and Common Disagreement

In our economy, returns can be correlated even if dividend fundamentals are weakly linked, or even uncorrelated. Due to the market-wide information component and the endogenous optimal risk sharing across agents, the correlation between returns is stochastic and potentially linked to relevant risk factors that pay a premium in equilibrium. The relevant risk factors in our economy are the dividend processes \( D_1(t) \), \( D_2(t) \), and the disagreement process \( \Psi(t) \). To the extent that stock volatilities and correlations co-move with these variables via the stochastic discount factor, our model naturally implies non-zero variance and covariance risk premia in equilibrium.

As shown in Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2009), an endogenous stock return co-movement arises through the market clearing mechanism between stocks in Lucas-type economies with weakly correlated multiple trees and potentially rare disasters, but with no heterogeneity in beliefs. In contrast to these papers, the main economic driver of correlation and correlation risk premia in our setting is the joint effect of market-wide information, which generates a common component in dividend beliefs, and the endogenous risk sharing between agents. Intuitively, since market-wide information tends to generates a co-movement in dividend beliefs even when dividends are uncorrelated, a larger degree of market-wide information in our economy (e.g., when parameter \( \beta \) is small) increases the degree of stock return correlation through the simple market clearing adjustment process between different stocks. If individual agents dividend beliefs across stocks feature a higher co-movement, all agents will perceive aggregate bad dividend states in the future as more likely and will realize that the potential for diversification is smaller. Thus, they will require a higher risk premium ex-ante for investing in all stocks: they are more exposed to the aggregate consumption risk. This first correlation channel is driven by belief co-movement and would arise also in a partial information economy with market-wide signals and homogenous investors.
The second channel through which co-movement in beliefs and stock returns are related is linked to the optimal risk-sharing among agents. In an economy with disagreement, agents face relative consumption risk. Since agents have different expectations, they have different optimal consumption behavior. In equilibrium, the pessimistic agent consumes a higher fraction of aggregate consumption in bad dividend states. Therefore, she will tend to reduce the exposure to stocks with dividend processes that correlate more with aggregate consumption. Those stocks are bought by the optimistic investor in exchange for a premium, which further lowers stock prices. In the presence of a market-wide signal, dividend streams across stocks are perceived as correlated and the equilibrium price adjustment needed for an ex-ante optimal risk sharing across agents requires a higher risk premium on both stocks. This feature generates a second channel for stock return co-movement, which is absent in a standard Lucas economies with homogeneous agents. Using the correlation expressions implied by the pricing formulas in Lemma 3, we formalize more rigorously the previous statements, providing an expression for stock co-movement as a function of market-wide uncertainty and belief disagreement. Consider the equilibrium dynamics of the returns of stock \(i = 1, 2\):

\[
\frac{dS_i(t)}{S_i(t)} = m_{S_i}^A(t)dt + \sigma_{S_i,D_1}(t)dW_{D_1}^A(t) + \sigma_{S_i,D_2}(t)dW_{D_2}^A(t) + \sigma_{S_i,z}(t)dW_z^A(t).
\]

The conditional variance and covariance of the returns on stock 1 and 2 follow in the next Lemma.

**Lemma 4.** The conditional covariance of the returns of stock \(i\) and stock \(j\), \(1 \leq i, j \leq 2\), is given by:

\[
\text{Cov}_t \left( \frac{dS_i}{S_i}, \frac{dS_j}{S_j} \right) = \sigma_{S_i,D_1}(t)\sigma_{S_j,D_1}(t)dt + \sigma_{S_i,D_2}(t)\sigma_{S_j,D_2}(t)dt + \sigma_{S_i,z}(t)\sigma_{S_j,z}(t)dt,
\]

with coefficients, \(\sigma_{S_i,D_1}(t), \sigma_{S_i,D_2}(t), \sigma_{S_i,z}(t), i = 1, 2\), given explicitly in the Appendix.

The equilibrium loadings \(\sigma_{S_i,D_1}(t), \sigma_{S_i,D_2}(t), \text{ and } \sigma_{S_i,z}(t)\) of stock returns on dividend and market-wide shocks are stochastic, since they are functions of the degree of disagreement about dividends and the market-wide indicator \((\Psi_{D_1}(t), \Psi_{D_2}(t) \text{ and } \Psi_z(t))\). Figure\(4\) depicts the effect of \(\Psi_{D_1}(t)\) and \(\Psi_z(t)\) on the correlation of stock returns for different model parameter choices. In all graphs, we set \(\Psi_{D_2}(t) = 0\) for simplicity, which is a conservative assumption for the implied correlation level.

[Insert Figure 4 approximately here.]

As expected, stock return correlation increases in all panels of Figure\(4\) as a function of \(\Psi_{D_1}(t)\) and \(\Psi_z(t)\). In absence of disagreement, stock return correlation is non-zero, but very small. As in Cochrane, Longstaff, and Santa-Clar (2008) and Martin (2009), this correlation arises due to a pure market clearing effect and in the absence of portfolio adjustments between agents. When disagreement increases, the endogenous optimal risk sharing among optimistic and pessimistic agents generates a substantial amount of additional co-movement in returns. Coeteris paribus, when \(\alpha_{D_1} = \alpha_{D_2} > 0\), the market-wide disagreement tends to have a larger positive impact than the dividend specific disagreement. This feature is particularly evident in the top right panel of Figure\(4\), in which \(z(t)\) is an informative signal about aggregate dividends \((\alpha_{D_1} = \alpha_{D_2} = 0.45)\), where the steepness of the correlation surface in the \(\Psi_z(t)\)
direction is larger than in the $\Psi_{D_i}(t)$ direction. The results highlights the important link between disagreement on market-wide information and stock returns co-movement. Consistent with the intuition derived earlier for the co-movement of disagreement, stock return correlation increases when the market-wide indicator is more informative about aggregate dividends ($\beta$ is smaller), when the average level of the market-wide uncertainty is larger (i.e. $\bar{\sigma}_{m_z}$ is larger) and when heterogeneity in subjective uncertainty is larger (i.e. $\Delta \sigma_{m_z}$ is higher). In such settings, the co-movement in beliefs across assets is larger and the impact of the risk-sharing among optimistic and pessimistic agents on stock correlations is amplified.

**Remark:** It is interesting to discuss the potential implications of the co-movement in relation to the 2007-2008 credit crisis, at whose heart lies an apparent puzzle. At the beginning of the crisis, several commentators argued that the mortgage sector was small relative to the overall economy to trigger a fully fledged recession; the logic of the argument hinges on the idea that financial contagion is triggered by correlated defaults. Indeed, the number of defaults cannot have been the only driver of the credit crisis. Our theoretical setting suggests the distinct role that can be played by the spreading of uncertainty across sectors, as documented in Section B.1. If uncertainty in one sector has implications for the level of future market-wide uncertainty, an increase in uncertainty in that sector might tend to rise the correlation of stock returns. During this crisis, as in previous ones, the overall economic uncertainty increased, in this case spreading from the financial sector to the real sector. According to our model, this higher uncertainty rises the co-movement of both firm-specific disagreement and stock returns.

These features of our model are related to, but distinct from, the work of Ribeiro and Veronesi (2002) who generate an endogenous time varying co-movement of stock returns in a model with learning and a multivariate dividend process driven by a common business cycle indicator. Cochrane, Longstaff, and Santa-Clara (2008) study a Lucas-type economy, in which the equilibrium correlation process between stocks is a deterministic function of the dividend share: If a stock has a small share of the overall endowment, then it will have low correlation with other stocks. Martin (2009) considers Lucas orchards together with rare disasters. In his economy, small stocks co-move endogenously, despite independent fundamentals or a negligible dividend share. Disasters spread in the economy in an asymmetric fashion; If a large firm experiences a disaster, the price of the other small asset experiences a large price drop. Vice versa, if a small firm suffers a disaster, the price of the other stocks jumps up. In contrast to these papers, in our economy contagion-like effects might emerge exclusively via the spreading of disagreement from one firm to the other.

**B. The Option Implied Volatility Smile**

Bakshi, Kapadia, and Madan (2003) emphasize that the differential pricing of index and individual options is directly related to the different risk-neutral skewness of index and single stock returns: The more negative the risk-neutral skewness, the steeper the implied volatility smile. In our model, the risk-neutral skewness of single stock and index

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15The overall issuer weighted annual default rate – including both investment grade and speculative grade entities – was the lowest in 2007 since 1981, see Standard & Poor’s (2008).

16In their model, excess co-movement arises because of the increasing uncertainty in bad times. Since there is no country-specific component in the uncertainty, contagion like spreading of uncertainty from one market to the other is not possible.
returns is directly related to the equilibrium price of states associated with (i) low future dividends of a single firm or (ii) low future dividends of both firms. The optimal risk sharing between optimistic and pessimistic agents implies a lower price for contingent claims with positive exposure to future dividends. This price is proportional to a stochastically weighted marginal utility of optimistic and pessimistic agents in the economy, with a weight that is a function of agents’ relative consumption share. Since agents have decreasing marginal utility and the equilibrium consumption share of the pessimist (optimist) is larger in low (high) dividend states, the price of all states tends to be lower in the economy with heterogeneity in beliefs. However, note that the equilibrium state price adjustment associated with low dividend states tends to be larger than the one associated with high dividend states, which yields an endogenous negative risk-neutral skewness of stock returns. In our model, this asymmetry in the equilibrium risk-neutral distribution follows from the fact that the agents’ marginal utility in the economy tends to be larger in low dividend states. It then follows that in order to reallocate a given amount of consumption across agents in bad dividend states, a larger state price adjustment is needed. Due to the assumption that agents have power utility, the marginal utility is convex, and hence the state price of a bad aggregate dividend state is proportionally lower that the average state price of a bad dividend of either one of the two firms in the economy. Therefore, index returns will feature an even more pronounced negative risk-neutral skewness than the returns of each individual stock.

Given the economic mechanism generating the risk-neutral skewness, we expect the differences in risk-neutral skewness of index and individual stock returns to be increasing in the degree of heterogeneity in beliefs ($\Psi_{D_1}(t), \Psi_{D_2}(t)$), the informativeness of the market-wide signal (parameters $\alpha_{D_1}, \alpha_{D_2}$), the average market-wide uncertainty ($\bar{\sigma}_{\mu}$) and the heterogeneity in subjective uncertainty ($\Delta\sigma_{\mu}$). Figure 5 confirms this intuition. First, the difference in (negative) risk-neutral skewness of index and single stock returns is increasing in both $\Psi_{D_1}(t)$ and $\Psi_{D_2}(t)$. The increase is more pronounced in the $\Psi_{D_2}(t)$ direction, as in this case the larger disagreement about market-wide signals generates a higher co-movement of stock returns and a lower price of bad aggregate dividend states. As we increase parameters $\alpha_{D_1}$ and $\alpha_{D_2}$ from 0.1 to 0.45 (left and right panels of Figure 5), the market-wide signal is more informative for aggregate dividend growth, dividend beliefs load more on market-wide signal shocks, and we obtain, as expected, a more pronounced index risk-neutral skewness for any choice of $\Psi_{D_1}(t)$ and $\Psi_{D_2}(t)$. As Figure 5 shows, this effect can be quantitatively large. In a related way, as $\bar{\sigma}_{\mu}$ or $\Delta\sigma_{\mu}$ increase the dynamics of individual dividend beliefs and their disagreement indices load more on shocks to the market-wide signal. These features imply a larger index (negative) risk-neutral skewness, even if the effect is smaller than the one obtained for the comparative statics of $\alpha_{D_1}$ and $\alpha_{D_2}$.

Consistent with the findings in Bakshi, Kapadia, and Madan (2003), the structure of the risk-neutral co-skewness of index and stock returns in our economy directly implies index option implied volatility smiles that are more negatively skewed than those of individual stocks, which is confirmed empirically in options data. To highlight this point, Figure 6 plots the difference between the option implied volatility and the expected volatility of the underlying, both for the index and the individual stocks, as a function of the degree of the moneyness of the options. As expected, index
option smiles are steeper than individual smiles. According to the previous results for the risk-neutral skewness, the
difference in slope can be large and is increasing in the informativeness of the market-wide signal (parameters $\alpha_{D_1}$,
$\alpha_{D_2}$), the average market-wide uncertainty ($\bar{\sigma}_{\mu_i}$), and the heterogeneity in subjective uncertainty ($\Delta\sigma_{\mu_i}$).

[Insert Figure 6 approximately here.]

C. The Correlation Risk Premium

There are two ways to identify the correlation risk premium. The first one is based on at-the-money option spreads,
the second is based on correlation swaps.

Figure 6 shows that the average difference between the option implied volatility and the realized volatility of the
underlying can be larger for index than for individual options, consistent with the data. The at-the-money value of
this difference is often used as a proxy for the volatility risk premium; see, e.g., Carr and Wu (2009). As expected,
the volatility risk premium for the index and the individual stock depends on the structure of the market-wide uncertainty.
For example, in the right column of Figure 6, we obtain proxies for the index (individual stock) volatility risk premium ranging between 2% and 3% (1.2% and 1.6%). By construction, the difference between the
index and individual stock volatility is directly related to the degree of correlation between the returns of the stocks
in the index basket. Therefore, differences between index and individual volatility risk premia can be easily mapped
onto an endogenous correlation risk premium. To the extent that stocks correlation are stochastic and functions
of the degree of market-wide disagreement, correlation risk is priced in our economy. Therefore, trading strategies
with exposure to changes in correlation have to pay a risk premium for their implicit exposure to variations in the
heterogeneity in beliefs across agents. If the econometrician can only observe stock price correlations (as opposed to
the correlation in the difference in beliefs), the correlation of stock returns will emerge as a risk factor explaining (in
reduced-form regressions) the spread between the volatility risk premia of index and individual options. A similar
reduced-form argument is proposed in Driessen, Maenhout, and Vilkov (2009), who explicitly explain the difference
of index and individual variance risk premia by the presence of a substantial correlation risk premium.

While the first method is used quite broadly, it does not provide a pure measure of the economic premium associated
to correlation risk. A short position in the option on the index and a long position in options on the individual
names constituting the index is also exposed to changes in the volatility and in the underlying assets. A theoretically
more demanding, but preferable approach to identify the correlation risk premium is based on the implied price
of a correlation swap, which is provides a pure exposure to changes in correlations. For sake of presentation, it is
convenient to consider first a variance swap. A long variance swap pays the difference between the realized variance
over some time period and the variance swap rate $(RV_i(t, T) - SW_i(t, T)) L_i$, where $L_i$ is the notional dollar value
of the contract of firm $i$, $RV_i(t, T)$ is the realized variance of the stock price of firm $i$ over some time period, and
$SW_i(t, T)$ denotes the variance swap rate. Let $\mathbb{Q}$ denote the risk-neutral probability measure from the viewpoint of
agent A. Since a swap has a market value of zero at initiation, $SW_i(t, T) = E^\mathbb{Q}_T (RV_i(t, T))$. Since the payoff of the

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swap is the realized volatility, the price of the swap must satisfy $SW_i(t, T) = E_t^\mathbb{P} \left( \frac{\Delta V(t)}{\sigma(t)} RV_i(t, T) \right)$ under the physical measure, $\mathbb{P}$. Equivalently, $SW_i(t, T) = E_t^\mathbb{P} (RV_i(t, T)) + \text{Cov}_t^\mathbb{P} \left( \frac{\Delta V(t)}{\sigma(t)} RV_i(t, T) \right)$, where $\text{Cov}_t^\mathbb{P} \left( \frac{\Delta V(t)}{\sigma(t)} RV_i(t, T) \right)$ is the variance risk premium, which can be obtained empirically as the difference between the variance swap rate and the expected variance under the physical measure:

$$VARP_i(t) = SW_i(t, T) - E_t^\mathbb{P} (RV_i(t, T)). \quad (11)$$

Equation (11) highlights how variance swaps are exposed to disagreement risk. The conditional covariance between the stochastic discount factor and the realized variance is positive in our economy. The variance swap pays off the most, when disagreement is high. The variance risk premium on the index is larger than the average individual variance risk premia due to the covariance risk premium term, which is non-zero in our economy. There are two main reasons for this. First, the covariance under the physical and risk-neutral dynamics are different, i.e. $\text{Cov}_t^\mathbb{Q}(S_1, S_2) \neq \text{Cov}_t^\mathbb{P}(S_1, S_2)$. Second, the dynamics of the economic fundamentals are different under the physical and risk-neutral measure, to this end, the drift adjustment leads to a non-zero difference: $\text{Cov}_t^\mathbb{Q}(S_1, S_2) - E_t^\mathbb{Q} \left( \text{Cov}_t^{Q}(S_1, S_2) \right) \neq \text{Cov}_t^\mathbb{P}(S_1, S_2) - E_t^\mathbb{P} \left( \text{Cov}_t^{P}(S_1, S_2) \right)$.

In our economy, the covariance risk premium is inherently counter-cyclical due to its dependence on the disagreement about the business cycle indicator. Negative shocks in the underlying fundamentals cause disagreement to increase which increases the correlation among the different stocks and implies a higher covariance risk premium in times of high disagreement.

To study the model implications for the correlation risk premium, as an empirical counterpart of the variance swap rate we use the standard industry approach and synthesize the variance swap rate from plain (listed) vanilla option prices. Under the assumption of no arbitrage and a continuous swap rate process, the following relation is exact (see e.g. Carr and Madan, 1998, Britten-Jones and Neuberger, 2000 and Carr and Wu, 2009):

$$SW_i(t) = E_t^\mathbb{Q} (RV(t, T)) = \frac{2}{(T-t) B(t, T)} \int_0^\infty \frac{P(K, T)}{K^2} dK, \quad (13)$$
where $B(t, T)$ is the price of a zero coupon bond with maturity $T$ and $P(K, T)$ is an out-of-the-money put option with strike $K$ and maturity $T$. We can now calculate the volatility risk premium for firm $i$ as $\text{VOLRP}_i(t) = \left( \sqrt{\text{SW}_i(t, T)} - \sqrt{\text{RV}_i(t, T)} \right) \times 100$, and for the index.

Figure 7 summarizes the results with respect to the volatility risk premium for an individual firm (left panels) and the correlation risk premium (right panels) as a function of belief disagreement about firm 1 and the common signal.

Both the firm-specific and common disagreement increase the volatility risk premium of firm 1 and the correlation risk premium. Increasing the firm-specific and common disagreement from zero to 0.3 increases the volatility risk premium of firm 1 from 0.2% to 1.6% for $\alpha_{D_1} = 0.1$ and to 0.8% for $\alpha_{D_1} = 0.45$. The correlation risk premium increases from zero to 1% for $\alpha_{D_1} = 0.1$ and 1.6% for $\alpha_{D_1} = 0.45$. These features are consistent with our previous reasoning: as $\alpha$ increases relative to $\beta$, the higher firm-specific and common disagreement increase correlation across stocks and, in addition, generate a higher correlation risk premium.

D. Simulated Option Trading Strategies, Disagreement and Volatility Risk Premia

To investigate variance and correlation risk premia, it is useful to consider factor mimicking portfolios that are exposed to the risk factors that we intend to study. First, we consider portfolios that are exposed to time-varying variance and correlation. Then, we study the extent to which the excess returns of these portfolios can be explained by their exposure to the risk factors we propose, i.e. differences in beliefs after controlling for a host of other potential explanatory variables.

To capture variance risk premia, a well-known trading strategy is a straddle, which involves a long position in a call and a put with identical moneyness and underlying. Another possibility is the dynamic delta-hedging of a long call or put position. The advantage of straddle portfolios with respect to delta-hedged portfolios is that they benefit from the difference in volatility risk premia of both calls and puts, and they involve very liquid at-the-money options; see, e.g., Bondarenko (2003).

We focus on straddle portfolios in the sequel. A straddle generates positive expected

\[ RV(t, t + 30) = \frac{365}{30} \sum_{i=1}^{30} R_i(t_n)^2, \]

where we define a set of dates $t = t_0 < t_1 < \cdots < t_N = T$ and $R_i(t_n) = \log(S_i(t_n, T)/S_i(t_{n-1}, T))$.

One might argue that since stock trading is cheaper than options trading, an advantage of delta-hedging relative to a straddle position is given by the the potentially lower transaction costs. However, in practice dynamic delta-hedging is expensive because strikes have to be rolled over time, which increases transaction costs. Moreover, the choice of the appropriate delta might be cumbersome. Branger and Schlag (2004) show that delta-hedged errors are not zero when either the incorrect model is used or rebalancing is discrete. For instance, in presence of jumps or stochastic volatility Black-Scholes (1973) model-implied deltas generate delta-hedged returns that may be biased. Bakshi and Kapadia (2003a) find that delta-hedged index call and put portfolios statistically under-perform zero, with larger losses for at-the-money options; in addition, the under-performance is worst in periods of higher volatility. These findings are compatible with a negative index volatility risk premium.

Straddle portfolios have been analyzed extensively in the literature; see, among others, Coval and Shumway (2001) and Driessen and Macnhout (2007).
excess returns if the variance risk premium is positive, i.e. if the integrated expected future variance of the underlying returns is larger than the variance swap rate. As shown above, due to the positive co-movement of stock variances and correlations with the stochastic discount factor in our economy, variance risk premia tend to be negative in equilibrium.

To capture correlation risk premia, a well-known strategy is a dispersion trade, i.e. a portfolio consisting of (i) a short straddle portfolio of index options and (ii) a long straddle portfolio of individual stock options. The short index straddle earns the index volatility risk premium. The long individual straddles pay an average of the individual volatility premia. The absolute index volatility risk premium is, on average, larger than the one of individual stocks: the difference is the correlation risk premium. Thus, if correlation risk is priced, dispersion trades can generate (unconditionally) positive expected excess returns. The naive dispersion strategy can be enhanced when, instead of purchasing the whole basket of constituents, the agent selects a subset of individual stock options with the lowest volatility risk premium. These stocks have the smallest difference between the implied volatility and expected future realized volatility. Since the expected future volatility is unobservable, correlation option traders often proxy it by the current realized volatility, since realized volatilities feature a substantial degree of autocorrelation. However, our model suggests a simple alternative: the cheapest options are those of stocks with the lowest degree of dividend disagreement, since the variance risk premia are increasing in the degree of heterogeneity in beliefs. When belief disagreement is large for one firm, the volatility risk premium for that particular firm tends to be large as well, since investors want to be compensated also for holding the disagreement risk. Therefore, if our model predictions are correct, a portfolio of short index straddles and long individual straddles of stocks having the lowest disagreement should yield excess returns. This idea provides a natural sorting device. If the model is correct, the sorted portfolios provides sorted expected excess returns.

Using the calibrated model parameters, we simulate the excess returns of dispersion trades consisting of a portfolio of both individual and index option straddles. We simulate an economy with two stocks having equal weight within the index basket. To compute option returns, we make use of the pricing expressions provided in Proposition. The parameters used for the simulation are summarized in Table. To be consistent with the main features of our data set, we simulate return time series of 11 years and 5 months (2,877 days).

At the beginning of each month, at-the-money index straddles are sold and at-the-money straddles on one of the two individual stocks are bought, in a way that makes the portfolio vega neutral with respect to the individual stock volatility. To construct the individual straddle, we consider the stock with the lower belief disagreement at the beginning of the month. The position is held for one month and it is dynamically delta hedged within the month, by buying a corresponding amount of the individual stock and investing the remainder in the risk-free bond. In our Monte Carlo simulation, we generate 137 monthly observations on gains and losses from the dispersion trade.

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20 The assumption of an economy with two stocks is for simplicity, since the computational costs for an economy with more stocks are very large. However, the main intuition of our findings about option strategies will remain in an economy with more than two stocks.

21 Delta neutralizing the portfolio poses some difficulties for the trade. Constant rolling of positions and dynamically hedging the basket with stocks exposes the trader to spread risk and transaction costs. Even if straddles have a delta of almost zero at initiation, market movements can cause the delta to change. A more direct approach would be to short variance swaps on the index, since they do not require to delta hedge the position in order to obtain a pure volatility exposure. Since we do not have data on variance swaps in our empirical tests of the model, we consider dispersion trades based on plain options only.
We also compute the returns of a simple short index put strategy, which is well-known to generate high returns and attractive Sharpe ratios in the data. This strategy is a natural benchmark of comparison for the dispersion strategy.

Table 3 reports summary statistics for the returns of the different trading strategies considered. We find that the dispersion trade yields large risk-adjusted returns: Its annualized Sharpe ratio is as large as approximately 1.9. The average return is slightly higher than the one of the short-index put, but the Sharpe ratio of the latter strategy is much lower (0.607) due to the very high volatility of its returns. The returns of the dispersion trade also feature a much lower negative skewness and a slightly lower kurtosis than those of the short-index put strategy.

In our model, each contingent claim is priced at its fair price and excess returns arise exclusively because of some exposure to systematic risk factors. In a standard CAPM-type of setting, the market return is the only relevant risk factor. However, in our model priced disagreement risk is an additional risk factor generating excess returns. Therefore, it is natural to ask whether these risk factors can jointly explain part of the excess returns of the different trading strategies. We construct a proxy of common-disagreement in our economy as the weighted average of the two firm-specific disagreement processes \( \Psi_{D_1}(t) \) and \( \Psi_{D_2}(t) \) in the economy. We will follow the same approach in our empirical study later. We then regress the excess returns of the different trading strategies on the market return and the common disagreement component. The estimated coefficients are summarized in the last lines of Table 3.

The short-index put strategy has an estimated market beta coefficient which is statistically significant at the 10% level, which is an indication of the well-known exposure of the returns of this strategy to large drops in the market; see also Broadie, Chernov, and Johannes (2007). Interestingly, the short index put has a statistically significant exposure to the common disagreement component, indicating that this trading strategy might yield excess returns also because of an existing exposure to a risk factor that has not been analyzed in the previous empirical literature. In contrast to the short-index put, the dispersion trade does not yield a significant \( \beta \) exposure to market risk, but it implies, as expected from our model, a significant estimate of \( \beta \) with respect to the common disagreement proxy.

There are two important learning points from this exercise: (a) a long-short position in index and individual options embedded in the dispersion strategy is not exposed to market risk. Thus, it should not be surprising that previous empirical studies have found small CAPM-betas. (b) This, however, does not imply that the strategy is riskless and that dispersion trades positive excess returns are indication of misspricing. Indeed, the opposite is true: the dispersion trade is exposed to differences in beliefs. Since differences in beliefs are priced in equilibrium, dispersion strategies generate positive expected excess returns and, by no means, these are risk free.

In what follows, we take these results to the data and conduct a rigorous study of the relation between excess returns of correlation trades using option trading strategies and common disagreement.
III. Data

In order to test empirically the main predictions of our model, we build a panel data set consisting of option and stock prices on all constituents of the S&P 100 index, the time series of index spot and option prices, and analysts’ forecasts of future earnings from the I/B/E/S database. The sample period of our study goes from January 1996 to June 2007.

A. Options Data

We use option information from the OptionMetrics Ivy DB database, which is the most comprehensive database available. Data runs from January 1996 to June 2007. Our index option sample contains trades and quotes of S&P 100 index options traded on the Chicago Board Options Exchange (CBOE). The S&P 100 is a capitalization-weighted index with quarterly re-balancing. Options on the index are European style and expire on the third Friday of the contract month. Our sample also consists of trades and quotes of CBOE options on all constituents of the S&P 100. Individual stock options are American style. They usually expire on the Saturday following the third Friday of the contract month. Therefore, time to maturity is defined as the number of calendar days between the last trading date and expiration date. We apply a number of data filters to circumvent the problem of large outliers. First, we eliminate prices that violate arbitrage bounds, i.e., call prices are required not to fall outside the interval \((Se^{-rd} - Ke^{-r\tau}, Se^{-rd})\), where \(S\) is the price of the underlying asset, \(K\) is the strike price, \(d\) is the dividend yield, \(r\) is the risk-free rate, and \(\tau\) is the time to maturity. Second, we eliminate all observations for which (i) the ask is lower than the bid price, (ii) the bid is equal to zero, or (iii) the spread is lower than the minimum tick size (equal to USD 0.05 for options trading below USD 3 and USD 0.10 in any other case). To mitigate the impact of stale quotes we eliminate from the sample all observations for which both the bid and the ask are equal to the one on the previous day. We focus on short-term options, which are known to be the most liquid, with a time to maturity between 14 and 31 days.

B. Stock Returns Data

Stock data is retrieved from the CRSP database. To calculate the realized volatility, we use daily returns from CRSP for single-stocks and from OptionMetrics for the index. We calculate the realized volatility over 21-day windows, requiring that the stock has at least 15 non-zero return observations.

C. The Index of Uncertainty and Differences in Beliefs

To obtain a proxy of firm-specific belief disagreement \(\Psi_{D,t}(t) = (m_{D,t}^A(t) - m_{D,t}^B(t)) / \sigma_{D,t}\), we follow the approach developed in Buraschi, Trojani, and Vedolin (2008). We use analysts’ forecasts of earnings per share from the Institutional Brokers Estimate System (I/B/E/S) database and compute for each firm the mean absolute difference

\[\text{CRSP data only runs until December 2006. For the remaining six months, we rely on stock prices from OptionMetrics.}\]
of analysts’ earning forecasts. This provides an empirical counterpart for the numerator \((m^{D_i}_1(t) - m^{B_i}_1(t))\). This process is then scaled by an indicator of earnings uncertainty to produce an empirical counterpart for \(\Psi_{D_i}(t) = (m^{D_i}_1(t) - m^{B_i}_1(t)) / \sigma_{D_i}\).

With regards to the common signal, we consider the case \(z(t) = D_1(t) + D_2(t)\) and compute a single index of common disagreement from the entire panel of firms. In order to obtain an index for the common belief disagreement \(\Psi_z(t)\), we estimate a dynamic common component using a dynamic factor analysis applied to the panel of analysts’ earning forecasts in the I/B/E/S database.\footnote{Factor analysis has mainly been implemented for forecasting measures of macroeconomic activity and inflation; see, e.g., Stock and Watson (2002a, 2002b, 2004). More recently, it has been used in financial applications; see Ludvigson and Ng (2007, 2009).}

The empirical evidence in Section B.1 highlights the existence of a substantial degree of commonality in disagreement across firms and sectors, with the presence of dynamic lagged effects in the multivariate dynamics of individual disagreement proxies.\footnote{See also, for instance, Buraschi, Trojani, and Vedolin (2008) for a related example in connection with the 2007 credit crisis.} Uncertainty shocks affecting analyst forecasts in one firm, or in one industry, often propagate to other firms in the same sector, and to other sectors, both contemporaneously and also with lags. To take into account these dynamics, we use a methodology that allows for serial correlation across differences in beliefs. Principal component techniques are not able to address this issue since they can only capture contemporaneous correlations. We use therefore a method proposed by Forni, Hallin, Lippi, and Reichlin (2000) which is based on the estimation of a generalized dynamic factor model specification. The specification allows for lead and lag correlation across different observable economic variables. First, we estimate the serial covariance \(\Gamma\) from the panel of firm-specific DiB and calculate the spectral density \(\Sigma(\omega)\). Second, from the eigenvalue structure of \(\Sigma(\omega)\), we reduce the dimensionality to \(q\) using a spectral single-value decomposition. The aggregation procedure minimizes the “idiosyncratic to common” factor variance ratio. Third, given that these \(q\) dynamic factors are assumed to drive the dynamics of \(\Psi_{D_i}(t)\), we estimate the dynamic factor model using a linear filter. Last, we project the factors onto a 1-dimensional space using generalized principal component. This procedure identifies a common dynamic factor that uses information from the entire serial correlation structure of the original panel and it can handle the large scale nature of the problem.\footnote{See the Appendix B and Forni, Hallin, Lippi, and Reichlin (2000, 2005) for details. In a first step, we estimate the spectral density matrix of the common and idiosyncratic components using a dynamic principal component procedure. From the estimated spectral density matrices, we obtain the estimated covariances of the common component by Fourier transform. In a second step, the factor space is estimated. Generalized principal components are computed as particular linear combinations of the observable disagreement processes that have the smallest idiosyncratic-common variance ratio. In the estimation of the common component, we also weight each individual disagreement proxy by its market capitalization. As a robustness check, we also used equal weights, but the results remained quantitatively the same.}

Additional technical details on the estimation method are provided in the Appendix.

\section*{D. Other Control Variables}

In order to isolate the additional explanatory power of common and firm specific belief disagreement, we consider several control variables in our regressions. A first natural variable is the market volatility. We calculate market volatility from CRSP and OptionMetrics as the 21-day historical realized volatility of returns. A higher market volatility tends to increase the index option implied volatility. Moreover, the nontrivial co-movement between individual stock volatility and market volatility is likely to affect also the implied volatility of individual options. Thus, controlling for market volatility allows us to study the impact of disagreement on variance risk premia, in
distinction from the indirect effect that might arise because volatilities and belief disagreement proxies are likely related. Option implied volatilities are also related to firm specific CAPM Beta. Chang, Christoffersen, Jacobs, and Vainberg (2009), for instance, find a significant relation between option implied volatilities and estimated betas. Therefore, in order to control also for such CAPM-type of effects we calculate monthly conditional betas, using historical returns over a window of 180 days, and include them as explanatory variables in our regressions.

We control for option liquidity effects using as a liquidity proxy the ratio between option trading volume and shares in the underlying outstanding. In separate analysis not included in the tables we also consider the Pastor and Stambaugh (2003) measure of liquidity in the underlying equity market.

In addition, since jump risk can affect the underlying distribution of stock returns and hence, potentially, the volatility risk premium, even in absence of disagreement risk, we explicitly control for differences in returns risk neutral skewness. Bakshi, Kapadia, and Madan (2003) give a theoretical explanation for how skewness is related to the implied volatility: The more negative the risk-neutral skewness the steeper the implied volatility function even in absence of disagreement risk. As a proxy for risk neutral skewness we use the difference between the implied volatility of a put with 0.92 strike-to-spot ratio (or the closest available) and the implied volatility of an at-the-money put, dividend by the difference in strike-to-spot ratios.

We also account for other standard risk factors in the literature using the two Fama and French (1993) factors and the momentum factor. These data are available from Kenneth French’s web page. Business-cycle effects are captured by macro factors, proxied by the market price-earning ratio, industrial production, housing start number, the producer price index, and non-farm employment. We estimate a latent macro factor using dynamic factor analysis applied to time series of industrial production, housing start number, the producer price index, non-farm employment, and the S&P 500 P/E ratio. We retrieve S&P 500 price-earnings data from the S&P webpage, and the other macro variables from FRED.

We summarize the sample moments of the most important variables in Table 4. Both the index and the average individual volatility risk premia are negative, but the index volatility risk premium is twice as large as the average individual risk premium. A test of the null hypothesis that average index implied and realized volatilities are equal is strongly rejected by means of a standard t-Test based on Newey-West (1987) autocorrelation consistent standard errors with 22 lags. For 25% of the individual firms in our sample, the null hypothesis of a zero volatility risk premium cannot be rejected at the 5% confidence level.

We also applied different lengths of the estimation window. We follow Lewellen and Nagel (2006) and use shorter data windows based on daily returns.

Since the price-earning ratio for the S&P 100 does not exist, we use price-earnings data from the S&P 500.

Driessen, Maenhout, and Vilkov (2009) find that 2/3 of firms in their sample has a volatility risk premium not statistically different from zero using average model-free implied and realized volatility. Using a subset of 25 firms and a short time period (January 1991 to December 1995), Bakshi and Kapadia (2003b) find an average difference between realized and implied volatility of single stock options of -1.5%. Carr and Wu (2009) use a sample of 35 firms and find a slightly stronger evidence of a variance risk premium in individual options. Duarte and Jones (2007) study the largest set of options with 5,156 stocks and find no evidence of a non zero average volatility risk premium. However, they find strong evidence of a conditional risk premium that covaries positively with market volatility. Goyal and Saretto (2009) find a positive average volatility risk premium in individual options.
IV. Empirical Analysis

We test the main predictions of our model in four different ways. First, we run panel regressions to study the relation between belief disagreement, the volatility risk premia of individual and index options, and the covariance risk premium embedded in index options after controlling for other systematic factors. This analysis allows us to quantify empirically the link between variance and covariance risk premia and the degree of dispersion in beliefs as suggested by our model. Second, an implication of our model is that belief disagreement is priced both in the time series and cross-section. Therefore, we consider option-based dispersion trading strategies that are exposed to belief disagreement and quantify the excess returns when these portfolios are sorted according to their exposure to a common factor of difference in beliefs. To examine the cross-section, we sort the same portfolio with respect to their cross-sectional exposure to firm-specific difference in beliefs. Then, we examine the monthly portfolio excess returns of these strategies both in the time series and cross-section and ask whether they are related to belief disagreement, while controlling for other potentially relevant stock and option characteristics.

A. Volatility and Covariance Risk Premia

For each individual firm $i$, denote by $RV_{i,t} - IV_{i,t}$ the difference between the realized volatility and the option implied volatility at time $t$, which we take as a proxy for the latent volatility risk premium. We consider the following regression for individual volatility risk premia:

$$\text{Volatility Risk Premium} = \beta_0 + \beta_1 \text{DIB}_{i,t} + \beta_2 \text{DIT}_{i,t} + \sum_{j=3}^{7} \beta_j \text{Control}(j)_{i,t} + \sum_{k=1}^{2} \gamma_k \text{Control}_{k,t} + \epsilon_{i,t},$$

(14)

where $\text{DIB}_{i,t}$ is the proxy of belief disagreement of each individual firm $i$ at time $t$, $\text{DIT}_{i,t}$ is the common disagreement proxy, $\text{Control}_{i,t}$ are control variables of each firm $i$ at time $t$, and $\text{Control}_{k,t}$ are time-series determinants, such as market volatility and a macro factor. For the index volatility risk premium, we consider the following regression:

$$\text{Index Volatility Risk Premium} = \beta_0 + \beta_1 \text{DIB}_{t} + \sum_{k=1}^{7} \beta_k \text{Control}_{k,t} + \epsilon_{t}.$$  

(15)

where $RV_t$ and $IV_t$ are the index realized and implied volatility at time $t$. Finally, we exploit relation (12) in order to compute from individual and index volatility risk premia a proxy for the implied covariance risk premium, denoted by $\text{COVRP}_t$. The corresponding regression is:

$$\text{COVRP}_t = \beta_0 + \beta_1 \text{DIB}_{t} + \sum_{k=2}^{3} \beta_k \text{Control}_{k,t} + \epsilon_{t}.$$  

(16)

In the context of our model, belief disagreement increases the absolute size of the volatility risk premia of index and individual options, proxied by the difference of realized and implied volatility, and the covariance risk premium.

\footnote{Equation (12) assumes constant index weights $\omega_i$ during the lifetime of the option.}
Since these risk premia tend to be negative, we expect a negative estimated coefficient for the disagreement proxies in our regressions.

The results in Table 5 indicate as expected a negative relation between disagreement, individual volatility risk premia (column (1)-(5)), index volatility risk premia (column (6)-(8)) and covariance risk premia (column (9)). In the regressions for individual volatility risk premia, the coefficient of both firm specific and common disagreement proxies are highly statistically significant, irrespective of the control variables used.

In column (1) (column (6)), a regression including the disagreement proxies, a proxy for jump risk, the market volatility and the common macro factor explains 26% (22%) of the variation in individual (index) volatility risk premia. The contribution of the disagreement to variations in volatility risk premia is economically significant: In a regression without belief disagreement proxies, the fraction of variance in volatility risk premia explained is less than 10%.

Thus, heterogeneity in beliefs has an economically relevant impact on both the cross sectional properties of individual volatility risk premia - this impact is captured by the idiosyncratic disagreement proxies - and the time series dynamics of volatility risk premia. This last effect is well captured by our empirical proxy for common disagreement.

Market volatility and the common macro factor are statistically significant and the result is robust to the choice of other control variables. Market volatility tends to increase the absolute size of volatility risk premia, indicating that a part of the volatility risk premium can be related to a priced market volatility factor embedded in index and individual option returns; see, also, Buraschi and Jackwerth (2001) and Bakshi and Kapadia (2003b). The macro factor has a positive association with the volatility risk premia: Intuitively, good business cycle conditions and a smaller macroeconomic uncertainty lower the implied volatility of options and imply a less negative volatility risk premium. Despite the significant effect of the macro factor, it appears that market-wide uncertainty reflected by the common disagreement proxy has an additional significant role in explaining volatility risk premia. The proxy for jump risk is marginally statistically significant, especially in the regression for individual firms, and it tends to decrease the absolute size of the volatility risk premium. Moreover, its statistical significance depends on the choice of the other control variables used. When considering also CAPM-beta and liquidity effects (see column (2)), we find marginally significant parameter estimates and a smaller economic impact of these variables on individual volatility risk premia: While firm-specific DiB and Common DiB predict on average an increase of 0.8% in the absolute size of the individual volatility risk premium, the average increase implied by the CAPM Beta is 0.6%. On the one hand, this confirms the importance of the traditional CAPM beta to explain the cross-section of the volatility risk premia, on the other hand the results highlight the role of uncertainty as an important explanatory factor.

Finally, column (1) of Table 6 shows that the common disagreement proxy has a large statistically and economically significant impact on the (negative) covariance risk premium embedded in index options: A one percent increase in Common DiB implies an increase of almost 50% of the average correlation risk premium. Since the common disagreement proxy is counter-cyclical, this finding highlights a large negative correlation premium during periods of high uncertainty and high market volatility. This premium compensates investors ex-ante for the potentially
higher average correlation between stock returns during market turmoils or financial crises, and for the resulting lower ex-post degree of diversification.

B. Option Trading Strategies Exploiting the Volatility and Correlation Risk Premium

Previous results show that variance and covariance risk premia embedded in index and individual options are explained to an economically relevant degree by our proxies for idiosyncratic and common belief disagreement. Intuitively, option trading strategies that create exposure to changes in volatility or correlation should yield on average to the position holder the variance or correlation premium, respectively. Since these premia are related, at least in the context of our model, to the degree of disagreement in the economy, such strategies are likely to generate also an implicit exposure to unexpected changes in the level of disagreement.

Our model predicts that disagreement is a priced risk factor in equilibrium. To investigate this property empirically, we consider the returns on factor mimicking portfolio and study their exposure to these potential risk factors. First, we look at the returns of a portfolio exposed to volatility (straddles); then, we consider the returns of a portfolio exposed to correlation (dispersion). If difference in beliefs is priced and helps to explain correlation (volatility) then High-minus-Low dispersion (straddle) portfolios sorted with respect to difference in beliefs will generate positive excess returns. In this investigation, the cross-sectional role of the uncertainty is captured by a firm-specific measure of differences in beliefs, while the time-series role of uncertainty is captured by a common dynamic factor of disagreement.

B.1. Trading the Cross-Section of Individual Volatility Risk Premia: Option Mispricing or Return for Disagreement Risk?

Goyal and Saretto (2009) study in detail option trading strategies that take advantage of the cross sectional differences in volatility risk premia of different stocks. They find that a portfolio long (short) the options with the largest positive (negative) difference between realized and implied volatility yields economically significant returns. They interpret these findings as evidence of option mispricing. Since we find that the cross section of individual volatility risk premia is related to the degree of disagreement, in the context of our model trading strategies similar to those studied in Goyal and Saretto (2009) are implicitly exposed to changes in disagreement. Therefore, it is interesting to investigate whether the excess returns of these strategies can be explained by variations in our proxy of common disagreement. This is important since, in this case, positive excess returns would not necessarily be evidence of option mispricing. To test this hypothesis, we run the same exercise as in Goyal and Saretto (2009) and construct each month the following portfolios. First, we build quintile straddle portfolios sorted according to the difference between realized and implied volatility of individual stocks. The lowest (highest) quintile consists of those stocks with the largest (smallest) volatility risk premium. Finally, we consider a spread portfolio being long the lowest quintile portfolio and short the highest quintile portfolio. We then study the risk-return profile of the returns of this strategy and, in particular, whether it generates exposure to unexpected changes in the common disagreement factor.
A more direct way of constructing option portfolios with exposure to disagreement risk can exploit the cross sectional differences in firm specific disagreement across different stocks. With this objective in mind, we build quintile straddle portfolios sorted according to the degree of firm specific disagreement of individual stocks. The lowest (highest) quintile consists of those stocks with the smallest (highest) degree of firm-specific disagreement. We then consider again a spread portfolio being long the lowest quintile portfolio and short the highest quintile portfolio. Finally, we study the risk-return profile of the returns of this strategy and whether it generates excess returns reflecting exposure to unexpected changes in the common disagreement factor.

Table 7 reports summary statistics of the returns of the different quintile portfolios, as well as the return of the spread portfolio 1-5, formed by taking a long position in the straddles in quintile one and by shorting the straddles in quintile five. Panel A (Panel B) presents the results for the sorting procedure based on cross sectional differences in variance risk premia (firm specific disagreement).

Both Panel A and B present a strikingly monotonically decreasing pattern in average returns from the first to the fifth quintile: Average returns in Panel A (B) decrease from about 5.2% (7.2%) to about -10.3% (8.7%). Similarly, annualized Sharpe ratios decrease from about 0.49 (0.61) to about -1.23 (-1.56). Therefore, the spread portfolio 1-5 leads to an annual average return of 15.5% (15.9%), a standard deviation of 0.35 (0.38) and a Sharpe ratio of 1.51 (1.43). Thus, in terms of pure annual mean-variance tradeoff both spread portfolios deliver a similarly attractive risk-return profile. Are these returns due to option mispricing or to a compensation for exposure to some systematic risk factor? In order to answer this question, we regress the returns of each straddle portfolio in Panels A and B of Table 7 on a set of risk factors including the return of the market portfolio, the two Fama and French factors, a momentum factor, a liquidity factor, the VIX volatility index and our proxy for common disagreement. We find that while the spread portfolios in Panel A and B have to a large extent a non significant exposure to standard risk factors, they have a substantial statistically significant positive exposure to the common disagreement proxy and the liquidity factor. The exposure to the common disagreement proxy of the spread straddle portfolios is significantly related to the positive exposure in the long side of this strategy, i.e., the low quintile portfolio, and less so to the short position in the strategy, i.e., the high quintile portfolio. Moreover, while the liquidity factor seems to explain part of the returns in spread portfolios, it has no significant impact on the returns of the different quintile portfolios. Interestingly, the main summary statistics and estimated exposures to the common disagreement for the spread portfolios are similar, both for the volatility risk premium and the disagreement sorting procedure. This feature is induced by the positive cross sectional relation between disagreement and volatility risk premia, documented in the previous section. Overall, both sorting procedures create a similar positive exposure to the systematic common disagreement factor and a corresponding compensation, in terms of average excess returns, for this risk. The positive exposure indicates that the strategy generates excess returns when common disagreement is high, by gaining more on the long side of the trade (the long low quintile portfolio) than on its short part (the short high quintile portfolio). On the other hand, the strategy tends to loose money on average when common disagreement is low. These return
patterns are remarkably consistent with the calibrated model implied straddle returns simulated in Table 3. Overall, given the counter-cyclical structure of the common disagreement, the spread portfolios studied in Table 7 will tend to earn money on average in periods of market distress or financial crises, but they will tend to earn less when financial markets recover from such periods. The positive skewness of the returns of the spread portfolios also suggests an asymmetric return pattern within and outside periods of financial distress, with potentially large positive returns during crisis periods and relatively moderate losses in phases of market recovery.

B.2. Trading the Average Stock Correlation: Correlation Risk Premia and Returns for Common Disagreement Risk

A natural way to form portfolios exposed to changes in correlation is by means of a dispersion trade. These are well-known trading strategies and are based on a short position in a straddle option on the index and a long position in a portfolio of option straddles on the individual constituent stocks. Such strategies aim to extract the correlation risk premium embedded in index options. Therefore, they come in handy to test empirically to what extent correlation premia are a compensation for exposure to a common disagreement risk factor, as predicted by our theoretical model.

The design of our empirical test is based on two observations. First, it has been noticed that in practice, trading a whole book of constituent volatilities can be very expensive in terms of implied transaction costs, or it can be difficult because of liquidity issues. Therefore, it is more convenient to trade only a sub basket of individual stock volatilities, by means of so-called proxy hedging. The art of such a trade is to find the cheapest options to form the basket according to some appropriate selection criterion. A successful choice of the cheap options allows the spread trader to partly disentangle excess return components for average correlation risk from those for average individual volatility risk, while at the same time producing attractive excess returns. Intuitively, different selection criteria for the cheap options might produce spread portfolios with similar correlation risk exposure, but with potentially quite different excess returns. Second, if a common factor driving the difference in beliefs is priced and helps to explain correlation risk premia, dispersion portfolio sorted by their exposure to this risk factor will highlight this economic link.

A. Sorting the Stocks in the Long Part of Dispersion Portfolios.

We adopt four different criteria to select into five different quintiles the individual options for the long position of the dispersion portfolio. First, we sort individual options based on firm-specific disagreement, since in our model options with the lowest disagreement are expected to have larger volatility risk premia. The following three sorting procedures are based on the results of the extant literature and can be used to benchmark our results. In the second sorting procedure we use the CAPM beta of the underlying stock to populate the five quintiles sub-samples. To the

30Driessen, Maenhout, and Vilkov (2009) find that their correlation trading strategy does not yield significant abnormal returns after transaction costs and margins. In their study, they employ a trading portfolio consisting of all constituents of the S&P 100, which is in general very costly. The industry solution to this problem is to reduce the number of individual stocks in the portfolio. A major reason for the large transaction costs of a portfolio of all index constituents is the so called spread risk, or slippage. Spread risk is due to illiquidity and market makers behavior: If an option is not actively traded, the market maker widens the volatility spread she is prepared to trade for it. In order to construct successfully a market or capitalization weighted basket of single stocks, a dispersion trade needs tight spreads that accurately reflect the true degree of correlation of an underlying single stock with the index. When a widening spread weakens this correlation link, the whole dispersion trade becomes less economically viable.
extent that individual option implied volatilities or volatility risk premia are positively related to the market risk exposure of the underlying stock, this is a natural choice. Third, we sort individual options based on the degree of liquidity of the underlying stock, since more liquid stocks are likely to generate lower option implied volatilities, coeteris paribus. Fourth, we sort individual options based on the difference between realized and implied volatility, which is a direct market-based proxy for volatility risk premia. We label this sorting procedure the “naïve sorting”.

To summarize, we use the following sorting criteria for the long position of the dispersion strategy:

2. “Beta Sorting”: The dispersion trader calculates monthly CAPM betas and chooses the options of the firms in the quintile with the lowest estimated beta exposure.
3. “Liquidity Sorting”: The dispersion trader selects the options in the highest liquidity quintile.
4. “Naïve sorting”: The dispersion trader selects the options in the highest quintile with respect to the difference between realized and implied option volatility.

In developing the different dispersion strategies, we fix the portfolio weights of the individual options by hedging away the individual volatility risk with a vega-neutral position. We then fix delta-neutral portfolio weights for the individual stocks underlying these options and invest the rest of the wealth in the risk-free asset. The resulting dispersion portfolio shorts the index straddle and invests the remainder of the total wealth in individual straddles, individual stocks and the risk-free bond.

In order to construct a time series of returns for the dispersion portfolios, we circumvent microstructure biases and initiate all option portfolio strategies on Tuesdays, as opposed to the first trading day (Monday). Returns are computed using as a reference beginning price the average of the closing bid and ask quotes on the previous day. As a reference final price for all options in the portfolio, we compute the realized value of the terminal payoff at the option’s expiration date. After expiration of each option, a new option is selected and a new monthly return is calculated following the above procedure. We use equally weighted monthly returns on calls and puts. Such a procedure is repeated for each month.

Table 8 provides summary statistics for the returns of the different dispersion portfolios, a short index put strategy on the S&P 500 and the return of the S&P 500 index itself. The DiB sorted portfolio features an attractive unconditional mean-variance tradeoff, with an average return of 16.54% and an annualized Sharpe ratio of 1.51. This compares to a return (Sharpe ratio) of 9.52% (0.79) based on the naïve sorting. The compensation for exposure to the disagreement factor is potentially large and all other dispersion strategies are mean-variance dominated by the DiB dispersion portfolio.

In order to understand the dynamics of these sorted returns, we regress the dispersion portfolio returns on monthly innovations to the common DiB proxy (denoted by $\epsilon^{DiB}$) and the traditional variables that are found to affect the

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31 Our results in Table 5 indicate a positive link between Market Beta and volatility risk premia of individual options.

32 These statistics are comparable to those in Goyal and Saretto (2009), who report a monthly return of 22% for a straddle portfolio and 2.6% for delta-hedged puts. Similarly, Coval and Shumway (2001) report a 3% return per week for a zero-beta at-the-money straddle portfolio on the S&P 500.

The estimated regression coefficients in Table 8 show that, in contrast to the short index put portfolio, which has a large and statistically significant beta on the S&P500 equal to 10.29 (see Bondarenko, 2003, and Broadie, Chernov, and Johannes, 2009, for an in-depth discussion on this strategy), all dispersion portfolios have no apparent exposure to market risk. Moreover, none of the dispersion portfolio returns can be explained by the traditional risk factors which are used to explain the cross-section of stock returns. At the same time, we find that all dispersion portfolios have a large and statistically significant exposure to the common disagreement factor. Consistent with the model-implied predictions of Table , the impact of the Common DiB on the average return of dispersion strategies is large. For instance, a one standard deviation change in the common disagreement proxy implies an increase of the average returns of the DiB sorted dispersion portfolio of about 3.5%.

This is very interesting as this shows, on the one hand, that the excess returns from dispersion trades cannot be easily explained using traditional economic risk factors and, on the other hand, that they feature a distinctive exposure to common disagreement risk. These findings are consistent with our model’s intuition that correlations risk premia are to a good extent compensation for a priced common disagreement factor.

[Insert Table 8 approximately here.]

B. Double Sorting the Stocks in the Long Part of Dispersion Portfolios.

The statistics in Table 8 show that the sorting procedure based on cross-sectional differences of firm specific disagreement for the long side of the dispersion strategy generates a more favorable mean-variance tradeoff. Our empirical also reveal that, in addition, firm specific disagreement tends to be strongly correlated in the cross-section, leading to a large common component in our empirical proxies of heterogeneity in beliefs.

Although these results shed new light on the components driving the correlation risk premium, their limitation is that a single sort makes it hard to isolate the two distinct roles of the firm-specific and the common disagreement component. Similarly, it would be desirable to isolate the distinct role of the common disagreement from other common factors, such as liquidity risk, as considered in Table 8. One way to overcome this difficulty is to study double-sorted dispersion portfolios. To this end, at the beginning of each month we double sort the straddle portfolios in the long side of the dispersion trade with respect to exposures to pairs of different potential risk factors.

In order to keep the straddle portfolios in the long side of the dispersion trade well populated, we first sort stocks into terciles, rather than quintiles, according to their exposure to the following firm-specific characteristics: firm-specific

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33 Monthly changes in the common disagreement have a mean of virtually zero, a standard deviation of 0.7% and a negligible autocorrelation of -8%. As a robustness check, we also applied an AR(2) specification to measure innovations in the common disagreement. However, the main results remain quantitatively the same.

34 As a robustness check, we also considered regressions including monthly changes in the VIX, as a proxy for shocks to market volatility risk, finding no significant exposure to this variable.
disagreement, CAPM beta sensitivity, and liquidity. We select those stocks that belong to the first tercile. Then, within these terciles we sort according to their exposure to systematic risk factors: the common disagreement proxy, market risk, size, book-to-market, momentum, and aggregate liquidity. Thus, when moving across terciles in the systematic risk factor dimension, we can quantify the importance of exposure to common risk factors for the excess returns of a dispersion trade, while controlling for stock specific characteristics.

Table 9 reports summary statistics for the returns of dispersion portfolios based on the above double sorting procedure. For each dispersion trade and each tercile along the exposure to the common risk factors dimension, we always use the lowest tercile of stocks with respect to firm-specific characteristics in order to construct the long part of the dispersion trade (i.e. options with the theoretical highest expected returns). Panel A (B and C) uses as a firm specific sorting criterion the degree of firm specific disagreement (the CAPM beta sensitivity and the individual exposure to the common liquidity factor, respectively). Within each Panel, Columns 1 to 3 correspond to returns of dispersion portfolios using the different terciles along the risk factor dimension for building the long part of the strategy.

The first striking result of our double sorting exercise is that the sorting with respect to the common DiB risk factor in Panel A produces the most consistent monotonically increasing risk reward tradeoff, e.g., in terms of Sharpe ratios, when going from the first to the third tercile. For instance, in Panel A the Sharpe ratio of the dispersion portfolio for the first common DiB tercile is 1.18, against the Sharpe ratios of 0.93 and 0.78 in the second and third tercile, respectively (first row). This shows that even within the first tercile of options on stocks with the lowest firm-specific DiB, knowing their exposure to the common DiB risk factor improve excess returns both in absolute and mean-variance terms. This suggests that the excess returns of the cross-sectional sorting of individual stocks with respect to individual DiB is in large part due to the role of the individual stock exposure to common DiB, consistent with the large common component and co-movement of individual DiB, as found earlier.

When we condition on other firm-specific characteristics, such as CAPM beta for instance, we find that among the six exposures to systematic factors, the one to the common DiB factor has the highest impact on both expected excess returns and Sharpe ratios. The first three columns of Panel B show that sorting with respect to CAPM Beta and Book-to-Market generate an excess return of 3.81% and an annual Sharpe ratio of 0.29 (Panel B, fourth row, first and third column). On the other hand, the second sort with respect to the exposure to the common DiB generates an excess return of 4.21% and an annual Sharpe ratio of 0.37 (Panel B, fourth row, first and third column). The Sharpe ratios of terciles across the other common risk factors are lower and their degree of variability from the first to the third terciles is also smaller.

Overall, the results shown in Table 9 document that firm-specific DiB risk helps explaining an important part of the excess returns of the long side of dispersion trades. The analysis based on double sorting the factor mimicking portfolios show that a good fraction of these large expected excess returns derives from a conditional exposure of the dispersion trade to a common disagreement factor, which we interpret as exposure to a common uncertainty factor.
Section B.2 documents that dispersion portfolios can produce attractive excess returns. However, these strategies have a significant non-zero beta with respect to a proxy of systematic disagreement risk. These results are also confirmed by the regression results in Table 8 when we study the link between these excess returns and unexpected shocks in the common disagreement proxy and are consistent with the model-implied regressions in Table 3.

To understand in more details the extent to which one can interpret the returns of dispersion portfolios as compensation for systematic risk or mispricing, we study the characteristics of the periods in which the dispersion strategy precipitate important losses. Do these periods occur randomly or is the occurrence linked to unusual realizations of some specific explanatory factor? The model suggest that large ex-post levels of common disagreement - linked to a high ex post degree of average stock correlation - should generate large portfolio losses in the short part of the dispersion strategy. This phenomenon is supported by the preliminary example discussed in the Introduction, that shows that the largest losses of dispersion portfolios arise during crisis periods, characterized by larger than average increases in the common disagreement risk. The existence of such a hidden extreme risk component is partly suggested also by the negative skewness of the returns of dispersion strategies in Table B.2. However, the positive parameter estimates of the disagreement proxy for the short put index strategy in Table B.2 confirm that this risk is not well reflected by the conditional mean effect of the common disagreement on portfolio excess returns.

In order to understand more precisely the effect of disagreement on the returns of dispersion portfolios, we perform an additional set of regressions for the different quantiles of the portfolio return distribution, using the quantile regression technique developed in Koenker and Basset (1978). This methodology allows us to test systematically the hypothesis that the effect of common disagreement for the excess returns of dispersion strategies is not equal across different return quantiles. We can compute quantile regression parameter estimates for the 0.25, the 0.5, and the 0.75 quantiles of the returns of the DiB-sorted dispersion portfolio.

Figure 9 presents for the same regressors of Table B.2, the estimated quantile regressions along with their 95% confidence bands.

We find that the point estimates for systematic market risk, size, book to market, momentum and liquidity imply insignificant parameter estimates across all estimated quantiles of the return distribution. The point estimates for the common disagreement are economically large, positive and statistically significant for the upper (0.75) and the central (0.5) quantiles. They are not statistically different from zero for the low 0.25 quantile. The monotonically increasing quantile regression function suggests that the positive impact of disagreement for the strategy returns is a feature exclusively of those quantiles that are not associated with the worst 25% returns.

35 Quantile regression is an extension of the classical least squares estimation of the conditional mean to a collection of linear regression models for different conditional quantiles of the endogenous variable. Each quantile regression characterizes a particular point of the conditional distribution. Combining different quantile regressions thus provides a more complete description of this distribution.

36 Standard errors are computed by a bootstrap procedure. For brevity, we do not report the results for the Market Beta and Liquidity sorted strategies. They are available on request.
Since it is not meaningful to formally estimate a quantile regression model for very small quantiles due to insufficient data, we decide to directly analyze the link between returns and common disagreement during these rare events. In Figure 10 we plot the largest losses of the dispersion trade versus changes in the common disagreement. Overall, we find a distinct pattern of low quantile returns that arise together with large positive variations in the upper quantiles of the common disagreement proxy: While higher common disagreement tends to rise the probability of positive returns, it also tends to increase the probability of large negative returns in the tails of the return distributions, thus implying a higher risk of rare, but potentially very large, portfolio losses. The positive average excess returns of the dispersion portfolio compensates dispersion traders for the presence of this rare, but potentially important, hidden risk.

V. Robustness Checks

Our results support the hypothesis that belief disagreement is a priced risk factor for volatility risk premia of individual and index options, and for the correlation risk premia. We have also shown that an option trading strategy that exploit a correlation risk premium in options yields economically significant returns and, more importantly, that these trading strategies are exposed to disagreement risk. In this section, we assess the robustness of our results by studying (i) the impact of transaction costs and (ii) whether other sources of risk can capture the effect of belief disagreement.

A. Transaction Costs

A vast literature documents that transaction costs in options markets are quite large and in part responsible for some pricing anomalies. Constantinides, Jackwerth, and Perrakis (2008) find that transaction costs do not eliminate the abnormal profits of index put and call strategies for different degrees of moneyness. Santa-Clara and Saretto (2009) find that transaction costs and margin requirements severely impact the profitability of index option trading strategies that involve writing out-of-the-money puts. The intuition is that such frictions constrain arbitrageurs in supplying liquidity to the market. Therefore, trading strategies that involve providing liquidity to the market (writing options) have an exceptionally good performance. Goyal and Saretto (2009) find that such profits are higher for illiquid than for liquid stock options. Moreover, liquidity considerations reduce, but do not eliminate, the economically important profits of their portfolios of individual options. Driessen, Maenhout, and Vilkov (2009) find that transaction costs and margin requirements significantly reduce the profitability of their correlation trading strategy, because of the larger bid-ask spread arising for individual stock options.

Intuitively, one could expect that our results should not be completely explained by transaction costs or bid-ask spreads, because we focus on buy-and-hold strategies that involve little trading: In our portfolios, options are traded only at the beginning of each trading period. Moreover, our strategies do not involve the writing of out-of-the-money options, which are particularly prone to trading frictions.
The average bid-ask spread for the index options is approximately 6.23% and for the individual options it amounts to 8.29%. We study the impact of these spreads on the performance of our trading strategies. In the previous analysis, we have used mid quotes calculated from the bid-ask spread. Now, we calculate bid returns when options are written and ask returns when options are bought. The results are reported in Table 10. We find that bid-ask spreads lower the return of the dispersion portfolio strategy by approximately 42%, from a 19% monthly return to 11%. In line with the literature, the impact on the other strategies is small. The decrease in the average return of the short index put strategy is 13%, from a 37% to a 32% monthly return. The annualized Sharpe ratio of the straddle strategy is above one, even after the inclusion of transaction costs considerations, and the common DiB proxy is the only systematic factor having a significant explanatory power for the dispersion portfolio returns.

B. Fundamental Uncertainty and Earning Announcements

When firms announce earnings every quarter, they reveal firm fundamentals which were - to some extent - unknown to investors prior to the announcement. This uncertainty might be related to investor’s expectations about future firm fundamentals, such as earnings. Empirical evidence has shown that volatility risk premia tend to be high prior to an earnings announcement. It is therefore an interesting question, whether the results implied by our proxy of belief disagreement are affected by the introduction of an uncertainty measure.

Ederington and Lee (1996) and Beber and Brandt (2006) document a strong decrease in implied volatility subsequent to major macroeconomic announcements in U.S. Treasury bond futures. While the first authors document that the implied volatility falls around announcements, the latter find in addition that skewness and kurtosis of the options returns distribution change after announcements. Dubinsky and Johannes (2006) find the same effect for earning announcements: Implied volatilities of single stock options increase prior to, and decrease subsequent to, an earning announcement. In particular, the risk-neutral volatility of price jumps deriving from earning announcements, which captures the anticipated uncertainty on the stock price embedded in an announcement, should be a priced risk factor. Using option prices, Dubinsky and Johannes (2006) develop an estimator of fundamental uncertainty surrounding announcement dates. They find no evidence of a price jump risk premium, but they find evidence of a jump volatility risk premium.

In the following, we estimate fundamental uncertainty using their term-structure estimator, defined as:

\[
\left( \sigma_{Q_{\text{time}}} \right)^2 = T_i \left( (\sigma_{T_i})^2 - (\sigma_{T_{i+1}, T_{i-1}})^2 \right),
\]

where \( \sigma_{T_i} \) is the Black-Scholes implied volatility at time \( t \) of an at-the-money option with \( T_i - t \) days to maturity.

Frazzini and Lamont (2007) find that there is a premium around earnings announcement dates, which is large, robust, and related to the surging volume around these dates. A potential explanation for this finding is a surging difference

\[\text{To save space, we report results for the DiB-sorted strategies only. The results for the other strategies are available upon request.}\]

\[\text{Evidence of a risk premium for volatility jumps in index options is also obtained by Broadie, Chernov, and Johannes (2008).}\]

\[\text{This term-structure estimator is less noisy than a time-series estimator, as it does not depend on implied volatilities at different dates.}\]
of opinions about the meaning of the announcements (Kandel and Pearson, 1995). Since volume and returns move together during and after the announcement, the volume hypothesis can explain both the event-day returns and the post-event drift in returns.

In the next sections, we check whether (i) our results are robust to the inclusion of the uncertainty measure of Dubinsky and Johannes (2006) and (ii) the pricing effect of belief disagreement is subsumed by earning announcement effects.

**B.1. Fundamental Uncertainty**

Comparing our common belief disagreement proxy to the fundamental uncertainty proxy, we find that average belief disagreement is 50% higher. Moreover, it also features a larger unconditional time-variation according to the sample standard deviation. We add fundamental uncertainty to our regressions in Table 5 (columns (4) and (7), respectively). Fundamental uncertainty loads negatively on the volatility risk premium, which is an intuitive finding: The higher the fundamental uncertainty, the more negative the volatility risk premium. It is interesting that the estimated regression coefficient of belief disagreement is not affected by the inclusion of fundamental uncertainty. Moreover, the economic impact of belief disagreement on both individual and index volatility risk premia is larger than the one of fundamental uncertainty.

**B.2. Earning Announcements**

Quarterly earning announcement dates are from the I/B/E/S database. As noted in DellaVigna and Pollet (2009), before 1995 a high large fraction of earnings announcements was recorded with an error of at least one trading day. In the more recent years, the accuracy of the earnings date has increased substantially, and is almost perfect after December 1994. The variable earning announcement is a dummy variable having value of 1 if there was an earning announcement the previous month. The variable interaction is belief disagreement multiplied by this dummy variable. Regression results are summarized in Table 5, column (2). Earning announcements have a negative and highly significant impact on volatility risk premia, but there is no evidence of a significant interaction with disagreement. The inclusion of the additional variable does not affect the significance of the belief disagreement coefficient. Thus, belief disagreement has likely a significant impact on volatility risk premia independently of earning announcements.

**C. Net-Buying Pressure**

An alternative hypothesis for explaining volatility risk premia of single-stock and index options is the demand-based theory in Bollen and Whaley (2004). As they argue, buying pressure in index put options drives the slope of the implied volatility, while buying pressure in calls on single-stocks drives the shape of the individual implied volatility surfaces. Motivated by this evidence, we consider buying pressure an an additional explanatory variable in our regressions. We follow Bollen and Whaley (2004) and define net buying pressure as the the number of contracts
traded during the day at prices higher than the prevailing bid/ask quote midpoint, minus the number of contracts traded during the day at prices below the prevailing bid/ask quote midpoint, multiplied by the absolute value of the option’s delta, and scaled by the total trading volume across all option series. Results are reported in Table 5 (columns (3) and (6)) and Table 6 (column (2)). Apart from the coefficient of demand pressure in puts for the index, all estimated coefficients related to net-buying pressure are insignificant. Demand pressure in index puts loads negatively on the volatility risk premium: Higher demand pressure in index puts increases the implied volatility and the resulting volatility risk premium. As in the previous robustness checks, the estimated regression coefficient of belief disagreement is not affected by the inclusion of variables related to net-buying pressure.

D. Sentiment

Shiller (1984) and Shleifer and Summers (1990) argue that the dynamic interplay of noise traders and rational arbitrageurs leads to price formation. Correlated trading activity of these noise traders therefore induce return correlation and arbitrage forces may not fully absorb these correlated demand shocks. In these models, investors’ sentiment affects return correlation. In the clientele-based models of Barberis, Shleifer, and Wurgler (2005) different investor groups restrict themselves to trading within different habitats, or groups of stocks. Thus, stock returns reflect changes in the systematic time-varying preference of important investor groups. Empirically, Kumar and Lee (2006) find corroborating evidence that retail investor sentiment has incremental power in explaining return comovement. Poteshman (2001)?, Pteshman and Serbin (2003)?, Mahani and Poteshman (2004)?, and Han (2008) among many others show that investor sentiment has a significant impact in option markets. To test the impact of investor sentiment on the correlation risk premium, we add investor sentiment which is calculated as the first principal component from trading volume as measured by NYSE turnover, the dividend premium, the closed-end fund discount, the number and first-day returns on IPOs and the equity share in new issues (see Baker and Wurgler, 2007) to our regressions. The results are reported in Table 6 (column 3). Indeed, the estimated coefficient for sentiment is significant and enters with the expected negative sign. The market volatility is not significant anymore which could hint that the sentiment index is picking up some of the counter-cyclicality of the market volatility. However, the addition of the sentiment index to the regression does not deteriorate neither the economic or statistical significance of the common disagreement proxy.

VI. Conclusion

We investigate theoretically and empirically the relation between market wide uncertainty, heterogeneity in beliefs among investors and volatility risk premia of index and individual options. We first develop a multiple-trees Lucas (1978) economy in which uncertainty about firms’ growth opportunities induces agents to disagree about expected dividends and a market-wide business cycle signal. These features generate a priced common disagreement component.

40 All these data are available on Jeffrey Wurgler’s webpage.
driving stock return co-movement, which is linked to the volatility risk premia of individual and index options. The main model implications can be summarized as follows.

First, disagreement about the market wide signal can generate substantial stock returns co-movement, even when underlying fundamentals are weakly related. In standard multiple-trees Lucas economies, as for instance Cochrane, Longstaff, and Santa-Clara (2008), stock returns can co-move because of the diversification effect generated by the pure market clearing mechanism between stocks. In the economy with heterogenous beliefs, these co-movement channels are amplified by the optimal equilibrium risk sharing between investors: Greater subjective uncertainty or higher disagreement on the market-wide signal imply a larger correlation of beliefs and a stronger co-movement of stock returns.

Second, the priced disagreement component can explain a good fraction of the volatility risk premium of index and individual options. In the cross section, the wedge between the index and individual implied volatility smiles is driven by a correlation risk premium. This premium is generated by the endogenous optimal risk-sharing among investors and increases with the degree of market-wide uncertainty and disagreement about the business-cycle indicator. In the time series, portfolio strategies with exposure to changes in correlations or heterogeneity of beliefs generate similarly attractive excess returns as compensation for priced disagreement risk. For instance, dispersion portfolios shorting index straddles and being long individual option straddles with the smallest degree of dividend disagreement produce much larger Sharpe ratios than standard short index put strategies widely studied in the literature. In contrast to the short index put strategy, these large excess returns of the dispersion portfolio are explained by an exposure to unexpected changes in the common disagreement component, but they have no significant exposure to market risk.

We study empirically the main predictions of our model using a panel data set consisting of firm specific earning forecasts and option price data for the S&P 100 index and its individual stocks. We first construct firm specific disagreement proxies using the cross-section of individual analysts’ forecasts. We then construct a common disagreement proxy out of the cross section of firm specific proxies using using dynamic factor analysis. We find a large counter-cyclical common component in the time series dynamics of the cross section of firm specific disagreement proxies, which we use as a proxy for market-wide disagreement in our empirical study.

Our empirical analysis highlights a number of findings that are novel to the literature. First, firm specific and market wide disagreement are the major determinants, in terms of statistical and economic significance, of both individual and index volatility risk premia. Market wide disagreement is also the major determinant of option-implied correlation risk premia, and it explains part of the differential pricing patterns of index and individual stock options. These results are robust to the choice of several other control variables in the literature. Second, option trading strategies creating straddle portfolios based on cross sectional differences in variance risk premia generate attractive excess returns. There returns are explained to a large extent by a systematic exposure to the common disagreement proxy. For instance, dispersion portfolios being short index straddles and long individual straddles with the lowest degree of dividend disagreement produce larger Sharpe ratios than standard short index put strategies. At the same time, these excess returns are explained by a significant exposure to unexpected changes in the common disagreement component, but no exposure to market risk and a number of other systematic risk factors like, e.g., size,
book-to-market and momentum. Thus, the correlation risk premium structurally embedded in dispersion portfolios is large, time-varying, and it is explained by an economically important exposure to common disagreement risk. Our findings show that this risk is particularly large during financial crises or market turmoils - i.e., periods of surging correlations or volatilities and high common disagreement - where it causes the largest losses of dispersion-type portfolio strategies. Finally, we find that our results are robust to the presence or realistic amounts of transaction costs and cannot be explained by competing theories of volatility risk premia linked to fundamental uncertainty, earnings announcement effects or net-buying pressure in option markets.

There are several potentially interesting avenues for future research. For instance, the robustness section has revealed some nontrivial link between disagreement and earning announcements. Frazzini and Lamont (2007) find a large announcement premium between 7% and 18%. They propose as a potential explanation for the premium the higher trading volume generated by the buying pressure of some investors at the announcement date. It would be interesting to study how news and earning announcements affect investors’ beliefs. To this end, one could model a potentially correlated signal/news channel for each firm by including a distinct deterministic jump component that reflects the deterministic structure of earning and dividend announcement dates. In such an economy, trading volume can rise endogenously as disagreement increases around announcement dates. Thus, it could be interesting to study which fraction of the announcement premium in Frazzini and Lamont (2007) can be explained by a higher ex-ante risk premium due to higher heterogeneity in beliefs prior to announcement dates.
References


Appendix

Appendix A Technical Proofs

Appendix A. Equilibrium

For completeness, we derive all equilibrium quantities in this Appendix. However, the proofs follow grossly Basak (2005) and Buraschi and Jitsev (2006). The time-separability of the utility function and the different goods allow for such a simple form. (i) Dynamics of the stochastic weighting process λ: Itô’s Lemma applied to \( \eta(t) = \xi^A(t)/\xi^B(t) \) gives:

\[
\text{d} \eta(t) = \frac{d \xi^A(t)}{\xi^B(t)} - \frac{\xi^A(t)}{(\xi^A(t))^2} d \xi^B(t) + \frac{1}{2} \left( \frac{d \xi^B(t)}{\xi^B(t)} \right)^2 - \frac{1}{(\xi^B(t))^2} d \xi^B(t) d \xi^A(t).
\]

Since markets are complete, there exists a unique stochastic discount factor for each agent. Absence of arbitrage implies for \( n = A, B \):

\[
\frac{d \xi^n(t)}{\xi^n(t)} = -r(t) dt - \theta^n(D_1(t), D_2(t), z(t))' dW^n_y,
\]

where \( \theta^n = (\theta^n_{D_1}, \theta^n_{D_2}, \theta^n_z) \)' is the vector of market prices of risk perceived by agent \( i \). It then follows,

\[
\text{d} \eta(t) = \frac{\xi^A(t) d \xi^A(t)}{\xi^B(t) \xi^A(t)} - \frac{\xi^A(t) d \xi^B(t)}{\xi^B(t) \xi^B(t)} + \frac{\xi^A(t) d \xi^B(t)}{\xi^B(t) \xi^B(t)} - \frac{1}{(\xi^B(t))^2} d \xi^B(t) d \xi^A(t),
\]

\[
= \eta(t) \left( -r(t) dt - \theta^A_1(t) dW_A(t) - \theta^B_1(t) dW_1(t) - (r(t) dt - \theta^A_2(t) dW_A(t) - \theta^B_2(t) dW_2(t)) \right) + \left( \theta^A_1(t) dW_A(t) - \theta^B_1(t) dW_1(t) \right) dt.
\]

The price of the stock in our economy follow the dynamics:

\[
\text{d} S_i(t) = S_i(t) (\mu^A_i(t) dt + \sigma_{S_i} dW^A_i(t) + \sigma_{S_i} dW^B_i(t) + \sigma_{S_1} dW^z_i(t)),
\]

where \( S_i(t) \) is the price of the equity of firm \( i \), and the expected growth rates \( \mu^A_i(t) \) and the volatility coefficients \( \sigma_{S_i} \) of \( S_i \) and \( \sigma_{S_1} \) are determined in equilibrium. It is easily shown that the difference in the perceived rates of return have to satisfy the consistency condition:

\[
\mu^A_i(t) - \mu^B_i(t) = \sigma_i \left( \Psi_{D_1}(t), \Psi_{D_2}(t), \alpha_{D_1} \Psi_{D_1}(t) \frac{\sigma_{D_1}}{\sigma_z} + \alpha_{D_2} \Psi_{D_2}(t) \frac{\sigma_{D_2}}{\sigma_z} + \beta \Psi_z(t) \right),
\]

where \( i \) denotes security \( i \). The definition of market price of risk yields:

\[
\sigma_{s_1} \Psi^A_{D_1}(t) + \sigma_{s_2} \Psi^A_{D_2}(t) + \sigma_{s_3} \Psi^A_z(t) = \mu^A_i(t) - r(t).
\]

After some simple algebra, I obtain:

\[
\sigma_{s_1} \left( \theta^A_{D_1}(t) - \theta^B_{D_1}(t) \right) + \sigma_{s_2} \left( \theta^A_{D_2}(t) - \theta^B_{D_2}(t) \right) + \sigma_{s_3} \left( \theta^A_z(t) - \theta^B_z(t) \right) = \sigma_{s_1} \Psi_{D_1}(t) + \sigma_{s_2} \Psi_{D_2}(t) + \beta \Psi_z(t).
\]

\[
+ \sigma_n \left( \alpha_{D_1} \Psi_{D_1}(t) \frac{\sigma_{D_1}}{\sigma_z} + \alpha_{D_2} \Psi_{D_2}(t) \frac{\sigma_{D_2}}{\sigma_z} + \beta \Psi_z(t) \right).
\]

Since this equation has to hold for any \( \sigma_{s_1} \) and \( \sigma_{n} \), it follows:

\[
\theta^A_{D_1}(t) - \theta^B_{D_1}(t) = \Psi_{D_1}(t),
\]

\[
\theta^A_{D_2}(t) - \theta^B_{D_2}(t) = \left( \alpha_{D_1} \Psi_{D_1}(t) \frac{\sigma_{D_1}}{\sigma_z} + \alpha_{D_2} \Psi_{D_2}(t) \frac{\sigma_{D_2}}{\sigma_z} + \beta \Psi_z(t) \right).
\]

By construction, we also have:

\[
\text{d} W_{D_1}(t) = \frac{m_{D_1}(t) - \mu_{D_1}(t)}{\sigma_{D_1}} dt + d W^y_{D_1}(t),
\]

\[
\text{d} W_z(t) = \left( \alpha_{D_1} \frac{m_{D_1}(t) - \mu_{D_1}(t)}{\sigma_z} + \alpha_{D_2} \frac{m_{D_2}(t) - \mu_{D_2}(t)}{\sigma_z} + \beta \frac{m_z(t) - \mu_z(t)}{\sigma_z} + d W^y(t) \right).
\]

50
Therefore, after substituting in equation (A-1), we get:

\[
\frac{dy(t)}{\eta(t)} = -dW^{A}_{D_1}(t)\Psi_{D_1}(t) - dW^{A}_{D_2}(t)\Psi_{D_2}(t) - \theta^{A}_{D_1}(t)dW^{A}_{Z}(t) \\
+ \theta^{B}_{D_1}(t) \left( dW^{A}_{Z}(t) + \alpha_{D_1}\Psi_{D_1}(t)\frac{\sigma_{D_1}}{\sigma_{z}} + \alpha_{D_2}\Psi_{D_2}(t)\frac{\sigma_{D_2}}{\sigma_{z}} + \beta\Psi_{z}(t) \right) \\
+ \left( \theta^{A}_{D_2}(t) - \Psi_{D_1}(t) \right)^2 + \left( \theta^{A}_{D_2}(t) - \Psi_{D_2}(t) \right)^2 + \theta^{B}_{D_2}(t) \left( \theta^{B}_{D_2}(t) - \theta^{A}_{D_2}(t) \right) \\
- \theta^{A}_{D_1}(t) \left( \theta^{A}_{D_1}(t) - \Psi_{D_1}(t) \right) - \theta^{A}_{D_2}(t) \left( \theta^{A}_{D_2}(t) - \Psi_{D_2}(t) \right) dt,
\]

\[
= -dW^{A}_{D_1}(t)\Psi_{D_1}(t) - dW^{A}_{D_2}(t)\Psi_{D_2}(t) - dW^{A}_{Z}(t) \left( \alpha_{D_1}\Psi_{D_1}(t)\frac{\sigma_{D_1}}{\sigma_{z}} + \alpha_{D_2}\Psi_{D_2}(t)\frac{\sigma_{D_2}}{\sigma_{z}} + \beta\Psi_{z}(t) \right).
\]

(ii) Representative investor optimization and optimal consumption policies: The representative agent in the economy faces the following optimization problem:

\[
\sup_{c_{D_1}^{A}(t)+c_{D_2}^{B}(t)\equiv D_{i}(t)} U'(c_{D_1}^{A}(t), c_{D_2}^{B}(t), \lambda(t)) = \frac{c_{D_1}^{A}(t)^{1-\gamma}}{1-\gamma} + \lambda(t)\frac{c_{D_2}^{B}(t)^{1-\gamma}}{1-\gamma},
\]

(A-2)

where \(\lambda(t) > 0\). Optimality of individual consumption plans implies that the stochastic weight takes the following form:

\[
\lambda(t) = u'(c_{D_1}(t))/u'(c_{D_1}(t)) = y_{A}\xi^{A}(t)/yb\xi^{B}(t),
\]

where \(u'(c_{D_1}(t)) = c_{D_1}(t)^{-1/\gamma}\) is the marginal utility function, which is assumed identical across agents. The first order condition for agent \(A\) is:

\[
e^{-\rho t}c_{D_1}(t)^{-\gamma} = y_{A}\xi^{A}(t).
\]

The first order condition for agent \(B\) is:

\[
\eta(t)e^{-\rho t}c_{D_1}(t)^{-\gamma} = yb\xi^{A}(t).
\]

The aggregate resource constraint can now be easily derived as:

\[
\left( \frac{yb\xi^{A}(t)e^{\rho t}}{\eta(t)} \right)^{-1/\gamma} + \left( y_{A}\xi^{A}(t)e^{\rho t} \right)^{-1/\gamma} = D_{i}(t).
\]

Thus, the solutions for the individual state price densities are:

\[
\xi^{A}(t) = e^{-\rho t} \frac{1}{y_{A}} D_{i}(t)^{-\gamma} \left( 1 + \lambda(t)^{1/\gamma} \right)^{\gamma}, \quad \xi^{B}(t) = e^{-\rho t} \frac{1}{yb} D_{i}(t)^{-\gamma} \left( 1 + \lambda(t)^{1/\gamma} \right)^{\gamma} \lambda(t)^{-1}.
\]

To solve for the optimal consumption policy of each agent, I plug in the functional forms for the individual state price densities:

\[
c_{D_1}^{A}(t) = (y_{A}\xi^{A}(t)e^{\rho t})^{-1/\gamma} = D_{i}(t) \left( 1 + \lambda(t)^{1/\gamma} \right)^{-1}.
\]

Good's market clearing, finally implies:

\[
c_{D_1}^{B}(t) = D_{i}(t) - c_{D_1}^{A}(t) = D_{i}(t)\lambda(t)^{1/\gamma} \left( 1 + \lambda(t)^{1/\gamma} \right)^{-1}.
\]

**Appendix B. Joint Laplace Transform**

To save space, we defer all calculations to a separate technical Appendix, which is available on the authors' webpage.

**Appendix C. Stock Price Volatility, Correlation and Skewness**

The price of the stock satisfies a diffusion process which is given by:

\[
\frac{dS_{1}}{S_{1}} = \mu_{1}^{A}(t)dt + \sigma_{1}^{A}(t)dW^{A}_{D_1}(t) + \sigma_{1}^{A}(t)dW^{A}_{D_2}(t) + \sigma_{1}^{A}(t)dW^{A}_{Z}(t).
\]
Using the following derivatives,

The diffusion term is characterized by:

\[ dS_1(t) - S_1(t) \mu_{D_1}(t) dt = \frac{\partial S_1}{\partial D_1} \left( dD_1(t) - E_{D_1} \left( dD_1(t) \right) \right) + \frac{\partial S_1}{\partial m_{D_1}} \left( dm_{D_1}(t) - E_{m_{D_1}} \left( dm_{D_1}(t) \right) \right) + \frac{\partial S_1}{\partial \Psi} \left( d\Psi(D_1(t)) - E_{\Psi} \left( d\Psi(D_1(t)) \right) \right) + \frac{\partial S_1}{\partial D_2} \left( dD_2(t) - E_{D_2} \left( dD_2(t) \right) \right) + \frac{\partial S_1}{\partial \Psi} \left( d\Psi(D_2(t)) - E_{\Psi} \left( d\Psi(D_2(t)) \right) \right) = \frac{\partial S_1}{\partial D_1} D_1 \sigma_{D_1} dW_{D_1}^A(t) + \frac{\partial S_1}{\partial m_{D_1}} \left( \frac{\gamma_{D_1}^A}{\sigma_{D_1}} dW_{m_{D_1}}^A(t) + \frac{\gamma_{D_1}^D D_2}{\sigma_{D_1}} dW_{D_2}^A(t) + \left( \alpha_{D_1} \gamma_{D_1}^A + \alpha_{D_2} \gamma_{D_2}^A D_1 + \beta \gamma_{D_1}^A \right) dW_{\Psi}^A(t) \right) + \frac{\partial S_1}{\partial \Psi} \left( \frac{\gamma_{D_2}^A}{\sigma_{D_2}} dW_{\Psi}^A(t) + \left( \alpha_{D_1} \gamma_{D_2}^A - \beta \gamma_{D_1}^A \right) dW_{\Psi}^A(t) \right) + \frac{\partial S_1}{\partial D_2} \left( \frac{\gamma_{D_2}^A D_2 - \gamma_{D_1}^A D_1}{\sigma_{D_2}} dW_{D_2}^A(t) + \left( \alpha_{D_1} \gamma_{D_2}^A - \beta \gamma_{D_2}^A \right) dW_{D_2}^A(t) \right) + \frac{\partial S_1}{\partial \Psi} \left( \frac{\gamma_{D_2}^A}{\sigma_{D_2}} dW_{\Psi}^A(t) + \left( \alpha_{D_1} \gamma_{D_2}^A - \beta \gamma_{D_2}^A \right) dW_{\Psi}^A(t) \right) + \frac{\partial S_1}{\partial \Psi} \left( \frac{1}{\sigma_{\Psi}} \right) \left( \alpha_{D_1} \gamma_{D_2}^A - \beta \gamma_{D_1}^A \right) dW_{\Psi}^A(t) \right) \]

where

\[ \sigma_{D_1, D_2}(t) = \frac{1}{S_1(t)} \left( \frac{\partial S_1}{\partial D_1} D_1 \sigma_{D_1} + \frac{\partial S_1}{\partial m_{D_1}} \frac{\gamma_{D_1}^A}{\sigma_{D_1}} \sigma_{D_1} + \frac{\partial S_1}{\partial \Psi} \left( \frac{\gamma_{D_1}^A}{\sigma_{D_1}} - \beta \gamma_{D_2}^A \right) \sigma_{D_1} + \frac{\partial S_1}{\partial D_2} \left( \frac{\gamma_{D_1}^A D_2 - \gamma_{D_2}^A D_1}{\sigma_{D_1} \sigma_{D_2}} \sigma_{D_2} + \frac{\gamma_{D_2}^A}{\sigma_{D_2}} \right) \right) \]

\[ \sigma_{D_2, \Psi}(t) = \frac{1}{S_1(t)} \left( \frac{\partial S_1}{\partial m_{D_1}} \frac{\gamma_{D_1}^A}{\sigma_{D_2}} \sigma_{D_2} + \frac{\partial S_1}{\partial D_1} \left( \frac{\gamma_{D_1}^A D_2 - \gamma_{D_2}^A D_1}{\sigma_{D_1} \sigma_{D_2}} \sigma_{D_1} + \frac{\gamma_{D_2}^A}{\sigma_{D_2}} \right) \right) \]

\[ \sigma_{\Psi}(t) = \frac{1}{S_1(t)} \left( \frac{\partial S_1}{\partial m_{D_1}} \left( \alpha_{D_1} \gamma_{D_2}^A + \alpha_{D_2} \gamma_{D_2}^A D_1 + \beta \gamma_{D_1}^A \right) \right) \]

Using the following derivatives,

\[ \frac{\partial S_1}{\partial D_1} = \int_t^\infty e^{-\delta(u-t)} F_{m_A} \left( m_A, t; \epsilon_{D_1}, \epsilon_{D_2} \right) G \left( t, u, 1; \epsilon_{D_1}, \epsilon_{D_2} \right) du, \]

\[ \frac{\partial S_1}{\partial m_{D_1}} = D_1 \int_t^\infty e^{-\delta(u-t)} A(u-t) F_{m_A} \left( m_A, t; \epsilon_{D_1}, \epsilon_{D_2} \right) G \left( t, u, 1; \epsilon_{D_1}, \epsilon_{D_2} \right) du, \]

\[ \frac{\partial S_1}{\partial \Psi} = D_1 \int_t^\infty e^{-\delta(u-t)} \left( B_{\Psi} + 2C_{\Psi} \right) F_{m_A} \left( m_A, t; \epsilon_{D_1}, \epsilon_{D_2} \right) G \left( t, u, 1; \epsilon_{D_1}, \epsilon_{D_2} \right) du, \]
we can easily compute the stock volatility which is given by \((\sigma_{1s}^2 + \sigma_{1s1}^2 + \sigma_{1s2}^2 + \sigma_{1s12}^2)^{1/2}\). The corresponding coefficients for the volatility of stock 2 are:

\[
\begin{align*}
\sigma_{s_{2s}}(t) &= \frac{1}{S_{2}(t)} \left( \frac{\partial S_{2}}{\partial \Psi_{D_{2}}} \frac{\gamma_{D_{1s}} - \gamma_{D_{1}}}{\sigma_{D_{1}}} + \frac{\partial S_{2}}{\partial \Psi_{D_{1}}} \frac{\gamma_{D_{2s}} - \gamma_{D_{1s}}}{\sigma_{D_{1s}}} \right) + \frac{\partial S_{2}}{\partial \Psi_{z}} \frac{\gamma_{D_{1s}} - \gamma_{D_{1}}}{\sigma_{D_{1s}}} + \frac{\partial S_{2}}{\partial \Psi_{z}} \frac{\gamma_{D_{2s}} - \gamma_{D_{1s}}}{\sigma_{D_{1s}}} \\
\sigma_{s_{2s}}(t) &= \frac{1}{S_{2}(t)} \left( \frac{\partial S_{2}}{\partial D_{2}} \frac{\gamma_{D_{1s}} - \gamma_{D_{1s}}}{\sigma_{D_{1s}}} + \frac{\partial S_{2}}{\partial D_{2}} \frac{\gamma_{D_{2s}} - \gamma_{D_{1s}}}{\sigma_{D_{1s}}} \right) + \frac{\partial S_{2}}{\partial \Psi_{D_{1}}} \frac{\gamma_{D_{1s}} - \gamma_{D_{1s}}}{\sigma_{D_{1s}}} + \frac{\partial S_{2}}{\partial \Psi_{z}} \frac{\gamma_{D_{2s}} - \gamma_{D_{1s}}}{\sigma_{D_{1s}}} \right), \\
\sigma_{s_{2s}}(t) &= \frac{1}{S_{2}(t)} \left( \frac{\partial S_{2}}{\partial \Psi_{D_{1}}} \frac{\alpha_{D_{1}} \gamma_{D_{1s}} + \alpha_{D_{2}} \gamma_{D_{1s}}}{\sigma_{D_{1s}}} + \frac{\partial S_{2}}{\partial \Psi_{z}} \frac{\alpha_{D_{1}} \gamma_{D_{1s}} + \alpha_{D_{2}} \gamma_{D_{1s}}}{\sigma_{D_{1s}}} \right) \\
&+ \frac{\partial S_{2}}{\partial \Psi_{D_{1}}} \frac{\alpha_{D_{1}} \gamma_{D_{1s}} - \gamma_{D_{1}}}{\sigma_{D_{1s}}} + \frac{\partial S_{2}}{\partial \Psi_{z}} \frac{\alpha_{D_{1}} \gamma_{D_{1s}} - \gamma_{D_{1}}}{\sigma_{D_{1s}}} \right). \\
\end{align*}
\]

The correlation between stock 1 and stock 2 can be calculated as follows:

\[
\text{corr} \left( \frac{dS_{1}}{S_{1}}, \frac{dS_{2}}{S_{2}} \right) = \frac{\text{Cov} \left( \frac{dS_{1}}{S_{1}}, \frac{dS_{2}}{S_{2}} \right)}{\sqrt{\text{Var} \left( \frac{dS_{1}}{S_{1}} \right) \text{Var} \left( \frac{dS_{2}}{S_{2}} \right)}},
\]

where

\[
\text{Cov} \left( \frac{dS_{1}}{S_{1}}, \frac{dS_{2}}{S_{2}} \right) = E \left( \frac{dS_{1}}{S_{1}} \right) E \left( \frac{dS_{2}}{S_{2}} \right) - E \left( \frac{dS_{1}}{S_{1}} \right) E \left( \frac{dS_{2}}{S_{2}} \right),
\]

Assume that the stock return is related to the market return via the following two factor representation:

\[
r_{i}(t) = \beta_{i} r_{M}(t) + \beta_{\Psi} \Psi(t) + \epsilon_{i}(t),
\]

where \(r_{i}\) is the return on the stock price of firm \(i\), \(r_{M}\) is the return on the market, and \(\Psi\) is the common disagreement. The skewness of \(r_{i}\) is defined as:

\[
SKEW(r_{i}) = E \left( (r_{i} - E(r_{i}))^3 \right).
\]

Plugging relation (A.3) into this definition yields:

\[
SKEW(r_{i}) = E \left( \beta_{i}^3 r_{M}^3 + \beta_{\Psi}^3 \Psi^3 + \epsilon_{i}^3 + 3 \beta_{i} \beta_{\Psi} \beta_{M} \Psi r_{M} + 3 \beta_{i} \beta_{\Psi}^2 \Psi^2 r_{M} + 3 \beta_{i} \beta_{M} r_{M} \epsilon_{i}^2 \\
+ 3 \beta_{\Psi} \Psi \epsilon_{i}^2 - 3 \beta_{i}^2 \beta_{M}^2 m_{M} - 3 \beta_{i} \beta_{\Psi}^2 \Psi^2 m_{M} - 3 \beta_{i} \beta_{\Psi} \beta_{M} m_{M} + 3 \beta_{i} \beta_{M} \Psi m_{M} + 3 \beta_{i} \beta_{\Psi} m_{M}^2 - m_{M}^3 \right),
\]

\[
= \beta_{i}^3 \text{SKEW}(r_{M}) + \beta_{\Psi}^3 \text{SKEW}(\Psi) - \text{COSKEWS} + \text{COSKEWS},
\]

where \(\text{COSKEWS} = m_{M}\). The last equality comes from the fact that \(\epsilon_{i} \sim N(0,1)\) and that all co-skewness terms with \(\epsilon_{i}\) are zero, because we assume independence between \(\Psi, m_{M}\) and \(\epsilon_{i}\).

\[
\text{COSKEWS} = 3 \beta_{\Psi} \beta_{i}^2 \beta_{M} \Psi \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Psi^2 q(\Psi, r_{M}) dr_{M} d\Psi - 3 \beta_{i} \beta_{\Psi}^2 \beta_{M} \Psi \int_{-\infty}^{\infty} m_{M} \Psi^2 q(r_{M}, m_{M}) dr_{M} d\Psi \\
- 3 \beta_{i} \beta_{\Psi} \Psi m_{M} \int_{-\infty}^{\infty} m_{M}^2 q(r_{M}, m_{M}) dr_{M} + 3 \beta_{i} \Psi \int_{-\infty}^{\infty} m_{M}^2 q(r_{M}, m_{M}) dr_{M} d\Psi \\
+ 3 \beta_{\Psi} \Psi \int_{-\infty}^{\infty} m_{M}^2 q(m_{M}) dr_{M} d\Psi.
\]

\(\Psi\) For simplicity, we omit the standardization by the variance to the power 3/2.
To synthesize the risk-neutral skewness, we follow Bakshi and Madan (2000) that the entire collection of twice-differentiable payoff functions with bounded expectation can be spanned algebraically. Applying this result to the stock price $S_1(t)$, we get

$$G(S_1) = G(\tilde{S}_1) + \left(1 - \tilde{S}_i \right) \tilde{S}_{i-1} \left(\tilde{S}_i \right) + \int_{\tilde{S}_i}^{\infty} G S_{i1} (K) (S_1 - K)^+ dK + \int_{0}^{S_1} G S_{i1} (K) (K - S_1)^+ dK,$$

where $G_{i1}$ is the partial derivative of the payoff function $G(S_1)$ with respect to $S_1$ and $G S_{i1}$ the corresponding second-order partial derivative. By setting $S_1 = S_1(t)$, we obtain the final formula for the risk-neutral skewness of the stock, after mimicking the steps in Bakshi, Kapadia, and Madan (2003) (Theorem 1, p. 137).

Let $s(t, T) = \ln(S_1(t + T))$ be the firm value return between time $t$ and $T$. The risk-neutral skewness of $s(t, T)$ of stock $S_1$ is given by

$$skew_{S_1}(t, T) = \frac{e^{T R(t, T)} - 2 \mu(t, T) e^{T R(t, T)} + 2 \mu(t, T)^2}{(e^{T R(t, T)} - \mu(t, T))^2},$$

where

$$R(t, T) = \int_{S_1(t)}^{\infty} 2 \left(1 - \ln \left(\frac{K}{\pi_{1i}(t)}\right)\right) \left(S_1(K) - K\right)^+ dK + \int_{0}^{S_1(t)} 2 \left(1 + \ln \left(\frac{S_1(t)}{K}\right)\right) \left(K - S_1(T)\right)^+ dK,$$

$$W(t, T) = \int_{S_1(t)}^{\infty} 6 \ln \left(\frac{S_1(t)}{K}\right) - 3 \left(\ln \left(\frac{K}{\pi_{1i}(t)}\right)\right)^2 \left(S_1(K) - K\right)^+ dK,$$

$$- \int_{0}^{S_1(t)} 6 \ln \left(\frac{S_1(t)}{K}\right) - 3 \left(\ln \left(\frac{S_1(t)}{K}\right)\right)^2 \left(K - S_1(T)\right)^+ dK,$$

and

$$X(t, T) = \int_{S_1(t)}^{\infty} 12 \left(\ln \left(\frac{K}{\pi_{1i}(t)}\right)\right)^2 - 4 \left(\ln \left(\frac{K}{\pi_{1i}(t)}\right)\right)^3 \left(S_1(T) - K\right)^+ dK,$$

$$- \int_{0}^{S_1(t)} 12 \left(\ln \left(\frac{S_1(t)}{K}\right)\right)^2 - 4 \left(\ln \left(\frac{S_1(t)}{K}\right)\right)^3 \left(K - S_1(T)\right)^+ dK,$$

$$\mu(t, T) = E_t \left(\ln \left(\frac{S_1(t + T)}{S_1(t)}\right)\right) \approx e^{T R(t, T)} - \frac{e^{T R(t, T)}}{2} R(t, T) - \frac{e^{T R(t, T)}}{6} W(t, T) - \frac{e^{T R(t, T)}}{24} X(t, T).$$

The skewness of stock 2 and of the index are equivalent.

This concludes the discussion about the stock volatility and skewness.

**Appendix B Construction of the Common DiB**

When the number of belief disagreement processes, $N$ is very large, an estimation of the common factor via the Likelihood function is computational infeasible (see Ludvigson and Ng, 2007). This high dimensionality has motivated the work on alternative methods to estimate dynamic factor models. Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986) show that if $n$ goes to infinity, the factors are estimated consistently using principal components, see Brillinger’s (2001) dynamic principal components. The theory of applying this theory, based on frequency domain methods is developed by Forni, Hallin, Lippi, and Reichlin (2000). In the following, we briefly summarize the estimation procedure, for technical details and a more rigorous presentation the reader is referred to Forni, Hallin, Lippi, and Reichlin (2000).

The estimation procedure is derived in two steps. The first step consists of estimating the spectral density matrix and the covariances of the common components. We start by estimating the spectral density matrix $\Sigma(\omega)$ of $D(t) = (D_{1i}, D_{2i}, \cdots, D_{Ni})^\prime$. The estimation of $\Sigma(\omega)$ is obtained by using a Bartlett lag-window size of $M = 2$. We use a heuristic rule, which sets the window size equal to $\text{round} (\sqrt{T}/4)$, see Forni, Hallin, Lippi, and Reichlin (2000). The estimation is done via the discrete Fourier transform:

$$\hat{\Sigma}_D (\omega) = \frac{1}{2\pi} \sum_{k=-M}^{M} w_k \hat{\Gamma}_k \exp(-i\omega k),$$

where $\hat{\Gamma}_k$ denotes the sample autocovariance matrices, and $w_k = 1 - |k|/(M + 1)$ the weights.
We then perform the dynamic principal component decomposition (see Brillinger, 2001). For each frequency of the grid, we compute the eigenvalues and eigenvectors of $\Sigma(\omega)$. By ordering the eigenvalues in descending order for each frequency and collecting values correspondingly to different frequencies, the eigenvalues, $\lambda_j$ and eigenvectors, $U_j(\omega)$, are obtained. The eigenvalue $\lambda_j$ can be interpreted as the spectral density of the $j$-th principal component. To determine the optimal number of common factors, we study the contribution of the $j$-th principal component to the total variance:

$$c_j = \int_{-\pi}^{\pi} \lambda_j(\omega)d\omega / \sum_{j=1}^{N} \int_{-\pi}^{\pi} \lambda_j(\omega)d\omega.$$ 

Forni, Hallin, Lippi, and Reichlin (2000) show that there exist a linkage between the number of common factors and the evanlues of the spectral density matrix. In practice, however, there does not exist a formal testing procedure to distinguish very slowly diverging eigenvalue and a bounded one, therefore, we follow the heuristic procedure applied by Cristadoro, Forni, Hallin, and Veronese (2005), by imposing the criteria that the dynamic common factors should account for a certain percentage of the total variability in the data across all frequencies, and the number of dynamic common factors is set equal to the number of largest dynamic eigenvalues that together capture this variance ratio. In our case, the number of common components is set to $q = 2$. Let $\Lambda_q(\omega)$ be a diagonal matrix, having as elements the eigenvalues, $\lambda_1(\omega), \ldots, \lambda_q(\omega)$ and let $U(\omega)$ be the $[n \times q]$ matrix of the eigenvectors, $U_1(\omega), \ldots, U_2(\omega)$. Then the estimate of the spectral density matrix of the common components, $\zeta(t) = (\zeta_1(\cdots, \zeta_N))'$:

$$\hat{\Sigma}_c(\omega) = U(\omega)\Lambda(\omega)U(\omega),$$

where the tilde denotes a conjugate. The spectral density of the idiosyncratic component is obtained as the difference of the spectral density matrices of the common component and the $D$.

The second step consists of estimating the factor space. Given the estimated covariance matrices in the first step, we can now estimate the factors as a linear combination of the observable variables, $D_{jkt}$, $j = 1, \ldots, N$. To this end, we take the first $r$ generalized principal components of the estimated spectral density matrix of the common components, $\Gamma_k$ with respect to the diagonal matrix having on the diagonal the variances of the idiosyncratic components. Estimates of the common components are derived by projecting the common components on the space spanned by the first $r$ generalized principal components. To this end, we compute the generalized eigenvalues, $\mu_j$, solving $\det(\Gamma_k - \mu I) = 0$ along with the generalized eigenvectors, $V_j$, satisfying $V_j\Gamma_k = \mu V_j$. Then we take the eigenvectors corresponding to the largest $r$ eigenvalues. The estimated factors are then $V_j t = V_j \Psi_j t$. To determine the number of static factors, $r$, we rely on Forni, Hallin, Lippi, and Reichlin (2005), who propose to use $r = q(k + 1)$. Using the generalized principal components and the covariances estimated previously, we can now estimate the common component $\Psi_k$. The estimate of $\Psi_k$ is given by:

$$\hat{\Psi}_{t+h} = \hat{V}_D(h)V \left( V^\top \Gamma(0)V \right)^{-1} V^\top \Psi_t.$$ 

**Appendix C Dispersion Trading**

**Appendix A. The Basics**

Consider an index with $n$ stocks. $\sigma_i$ is the volatility of stock $i$, $\omega_i$ is the weight of stock $i$ in the index, and $\rho_{ij}$ is the correlation between stock $i$ and stock $j$. The index itself has the following volatility:

$$\sigma^2_{\text{index}} = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2 + \sum_{i \neq j} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}.$$ 

The average index variance is:

$$\bar{\sigma}^2_{\text{index}} = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2.$$ 

We can now compare this average number to the actual index volatility. We define a dispersion spread, $D$, as:

$$D = \sqrt{\sum_{i=1}^{n} \omega_i^2 \sigma_i^2 - \bar{\sigma}^2_{\text{index}}} = \sqrt{\sigma^2_{\text{index}} - \bar{\sigma}^2_{\text{index}}}.$$ 

The upper bound for the dispersion spread is now simply the average basket volatility and the lower bound is zero. A trading strategy which bets on the dispersion spread has two legs. If the investor is long dispersion, then she is long the volatility of the constituents and short index volatility. So, it would be desirable to have lots of volatility on the constituents and no volatility.
on the index. One of the main drivers is the exposure of this strategy to correlation. Considering the average correlation, it is easy to see that if one is long dispersion then one is short in correlation:

$$\rho = \frac{\sigma^2_{\text{Index}} - \sum_{i=1}^{n} \omega_i^2 \sigma_i^2}{\sum_{i \neq j} \omega_i \omega_j \sigma_i \sigma_j}.$$  

**Appendix B. P&L of a Dispersion Trade**

For simplicity reasons, we study a case with constant volatility. So, consider a delta-hedged portfolio which is long the stock options and short the index options. Remember, that the P&L of a delta-hedged option Π in a Black and Scholes framework is (see Hull, 2002)

$$\text{P&L} = \theta \left( \frac{dS}{S \sqrt{dt}} \right)^2 - 1,$$

where the θ of the option is defined as the options sensitivity with respect to a change in the time to maturity.

Let the term \( n = \frac{\delta S}{S \sqrt{dt}} \) represent the standardized move of the underlying stock S on the considered time period. Then, the P&L of the index can be written as

$$\text{P&L} = \theta_t \left( \frac{\omega_i \sigma_i}{\sigma_{\text{Index}}} \right)^2 - 1,$$

$$= \theta_t \left( \sum_{i=1}^{n} \omega_i n_i \frac{\sigma_i}{\sigma_{\text{Index}}} \right)^2 - 1,$$

$$= \theta_t \left( \sum_{i=1}^{n} \left( \omega_i n_i \frac{\sigma_i}{\sigma_{\text{Index}}} \right)^2 + \sum_{i \neq j} \omega_i \omega_j \frac{\sigma_i \sigma_j}{\sigma_{\text{Index}}} n_i n_j \right),$$

$$= \theta_t \sum_{i=1}^{n} \frac{\omega_i^2 \sigma_i^2}{\sigma_{\text{Index}}^2} (n_i^2 - 1) + \theta_t \sum_{i \neq j} \frac{\omega_i \omega_j \sigma_i \sigma_j}{\sigma_{\text{Index}}^2} (n_i n_j - \delta_{ij}).$$

Hence, a dispersion trade being short the index options and being long the individual options has the following P&L:

$$\text{P&L} = \sum_{i=1}^{n} \text{P&L}_i - \text{P&L}_t,$$

$$= \sum_{i=1}^{n} \theta_i (n_i^2 - 1) + \theta_t (n_t^2 - 1).$$

The short and long positions in the options are reflected in the sign of the \( \theta_i \). A long (short) position means a positive (negative) \( \theta \).

**Appendix C. Weighting Scheme for Dispersion Trading**

Since trading all constituents would be far too expensive, the investor has to ask herself which stock she should pick and then, how to weight them. There are the following weighting schemes, which are employed in the industry:

1. **Vega Hedging:**
   The investor will build her dispersion such that the vega of the index equals the sum of the vegas of the constituents.

2. **Gamma Hedging:**
   The gamma of the index is worth the sum of the gamma of the components. As the portfolio is already delta-hedged, this weighting scheme protects the investor against any move in the stocks, but leaves her with a vega position.

3. **Theta Hedging:**
   This strategy results in a short vega and a short gamma position.\(^{43}\)

\(^{43}\)Adding stochastic volatility yields analogous expressions with some additional terms which account for the Vega, Volga, and Vanna of the option.

\(^{44}\)To this end, remember that the relationship between the option’s theta and gamma is as follows:

$$\theta \approx -\frac{1}{2} \Gamma S^2 \sigma^2,$$

where \( S \) is the underlying spot price.
Table 1
Largest Losses of Dispersion Trade, Realized Correlation, and Uncertainty

This table reports the largest losses on a dispersion traded implemented using at-the-money straddles of index options on the S&P 100 and at-the-money straddles on the constituents options. Realized correlation is calculated from the average firm-by-firm correlation using daily return data over a two month window. Common DiB is calculated from the cross-section of individual disagreement proxies calculated from forecasts on future earnings. The time period runs from January 1996 to June 2007.

<table>
<thead>
<tr>
<th>Date</th>
<th>Description</th>
<th>Loss</th>
<th>% Common DiB</th>
<th>% S&amp;P 100 Corr</th>
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<tr>
<td>August 1998</td>
<td>LTCM &amp; Russian Default</td>
<td>-122%</td>
<td>+33%</td>
<td>+106%</td>
</tr>
<tr>
<td>February 2001</td>
<td>DotCom Bubble</td>
<td>-111%</td>
<td>+10%</td>
<td>+46%</td>
</tr>
<tr>
<td>September 2001</td>
<td>Terrorists’ Attacks</td>
<td>-141%</td>
<td>+25%</td>
<td>+40%</td>
</tr>
</tbody>
</table>
Table 2  
Choice of Parameter Values and Benchmark Values of State Variables

This table lists the parameter values used for all figures in the paper. We calibrate the model to the mean and volatility of the dividends on the S&P 500. The average growth rate for the period 1996-2006 is 5.93% and the volatility is 3.52%. The initial values for the conditional variances are set to their steady-state variances.

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<tbody>
<tr>
<td>Relative risk aversion for both agents</td>
<td>$\gamma$</td>
<td>2.00</td>
</tr>
<tr>
<td>Time Preference Parameter</td>
<td>$\rho$</td>
<td>0.02</td>
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</table>
Table 3
Moments of Simulated Option Strategies

This table reports summary statistics of the simulated strategy returns, average, standard deviation, skewness, kurtosis and Sharpe ratio. The DiB sorted straddle portfolio is long the stock with the lower disagreement and short the stock with the higher disagreement. The DiB sorted dispersion portfolio is formed by investing 100% of the wealth in shorting index straddles and investing a fraction of wealth into the options of the firm with the lowest belief disagreement such that the portfolio is vega neutral. The remainder is invested in the individual stock of this particular firm such that the portfolio is delta neutral. The index consists of two equally weighted stocks. We simulate 2,877 trading days and 1,000 simulation runs. All options are at-the-money with a maturity of 28 trading days. The Alpha, MRKT Beta, and Common DiB Beta are estimated running the following regression on the realized returns:

\[ r_i(t) - r_f(t) = \alpha + \beta^{MRKT}MRKT(t) + \beta^{DiB}DiB(t) + \epsilon(t), \]

where \( r_i(t) \) is the return of the trading strategy at time \( t \), \( r_f(t) \) is the risk-free rate, \( MRKT(t) \) is the index excess return, and \( \Delta DiB(t) \) is the monthly change in the common disagreement. The market excess return is defined as the return on the index (i.e. the equal weighted sum of both stocks) and the common disagreement is a weighted average of the firm-specific disagreement proxies.

<table>
<thead>
<tr>
<th></th>
<th>DiB Sorted Straddle</th>
<th>DiB Sorted Dispersion</th>
<th>Index</th>
<th>Short Index Put</th>
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<tr>
<td>Return</td>
<td>0.081</td>
<td>0.127</td>
<td>0.010</td>
<td>0.117</td>
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<tr>
<td>Standard Deviation</td>
<td>0.273</td>
<td>0.231</td>
<td>0.038</td>
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<tr>
<td>Skewness</td>
<td>1.102</td>
<td>-3.127</td>
<td>-7.423</td>
<td>-8.239</td>
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<tr>
<td>Kurtosis</td>
<td>6.928</td>
<td>7.532</td>
<td>5.819</td>
<td>8.477</td>
</tr>
<tr>
<td>Annualized Sharpe Ratio</td>
<td>1.027</td>
<td>1.867</td>
<td>0.683</td>
<td>0.607</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.087**</td>
<td>0.087**</td>
<td>0.092*</td>
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</tr>
<tr>
<td>t-Stat</td>
<td>1.72</td>
<td>1.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRKT Beta</td>
<td>1.837</td>
<td>-0.932</td>
<td>4.293*</td>
<td></td>
</tr>
<tr>
<td>t-Stat</td>
<td>1.03</td>
<td>-1.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common DiB Beta</td>
<td>2.927**</td>
<td>4.192***</td>
<td>3.191*</td>
<td></td>
</tr>
<tr>
<td>t-Stat</td>
<td>2.08</td>
<td>2.96</td>
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</tbody>
</table>
Table 4
Summary Statistics of Most Important Variables

We report summary statistics of the main variables used in the analysis. The data runs from January 1996 to June 2007, with monthly frequency. Risk Premium is the difference between the realized and implied volatility. The realized volatility is calculated from stock return data retrieved from the CRSP database. It is calculated over a 21-day window, requiring that there are at least 15 nonzero observations per window. The implied volatility is calculated from option prices taken from the OptionMetrics database. Dispersion Individual is defined as the ratio of the mean absolute difference of analysts’ forecasts and the standard deviation of these forecasts, retrieved from the I/B/E/S database. Dispersion Common is a common component estimated by dynamic factor analysis from the individual disagreement series. Market Volatility is defined as the historical volatility over a 21-day window. Corr Individual (Index) is the time-series average correlation with the individual (index) volatility risk premium.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>StDev</th>
<th>0.25 percentile</th>
<th>0.75 percentile</th>
<th>Corr Individual</th>
<th>Corr Index</th>
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<tbody>
<tr>
<td>Risk Premium Individual</td>
<td>-0.0110</td>
<td>0.0337</td>
<td>-0.0356</td>
<td>-0.0041</td>
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<td>Risk Premium Index</td>
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<td>-0.0475</td>
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<td>DiB Individual</td>
<td>0.3089</td>
<td>0.2038</td>
<td>0.1612</td>
<td>0.3787</td>
<td>0.5973</td>
<td>-</td>
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<tr>
<td>Common DiB</td>
<td>0.1152</td>
<td>0.0419</td>
<td>0.0264</td>
<td>0.0486</td>
<td>-</td>
<td>0.5252</td>
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<tr>
<td>Market Volatility</td>
<td>0.1663</td>
<td>0.0752</td>
<td>0.1070</td>
<td>0.2070</td>
<td>-0.0160</td>
<td>-0.6262</td>
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</table>
Using data from January 1996 to June 2007, we run regressions from the volatility risk premium of individual and index options on a number of determinants. The volatility risk premium is defined as the difference between the options’ 21 day realized and implied volatility. DiB is our proxy for difference in beliefs for each firm, defined as the mean absolute difference among analysts forecasts standardized, Common DiB is our proxy for difference in beliefs for the market calculated using dynamic factor analysis, Market Vola is the 21 day realized volatility of the index. Skewness is measured as the difference between the implied volatility of a put with 0.92 strike-to-spot ratio (or the closest available) and the implied volatility of an at-the-money put, dividend by the difference in strike-to-spot ratios. CAPM Beta is estimated from a regression using a window of 180 daily returns. Liquidity is the ratio between trading volume and shares outstanding. Earning Announcement is a dummy variable which takes the value of 1 if there is an earning announcement scheduled for the respective month and zero else. Interaction is the variable Earning Announcement multiplied by DiB. Fundamental Uncertainty is defined as in equation (17). DP is demand pressure and is defined as the difference between the number of contracts traded during the day at prices higher than the prevailing bid/ask quote midpoint and the number of contracts traded during the day at prices below the prevailing bid/ask quote midpoint, times the absolute value of the option’s delta and then scale this difference by the total trading volume across all option series. Macro Factor is a common component estimated via dynamic factor analysis from Industrial production, Housing Starts, S&;P 500 P/E ratio, and, Producer Price index (PPI). We use logarithmic changes over the past twelve months. * denotes significance at the 10% level, ** denotes significance at the 5% level and *** denotes significance at the 1% level. All estimations use autocorrelation and heteroskedasticity-consistent t-statistics reported in parenthesis below the estimated coefficient.

<table>
<thead>
<tr>
<th></th>
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<td>-0.043***</td>
<td>-0.042***</td>
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<td>-0.028***</td>
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<td>Common DiB</td>
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<td>-0.010**</td>
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<td>-0.019**</td>
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<td>(-2.53)</td>
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<tr>
<td>Market Vola</td>
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<td>-0.031**</td>
<td>-0.030**</td>
<td>-0.029**</td>
<td>-0.029**</td>
<td>-0.028**</td>
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<td>(-6.28)</td>
<td>(-5.37)</td>
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<td>Macro Factor</td>
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<td>0.039**</td>
<td>0.021**</td>
<td>0.013**</td>
<td>0.010**</td>
<td>0.012*</td>
<td>0.017***</td>
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<td></td>
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<td>(2.29)</td>
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<td>0.001*</td>
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<td>0.011</td>
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<td>0.008</td>
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<td>(1.17)</td>
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<td>0.020</td>
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<td>0.007</td>
<td>0.011</td>
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<td>0.029*</td>
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<tr>
<td></td>
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<td>(1.29)</td>
<td>(1.12)</td>
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<td>(1.17)</td>
<td>(1.89)</td>
<td>(1.32)</td>
<td>(1.17)</td>
</tr>
<tr>
<td>Earning Announc.</td>
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<td>-0.004***</td>
<td>-0.004***</td>
<td>-0.004***</td>
<td>-0.004***</td>
<td>-0.004***</td>
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<tr>
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<td>(-2.74)</td>
<td>(-2.74)</td>
<td>(-2.74)</td>
<td>(-2.74)</td>
<td>(-2.74)</td>
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<td>Interaction</td>
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<td>-0.018</td>
<td>-0.018</td>
<td>-0.018</td>
<td>-0.018</td>
<td>-0.018</td>
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<tr>
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<td>(-1.13)</td>
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<td>(-1.13)</td>
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<td>-0.019***</td>
<td>-0.019***</td>
<td>-0.019***</td>
<td>-0.019***</td>
<td>-0.019***</td>
<td>-0.019***</td>
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<td>(-2.99)</td>
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</tr>
<tr>
<td>Adj. $R^2$</td>
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</table>
Using data from January 1996 to June 2007, we run regressions from the correlation risk premium on a number of determinants. The correlation risk premium is approximated as the difference between the index volatility risk premium and a weighted average of the constituents volatility risk premia. Common DiB is our proxy for difference in beliefs for the market, Market Vola is the 21 day realized volatility of the index, Demand Pressure is defined as the ratio of the open interest for the out-of-money index put options to the open interest for the near and at-the-money index options. Sentiment is the first principal component from trading volume as measured by NYSE turnover, the dividend premium, the closed-end fund discount, the number and first-day returns on IPOs and the equity share in new issues. ★ denotes significance at the 10% level, ★★ denotes significance at the 5% level and ★★★ denotes significance at the 1% level. All estimations use autocorrelation and heteroskedasticity-consistent t-statistics reported in parenthesis below the estimated coefficient.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
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<td>-0.001**</td>
<td>-0.002**</td>
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<tr>
<td></td>
<td>(-2.71)</td>
<td>(-1.98)</td>
<td>(-2.01)</td>
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<td>Common DiB</td>
<td>-0.098**</td>
<td>-0.103**</td>
<td>-0.098**</td>
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<tr>
<td></td>
<td>(-2.12)</td>
<td>(-2.32)</td>
<td>(-1.99)</td>
</tr>
<tr>
<td>Market Vola</td>
<td>-0.041***</td>
<td>-0.057**</td>
<td>-0.012</td>
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<tr>
<td></td>
<td>(-2.35)</td>
<td>(-2.37)</td>
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<tr>
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<tr>
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<tr>
<td>Adj. $R^2$</td>
<td>0.08</td>
<td>0.08</td>
<td>0.13</td>
</tr>
</tbody>
</table>
Table 7

Returns on Straddle Trading Strategies Sorted on Volatility Risk Premia and Firm-Specific DiB

This table reports summary statistics of the strategy returns, average, standard deviation, kurtosis, skewness, and Sharpe ratio. Portfolios are formed according to the size of the volatility risk premium (Panel A) and the size of the firm-specific DiB (Panel B). Quintile 1 (5) consists of stocks with the lowest (largest) volatility risk premium (DiB). Option returns of single-stocks are sampled between January 1996 and June 2007 and in excess of the 1-month risk-free rate. All statistics are monthly, except the Sharpe ratios, which are annualized. The alpha, and beta coefficients are estimated from running the following least-squares regression:

\[ r_i(t) - r_f(t) = \alpha + \beta^{RMKT}t + \beta^{SMB}t + \beta^{HML}p(t) + \beta^{MOM}p(t) + \beta^{\nu \Delta VIX}t + \beta^{\nu \Delta Liq}t + \beta^{DiB}p(t) + \epsilon(t), \]

where \( r_i(t) - r_f(t) \) are the strategy returns in excess of the one-month Libor, \( RMKT \) is the value-weighted excess return on all NYSE, AMEX, and NASDAQ stocks, \( SMB \) is the size factor, \( HML \) is the book-to-market factor, and finally \( MOM \) is the momentum factor. \( \nu \Delta VIX \) are monthly changes of the VIX, \( \nu \Delta Liq \) are monthly changes of the aggregate liquidity measure from Pástor and Stambaugh (2003) and \( \nu \Delta DiB \) are monthly changes of the common disagreement factor.

### Panel A: Volatility Risk Premium Sorted

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<th>3</th>
<th>4</th>
<th>5</th>
<th>1-5</th>
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</thead>
<tbody>
<tr>
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<td>0.0219</td>
<td>0.0192</td>
<td>-0.0384</td>
<td>-0.1028</td>
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</tr>
<tr>
<td>StDev</td>
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<td>0.3432</td>
<td>0.2918</td>
<td>0.283</td>
<td>0.2884</td>
<td>0.353</td>
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<tr>
<td>Skewness</td>
<td>0.9173</td>
<td>1.0292</td>
<td>0.9273</td>
<td>0.832</td>
<td>0.5273</td>
<td>0.8325</td>
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<td>Sharpe Ratio</td>
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<td>0.2279</td>
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<td>-0.99</td>
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This table reports summary statistics of the strategy returns, average, standard deviation, kurtosis, skewness, and Sharpe ratio. The dispersion trades are formed by investing 100% of the wealth in shorting index straddles and investing a fraction of wealth into the options of firms with the lowest belief disagreement (DiB sorting), lowest CAPM beta exposure (Beta sorting), highest liquidity (Liquidity sorting), such that the portfolio is vega neutral. The naïve sorting is long options on the stock which have the smallest volatility risk premium. The rest is invested in the individual stocks such that the portfolio is delta neutral. The index put is an equally-weighted portfolio of index put options with Black-Scholes deltas ranging from -0.8 to -0.2. Option returns of single-stocks and the index are sampled between January 1996 and June 2007 and in excess of the 1-month risk-free rate. All statistics are monthly, except the Sharpe ratios, which are annualized. The alpha, and beta coefficients are estimated from running the following least-squares regression:

\[ r_i(t) - f(t) = \alpha + \beta^MMKT(t) + \beta^SMB(t) + \beta^HML(t) + \beta^MOM(t) + \beta^DiB e^{DiB(t)} + \epsilon(t), \]

where \( r_i(t) - f(t) \) are the strategy returns in excess of the one month Libor, \( MKT \) is the value-weighted excess return on all NYSE, AMEX, and NASDAQ stocks, \( SMB \) is the size factor, \( HML \) is the book-to-market factor, and finally \( MOM \) is the momentum factor. \( e^{DiB} \) are monthly changes of the aggregate liquidity measure from Pastor and Stambaugh (2003) and \( e^{DiB} \) are monthly changes of the common disagreement factor.
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Table 9: Returns on Double-Sorted Option Strategies

This table reports summary statistics of the strategy returns, average, standard deviation, kurtosis, skewness, and Sharpe ratio. The dispersion trades are formed by investing 100% of the wealth in shorting index straddles and investing a fraction of wealth into double-sorted stocks. Each month, we sort stocks into terciles, based on the common disagreement proxy, the market return, the size, book-to-market, momentum, and aggregate liquidity factor, within each tercile, we then sort according to the firm-specific disagreement. The firm-specific disagreement portfolios are then averaged over each of the six stock characteristic portfolios. Option returns of single-stocks and the index are sampled between January 1996 and June 2007 and in excess of the 1-month risk-free rate. All statistics are monthly, except the Sharpe ratios, which are annualized.

### Panel A: DiB-Sorted Terciles

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Table 10

Impact of Transaction Costs on Returns on Option Strategies

This table reports summary statistics of the strategy returns, average, standard deviation, and Sharpe ratio. The DiB sorted strategy is long the stocks in the lowest DiB quintile and short the index options. The index put is an equally-weighted portfolio of 1-month index put options with Black-Scholes deltas ranging from -0.8 to -0.2. We use bid prices when options are written and ask prices when options are bought. Option returns of single-stocks and the index are sampled between January 1996 and June 2007. All statistics are monthly, except the Sharpe ratios, which are annualized.

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<td>Short Index Put</td>
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<td>1.87</td>
</tr>
<tr>
<td>Liq Beta</td>
<td>-0.155</td>
<td>0.193</td>
<td>-0.227</td>
<td>0.201</td>
</tr>
<tr>
<td>t-Stat</td>
<td>-0.26</td>
<td>1.02</td>
<td>-0.76</td>
<td>1.02</td>
</tr>
</tbody>
</table>
The left panel plots the average volatility risk premium defined as the difference between the 30 day at-the-money implied volatility and the 30 day realized volatility for different sectors together with the corresponding sector disagreement proxies. The right panel plots the Hodrick-Prescott filtered index volatility risk premium defined as the difference between the end-of-month VIX and the annualized 30 day realized volatility on the S&P 500 together with the Hodrick-Prescott filtered common disagreement proxy. Realized volatility is the square root of the sum of squared daily log returns on the S&P 500 over the month. Both volatility measures are of monthly basis and are available at the end of each observation month. The shaded areas represent financial or economic crisis defined according to the NBER.
This figure plots the conditional correlation between the financial and consumer services sector disagreement proxies (blue line (−)) and between the consumer services and transportation sector (red line (− ∘ −)) from September 1998 to December 2008. Conditional correlations are calculated using a window of three years using monthly DiB data.

**Figure 2. Conditional DiB Correlation**
Instantaneous Correlation between $\Psi_{D_1}$ and $\Psi_{D_2}$

Instantaneous Correlation between $\Psi_{D_1}$ and $\Psi_z$

Figure 3. Uncertainty Correlation

The upper left panel plots the instantaneous correlation between the disagreement about firm 1, $\Psi_{D_1}$, and firm 2, $\Psi_{D_2}$, as a function of the weights $\alpha_{D_1}$ and $\alpha_{D_2}$ for different levels of difference in subjective uncertainty ($\sigma_{\mu_1} - \sigma_{\mu_2} \equiv \Delta \sigma_{\mu_z}$). The upper right panel plots the instantaneous correlation between the disagreement about firm 1, $\Psi_{D_1}$, and the signal, $\Psi_z$, as a function of the weights $\alpha_{D_1}$ and $\alpha_{D_2}$ for different levels of difference in subjective uncertainty ($\sigma_{\mu_1} - \sigma_{\mu_2} \equiv \Delta \sigma_{\mu_z}$). The lower panels plot the same correlations but for different levels of average economic uncertainty, ($\bar{\sigma}_{\mu_z}$) and $\Delta \sigma_{\mu_z}$ is fixed to 0.01. The parameters chosen are summarized in Table 2.

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Figure 4. Stock Return Correlation

This figure plots the return correlation of stock 1 and stock 2 as a function of belief disagreement $\Psi_{D_1}$ and $\Psi_z$. The parameters chosen are summarized in Table 2.
Difference of Index and Individual Risk-Neutral Skewness
\( \alpha_{D_1} = \alpha_{D_2} = 0.1 \)

\( \alpha_{D_1} = \alpha_{D_2} = 0.45 \)

\( \Delta \tilde{\sigma}_{\mu_z} = 0.01 \)

\( \Delta \tilde{\sigma}_{\mu_z} = 0.02 \)

\( \Delta \sigma_{\mu_z} = 0.01 \)

\( \Delta \sigma_{\mu_z} = 0.02 \)

\( \sigma_{\mu_z} = 0.1 \)

\( \sigma_{\mu_z} = 0.02 \)

Figure 5. Risk-Neutral Skewness for Individual Stock and Index Options
These figures plot the risk-neutral skewness of the returns of stock 1 (left panel) and the index (right panel). The parameters chosen are summarized in Table 2.
This figure plots the volatility risk premia for the individual stock and the index. In the left panel, we plot the risk premia for high disagreement, i.e. $\Psi_1 = \Psi_2 = \Psi_3 = 0.3$ and low disagreement, i.e. $\Psi_1 = \Psi_2 = \Psi_3 = 0.1$. The right panel plots the risk premia for high disagreement, i.e. $\Psi_1 = \Psi_2 = \Psi_3 = 0.3$ and $\Psi_3 = 0$ and low disagreement, i.e. $\Psi_1 = \Psi_2 = 0.1$ and $\Psi_3 = 0$. The volatility risk premium is defined as the difference between the implied volatility and the square root of the integrated variance $\int_t^{T} \sigma^2(s) ds$ under the physical measure. The parameters chosen are summarized in Table 2. Moneyness, is defined as $\ln(S_T/K)$. 

Figure 6. Index and Individual Volatility Risk Premia
Figure 7. Volatility and Covariance Risk Premia

The left panel plots the volatility risk premium for firm 1 as a function of the disagreement about the growth rate of firm 1, $\Psi_{D_1}$, and the disagreement about the signal, $\Psi_z$. The volatility risk premium is calculated as the difference between the 30 day realized volatility and the volatility swap rate. The 30 day realized volatility is calculated from running 10,000 simulations and averaging. The parameters chosen are summarized in Table 2.
Figure 8. Summary Statistics of Quintile DiB Sorted Portfolios

This figure plots the mean (□), standard deviation (★), and the annualized Sharpe ratio (○) of the quintile DiB sorted dispersion portfolios.
Figure 9. Estimated Coefficients of Quantile Regressions

This figure plots estimated coefficients for the 0.25, 0.5, and 0.75 quantile from a regression of the dispersion strategy (DiB Sorted) onto the different risk factors. The red line indicates a two standard deviation change. Standard errors are bootstrapped using 1,000 iterations.
Figure 10. Maximum Draw-Downs of Dispersion Trade versus Common DiB

This figure plots the maximum draw-downs of the dispersion trading strategy (lower quintile of DiB sorted strategy) versus changes in the common disagreement proxy. Gains/Losses and changes in Common DiB are monthly numbers. The numbers are in percentage terms, i.e. -1 corresponds to a -100% loss.